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Nonlinear thermoelastic response of laminated composite conical panels

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Abstract. Nonlinear thermoelastic static response characteristics of laminated composite conical panels are studied employing finite element approach based on first-order shear deformation theory and field consistency principle. The nonlinear governing equations, considering moderately large deformation, are solved using Newton-Raphson iterative technique coupled with the adaptive displacement control method to efficiently trace the equilibrium path. The validation of the formulation for mechanical and thermal loading cases is carried out. The present results are found to be in good agreement with those available in the literature. The adaptive displacement control method is found to be capable of handling problems with multiple snapping responses. Detailed parametric study is carried out to highlight the influence of semi-cone angle, boundary conditions, radius-to-thickness ratio and lamination scheme on the nonlinear thermoelastic response of laminated cylindrical and conical panels.

Keywords: laminated; conical panel; thermoelastic; postbuckling; field consistency.

1. Introduction

Conical shell panels have a wide range of engineering applications, particularly in aircrafts, space vehicles, marine and power plants. Many structures comprise at least a few components with this geometrical profile such as turbine blades or aircraft fuselage.

The postbuckling analysis based on finite element formulation for composite laminated cylindrical panels under different loading and boundary conditions has been performed extensively. The postbuckling behaviour of generally layered anisotropic composite cylindrical panels under compressive loading has been studied by Zhang and Matthews (1983). Hui (1985) has used Donell type equilibrium and compatibility equations, and Koiter's theory of elastic stability for studying initial postbuckling behaviour of symmetrically laminated thin cross-ply cylindrical panels under

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axial compression with simply supported edges. Postbuckling analysis of deep curved composite panels with different lay-ups under axial compression with mixed boundary conditions has been carried out by Laschet and Jeusette (1990) employing three-dimensional degenerated isoparametric multilayer finite element. It has been concluded that the presence of clustered bifurcation points is a source of numerical problems which are difficult to treat (Laschet and Jeusette 1990). Huang and Tauchert (1991) have investigated thermally-induced large-deflection behaviour of laminated cylindrical and doubly-curved panels using first-order shear deformation theory and von Kármán type kinematics. Arc length and Riks methods have been used to trace the post-critical equilibrium paths. Tsai and Palazotto (1991) have studied nonlinear and multiple snapping responses of cylindrical panels employing modified Riks method to trace the multiple equilibrium paths. Kweon and Hong (1993) have investigated postbuckling behaviour of composite laminated cylindrical panels with various stacking sequences under compression employing nonlinear finite element method and an improved load-increment method based on arc-length scheme. The linear and geometrically nonlinear analyses of laminated composite shells have been carried out by Yoo and Cho (2000) using an improved degenerated shell element. The assumed transverse shear strain method is used to eliminate the shear locking and the reduced integration method to eliminate the membrane locking. Oh and Lee (2001) have studied the snapping response of laminated cylindrical panels subjected to thermal and pressure loading using layerwise finite elements and von Kármán nonlinear strain displacement relations with cylindrical arc-length method. A predictor-corrector algorithm is presented for tracing the geometrically nonlinear path of cylindrical shells (Lopez 2001). The predictor step is carried out by an asymptotic extrapolation based on residual error minimization and corrector step is defined imposing the minimum distance between the approximate solution point and the solution curve. Lee *et al.* (2002) have studied the thermal postbuckling behavior of patched laminated panels under uniform and non-uniform temperature distribution using finite element based on Hellinger-Reissner principle. The postbuckling behaviour of thin, imperfect laminated composite panels has been studied using finite element secant matrices based on Marguerre shallow shell theory (Jayachandran *et al.* 2004). The results exhibiting snap-through behaviour are obtained using the arc length and the minimum residual displacement methods. An element based 9-node resultant shell element has been presented for isotropic and anisotropic laminated shells based on first-order shear deformation theory (Han *et al.* 2004). The element is derived using assumed natural strain method to eliminate membrane and shear locking problems. The arc length method is used to trace complex equilibrium paths for thin shell panels. A method based on simultaneous control of applied loads and displacements at one or more points has been proposed by Kwon *et al.* (2005) to analyze postbuckling phenamena including snap-through and snap-back. The difficulties in tracing the equilibrium paths due to numerical instabilities have been overcome by employing relaxation factors.

Limited attention is paid to the study of nonlinear response of conical shells/panels under different loading conditions. A theoretical analysis based on Donnell-type shell equations with the effect of nonlinear prebuckling deformations taken into consideration is performed by Tani (1985) on the buckling of clamped truncated isotropic conical shells under combined uniform pressure, axial load and uniform heating. A theory for nonlinear bending of symmetrically laminated, cylindrically orthotropic, shallow conical shells subjected to an axisymmetrically distributed load including transverse shear effects is presented by Ren-Huai (1996). Elastic buckling and postbuckling behaviour of widely-stiffened conical shells under axial compression is studied by Spagnoli and Chryssanthopoulos (1999) using ABAQUS finite element package. Different buckling modes in axially stiffened conical shells are studied through a linear eigenvalue finite element analysis approach by Spagnoli (2001). Patel *et al.* (2005) have studied the nonlinear thermoelastic buckling/postbuckling characteristics of laminated circular conical/cylindrical shells subjected to uniform temperature rise employing semi-analytical finite element approach and adaptive displacement control method. However, the application of the adaptive displacement control method for problems involving multiple snapping responses is unexplored.

To optimally exploit the strength and load-carrying capacity of laminated composite conical shell panels at elevated temperature, accurate prediction and understanding of their nonlinear thermoelastic response characteristics are very important. Neglecting the possible postbuckling strength of these composite panels constitutes a severe design limitation when weight saving is of prime importance. To the best of the authors' knowledge, there are no studies available on the nonlinear thermoelastic response characteristics of laminated conical panels. In the present work, nonlinear thermoelastic static response characteristics of laminated composite conical panels are studied employing field consistent finite element based on first-order shear deformation theory. The nonlinear governing equations are solved using Newton-Raphson iteration technique coupled with the adaptive displacement control method. The present results for mechanical and thermal loading cases of cylindrical panels are found to be in good agreement with those available in the literature. Detailed parametric study is carried out to highlight the influence of semi-cone angle, boundary conditions, radius-to-thickness ratio and lamination scheme on the nonlinear thremoelastic response of the laminated conical panels.

2. Formulation

A laminated composite circular conical panel is considered with the co-ordinates x along the meridional direction, y along the circumferential direction and z along the thickness direction normal to panel wall having origin at the middle-surface of the panel as shown in Fig. 1. The displacements u, v, w at a point (x, y, z) are expressed as functions of the middle-surface displacements u_0, v_0, w_0 and the independent rotations θ_x and θ_y of the normal in the xz and yz planes, respectively, as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

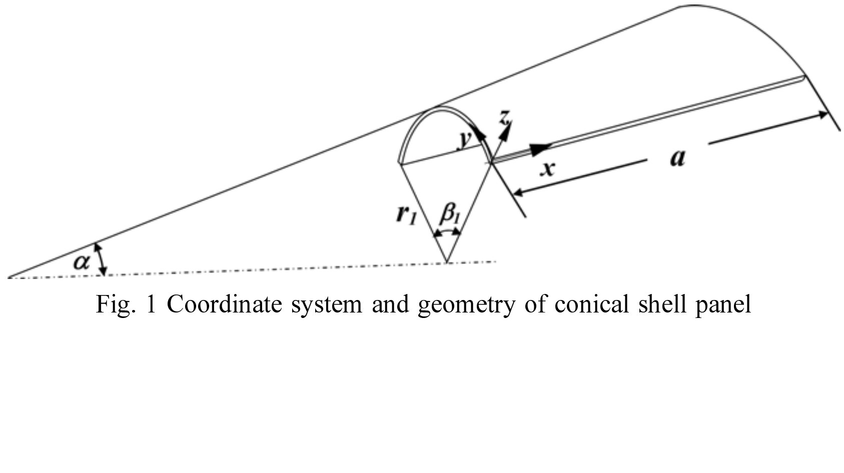

Fig. 1 Coordinate system and geometry of conical shell panel

The strain-displacement relations are based on kinematic approximations: (i) small strains, (ii) moderately large deformation; and (iii) thin shell ($z/r << 1$) such that $1+z/r \approx 1$, however, transverse shear deformation is important due to larger E/G ratio for composites. Green's strains can be written in terms of middle-surface deformations as

$$\{\boldsymbol{\varepsilon}\} = \begin{Bmatrix} \boldsymbol{\varepsilon}_\mathbf{p}^\mathbf{L} \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} \mathbf{z}\boldsymbol{\varepsilon}_\mathbf{b} \\ \boldsymbol{\varepsilon}_\mathbf{s} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{\varepsilon}_\mathbf{p}^\mathbf{NL} \\ \mathbf{0} \end{Bmatrix} \tag{2}$$

where membrane $\{\boldsymbol{\varepsilon}_\mathbf{p}^\mathbf{L}\}$, bending $\{\boldsymbol{\varepsilon}_\mathbf{b}\}$, transverse shear $\{\boldsymbol{\varepsilon}_\mathbf{s}\}$ and nonlinear $\{\boldsymbol{\varepsilon}_\mathbf{p}^\mathbf{NL}\}$ strain vectors in Eq. (2) are written as (Kraus 1976)

$$\{\boldsymbol{\varepsilon}_\mathbf{p}^\mathbf{L}\} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{u_o \sin\alpha}{r} + \dfrac{\partial v_o}{\partial y} + \dfrac{w_o \cos\alpha}{r} \\ \dfrac{\partial u_o}{\partial y} - \dfrac{v_o \sin\alpha}{r} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix};\ \{\boldsymbol{\varepsilon}_\mathbf{b}\} = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\theta_x \sin\alpha}{r} + \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} - \dfrac{\theta_y \sin\alpha}{r} \end{Bmatrix};\ \{\boldsymbol{\varepsilon}_\mathbf{s}\} = \begin{Bmatrix} \theta_x + \dfrac{\partial w_o}{\partial x} \\ \theta_y + \dfrac{\partial w_o}{\partial y} - \dfrac{v_o \cos\alpha}{r} \end{Bmatrix} \tag{3a}$$

$$\{\boldsymbol{\varepsilon}_\mathbf{p}^\mathbf{NL}\} = \begin{Bmatrix} \dfrac{1}{2}\left[\left(\dfrac{\partial u_o}{\partial x}\right)^2 + \left(\dfrac{\partial v_o}{\partial x}\right)^2 + \left(\dfrac{\partial w_o}{\partial x}\right)^2\right] \\ \dfrac{1}{2}\left[\left(\dfrac{\partial u_o}{\partial y} - \dfrac{v_o \sin\alpha}{r}\right)^2 + \left(\dfrac{\partial v_o}{\partial y} + \dfrac{u_o \sin\alpha}{r} + \dfrac{w_o \cos\alpha}{r}\right)^2 + \left(\dfrac{\partial w_o}{\partial y} - \dfrac{v_o \cos\alpha}{r}\right)^2\right] \\ \dfrac{\partial u_o}{\partial x}\left(\dfrac{\partial u_o}{\partial y} - \dfrac{v_o \sin\alpha}{r}\right) + \dfrac{\partial v_o}{\partial x}\left(\dfrac{\partial v_o}{\partial y} + \dfrac{u_o \sin\alpha}{r} + \dfrac{w_o \cos\alpha}{r}\right) + \dfrac{\partial w_o}{\partial x}\left(\dfrac{\partial w_o}{\partial y} - \dfrac{v_o \sin\alpha}{r}\right) \end{Bmatrix} \tag{3b}$$

where r, the radius of parallel circle, is a function of x coordinate; and α is semi-cone angle.

The stress resultant vector $\{\mathbf{N}\} = \{N_{xx}\ N_{yy}\ N_{xy}\}^T$ and the moment resultant vector $\{\mathbf{M}\} = \{M_{xx}\ M_{yy}\ M_{xy}\}^T$ can be expressed in terms of the membrane strains $\{\boldsymbol{\varepsilon}_\mathbf{p}\} = \{\boldsymbol{\varepsilon}_\mathbf{o}^\mathbf{L}\} + \{\boldsymbol{\varepsilon}_\mathbf{o}^\mathbf{NL}\}$ and the bending strains $\{\boldsymbol{\varepsilon}_\mathbf{b}\}$ through the constitutive relation

$$\begin{Bmatrix} \{\mathbf{N}\} \\ \{\mathbf{M}\} \end{Bmatrix} = \begin{bmatrix} [\mathbf{A}] & [\mathbf{B}] \\ [\mathbf{B}] & [\mathbf{D}] \end{bmatrix} \begin{Bmatrix} \{\boldsymbol{\varepsilon}_\mathbf{p}\} \\ \{\boldsymbol{\varepsilon}_\mathbf{b}\} \end{Bmatrix} - \begin{Bmatrix} \{\overline{\mathbf{N}}\} \\ \{\overline{\mathbf{M}}\} \end{Bmatrix} \tag{4}$$

where [**A**], [**D**] and [**B**] are the matrices of extensional, bending and bending-extensional coupling stiffness coefficients. $\{\overline{\mathbf{N}}\}$ and $\{\overline{\mathbf{M}}\}$ are the thermal stress and moment resultants, respectively.

The transverse shear stress resultant vector, $\{\mathbf{Q}\} = \{Q_{xz}\ Q_{yz}\}^T$, is related to the transverse shear strain $\{\boldsymbol{\varepsilon}_\mathbf{s}\}$ as

$$\{\mathbf{Q}\} = [\mathbf{E}]\{\boldsymbol{\varepsilon}_\mathbf{s}\} \tag{5}$$

where $[\mathbf{E}]$ is the matrix of transverse shear stiffness coefficients.

For a laminated shell of thickness h, consisting of N layers with stacking angles θ_i ($i = 1, \ldots, N$) and layer thicknesses h_i ($i = 1, \ldots, N$), the expressions to compute the stiffness coefficients and thermal stress/moment resultants available in the literature (Jones 1975) are used.

The potential energy $U_1(\boldsymbol{\delta})$, consisting of strain energy and potential energy of transverse load, is given by

$$U_1(\delta) = \frac{1}{2}\int_A\left[\begin{Bmatrix}\varepsilon_p\\ \varepsilon_b\end{Bmatrix}^T\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B} & \mathbf{D}\end{bmatrix}\begin{Bmatrix}\varepsilon_p\\ \varepsilon_b\end{Bmatrix} + \{\varepsilon_s\}^T[\mathbf{E}]\{\varepsilon_s\} - \begin{Bmatrix}\varepsilon_p\\ \varepsilon_b\end{Bmatrix}^T\begin{Bmatrix}\overline{\mathbf{N}}\\ \overline{\mathbf{M}}\end{Bmatrix}\right]dA - \int_A q w_o dA \tag{6}$$

where δ is the vector of degrees of freedom associated to the displacement field in a finite element discretisation and q is the distributed load.

The potential energy $U_2(\delta)$ of initial state of in-plane stress resultants $\{\mathbf{N^0}\} = \{N^0_{xx}\ N^0_{yy}\ N^0_{xy}\}^T$ is written as

$$U_2(\delta) = \int_A \{\varepsilon_p^{NL}\}^T\{\mathbf{N^0}\}dA \tag{7}$$

Following the procedure of Rajasekaran and Murray (1973), the total potential energy $U(\delta)[=U_1(\delta)+U_2(\delta)]$ can be expressed as

$$\mathbf{U}(\delta) = \{\delta\}^T\left[\frac{1}{2}[[\mathbf{K}]-[\mathbf{K_T}]+[\mathbf{K_G}]]+\frac{1}{6}[\mathbf{N}_1(\delta)]+\frac{1}{12}[\mathbf{N}_2(\delta)]\right]\{\delta\} - \{\delta\}^T\{\mathbf{F_M}\} - \{\delta\}^T\{\mathbf{F_T}\} \tag{8}$$

where $[\mathbf{K}]$ is the linear stiffness matrix, $[\mathbf{N_1}]$ and $[\mathbf{N_2}]$ are nonlinear stiffness matrices linearly and quadratically dependent on the field variables, respectively. $[\mathbf{K_T}]$ and $[\mathbf{K_G}]$ are geometric stiffness matrices due to thermal and initial stress resultants. $\{\mathbf{F_M}\}$ and $\{\mathbf{F_T}\}$ are mechanical and thermal load vectors.

The minimization of total potential energy $U(\delta)$ given in Eq. (8) with respect to vector of degrees of freedom δ leads to the governing equation of shell

$$\left[[\mathbf{K}]-[\mathbf{K_T}]+[\mathbf{K_G}]+\frac{1}{2}[\mathbf{N}_1(\delta)]+\frac{1}{3}[\mathbf{N}_2(\delta)]\right]\{\delta\} = \{\mathbf{F_M}\}+\{\mathbf{F_T}\} \tag{9}$$

Eq. (9) can be employed to carry out linear/nonlinear static and eigenvalue buckling analyses by neglecting the appropriate terms as

Linear Static Analysis:

$$[\mathbf{K}]\{\delta\} = \{\mathbf{F_M}\}+\{\mathbf{F_T}\} \tag{10}$$

Nonlinear Static Analysis:

$$\left[[\mathbf{K}]-[\mathbf{K_T}]+\frac{1}{2}[\mathbf{N}_1(\delta)]+\frac{1}{3}[\mathbf{N}_2(\delta)]\right]\{\delta\} = \{\mathbf{F_M}\}+\{\mathbf{F_T}\} \tag{11}$$

Eigenvalue Buckling Analysis:

$$[\mathbf{K}]\{\delta\} = \lambda[\mathbf{K}^*_\mathbf{G}]\{\delta\} \tag{12}$$

where, $[\mathbf{K}^*_\mathbf{G}]$ is the geometric stiffness due initial state of stress developed because of unit loading, λ is load multiplication factor.

It may be noted that for evaluating $[\mathbf{K}^*_\mathbf{G}]$, firstly the static analysis of the shell using Eq. (10) for unit loading is carried out. The resulting deformation field is used to calculate the initial state of stress resultants using Eq. (4) and in turn, for evaluating the $[\mathbf{K}^*_\mathbf{G}]$ matrix.

Equilibrium path is traced by solving Eq. (11) using Newton-Raphson iteration procedure coupled with the adaptive displacement control method (Patel *et al.* 2005) to efficiently trace the equilibrium path. The nonlinear solution before critical points is obtained using load increments; and when the tangent stiffness matrix becomes semi-positive or negative definite, the subsequent solution is obtained by the adaptive displacement control. The degree of freedom having the highest rate of change in the previous step is selected as a control parameter and is updated in each step. The current step size is based on the step size and the number of equilibrium iterations in the previous step. The equilibrium iterations are continued for each load/displacement step until the convergence criteria suggested by Bergan and Clough (1972) are satisfied within the specific tolerance limit of 0.001%.

3. Element description

A C^0 continuous, eight-noded serendipity quadrilateral shear flexible shell element with five nodal degrees of freedom (u_0, v_0, w_0, θ_x, θ_y) developed based on field consistency approach is employed. The field variables are expressed in terms of their nodal values using shape functions as

$$(u_0, v_0, w_0, \theta_x, \theta_y) = \sum_{i=1}^{8} N_i^0 (u_{0i}, v_{0i}, w_{0i}, \theta_{xi}, \theta_{yi}) \tag{13}$$

where, N_i^0 are the original shape functions for the eight-noded quadratic serendipity element. It can be noted that the derivatives of shape functions $N_{i,x}^0$ and $N_{i,y}^0$ required for defining the various strain components within the element are linear in x and quadratic in y; and quadratic in x and linear in y, respectively, as the original interpolation functions are of quadratic type (in x and y) for the eight-noded element.

If the interpolation functions for an eight-noded element are used directly to interpolate the five field variables u_0, v_0, w_0, θ_x, θ_y in deriving the membrane and transverse shear strains, the element will lock and show oscillations in the membrane and transverse shear stresses. Field consistency requires that the membrane and transverse shear strains must be interpolated in a consistent manner (Prathap *et al.* 1988). This is achieved by smoothening the original interpolation functions using least-square method to the desired form i.e., the functions that are consistent with the derivative functions ($N_{i,x}^0$ or $N_{i,y}^0$).

Using the smoothed interpolation functions, the constrained membrane and shear strain components are expressed as

$$\left(\frac{u_o \sin\alpha}{r} + \frac{\partial v_o}{\partial y} + \frac{w_o \cos\alpha}{r}\right) = \sum_{i=1}^{8} \left(\frac{N_{yi}^1 u_{0i} \sin\alpha}{r} + \frac{\partial N_i^0}{\partial y} v_{0i} + \frac{N_{yi}^1 w_{0i} \cos\alpha}{r}\right) \tag{14}$$

$$\left(\theta_x + \frac{\partial w_o}{\partial x}\right) = \sum_{i=1}^{8} \left(N_{xi}^1 \theta_{xi} + \frac{\partial N_i^0}{\partial x} w_{0i}\right) \tag{15a}$$

$$\left(\theta_y + \frac{\partial w_o}{\partial y} - \frac{v_o \cos\alpha}{r}\right) = \sum_{i=1}^{8} \left(N_{yi}^1 \theta_{yi} + \left(\frac{\partial N_i^0}{\partial y}\right) w_{0i} - \frac{N_{yi}^1 v_{0i} \cos\alpha}{r}\right) \tag{15b}$$

Here, the smoothed functions N_{xi}^1 (linear in x and quadratic in y) and N_{yi}^1 (quadratic in x and linear in y) are consistent with derivative functions $\partial N_i^0/\partial x$ and $\partial N_i^0/\partial y$. The other strain fields are expressed in terms of original shape functions (N_i^0) and their derivatives.

4. Results and discussion

The nonlinear thermoelastic static response characteristics of laminated composite conical panels subjected to uniform temperature rise are analyzed. The finite element formulation is verified with the results available in the literature before proceeding for the detailed parametric studies.

The material properties considered, unless otherwise specified, are

$$E_1 = 138 \text{ GPa}, E_2 = E_3 = 8.28 \text{ GPa}, G_{12} = G_{23} = G_{13} = 6.9 \text{ GPa}, \nu_{12} = \nu_{13} = 0.33, \nu_{23} = 0.373,$$
$$\alpha_1 = 0.18 \times 10^{-6} \text{ /°C}, \alpha_2 = \alpha_3 = 27 \times 10^{-6} \text{ /°C}$$

where E, G, ν and α are Young's modulus, shear modulus, Poisson's ratio and coefficient of thermal expansion, respectively. The subscripts 1, 2 and 3 are the principal material directions. All the layers are of equal thickness and the ply-angle is measured with respect to the meridional axis (x-axis). The first layer is the innermost layer of the panel.

The boundary conditions considered, unless otherwise specified, are:

Immovable simply supported (SSSS):

$$u_0 = v_0 = w_0 = \theta_y = 0 \quad \text{along curved edges}$$
$$u_0 = v_0 = w_0 = \theta_x = 0 \quad \text{along straight edges.}$$

Clamped (CCCC): $u_0 = v_0 = w_0 = \theta_x = \theta_y = 0$ along all edges.

The nonlinear response of isotropic ($E = 3.1 \times 10^3$ MPa and $\nu = 0.3$) cylindrical panel, hinged ($u_0 = v_0 = w_0 = \theta_x = 0$) along straight edges and free along curved edges, is analyzed for central point loading. The geometrical parameters of the panel are: $a = 504$ mm, $r_1 = 2540$ mm, $h = 6.35$ mm and 3.175 mm, $\beta_1 = 0.2$ rad, $\alpha = 0$. Based on convergence study, 5×5 grid mesh (meridional and circumferential directions) for $h = 6.35$ mm and 4×12 for $h = 3.175$ mm are found to be adequate to model one quarter of the panel. The results are shown in Figs. 2 and 3. It can be observed from these figures that the present results are in good agreement with those available in the literature. Further, the present adaptive displacement control method correctly captures the multiple snapping responses.

For validation of present approach for thermal loading case, immovable simply supported laminated cylindrical panel with the following geometrical parameters is considered:

$$r_1/a = 5;\ a/h = 200, 400, 800;\ a = 0.1 \text{ m};\ \beta_1 = 0.2 \text{ rad};\ \alpha = 0.$$

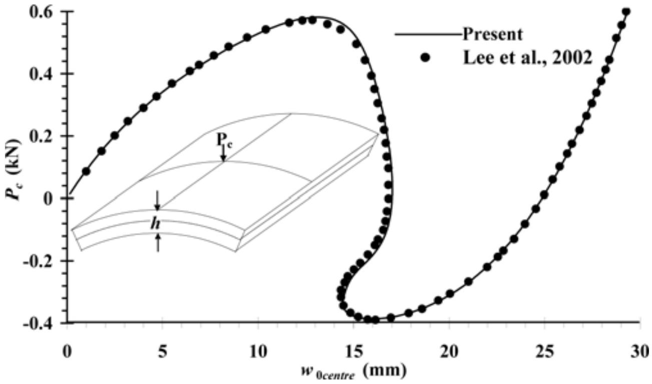

Fig. 2 Nonlinear response of isotropic cylindrical panel under central point load ($r_1/a = 5$, $\beta_1 = 0.2$ rad., $a/h = 80$)

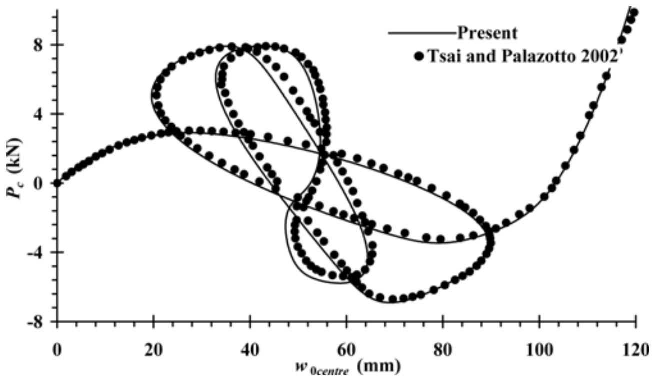

Fig. 3 Nonlinear response of isotropic shallow cylindrical panel under central point load (r_1/a = 5, β_1= 0.2 rad., a/h = 40)

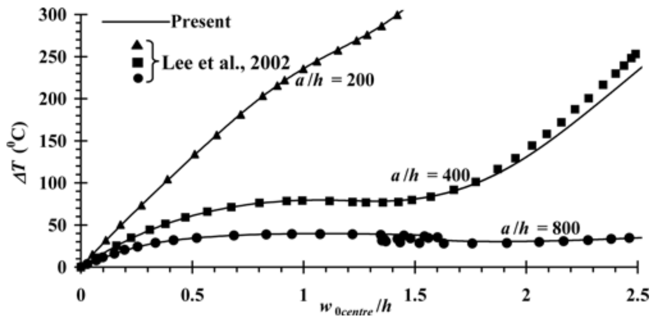

Fig. 4 Nonlinear response of laminated (0º/90º)$_S$ cylindrical panel subjected to uniform temperature rise (r_1/a = 5, β_1= 0.2 rad.)

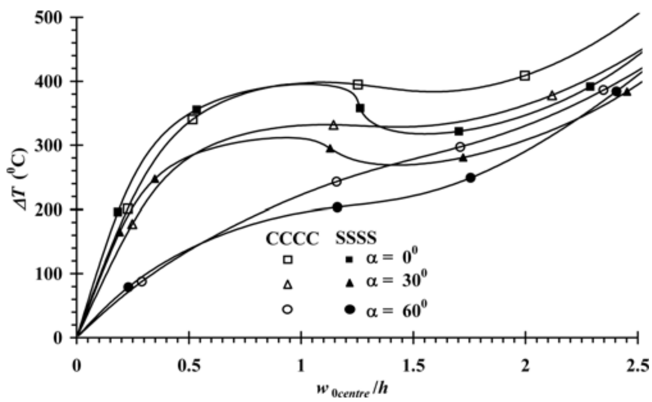

Fig. 5 Effect of boundary conditions on the nonlinear response of laminated (0º/90º)$_S$ conical panel (a/r_1=0.5, r_1/h = 400, β_1 = 0.6 rad.)

The results, using 5 × 5 converged mesh for quarter model, shown in Fig. 4 are in fairly good agreement with those of Lee *et al.* (2002).

The effect of semi-cone angle (α) and the boundary conditions on the response of four-layered cross-ply laminated (0º/90º)$_S$ conical panel (r_1/h = 400; L/r_1 = 0.5; α = 0º, 30º, 60º; h = 8 mm, β_1 = 0.6 rad) is studied. The results, obtained using 12 × 12 converged mesh, are shown in Fig. 5 as

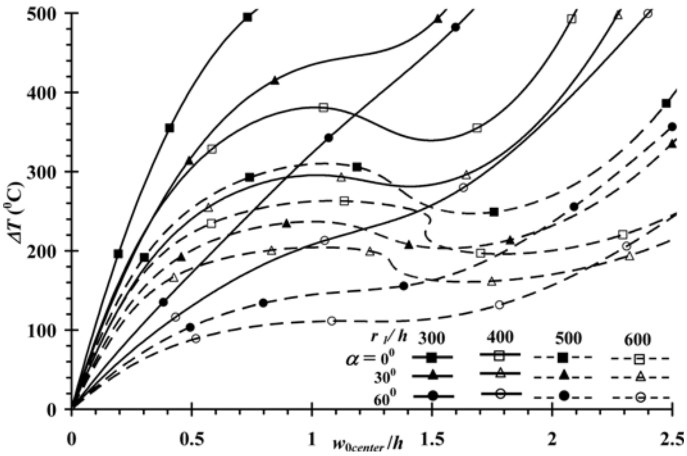

Fig. 6 Effect of radius-to-thickness ratio (r_1/h) on the nonlinear response of laminated $(0^o/90^o)_S$ conical panel ($a/r_1 = 0.5$, $\beta_1 = 0.5$ rad.)

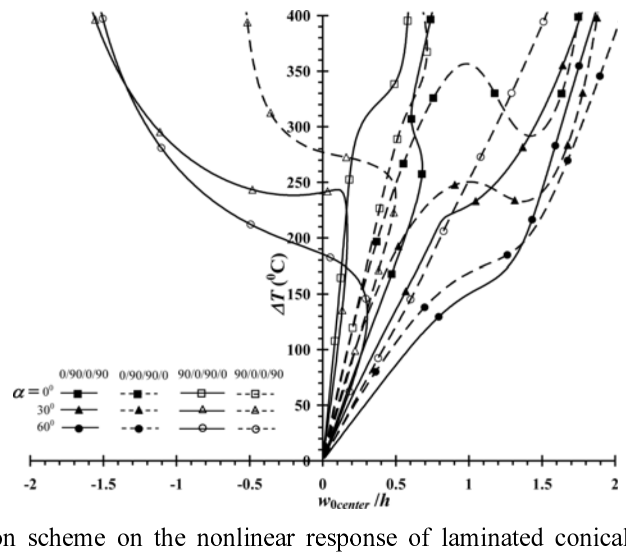

Fig. 7 Effect of lamination scheme on the nonlinear response of laminated conical panel ($a/r_1 = 0.5$, $r_1/h = 500$, $\beta_1 = 0.4$ rad.)

uniform temperature rise (ΔT) versus central transverse displacement parameter ($w_{0\text{centre}}/h$) curves. It can be inferred from this figure that with the increase in semi-cone angle (α), $w_{0\text{center}}/h$ increases for a particular value of temperature rise (ΔT). The panels with all edges simply supported (SSSS) exhibit significant snap-through type of response especially for $\alpha = 0^o$ and 30^o whereas panels with all edges clamped (CCCC) show mostly stable response. It can also be viewed from this figure that the initial equilibrium paths for both the boundary conditions are qualitatively identical whereas the response at greater displacement levels reveals significant sensitivity to boundary conditions and semi-cone angle (α).

The effect of radius-to-thickness ratio (r_1/h) on nonlinear response is studied considering immovable simply-supported four-layered cross-ply $(0^o/90^o)_S$ laminated panels ($L/r_1 = 0.5$; $\alpha = 0^o$, 30^o, 60^o; $h = 8$ mm, $\beta_1 = 0.5$ rad) and the results are shown in Fig. 6. It can be observed from this figure that even conical panels with α = 60° reveal snap-through type of response at larger r_1/h ratio.

The effect of lamination scheme is studied for immovable simply-supported panels (r_1/h = 500; L/r_1 = 0.5; α = 0°, 30°, 60°; h = 8 mm, β_1 = 0.4 rad). The results are shown in Fig. 7. Panels with $(0°/90°)_S$ lamination scheme show greater degree of snap-through behaviour whereas those with lamination $(90°/0°)_S$ scheme show stable response. The response of anti-symmetric lamination schemes $(0°/90°)_2$ and $(90°/0°)_2$ exhibit qualitatively similar behaviour for α = 0° (cylindrical panel) whereas behaviour is different for conical panels.

5. Conclusions

The nonlinear thermoelastic response characteristics of laminated composite conical panels subjected to uniform temperature rise are studied employing finite element approach. The influence of semi-cone angle, boundary conditions, radius-to-thickness ratio and lamination scheme is examined through a parametric study. It is brought out that the panels with all edges simply supported (SSSS) exhibit significant snap-through type of response compared to panels with all edges clamped (CCCC). The degree of snap-through increases with the increase in the radius to thickness ratio. Further, the symmetrically laminated panels with outer layer fibres in meridional direction reveal snap-through response whereas panels with outer layer fibres in circumferential direction show stable response.

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