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Modeling of biofilm growth and the related changes in hydraulic properties of porous media

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Abstract. Pore blocking is considered to dominate the hydraulic conductivity in solute transport processes. Biomass accumulation is effective in reducing the hydraulic conductivity of porous medium. In this paper, the sphere model and the cut-and-random-rejoin-type model were adopted to establish mathematical equations for hydraulic characteristics of porous media caused by biological clogging. A new mathematical correlation was proposed to address the coupling effect of hydraulic, biofilm growth fields on the basis of thorough review on Kozeny-Carman equation relevant researches. The time-dependent solution were investigated with the consideration of a series of different model factors. The study found that there are similar phenomena both in the sphere model and in the cut-and-random-rejoin-type model. When the pores of the porous media are filled with biofilms, the pore volume is continuously reduced, and the porosity of the porous media continues to decrease. Macroscopically, it is manifested as a decrease in permeability. The model image analysis shows that growth of biofilm in a porous medium reduces the total volume and the average size of the pores and directly affects the permeability of pores. But this effect is not permanent, the pores will not be completely blocked, and the permeability will not drop to zero.
Keywords: biofilm growth; microbial clogging; relative hydraulic conductivity; relative permeability

1. Introduction

The growth and propagation of microbial reduces the hydraulic conductivity *k* of porous media, which is a common phenomenon in nature and this phenomenon is referred to as biological clogging (Tang *et al*. 2005, 2018, Jeong *et al*. 2019). A number of investigations have been performed on evaluating the biological clogging (Dennis and Thrner 1998). The biological clogging phenomenon was first discovered in the penetration test to compare the permeability of sterile soil samples with bacterial soil samples, and found that the inherent bacteria in the soil can effectively reduce the permeability coefficient of the soil by 1-2 orders of magnitude (Allison 1947). Kalish *et al*. (1964) carried out a series of trails of bio-clogging involving the use of different microbial activities and observed that the k values of porous medium reduced in different degrees. Shaw *et al*. (1985) and Cunningham *et al*. (1991) conducted a column experiment with glass beads and reported that a remarkable reduction in *k* was observed with micro-organism adhesion and community aggregates on glass beads. Marshall *et al*. (1971) and Costerton *et al*. (1978) reached the same result from a micro perspective by scanning electron microscope (SEM). They found that microorganisms adhere to the surface of porous media by producing extracellular polymeric substances (EPS), which completely filled the pore spaces, and further reduced the permeability. Previous microbial studies confirmed the k in porous media column reactors decreased by factors of 5×10^{-4} and concluded that the reduction in permeability can be described as a function of the bulk biomass density for a given porous medium (Taylor and Jaffe 1990a, b, Childs and Collis-George 1950, Marshall 1958, Millington and Quirk 1959, Mualem 1976, Song *et al*. 2018). In the aspect of the change in unsaturated hydraulic properties of porous media due to biofilm presence, early researchers have proved that a moderate increase in retention, a reduction of saturated hydraulic conductivity by about one order of magnitude, and a reduction in hydraulic conductivity by about a factor of four in the tension range between 60cm and 600cm of water (Volk *et al*. 2016).

Soil is one of the most widely studied natural porous media materials in the development of human society. Due to the existence of a certain amount of pore space in porous materials, the pore characterization characteristics will change under various clogging materials, such as asphalt mixture pavement (Cheng and Hussain 2020, Cheng 2017, Lei and Cheng 2017, Tang *et al*. 2015). Soil is also a generally favorable habitat for the proliferation of microorganisms with micro-colonies (Atlas and Bartha 1997). The term ‘‘bio-barrier’’ is used to describe a biological technology that relies on Extraccllular Polymeric Substances (EPS) excreted by microorganisms to cause severe plugging and hydraulic conductivity decrease (Cunningham 1993, Rijnaarts *et al*. 1997). In this technology, microorganisms are injected into the soil,

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together with an appropriate substrate. Zhang *et al*. (2021) systematically studied the effects of bio-clogging on hydraulic conductivity of soils in terms of microbial adhesion, bio-logical clogging mechanism, and different experimental conditions. They found biofilm is resistant to various environments, biofilm is resistant to various environments, and demonstrated that a potential for using biofilm to create waste barriers.

Biofilms form in porous media when cells attach firmly to the soil surface. The cells produce EPS that act as a shield to prevent biofilm damage. Biofilm is the overall result of communities of bacteria attaching and developing on surfaces embedded in a matrix of EPS (Wingender *et al*. 1999). Cunningham (1993) performed experiments for a constant hydraulic head difference between the inlet and outlet of their columns, they observed the homogenous growth of a biofilm, which reduced the porosity of the media by 50-96%. Recently, some researchers have evaluated the effect of bacterial activity on hydraulic conductivity for the remodeled loess, and found the variation of hydraulic conductivity was divided into the unaffected stage, linear reduction stage and stable stage (Chen *et al*. 2021).

The simplified mathematical model is established to simulate the relationship between accumulated bio-volume and *K*s reductions in the extreme complexity of the microbial clogging process has been a research hotspot in recent years. These studies are based on the hypothesis that bacteria colonizing natural porous media cover the pore walls with continuous biofilms (Zhang *et al*. 2018, Kim *et al*. 2017, Zhao *et al*. 2019). Some researchers view porous media as bundles of straight, circular capillary tubes with a constant diameter and embedded in a solid matrix, assuming that the effective diameter of the capillary tubes decreases uniformly as a result of biological activity and using the Hagen-Poiseuille equation to evaluate the total cross-sectional flux (Okubo and Matsumoto 1979). More recently, researchers develop a number of mathematical models based on the concept of biofilm and on the assumption that the geometrically complex interstices in natural porous media may be approximated by much simpler configurations (e.g., voids in regular packings of spheres, cylindrical capillaries), several of these models account satisfactorily for the permeability reductions observed in yearlong experiments in sand columns (Taylor *et al*. 1990, Tang *et al*. 2019).

In the present study, the original Kozeny-Carman permeability model and biofilm growth function models were used as basis for developing the model which includes biological clogging possesses (biofilm fills the pores of porous media and reduces the permeability). The new mathematical model was used to predict the correlation between the permeability of porous media and the biofilm thickness, and provide a theoretical basis for the research of microbial clogging in saturated porous media.

2. Model description

2.1 Permeability models

The permeability of a porous medium saturated with water is defined by Darcy's law.

$$V = -\left(\frac{k\rho_w g}{\mu_w}\right)\left(\frac{dh}{dx}\right) \tag{1}$$

where V is the specific discharge or Darcy velocity (m·s^{-1}), k is the permeability (m^2), ρ_w is the water density (kg·m^{-3}), g is the acceleration of gravity (m·s^{-2}), μ_w is the water viscosity (kg· (mt)$^{-1}$), and h is the piezometric head (m). Permeability is considered an intrinsic property of the porous medium, depending on pore size distribution, pore shape, tortuosity, specific surface, and porosity (Bear 1979).

The literature describes many different modeling methods for dealing with single-phase permeability. Dullien (1979) has categorized the various modeling approaches into what he terms phenomenological flow models, models based on conduit flow, and techniques based on a more direct application of the Navier-Stokes equations to flow through a porous medium. The phenomenological approach is based upon experimental determination of the relation among the dimensionless parameters. Given that the most important parameter, i.e., biofilm thickness, is very difficult to measure, these relationships cannot be sufficiently defined. At the other end of the spectrum lies the possibility of solving numerically the Navier-Stokes equations at the pore level. For the present problem, this would require an inordinate amount of effort, particularly given the crude assumptions we shall be forced to accept regarding the geometry of the biofilm. Consequently, only conduit flow models will be considered here, i.e., models based on steady, laminar flow through bundles of capillary tubes (Dullien 1979).

The simplest approach based on the idea of conduit flow does not consider the irregular way in which different capillary sizes are interconnected with one another and are termed "geometrical" permeability models by Dullien (1979). The Kozeny-Carman model exemplifies the geometrical conduit approach (Kozeny 1927, Carman 1937, Fair and Hatch 1933). This theory assumes the porous medium to be equivalent to a conduit, the cross section of which has a complicated shape but a constant area. Borrowing from hydraulic theory, the conduit diameter is taken to be 4 times the hydraulic radius, defined as the flow cross-sectional area divided by the wetted perimeter. Combining the Hagen- Poiseuille equation for laminar flow in a conduit with Darcy's law yields the Kozeny-Carman equation (Tang *et al*. 2016).

$$k = c_0\left(\frac{n^3}{(1-n)^2M^2}\right) \tag{2}$$

where n is the porosity, M is the specific surface (m^2/m^3), and c_0 is a constant for which Carman (1937) suggests a value of 1/5.

2.1.1 Sphere model

Values for porosity and specific surface are readily obtained when assumptions regarding the structure of the porous medium are made. Here we assume that the solid phase can be represented by regular packings of uniform spheres. Graton and Fraser (1935) and Cadle (1965)

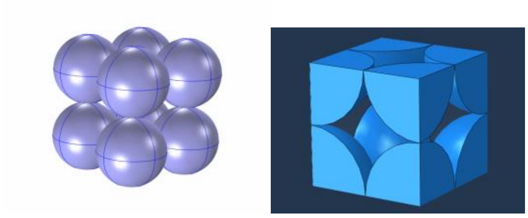

Fig. 1 The cubic arrangement of equal sphere

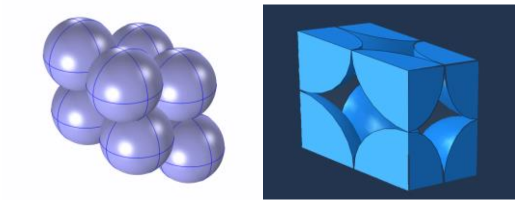

Fig. 2 The oblique hexagonal arrangement of equal sphere

Table 1 the value of porosity under different characterize the packing arrangement

Arrangent name / Parametes	Cubic	Ortho-rhombic	Tetragonal-spheroidal	Rhombohedral
m	6	8	10	12
α_m	1	$\frac{\sqrt{3}}{2}$	3/4	$\frac{1}{\sqrt{2}}$
n	47.64%	39.54%	30.19%	25.96%

analyzed the porosity of the various arrangements in which uniform spheres may be packed in a stable manner. In any given arrangement, each sphere tangentially contacts a certain number of neighboring spheres, and the number of contact point m can be used to characterize the packing arrangement. Table 1 shows the value of porosity under different characterize the packing arrangement. The cubic arrangement of equal sphere is shown in Fig. 1 and the oblique hexagonal arrangement of equal sphere is shown in Fig. 2. From geometric considerations, a general expression for porosity was obtained

$$n = 1 - \left(\frac{\pi}{6\alpha_m}\right) \quad (3)$$

and for specific surface

$$M = \frac{\pi}{\alpha_m d} \quad (4)$$

where α_m is a packing arrangement factor, and d is the sphere diameter. For the packing arrangements described above, $\alpha_6 = 1$, $\alpha_8 = \frac{\sqrt{3}}{2}$, $\alpha_{10} = 3/4$, $\alpha_{12} = \frac{1}{\sqrt{2}}$.

2.1.2 Random distribution model

Permeability models which consider the random nature of the interconnectedness of the pores are termed “statistical” by Dullien (1979). The cut-and-random-rejoin-type model introduced by Childs and Collis-George (1950) and later modified by Marshall (1958) and Millington and Quirk (1959) are examples of this approach. It is assumed that a porous medium contains pores of various radii which are randomly distributed in space and that when adjacent planes of the medium are brought into contact the overall hydraulic conductance across the plane depends statistically upon the number of pairs of interconnected pores and geometrically upon their configurations. The conductance of each pair of interconnected pores is controlled by the pore having the smaller radius, while unconnected pores are neglected. Under this assumption, the Hagen-Poiseuille equation combined with Darcy’s law can be made to yield a relationship between the permeability and the pore size distribution.

The cut-and-random-rejoin model proposed by Marshall can be expressed in integral form as

$$k = \frac{1}{4}\int_{r_0}^{R} r^2[n - \theta(r)]\, f(r)dr \quad (5)$$

in which r is a pore radius (m), r_0 is the minimum pore radius (m), R is the maximum pore radius (m), $\theta(r)$ is the unsaturated moisture content when all pores of radius r or smaller are filled, and $f(r)$ is the pore size distribution function defined by

$$d\theta = f(r)dr \quad (6)$$

Hence $f(r)dr$ represents the volume of full pores of radii $[r, r+dr]$ per unit volume of medium. The $f(r)$ is nonzero only in the interval $r_0 \le r \le R$. The earlier work of Childs and Coilis-George (1950) arrived at a result that is essentially equivalent to Eq. (5). In a later analysis, Millington and Quirk (1959) found

$$k = \frac{n^{-q}}{4}\int_{r_0}^{R} r^2[n - \theta(r)]\, f(r)dr \quad (7)$$

with the constant q taking a value of 2/3. If $q = 0$, then Eq. (7) reduces to the Eq. (5).

It is useful to note that the moisture content, porosity, and specific surface may also be expressed in terms of $f(r)$. From Eq. (6) we obtain

$$\theta(r) = \int_{r_0}^{r} f(\rho)\, d\rho \quad (8)$$

and in particular,

$$n = \int_{r_0}^{R} f(r)\, dr \quad (9)$$

assuming long tubular pores, specific surface can be found from a cross section normal to the pores as the ratio of total pore circumference to total area. If $g(r)dr$ is the number of pores having radii $[r, r+dr]$ intersecting a unit area of the medium, then

$$M = \int_{r_0}^{R} 2\pi r g(r)\, dr \quad (10)$$

$g(r)$ is related to $f(r)$ by

$$f(r)dr = \pi r^2 g(r)dr \tag{11}$$

Hence

$$M = 2\int_{r_0}^{R} \frac{f(r)}{r}dr \tag{12}$$

The foregoing equations allow us to determine porosity and specific surface and to make various estimates of the permeability for porous media represented as uniform spheres or bundles of capillary tubes. For the sphere model the problem of determining changes in permeability due to changes in biofilm thickness becomes one of specifying the variation of n and M with biofilm thickness for a given packaging arrangement. In the case of the cut-and-random-rejoin-type model, the problem is reduced to specifying the variation of $f(r)$ with biofilm thickness. Both avenues are explored below.

2.2 Treatment of the biofilm

2.2.1 Sphere model

Firstly, it is assumed that the porous medium is composed of spheres of equal diameter, which are filled in the arrangement described above. Suppose a biofilm develops in such a way that all spheres are covered by a layer of impermeable biofilm with a constant thickness L_f. Under the latter assumption, the growth of the biofilm effectively increases the volume of the solid phase and reduces the surface area of the solid phase. The expression of biofilm influence porosity n_b and specific surface M_b area can be derived from the geometry of the coated ball.

The volume of a solid sphere and its biofilm coating is given by Deb (1969) as

$$V_b^s = \frac{4\pi}{3}\left(\frac{d}{2}+L_f\right)^3 - m\left\{\frac{\pi L_f^2}{3}\left[3\left(\frac{d}{2}+L_f\right)-L_f\right]\right\} \tag{13}$$

while the bulk media volume V occupied by the sphere is

$$V = \alpha_m d^3 \tag{14}$$

Substituting these expressions into

$$n_b = 1 - \frac{V_b^s}{V} \tag{15}$$

and rearranging yields

$$n_b = 1 - \frac{\pi}{\alpha_m}\left[\frac{(2-m)}{12}\left(\frac{L_f}{R}\right)^3 + \frac{(4-m)}{8}\left(\frac{L_f}{R}\right)^2 + \frac{1}{2}\left(\frac{L_f}{R}\right) + \frac{1}{6}\right] \tag{16}$$

An expression for specific surface of the biofilm-fluid interface can be derived along similar lines. Deb (1969) gives the area of a film coating a sphere as

$$A_b^s = 4\pi\left(\frac{d}{2}+L_f\right)^2 - m\left[2\pi\left(\frac{d}{2}+L_f\right)L_f\right] \tag{17}$$

Substituting Eq. (14) and Eq. (17) into the definition of specific surface, i.e.,

$$M_b = \frac{A_b^s}{V} \tag{18}$$

results in the following

$$M_b = \frac{\pi}{d}\left[\frac{(2-m)}{2}\left(\frac{L_f}{R}\right)^2 + \frac{(4-m)}{2}\left(\frac{L_f}{R}\right) + 1\right] \tag{19}$$

The biofilm-affected permeability k_b for the sphere model is found by substituting n_b and M_b into the Kozeny-Carman relation given by Eq. (2), assuming c_0 does not change with biofilm growth. Hence

$$k_b = c_0\left(\frac{{n_b}^3}{(1-n_b)^2{M_b}^2}\right) \tag{20}$$

where n_b and M_b are given by Eqs. (16) and (19).

2.2.2 Cut and random rejoin type model

Considering the absence of biofilm, Taylor *et al*. (1990) used $f(r)$ to describe the changes in permeability for cut-and-random-rejoin-type conduit model of the porous medium. Assuming that an effectively impermeable biofilm of uniform thickness L_f coats all solids, biofilm growth will completely plug pores having a radius less than L_f, if they exist, and reduce the radius of those larger. $f_b(r)$ was expressed to describe the pore size distribution of the void space not filled by biomass. Pores with a new radius r previously have a radius $r+L_f$, and their volume is reduced by the ratio of conduit volume $r^2/(r+L_f)^2$, then the distribution of $f_b(r)$ and $f(r)$ is as follows (Taylor *et al*. 1990),

$$f_b(r) = \frac{r^2}{\left(r+L_f\right)^2} f\left(r+L_f\right) \tag{21}$$

which is defined for $r_0-L_f < r < R-L_f$. Other biofilm-affected parameters follow by substitution of $f_b(r)$ for $f(r)$ in the earlier equations. The biofilm-affected porosity, moisture content and specific as

$$n_b = \int_{r_{ob}}^{R-L_f} \frac{r^2}{\left(r+L_f\right)^2} f\left(r+L_f\right)dr \tag{22}$$

$$\theta_b(r) = \int_{r_{ob}}^{r} \frac{\rho^2}{\left(\rho+L_f\right)^2} f\left(\rho+L_f\right)d\rho \tag{23}$$

$$M_b = 2\int_{r_{ob}}^{R-L_f} \frac{r}{\left(r+L_f\right)^2} f\left(r+L_f\right)dr \tag{24}$$

where $r_{ob} = \max[r_0-L_f, 0]$ and the Millington and Quirk permeability model embodied in Eq. (7), become

$$k_b = \frac{{n_b}^{-q}}{4}\int_{r_{ob}}^{R-L_f} [n_b - \theta_b(r)]\frac{r^4}{\left(r+L_f\right)^2} f\left(r+L_f\right)dr \tag{25}$$

Taylor *et al*. (1990) confines the analysis to the case where is $f(r)$ given by a power function,

$$f(r) = \frac{\beta}{R}\left(\frac{r}{R}\right)^{\lambda-1} \tag{26}$$

in which β and λ are constants. Porosity is obtained from Eq. (9), the moisture content is obtained from Eq. (8), and the specific surface is found from Eq. (12) as

$$n = \frac{\beta}{\lambda}\left[1-\left(\frac{r_0}{R}\right)^{\lambda}\right] \tag{27}$$

$$\theta(r) = \frac{\beta}{\lambda}\left[\left(\frac{r}{R}\right)^{\lambda}-\left(\frac{r_0}{R}\right)^{\lambda}\right] \tag{28}$$

$$M = \frac{2\beta}{R(\lambda-1)}\left[1-\left(\frac{r_0}{R}\right)^{\lambda-1}\right] \tag{29}$$

Permeability formulae are obtained as follows: From the Kozeny-Carman equation Eq. (2),

$$k = (\lambda-1)^2\left[\frac{c_0 n^3 R^2}{4(1-n)^2\beta^2}\right]\left[1-\left(\frac{r_0}{R}\right)^{\lambda-1}\right]^{-2} \tag{30}$$

From the Millington and Quirk equation Eq. (7),

$$k = \frac{\beta^2 R^2}{4\lambda n^q}\left\{\begin{aligned}&\frac{1}{2+\lambda}\left[1-\left(\frac{r_0}{R}\right)^{2+\lambda}\right]\\ &-\frac{1}{2+2\lambda}\left[1-\left(\frac{r_0}{R}\right)^{2+\lambda}\right]\end{aligned}\right\} \tag{31}$$

By substituting Eq. (26) into the expressions of moisture content Eq. (23), porosity Eq. (22), specific surface Eq. (24) and permeability Eq. (20), Taylor deduced the expression of the influence parameters of biofilm, and introduce convenient notation (Taylor *et al*. 1990).

$$\theta_b(r) = \beta\left(\frac{L_f}{R}\right)^{\lambda}\left[I_2\left(\frac{r}{L_f},\lambda\right)-I_2\left(\frac{r_{ob}}{L_f},\lambda\right)\right] \tag{32}$$

$$n_b = \beta\left(\frac{L_f}{R}\right)^{\lambda}\left[I_2\left(\frac{R}{L_f}-1,\lambda\right)-I_2\left(\frac{r_{ob}}{L_f},\lambda\right)\right] \tag{33}$$

$$M_b = \frac{2\beta}{L_f}\left(\frac{L_f}{R}\right)^{\lambda}\left[I_1\left(\frac{R}{L_f}-1,\lambda\right)-I_1\left(\frac{r_{ob}}{L_f},\lambda\right)\right] \tag{34}$$

In which $I_n(u,\lambda)$ and $J(u,\lambda)$is introduced notation to facilitate calculation and $I_n(u,\lambda) = \int_0^u \frac{x}{(1+x)^{3-\lambda}}dx$; $J(u,\lambda) = \int_0^u \frac{x^4 I_2(x,\lambda)}{(1+x)^{3-\lambda}}dx$, Millington and Quirk permeability is obtained from Eq. (25) as

$$k_b = \frac{\beta^2 L_f^2}{4n_b^2}\left(\frac{L_f}{R}\right)^{2\lambda}\left\{\begin{aligned}&I_2\left(\frac{R}{L_f}-1,\lambda\right)\cdot\\ &\left[I_4\left(\frac{R}{L_f}-1,\lambda\right)-I_4\left(\frac{r_{ob}}{L_f},\lambda\right)\right]\\ &-\left[J\left(\frac{R}{L_f}-1,\lambda\right)-J\left(\frac{r_{ob}}{L_f},\lambda\right)\right]\end{aligned}\right\} \tag{35}$$

The Mualem-based model of Taylor (Taylor *et al*. 1990) assumes that the relationship between the volumetric moisture content $\theta_v(r)$ of the porous medium and the radius r of the largest pores filled with water is described by

$$\theta_v(r) = \frac{\gamma}{\lambda}\left[\left(\frac{r}{R}\right)^{\lambda}-\left(\frac{r_0}{R}\right)^{\lambda}\right] \tag{36}$$

where λ and γ are constants, R and r_0 are the maximum and minimum pore radii (In the cut-and-random-rejoin-type model, in the initial stage of microbial plugging, the original radius of uneven distribution of pores is between r_0 and R). Taylor *et al*. (1990) considered that the parameter λ can theoretically range from zero to infinity, larger values being associated with more homogeneous pore sizes.

On the basis of Eq. (21) and of the additional assumption that a biofilm of thickness L_f uniformly coats the internal surfaces in the clogged porous media, the Mualem-based model of Taylor (Taylor *et al*. 1990) yields relationships for the biovolume ratio φ and for the ratio k_b/k that can be expressed.

$$\begin{aligned}\varphi \equiv 1-\frac{n_b}{n_0} = 1-\lambda\left[1-\left(\frac{r_0}{R}\right)^{\lambda}\right]\left(\frac{L_f}{R}\right)^{\lambda}\Bigg[&I_2\left(\frac{R}{L_f}-1,\lambda\right)\\ &-I_2\left(\frac{r_{ob}}{L_f},\lambda\right)\Bigg]\end{aligned} \tag{37}$$

$$\frac{k_b}{k} = \left(\frac{L_f}{R}\right)^{2+2\lambda}\left(\frac{\left[I_3\left(\frac{R}{L_f}-1,\lambda\right)-I_3\left(\frac{r_{ob}}{L_f},\lambda\right)\right]^2}{\left\{\frac{1}{1+\lambda}\left[1-\left(\frac{r_0}{R}\right)^{1+\lambda}\right]\right\}^2}\right)(1-\varphi)^{\frac{1}{2}} \tag{38}$$

From the Eqs. (37) and (38) formulas and the Kozeny-Carman model described earlier, the same feature can be obtained that regardless of the value of R , the change in k_b/k is only related to the ratio of k_b/k.

3. Computational results

3.1 Sphere model with biofilm growth

Fig. 3 shows the relationship between porosity reduction n_b/n and L_f/R for various filling arrangements of a homogeneous sphere. It can be seen that under the condition of a certain biofilm thickness, the values of L_f/R depend on the filling arrangement. In the case of uniformity, n_b/n decreases with the number of contact points of the sphere, reflecting that the remaining pore space decreases with the increase of the density of the sphere. The correlation function in Fig. 3 is truncated when the porosity decreases to a certain extent, and the truncated range is from 0.04 to 0.21, this is due to the decomposition of the biofilm sphere model on the larger L_f/R porous medium. When the biofilm fills the narrow channel between the adjacent spheres and isolates the closed pore space from the adjacent pores, the pore space is no longer continuous. If there is an impermeable biofilm, the fluid is not allowed to flow through the medium. In this case, L_f/R value can be determined from purely geometric considerations. For

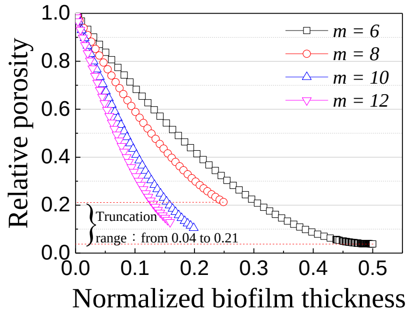

Fig. 3 Relative porosity as a function of the normalized biofilm thickness (Sphere model with biofilm growth)

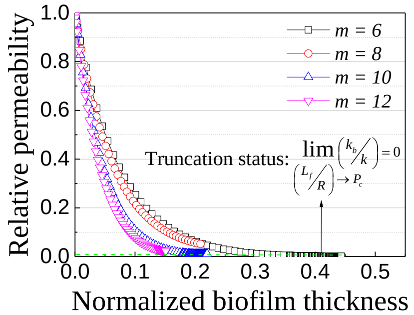

Fig. 4 Relative permeability as a function of the normalized biofilm thickness

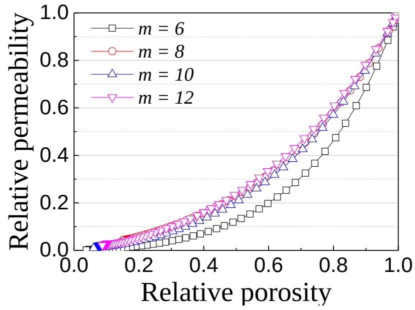

Fig. 5 Relative permeability as a function of relative porosity

example, with a cubic stacking arrangement, hole discontinuities can occur for $L_f/R \geq \sqrt{2}-1$.

Fig. 4 shows the relationship between relative permeability k_b/k and L_f/R for various filling arrangements of a homogeneous sphere. It can be seen that under a certain thickness of the biofilm, the value of the relative permeability also depends on the arrangement of the filling. Under uniform conditions, it decreases with the increase of the number of sphere contact points, reflecting that the fluid is difficult to flow through porous medium. At the same time, there is also a cut-off value for the relative porosity in Fig. 4. In fact, when the biofilm growth and model decomposition reach dynamic equilibrium, it is the cut-off state.

Fig. 5 shows the decrease in permeability based on the Kozeny-Carman permeability relationship, k_b/k as a correlation function image of n_b/n, all functions are drawn in a narrow range defined by the curves of m = 6 and m = 12, which indicates that it is insensitive to the filling

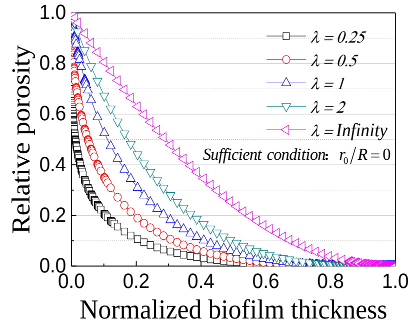

Fig. 6 Relative porosity as a function of the normalized biofilm thickness (Cut and random rejoin type model with biofilm growth)

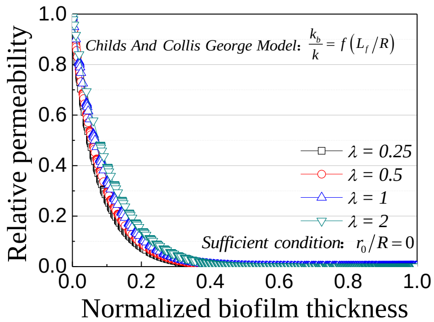

Fig. 7 Relative permeability as a function of the normalized biofilm thickness in Childs and Collis-George model

arrangement assumed in the spherical model. In addition, for $m \geq 8$, there is no practical difference between homogeneous and heterogeneous models (which m = 6 is the homogeneous model and $m \geq 8$ are all the heterogeneous models). As mentioned earlier, due to the beginning of the discontinuity of the pore space, the relationship of k_b/k can be ignored for smaller n_b/n values.

3.2 Cut and random rejoin type model with biofilm growth

Fig. 6 shows the functional relationship between n_b/n and the normalized biofilm thickness L_f/R. In this paper, it has been assumed that there is a certain inhomogeneity in the pore channel, then the initial radius of all pores in the porous media (which has not been affected by biological plugging) ranges from 0 to R, and the minimum value of r0 is 0, for r_0=0 and for different λ. When λ is very small, for a large range of pore diameter, most of the porosity is formed by pore diameter with radius less than R. Therefore, with L_f/R increasing from zero, n_b/n begins to decrease rapidly. For medium with relatively uniform pore diameter, the decrease is smaller. When r_0 is nonzero, the above relationship is the same. But with the increase of r_0/R, the curve with lower λ value approaches the curve with higher λ value as a whole.

Fig. 7 displays the relationship between relative permeability k_b/k and L_f/R in the Childs and Collis-George model. With the increase of the relative biofilm thickness L_f/R, the relative permeability k_b/k coefficient of porous media also decreases rapidly, which is basically consistent

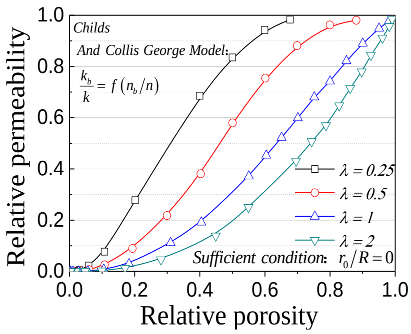

Fig. 8 Relative permeability as a function of relative porosity in Childs and Collis-George model

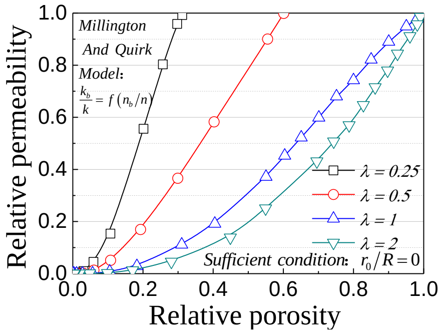

Fig. 9 Relative permeability as a function of relative porosity in Millington and Quirk model

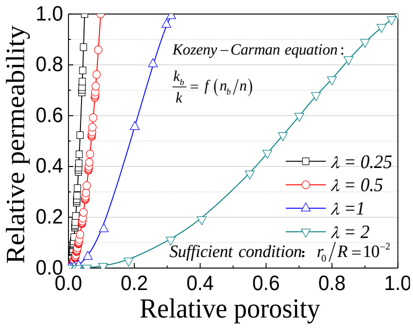

Fig. 10 Relative permeability as a function of relative porosity in Kozeny-Carman equation

with the results shown in the above spherical model in Fig. 7. However, the change of λ value has little effect on the relative permeability of porous media.

The Childs and Collis-George permeability reductions for $r_0 = 0$ are shown in Fig. 8. The permeability decreases monotonically with porosity for a given medium as the biofilm grows. Quantitatively, the k_b/k predicted by this model for relatively uniform pore sizes, i.e., $\lambda = 2$, is similar to that given by the sphere model. Millington and Quirk model is the general expression form of Childs and Collis-George model, and Fig. 9 shows the Millington and Quirk permeability reductions with $r_0 = 0$. An analysis of Eq. (35) in the vicinity of $L_f = 0$, however, shows k_b to be proportional to n_b^{-q}.

Finally, Fig. 10 shows the relationship between k_b/k based on the Kozeny-Carman equation and n_b/n for the case when $r_0/R = 10^{-2}$. Obviously, this result is quite different from the previous sphere model, In the probability model, all the intermediate variables in the Kozeny-Carman expression, including porosity n, specific surface M, are probability functions, which leads to large distortion of the final result. This is because this study found that the partial filling of biomass to the pore space will lead to the increase of permeability. From the intermediate results during the study period, it is shown that the decrease of M is much faster than that of n with the formation of biofilm. Therefore, the concept of a hydraulic radius defined with respect to specific surface in a porous medium having a wide pore size distribution is obviously inadequate.

4. Discussion

The above calculation results show that the correlation function between the spherical model of porous media and the permeability of Kozeny-Carman provides a reasonable result for the problem of reducing the permeability of biofilm to some extent. Compare Fig. 3 with Fig. 7, the permeability reduction predicted by the sphere model is quantitatively similar to the cut-and-random-rejoin-type model of Childs and Collis-George, with relatively uniform pore size (λ = 2), this is physically consistent with the spherical model, the uniform accumulation of uniform spheres results in uniform aperture. However, the sphere model cannot represent all kinds of apertures in porous media. After all, porous media is complex and diverse, and a single sphere model cannot reflect the sufficiency of the results.

The computational results of the cut-and-random-rejoin-type model show that, although all agents produce physically reasonable results for the medium with uniform pore size, only the permeability relationship based on Child and Collis-George seems to have universal applicability. The flow in porous media is usually laminar, so the results obtained by Kozeny-Carman equation are relatively unreasonable (Dullien 1979). Although the similarity between Millington and Quirk model and Kunze (Kunze *et al*. 1968) and these comparisons are made with matching factors and Kunze model will not solve the problem, it's generally agreed that the Millington and Quirk model is superior to Child and Collis-George. Finally, it is noted that the cut-and-random-rejoin-type is better than the spherical models, because the former is suitable for porous media with wide pore size, and when the pore space is full of biomass, they show appropriate asymptotic behavior, i.e., $k_b \rightarrow 0$ as $n_b \rightarrow 0$ and $L_f \rightarrow R$.

Most studies show that the permeability of porous media decreases with the decrease of porosity in the process of pore clogging. In fact, when the porosity is reduced to a certain extent, the permeability will eventually be truncated, and the microbial growth cannot last forever in the pores, and the pore permeability coefficient can only drop to a critical value. In this state, the growth and decline of organisms are in a dynamic equilibrium state. In short, the pores will not be completely blocked, but the migration capacity of large particles may be greatly reduced.

5. Conclusions

This article mainly studies the influence of biofilm growth in porous media on the relative permeability of the media and uses multiple models for analysis and comparison, including the original Kozeny-Carman equation model, Childs and Collis-George model, Millington and Quirk model, and a certain functional correlation between the biofilm thickness and permeability reduction is determined through analysis, $\frac{k_b}{k} = f\left(\frac{L_f}{R}\right)$. In the research process, the sphere model and Cut and random rejoin type model of porous media are used. The conclusions of this paper are as follows:

• In special cases, when the porous medium has a uniform pore diameter (sphere model or λ = 2), all the permeability models can get physical reasonable results. Moreover, the classical original Kozeny-Carman equation combined with the biofilm activity equation can get the correlation function relationship of the relative permeability of the porous medium under the corresponding biofilm growth, and when the medium has a uniform pore diameter, the given volume the permeability of biomass decreased the most.

• Under the condition of non-uniform pore size distribution in porous media, the classical original Kozeny-Carman model has considerable deviation. Only the models based on Childs and Collis-George and Mualem can provide physical results for large-diameter media. The asymptotic properties of the shear stochastic reconnection model show that when the pore volume is zero, the permeability is close to zero.

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