Numerical solution of singular integral equation for multiple curved branch-cracks

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Abstract. In this paper, numerical solution of the singular integral equation for the multiple curved branch-cracks is investigated. If some quadrature rule is used, one difficult point in the problem is to balance the number of unknowns and equations in the solution. This difficult point was overcome by taking the following steps: (a) to place a point dislocation at the intersecting point of branches, (b) to use the curve length method to covert the integral on the curve to an integral on the real axis, (c) to use the semi-open quadrature rule in the integration. After taking these steps, the number of the unknowns is equal to the number of the resulting algebraic equations. This is a particular advantage of the suggested method. In addition, accurate results for the stress intensity factors (SIFs) at crack tips have been found in a numerical example. Finally, several numerical examples are given to illustrate the efficiency of the method presented.

Keywords: curved branch-cracks; stress intensity factor; singular integral equation; numerical solution.

1. Introduction

The curved crack problems have got much attention by many researchers. A weakly singular integral equation with logarithmic kernel was developed to solve the problems (Cheung and Chen 1987). In the equation the dislocation distribution is taken as the unknown function, and the resultant force as the right hand term. Two types of singular integral equations were developed to the same problems. In the first type of equations the dislocation distribution is taken as the unknown function, and the traction as the right hand term (Savruk 1981, Dreilich and Gross 1985, Chen 1995). In the second type of equations the crack opening displacement (COD) is taken as the unknown function, and the resultant force function as the right hand term. Recently, a hypersingular integral equation was developed to solve the curved crack problems (Mayrhofer and Fischer 1992, Linkov and Mogilevskaya 1994, Mogilevskaya 2000, Martin 2000, Linkov 2002, Chen 2003). In the equation, the crack opening displacement (COD) is taken as the unknown function, and the traction as the right hand term.

In addition, several relevant publications are introduced. A concentric arc crack in a circular disk was studied (Xu 1995). A fundamental solution was obtained, which is used for the formulation of

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the singular integral equation. A new boundary integral equation formulation for plane elastic bodies containing cracks and holes was suggested (Chau and Wang 1999). Suppose that the inner cracks and the holes and the outer boundary are under traction boundary condition. In the resultant integral equation, the involved unknowns are the derivative of displacement with respect to the curve length, or derivative of COD with the curve length. Therefore, the kernels involved are not hyperpersingular, and are Cauchy type singular.

The interaction of a curved crack with a circular elastic inclusion was solved by using the complex variable function method, and computed results for SIFs were given (Cheeseman and Santare 2000). Crack problems with branches or kinks were solved (Burton and Phoenix 2000). The solution depends on an appropriate assumption of the dislocation density distribution along crack. A hypersingular integral equation was suggested to solve the edge crack problem of half-plane (Mogilevskaya 2000). Based on the hypersingular integral equation for curve crack problem, the perturbation method is used to solve the slightly curved crack problem (Martin 2000). Solution of multiple edge cracks in an elastic half plane was studied (Jin and Keer 2006). The solution depends on an appropriate distribution of dislocation along the prospective site of crack.

The line branch-crack problem was studied previously (Theocaris 1977). In the study, in order to balance the number of unknowns and equations, different discretization schemes were used to different branches. This is a difficult point in the branch crack problem. This difficult point was overcome in this paper, and the detail can be found in the following analysis.

In addition to the formulation of integral equations the numerical solutions for the equations are important to obtain the final solution. Recently, a curve length method was developed to solve the hypersingular integral equations in the curved crack problems numerically (Chen 2003). A striking advantage of the method is that the integral defined on a curve can be converted to a relevant integral defined on the real axis. Therefore, the known integration rules on the real axis can be used to the curved cracks.

For the branch crack problem, an extended finite element method was suggested to solve problems of the arbitrary branched and intersecting cracks (Daux *et al.* 2000). Recently, an algorithm for the computation of the stress field around a branched crack was suggested (Englund 2006). The algorithm is based on an integral equation with good numerical properties. Those papers devoted to the line crack case.

For a slightly curved crack, a treatment of the integral equations by Frobenius' method was suggested (Ballarini and Vallaggio 2006). The complex variable function method was used to formulate the multiple curved crack problems into hypersingular integral equations. The equations were solved numerically using the so-called curve length coordinate method (Nik Long and Eshkuvatov 2009).

A general formulation for evaluating the T-stress at tips of a curved crack was introduced. In the formulation, a singular integral equation with the distribution of dislocation along the curve was suggested (Chen and Lin 2008). A perturbation method was suggested for evaluating the T-stress at crack tips in a curved crack (Chen *et al.* 2008).

In this paper, the problem for multiple curved branch-cracks is studied. A singular integral equation is suggested to solve the problem. It is found that there was a difficult point in the numerical solution. We know that the integral equation is generally solved by discretization of the unknown function. However, after discretization the number of unknowns may not be equal to the number of the algebraic equations. This difficult point was overcome by assuming a point dislocation at the intersect point of the branch-cracks. Based on this assumption, the semi-open quadrature rule and the curve length method will lead to a final solution numerically. Finally, several numerical examples are given to illustrate the efficiency of the presented method.

2. Singular integral equation for curved branch-cracks

The fundamentals of the complex variable function method, which plays an important role in plane elasticity, are briefly introduced in what follows (Muskhelishvili 1953). In the method, the stresses (σ_x, σ_y, σ_{xy}), the resultant forces (X, Y) and the displacements (u, v) are expressed in terms of the complex potentials $\phi(z)$ and $\psi(z)$ such that

$$\begin{aligned} \sigma_x + \sigma_y &= 4\mathrm{Re}\Phi(z) \\ \sigma_y - i\sigma_{xy} &= 2\mathrm{Re}\Phi(z) + z\overline{\Phi'(z)} + \overline{\Psi(z)} \end{aligned} \tag{1}$$

$$f = -Y + iX = \phi(z) + z\overline{\phi'(z)} + \overline{\psi(z)} \tag{2}$$

$$2G(u+iv) = \kappa\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)} \tag{3}$$

where G is the shear modulus of elasticity, $\kappa = (3-\nu)/(1+\nu)$ in the plane stress problem, $\kappa = 3-4\nu$ in the plane strain problem, and ν is the Poisson's ratio. In Eq. (1) we denote $\Phi(z) = \phi'(z)$, $\Psi(z) = \psi'(z)$.

Except for the physical quantities mentioned above, from Eqs. (2) and (3) two derivatives in specified direction (abbreviated as DISD) are introduced as follows (Savruk 1981, Chen 1995)

$$J_1(z) = \frac{d}{dz}\{-Y+iX\} = \Phi(z) + \overline{\Phi(z)} + \frac{d\bar{z}}{dz}(z\overline{\Phi'(z)} + \overline{\Psi(z)}) = N + iT \tag{4}$$

$$J_2(z) = 2G\frac{d}{dz}\{u+iv\} = \kappa\Phi(z) - \overline{\Phi(z)} - \frac{d\bar{z}}{dz}(z\overline{\Phi'(z)} + \overline{\Psi(z)}) = (\kappa+1)\Phi(z) - J_1 \tag{5}$$

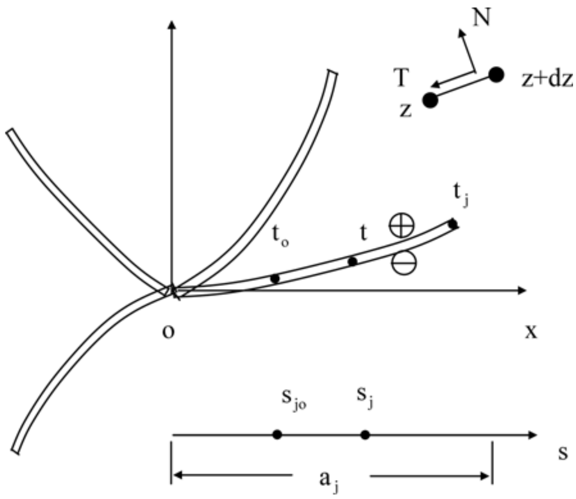

Fig. 1 The curved branch-cracks and the curve length coordinate

It is easy to verify that $J_1 = N + iT$ denotes the normal and tangential tractions along the segment $\overline{z, z+dz}$ (Fig. 1). Secondly, the J_1 and J_2 values depend not only on the position of a point "z", but also on the direction of the segment "$d\bar{z}/dz$". The symbol of derivative $d\{\ \}/dz$ is always defined as a derivative in a specified direction (DISD).

If the tractions applied on the curved crack are the same in magnitude and opposite in direction for the upper and lower crack faces, the complex potentials caused by a dislocation distribution $g_j'(t)$ $(t \in L_j,\ j = 1, 2, \dots N)$ along the curved branch-cracks and a point dislocation "D" can be expressed by (Savruk 1981, Chen 1995)

$$\phi(z) = -\frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}\ln(t-z)g_j'(t)dt + \frac{D}{2\pi}\ln z, \quad (\text{with } D = D_1 + iD_2)$$

$$\Phi(z) = \phi'(z) = \frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}\frac{g_j'(t)dt}{t-z} + \frac{D}{2\pi z}$$

$$\psi(z) = -\frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}\ln(t-z)\overline{g_j'(t)}d\bar{t} - \frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}\frac{\bar{t}g_j'(t)dt}{t-z} + \frac{\bar{D}}{2\pi}\ln z$$

$$\Psi(z) = \psi'(z) = \frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}\frac{\overline{g_j'(t)}d\bar{t}}{t-z} - \frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}\frac{\bar{t}g_j'(t)dt}{(t-z)^2} + \frac{\bar{D}}{2\pi z} \tag{6}$$

where

$$g_j'(t) = \frac{2Gi}{\kappa+1}\frac{d\{u_j(t)+iv_j(t)\}_\Delta}{dt}, \quad (t \in L_j, j = 1, 2, \dots, N) \tag{7}$$

In Eq. (7), $\{u_j(t)+iv_j(t)\}_\Delta (= (u_j(t)+iv_j(t))^+ - (u_j(t)+iv_j(t))^-)$ stands for the jump value of the displacements, and $(u_j(t)+iv_j(t))^+$, $(u_j(t)+iv_j(t))^-$ denotes the displacements at a point "t" of the upper (lower) face of the j-th branch-crack "L_j"(Fig. 1). In Eq. (6), the terms $D/2\pi z$ and $\bar{D}/2\pi z$ are assumed, which corresponds to a point dislocation at the origin. The reason for introducing this term will be illustrated later.

Clearly, from Eqs. (3), (6) and the single-valuedness condition of displacements, the following constraint equation is obtained (Savruk 1981, Chen and Hasebe 1995)

$$\sum_{j=1}^{N}\int_{L_j}g_j'(t)dt - D = 0 \tag{8}$$

In the problem, the boundary condition is expressed as

$$N_k(t_o) + iT_k(t_o) = \tilde{N}_k(t_o) + i\tilde{T}_k(t_o), \quad (t_o \in L_k, k = 1, 2, \dots, N) \tag{9}$$

where $\tilde{N}_k(t_o) + i\tilde{T}_k(t_o)$ is traction assumed along the k-th curved branch-crack (Fig. 1).

Simply taking the following steps: (a) substituting Eq. (6) into (4), (b) letting "z" approach "t_o" $(t_o \in L_k)$ and "$d\bar{z}/dz$" to "$d\bar{t}_o/dt_o$" in Eq. (4), (c) using the equations for limit values of the Cauchy type integral (Savruk 1981, Chen 1995), (d) using the boundary condition (9), we obtain the following integral equation

$$\frac{1}{\pi}\sum_{j=1}^{N}\int_{L_j}\frac{g_j'(t)dt}{t-t_o}+\frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}K_1(t,t_o)g_j'(t)dt+\frac{1}{2\pi}\sum_{j=1}^{N}\int_{L_j}K_2(t,t_o)\overline{g_j'(t)}d\bar{t}$$

$$+\frac{1}{2\pi}\left\{\frac{D}{t_o}+\frac{\bar{D}}{\bar{t}_o}+\frac{d\bar{t}_o}{dt_o}\left(-\frac{t_o\bar{D}}{\bar{t}_o^2}+\frac{D}{\bar{t}_o}\right)\right\}=\tilde{N}_k(t_o)+i\tilde{T}_k(t_o)\quad(t_o\in L_k,k=1,2,\ldots,N) \tag{10}$$

where the kernels are defined by

$$K_1(t,t_o)=\frac{d}{dt_o}\left\{\ln\frac{t_o-t}{\bar{t}_o-\bar{t}}\right\}=-\frac{1}{t-t_o}+\frac{1}{\bar{t}-\bar{t}_o}\frac{d\bar{t}_o}{dt_o}$$

$$K_2(t,t_o)=-\frac{d}{dt_o}\left\{\frac{t_o-t}{\bar{t}_o-\bar{t}}\right\}=\frac{1}{\bar{t}-\bar{t}_o}-\frac{t-t_o}{(\bar{t}-\bar{t}_o)^2}\frac{d\bar{t}_o}{dt_o} \tag{11}$$

In Eq. (11) the expression $d\{\ \}/dt_o$ should be defined as a DISD-derivative (Chen 1995). Note that only if the observation point t_o is on L_k (or $t_o\in L_k$) and the integration is performed along the curve "L_k" (or $t\in L_k$), one of the first integral in Eq. (10) is singular. Meantime, other integrals in Eq. (10) are regular.

The curve length method is used to solve the integral equations numerically (Chen 2003). The j-th curved branch-crack is mapped on the real axis "s_j" with a crack length "a_j" (Fig. 1). The mapping is expressed by the function $t(s_j)$. After mapping, the function $g_j'(t)$ is rewritten in the form

$$g_j'(t)=h_j(s_j)=\sqrt{\frac{s_j}{a_j-s_j}}H_j(s_j),\quad(0<s_j<a_j,\text{ where }H_j(s_j)=H_{j1}(s_j)+iH_{j2}(s_j)) \tag{12}$$

The assumed expression in Eq. (12) is obtained from the behavior of the dislocation distribution in the vicinity of a crack tip (Chen and Hasebe 1995).

When the observation point in on the j-th curved branch-crack (or $t_o\in L_j$), the first two integrals along the integration path L_j in Eq. (10) may be rewritten in the form

$$I_1=\frac{1}{\pi}\int_{L_j}\frac{g_j'(t)dt}{t-t_o}=\frac{1}{\pi}\int_0^{a_j}\frac{s_j-s_{jo}}{t-t_o}\frac{dt}{ds_j}\frac{H_j(s_j)}{s_o-s_{jo}}\sqrt{\frac{s_j}{a_j-s_j}}ds_j \tag{13}$$

$$I_2=\frac{1}{2\pi}\int_{L_j}K_1(t,t_o)g_j'(t)dt=\frac{1}{2\pi}\int_0^{a_j}K_1(t,t_o)\frac{dt}{ds_j}H_j(s_j)\sqrt{\frac{s_j}{a_j-s_j}}ds_j \tag{14}$$

where s_{jo} is obtained from the mapping $t_o=t(s_{jo})$ (Fig. 1). Clearly, I_1 is a singular integral and I_2 is a regular integral. Note that all integrals in Eq. (10) belong to the types shown by Eqs. (13) and (14).

In the following derivation, the semi-open quadrature rules for the singular integral and the regular integral are suggested (Boiko *et al.* 1981)

$$\int_0^a\frac{f(t)}{t-x_k}\sqrt{\frac{t}{a-t}}dt=\sum_{m=1}^{M}\frac{W_mf(t_m)}{t_m-x_k}$$

$$\int_0^a g(t)\sqrt{\frac{t}{a-t}}dt=\sum_{m=1}^{M}W_mg(t_m) \tag{15}$$

where

$$W_m = \frac{\pi a}{M}\sin^2\frac{m\pi}{2M} \quad (m = 1, 2, \ldots, M-1), \quad W_M = \frac{\pi a}{2M}$$

$$t_m = a\sin^2\frac{m\pi}{M}, \qquad (m = 1, 2, \ldots, M)$$

$$x_k = a\sin^2\frac{(k-0.5)\pi}{M}, \quad (k = 1, 2, \ldots, M) \tag{16}$$

where M is some integer. Here and after, we call t_m the abscissas, and x_k the collocation points (or the observation points).

After taking the following steps: (a) to use the integral Eq. (10) and the constraint equation (8), (b) to use the curve length method based on the mapping relation function $t(s_j)$ $(j = 1, 2, \ldots N)$, (c) to use the quadrature rues shown by Eq. (15), an algebraic equation is obtained. In the equation, the unknowns are the values of $H_j(s)$ $(j = 1, 2, \ldots N)$ at abscissas and D_1, D_2. It is proved that in the present formulation the number of equations is equal to the number of unknowns. Details for this formulation can be referred to (Chen and Hasebe 1995).

From obtained solution of the algebraic equation, the stress intensity factor (abbreviated as SIF) at the j-th crack tip can be evaluated by (Savruk 1981, Chen 1995)

$$(K_1 - iK_2)_j = -\lim_{t \to t_j} \sqrt{2\pi|t-t_j|}\, g_j'(t) = -(2\pi a_j)^{1/2} H_j(a_j) \tag{17}$$

At the intersecting point of branches, the sharp corner may have some singular stress distribution. However, the order of the singularity at the corner is generally less than that at the crack tip. Therefore, many researchers neglected the singularity (Savruk 1981, Chen and Hasebe 1995).

3. Numerical example

Some numerical examples are given to illustrate the results of the presented method.

Example 1

In the first example, a circular arc crack is considered as composed of two arc cracks (OA and OB) and the remote tension is $\sigma_x^\infty = \sigma_y^\infty = p$ (Fig. 2(a)). In the computation, $M = 5$, 11 and 21 in

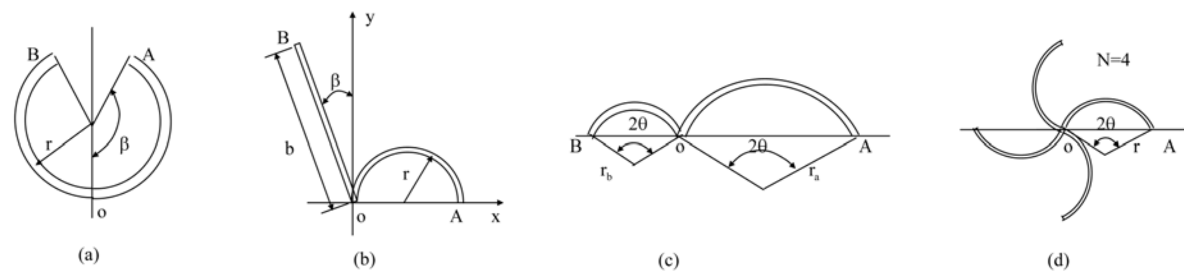

Fig. 2 Four cases of the curved branch-cracks: (a) a circular arc crack composed of two branches “OA” and “OB”, (b) the curved branch-cracks composed of a line crack and a half-circle crack, (c) the curved branch-cracks composed of two arc cracks, (d) many circular arc cracks in a stacked position

Table 1 Non-dimensional SIFs $F_{1A}(\beta)$ and $F_{2A}(\beta)$ for an arc crack (see Fig. 2(a) and Eq. (18))

$F_{1A}(\beta)$

β (degree)	15	30	45	60	75	90	105	120	135	150	165
$M = 5$	0.9748	0.9053	0.8059	0.6928	0.5788	0.4714	0.3736	0.2857	0.2065	0.1339	0.0659
$M = 11$	0.9748	0.9053	0.8059	0.6928	0.5788	0.4714	0.3736	0.2857	0.2065	0.1339	0.0658
$M = 21$	0.9748	0.9053	0.8059	0.6928	0.5788	0.4714	0.3736	0.2857	0.2065	0.1339	0.0658
*	0.9752	0.9056	0.8063	0.6935	0.5794	0.4717	0.3734	0.2851	0.2053	0.1319	0.0619
Exact**	0.9748	0.9053	0.8059	0.6928	0.5788	0.4714	0.3736	0.2857	0.2065	0.1339	0.0658

$F_{2A}(\beta)$

β (degree)	15	30	45	60	75	90	105	120	135	150	165
$M = 5$	0.1283	0.2426	0.3338	0.4000	0.4442	0.4714	0.4869	0.4949	0.4984	0.4997	0.5002
$M = 11$	0.1283	0.2426	0.3338	0.4000	0.4442	0.4714	0.4869	0.4949	0.4984	0.4997	0.5000
$M = 21$	0.1283	0.2426	0.3338	0.4000	0.4441	0.4714	0.4869	0.4949	0.4984	0.4997	0.5000
*	0.1285	0.2427	0.3340	0.4006	0.4449	0.4724	0.4877	0.4957	0.4991	0.5001	0.4999
Exact**	0.1283	0.2426	0.3338	0.4000	0.4441	0.4714	0.4869	0.4949	0.4984	0.4997	0.5000

*Using the hypersingular integral equation method (Chen 2003), Exact** (Murakami 1987).

Eq. (15) is used. The calculated results for the SIFs at the crack tip "A" are expressed as

$$K_{1A} = F_{1A}(\beta)\sqrt{\pi r \sin\beta}, \quad K_{2A} = F_{2A}(\beta)p\sqrt{\pi r \sin\beta} \tag{18}$$

and are listed in Table 1. From tabulated results we see that even $M = 5$ is taken, accurate results for SIFs are obtained. The difference between the results of $M = 5$ and those from an exact solution is negligible. In Table 1, a comparison result (marked with *) is also listed (Chen 2003). The result is obtained from the hypersingular integration equation with the number of abscissas $M = 155$. Thus, it is easy to see the advantage of the present approach.

Example 2

In the second example, one branch is a half-circular crack, and other is a line crack (Fig. 2(b)). The length of the line crack is $b = \pi r$, and the crack has a rotation with an angle β. The remote tension is $\sigma_x^\infty = \sigma_y^\infty = p$, and $M = 21$ in Eq. (15) is used. The calculated results for the SIFs at the crack tips "A" and "B" are expressed as

$$K_{1A} = F_{1A}(\beta)p\sqrt{\pi b}, \quad K_{2A} = F_{2A}(\beta)p\sqrt{\pi b}$$
$$K_{1B} = F_{1B}(\beta)p\sqrt{\pi b}, \quad K_{2B} = F_{2B}(\beta)p\sqrt{\pi b} \ \text{(with } b = \pi r\text{)} \tag{19}$$

and are plotted in Fig. 3. From Fig. 3 we see that in the assumed loading case ($\sigma_x^\infty = \sigma_y^\infty = p$), variation of the F_{1A} with respect to the rotation angle β is significant, for example, $F_{1A} = 0.4886$, $F_{2A} = -0.3222$ for $\beta = 0^\circ$ case, and $F_{1A} = 0.0325$, $F_{2A} = -0.1956$ for $\beta = 160^\circ$ case. Secondly, in the studied range of β, the SIFs for F_{1B} are generally in a higher level, and they are varying from 0.7449 to 0.9095. That is to say the propagation of crack may happen at the tip of the line crack "OB".

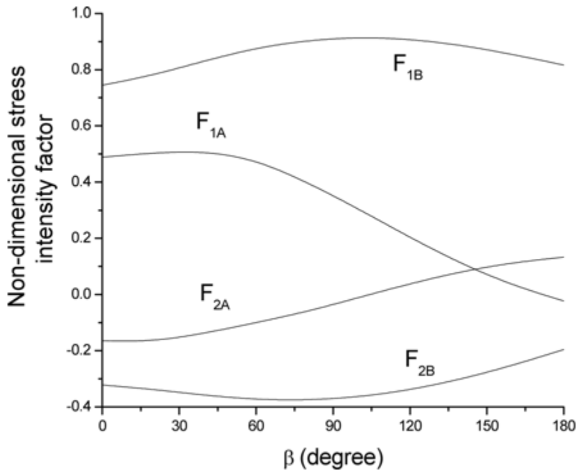

Fig. 3 Non-dimensional SIFs $F_{1A}(\beta)$, $F_{2A}(\beta)$, $F_{1B}(\beta)$ and $F_{2B}(\beta)$ for the multiple curved branch-cracks (see Fig. 2(b) and Eq. (19))

Example 3

In the third example, two arc cracks are in series with the radius r_a and r_b, respectively (Fig. 2(c)). Cracks have a spanning angle 2θ. The remote tension is $\sigma_x^\infty = \sigma_y^\infty = p$. $M = 21$ is used for the arc crack "OA", and $M = 21 * \sqrt{r_b/r_a}$ is used for the arc crack "OB". The calculated results for the SIFs at the crack tips "A" and "B" are expressed as

$$K_{1A} = F_{1A}(\theta) p\sqrt{\pi a}, \quad K_{2A} = F_{2A}(\theta) p\sqrt{\pi a} \text{ (with } a = 2\theta r_a)$$
$$K_{1B} = F_{1B}(\theta) p\sqrt{\pi b}, \quad K_{2B} = F_{2B}(\theta) p\sqrt{\pi b} \text{ (with } b = 2\theta r_b) \tag{20}$$

and are plotted in Fig. 4. From Fig. 4 we see that with the increase of the angle "θ", for example, the value of $F_{1A}(\theta)$ is gradually reduced, and the value of $|F_{2A}(\theta)|$ is gradually increased.

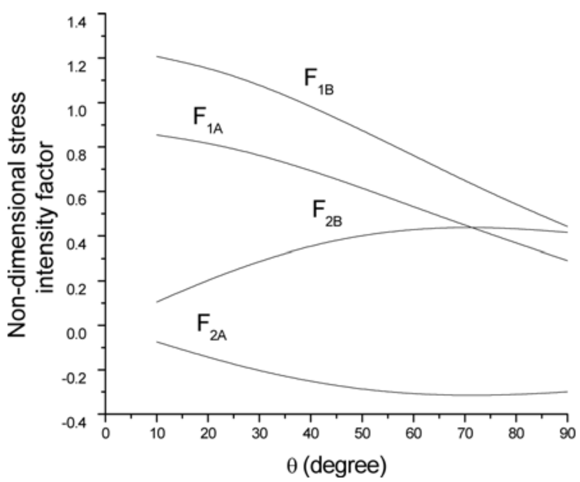

Fig. 4 Non-dimensional SIFs $F_{1A}(\theta)$, $F_{2A}(\theta)$, $F_{1B}(\theta)$ and $F_{2B}(\theta)$ for two multiple curved branch-cracks in series (see Fig. 2(c) and Eq. (20))

Example 4

In the fourth example, N arc cracks with a radius "r" are in a stacked position (Fig. 2(d)). Cracks have a spanning angle 2θ. The remote tension is $\sigma_x^\infty = \sigma_y^\infty = p$. $M = 21$ is used for all curved branch-cracks. For $N = 2, 3 \ldots$ to 10 and $\theta = 10^\circ, 20^\circ$,.. to 90°. The calculated results for the SIFs at the crack tip "A" are expressed as

$$K_{1A} = F_{1A}(\theta) p\sqrt{\pi a}, \quad K_{2A} = -F_{2A}(\theta) p\sqrt{\pi a}, \quad (\text{with } a = 2\theta r) \tag{21}$$

and are listed in Table 2. As in the previous example, we see that with the increase of the angle "θ", the value of $F_{1A}(\theta)$ is gradually reduced, and the value of $F_{2A}(\theta)$ is gradually increased. Secondly, when the N (the number of braches) is increased, SIFs are generally reduced. For example, in the case of $\theta = 30^\circ$, $F_{1A} = 0.8799$, $F_{2A} = 0.2404$ is for $N = 2$ case, and $F_{1A} = 0.5083$, $F_{2A} = 0.1931$ is for $N = 10$ case. This is somewhat called shielding effect in fracture analysis.

Table 2 Non-dimensional SIFs $F_{1A}(\beta)$ and $F_{2A}(\beta)$ for N arc crack in a stacked position (see Fig. 2(d) and Eq. (21))

$F_{1A}(\theta)$

$N =$	2	3	4	5	6	7	8	9	10
$\theta =$									
$\to 0$	1.0000	0.9415	0.8635	0.7952	0.7425	0.6972	0.6590	0.6263	0.5979
10.0	0.9861	0.9267	0.8494	0.7838	0.7299	0.6852	0.6476	0.6154	0.5875
20.0	0.9453	0.8836	0.8081	0.7448	0.6931	0.6503	0.6143	0.5835	0.5569
30.0	0.8799	0.8147	0.7423	0.6828	0.6344	0.5947	0.5613	0.5329	0.5083
40.0	0.7940	0.7248	0.6565	0.6019	0.5581	0.5223	0.4925	0.4672	0.4454
50.0	0.6926	0.6195	0.5563	0.5076	0.4692	0.4383	0.4126	0.3910	0.3724
60.0	0.5819	0.5056	0.4483	0.4061	0.3738	0.3481	0.3271	0.3095	0.2944
70.0	0.4678	0.3899	0.3391	0.3039	0.2779	0.2577	0.2414	0.2279	0.2165
80.0	0.3564	0.2791	0.2353	0.2073	0.1875	0.1727	0.1610	0.1514	0.1435
90.0	0.2529	0.1789	0.1430	0.1221	0.1082	0.0982	0.0905	0.0843	0.0801

$F_{2A}(\theta)$

$N =$	2	3	4	5	6	7	8	9	10
$\theta =$									
$\to 0$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10.0	0.0864	0.0939	0.0922	0.0885	0.0845	0.0808	0.0774	0.0744	0.0716
20.0	0.1681	0.1818	0.1780	0.1707	0.1629	0.1557	0.1490	0.1430	0.1377
30.0	0.2404	0.2579	0.2517	0.2407	0.2294	0.2189	0.2094	0.2008	0.1931
40.0	0.2993	0.3177	0.3083	0.2938	0.2793	0.2659	0.2539	0.2432	0.2336
50.0	0.3419	0.3574	0.3442	0.3263	0.3090	0.2933	0.2793	0.2669	0.2559
60.0	0.3663	0.3753	0.3573	0.3361	0.3164	0.2989	0.2836	0.2702	0.2583
70.0	0.3719	0.3708	0.3473	0.3229	0.3011	0.2824	0.2662	0.2523	0.2402
80.0	0.3592	0.3452	0.3154	0.2877	0.2642	0.2445	0.2280	0.2140	0.2020
90.0	0.3302	0.3008	0.2641	0.2331	0.2079	0.1875	0.1708	0.1568	0.1443

4. Conclusions

In reality, the crack may propagate in a curve configuration. Therefore, the problem for the curved crack is important in analysis. However, the investigations in literature for the curved branch-cracks are few. This paper provides some numerical technique to solve the problem.

It is proved that the curve length method is very effective for the problem of curved branch-cracks. In this case, one does not need to construct a boundary element and simply use the available quadrature rule.

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