Intelligent fuzzy weighted input estimation method for the input force on the plate structure

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Abstract. The innovative intelligent fuzzy weighted input estimation method which efficiently and robustly estimates the unknown time-varying input force in on-line is presented in this paper. The algorithm includes the Kalman Filter (KF) and the recursive least square estimator (RLSE), which is weighted by the fuzzy weighting factor proposed based on the fuzzy logic inference system. To directly synthesize the Kalman filter with the estimator, this work presents an efficient robust forgetting zone, which is capable of providing a reasonable compromise between the tracking capability and the flexibility against noises. The capability of this inverse method are demonstrated in the input force estimation cases of the plate structure system. The proposed algorithm is further compared by alternating bwtween the constant and adaptive weighting factors. The results show that this method has the properties of faster convergence in the initial response, better target tracking capability, and more effective noise and measurement bias reduction.

Keywords: Kalman Filter; recursive least square estimator; fuzzy logic; input force.

1. Introduction

The plate plays an important role in the mechanical structure as the beam does. The determination of excitation forces is a very important task in the structure design. However, the direct measurement approach from the excitation forces is not feasible in some practical physical and mechanical systems. The inverse technique has been studied extensively and various techniques have been developed. Some recent studies (Hollandsworth and Busby 1989) use modal methods to analyze the structure and dynamic programming to perform the inverse solution. The wave propagation response are used to spectral analysis of the structural dynamics to analytically establish the relation between the Fourier transforms of the responses and impacting force (Martin and Doyle

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1996). The experimental results for the impact of an aluminum plate are presented, and comparisons are made with finite element predictions and measurements from a force transducer (Doyle 1987). Doyle showed that an objective function based on the correlation of forces could be used to evaluate different guessed for the case of a simple beam (Doyle 1994). Doyle's work has been further developed and extended to include frame structures (Martin and Doyle 1996). Wang (1994) used the weighted total acceleration method to detect the vibration force acting on the concentrated-massed nonlinear beam. Recently, Huang (2001) adopted the conjugate gradient method (CGM) to estimate the force of the one-dimensional mass-spring-damper structure with the time-varying system parameters. The above researches used the batch form to process the measurement data. This method is time-consuming and is not a on-line procedure of the unknown input estimation.

In order to overcome these difficulties, an indirect estimation approach to the estimation of the excitation forces is frequently employed. Input force estimation is the process of determining the applied loadings from the measurements of the system responses. The input estimation method has recently been applied to both heat transfer and structural dynamic problems. Tuan *et al.* (1996, 1997) developed an input estimation algorithm to deal with one and two dimensional inverse heat conduction problems. A Kalman filter method is used to estimate a time wise variation of rod force source on the rod end with free boundaries (Ji and Liang 2000). Ma *et al.* (2003) first used the finite element method (FEM) to construct the system state equations of the beam structure, and then used the inverse method to determine the unknown excitation forces. Ma *et al.* (2004) presented an inverse method to estimate the impulsive loads on the lumped-mass structure systems. An inverse method to estimate the excitation forces from the dynamic responses of plate structure was therefore presented (Ma *et al.* 2003). Liu *et al.* (2000) and Ji *et al.* (2001) used the Kalman filter with the recursive least square method to estimate the input force of a plate. However, the plate was simplified to the system with a single degree of freedom. Deng (2006) presented the recursion relation algorithm to determine the input forces of beam structures and each individual node displacement.

According to the above developments, the estimator with a constant weighting factor is used to estimate the unknown time-varying inputs. However, the optimal constant weighting factor can only be obtained through complicated estimation process analysis (Tuan *et al.* 1997). In order to improve the robustness and efficiency of the estimator, Tuan *et al.* (1998) presented an adaptive robust weighted input estimation method for the one-dimensional inverse heat conduction problem. Lee *et al.* (2008) utilized the adaptive weighted input estimation method to inversely solve the burst load of the truss structure system. Chen *et al.* (2008) investigated the adaptive input estimation method applied to the inverse estimation of load input in the multi-layer shearing stress structure. The input estimates converge slowly in the initial time when the adaptive weighting function is used in the RLSE. However, the overall tracking performance of the estimator is good when the unknown input is time-varying regardless of the influence of the measurement noise interference.

In this paper, an intelligent fuzzy weighting function is used to replace the weighting factor, $\gamma(k)$, of the RLSE. Improving the weighting efficiency of the RLSE is essential, because the unknown input is time-varying and changes continuously. The adaptive weighting function takes any input variation into account. Therefore, the inverse method with quick target tracking and effective noise reduction is developed. This inverse method presents an efficient and robust estimation procedure to any unknown input situation. The presented work addresses an intelligent fuzzy weighted estimator based on the fuzzy logic system. The robustness and efficiency of this method will be demonstrated through two simulation case studies. The results are also compared with the ones using other algorithms. The reliability, adaptivity, and robustness of this method can therefore be verified.

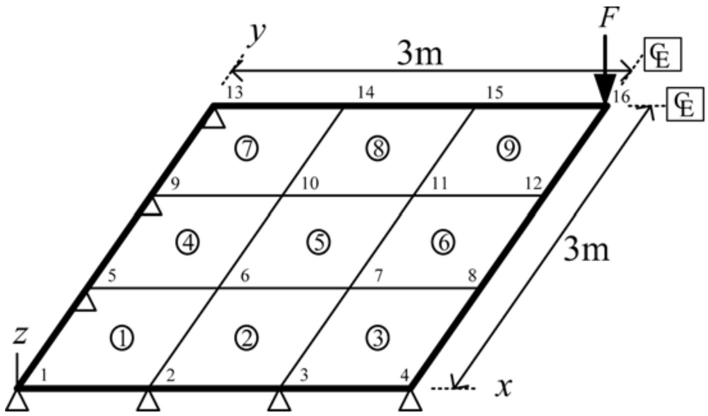

Fig. 1 Finite element model of the plate structure system (9 elements with 16 nodes)

2. Problem formulation

To illustrate the practicability and precision of the presented approach in estimating the unknown input force, the numerical simulation of the plate structure is investigated in this paper. As shown in Fig. 1, the plate is modeled as a structure system. Input estimation is based on the state-space analysis method. In this paper, the FEM is used to construct the state-space model of the plate structure system. The finite element model of a plate structure is considered to be a system with n degrees of freedom. Therefore, the differential equation presenting the motion of the system in terms of mass, stiffness and damping matrices is shown below

$$M\ddot{Y}(t) + C\dot{Y}(t) + KY(t) = F(t) \tag{1}$$

where M is then $n \times n$ mass matrix. C is the $n \times n$ damping coefficient matrix. K is the $n \times n$ stiffness matrix. $\ddot{Y}(t)$, $\dot{Y}(t)$, and $Y(t)$ are the $n \times 1$ acceleration, velocity, and displacement vectors, respectively. $F(t)$ is the $n \times 1$ input force vector. The matrices, M and K, can be obtained by using the FEM. The matrix C is a proportional damping model obtained by assembling the matrices M and K.

After converting to the state-space model, the state variables of the second order dynamic system with n degrees of freedom are represented by a $2n \times 1$ state vector, i.e., $X = [Y(t)\ \dot{Y}(t)]^T$. From Eq. (1), the continuous-time state equation and measurement equation of the structure system can be formulated as follows

$$\dot{X}(t) = AX(t) + BF(t) \tag{2}$$

$$Z(t) = HX(t) \tag{3}$$

where

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{n\times n} \\ M^{-1} \end{bmatrix}$$

$$H = [I_{2n\times 2n}]$$

$$X(t) = [X_1(t)\ \ X_2(t)\ \ \ldots\ \ X_{2n-1}(t)\ \ X_{2n}(t)]^T$$

A and B are both constant matrices composed of mass, damping and stiffness of the plate structure system. $X(t)$ is the state vector. $Z(t)$ is the observation vector and H is the measurement matrix.

There always exists the noise turbulence in the practical environment. This is the reason that any of the physical systems contains two portions: One is the deterministic portion, and the other is the random portion, which is distributed around the deterministic portion. Eqs. (2) and (3) do not take the noise turbulence into account. In order to construct the statistic model of the system state characteristics, a noise disturbance term, which can reflect these characteristics of the state, will need to be added to these two equations. Up to now, one of the random noise disturbances that can be completely resolved is the Gaussian white noise, which has been statistically illustrated in full by using the probability distribution function and the probability density function. Practically, any function corresponding to the functions mentioned above has the same effect. The characteristic function of the random variable is one example. Two most important characteristic values are the mean and the variance, which represent the statistic properties of the random process (Chan *et al.* 1979). Taking the above consideration into account, the continuous-time state equation is to be sampled with the sampling interval, Δt, to obtain the discrete-time statistic model of the state equation as shown below (Bogler 1987)

$$X(k+1) = \Phi X(k) + \Gamma[F(k) + w(k)] \tag{4}$$

where

$$X(k) = [X_1(k)\ \ X_2(k)\ \ \ldots\ \ X_{2n-1}(k)\ \ X_{2n}(k)]^T$$

$$\Phi = \exp(A\Delta t)$$

$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp\{A[(k+1)\Delta t - \tau]\}B d\tau$$

$$F(k) = [F_1(k)\ \ F_2(k)\ \ \ldots\ \ F_{n-1}(k)\ \ F_n(k)]^T$$

$$w(k) = [w_1(k)\ \ w_2(k)\ \ \ldots\ \ w_{n-1}(k)\ \ w_n(k)]^T$$

$X(k)$ is the state vector. Φ is the state transition matrix. Γ is the input matrix. Δt is the sampling interval. $w(k)$ is the processing error vector, which is assumed as the Gaussian white noise. Note that $E\{w(k)w^T(k)\} = Q\delta_{kj}$, and $Q = Q_W \times I_{2n\times 2n}$. Q is the discrete-time processing noise covariance matrix. δ_{kj} is the Kronecker delta function. When describing the active characteristics of the structure system, the additional term, $w(k)$, can be used to present the uncertainty in a numerical manner. The uncertainty could be the random disturbance, the uncertain parameters, or the error due to the over-simplified numerical model.

Generally speaking, the system state can be determined by measuring the output of the system. The measurement usually has a certain relationship with the system output. However, there is also the noise issue with the measurement. As a result, the discrete-time statistic model of the measurement vector can be presented below

$$Z(k) = HX(k) + v(k) \tag{5}$$

where

$$Z(k) = [Z_1(k)\ \ Z_2(k)\ \ \ldots\ \ Z_{2n}(k)]^T$$

$$v(k) = [v_1(k)\ \ v_2(k)\ \ \ldots\ \ v_{2n}(k)]^T$$

$Z(k)$ is the observation vector. $v(k)$ represents the measurement noise vector and is assumed to be the Gaussian white noise with zero mean and the variance, $E\{v(k)v^T(k)\} = R\delta_{kj}$, where $R = R_V \times I_{2n\times 2n}$. R is the discrete-time measurement noise covariance matrix. H is the measurement matrix.

3. The intelligent fuzzy weighting function in the RLSE input estimation method

The input estimation method consists of two parts; one is the Kalman filter and the other is the on-line least square algorithm. The input is the unknown time-varying input force. Using the Kalman filter requires an exact knowledge of the process noise variance Q and the measurement noise variance R, which depends on the sensor measurements. The Kalmen filter is used to generate the residual innovation sequence. The on-line recursive least square algorithm is derived by applying the residual sequence to compute the value of the input force. The detailed formulation of this technique can also be found in the paper by Tuan *et al.* (1996).

The equations of the Kalman filter are as follows

$$\bar{X}(k/k-1) = \Phi\bar{X}(k-1/k-1) \tag{6}$$

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi^T + \Gamma Q\Gamma^T \tag{7}$$

$$s(k) = HP(k/k-1)H^T + R \tag{8}$$

$$K(k) = P(k/k-1)H^T s^{-1}(k) \tag{9}$$

$$P(k/k) = [I-K(k)H]P(k/k-1) \tag{10}$$

$$\bar{Z}(k) = Z(k) - H\bar{X}(k/k-1) \tag{11}$$

$$\bar{X}(k/k) = \bar{X}(k/k-1) + K(k)\bar{Z}(k) \tag{12}$$

The equations of the recursive least square estimator are as follows

$$B(k) = H[\Phi M(k-1) + I]\Gamma \tag{13}$$

$$M(k) = [I-K(k)H][\Phi M(k-1) + I] \tag{14}$$

$$K_b(k) = \gamma^{-1}P_b(k-1)B^T(k)[B(k)\gamma^{-1}P_b(k-1)B^T(k) + s(k)]^{-1} \tag{15}$$

$$P_b(k) = [I - K_b(k)B(k)]\gamma^{-1}P_b(k-1) \tag{16}$$

$$\hat{F}(k) = \hat{F}(k-1) + K_b(k)[\bar{Z}(k) - B(k)\hat{F}(k-1)] \tag{17}$$

where $\bar{X}(k)$ denotes the state estimate. P is the state estimation error covariance. $s(k)$ is the covariance of the residual. $K(k)$ is the Kalman gain. $\bar{Z}(k)$ is the bias innovation produced by the measurement noise and input disturbance. $\hat{F}(k)$ is the estimated input vector. $P_b(k)$ is the error covariance of the input estimation process. $K_b(k)$ is the correction gain. γ is the weighting factor.

$B(k)$ and $M(k)$ are both the sensitivity matrices. From Eq. (16), the error covariance of the input estimation is increased by the weighting factor, $\gamma(k)$, which is a constant with the value within the interval, [0, 1].

The intelligent fuzzy weighting factor in this research is proposed based on the fuzzy logic inference system. The intelligent fuzzy weighting factor can be operated at each step based on the innovation from the Kalman filter. It plays the role as an adjustable parameter to control the bandwidth or gain magnitude of the recursive least square estimator. Furthermore, the weighting factor $\gamma(k)$ is employed to compromise between the upgrade of tracking capability and the loss of estimation precision. The relation as the following was derived by Tuan *et al*. (1996)

$$\gamma(k) = \begin{cases} 1 & |\bar{Z}(k)| \le \sigma \\ \dfrac{\sigma}{|\bar{Z}(k)|} & |\bar{Z}(k)| > \sigma \end{cases} \tag{18}$$

The weighting factor, $\gamma(k)$, as shown in Eq. (18) is adjusted according to the measurement noise and input bias. In the industrial applications, the standard deviation σ is set as a constant value. The magnitude of weighting factor is determined according to the modulus of bias innovation, $|\bar{Z}(k)|$. The unknown input prompt variation will make the modulus of bias innovation larger. In the meantime, the smaller weighting factor is obtained when the modulus of bias innovation is larger. Therefore, the estimator accelerates the tracking speed and produces larger vibration in the estimation process. On the contrary, the smaller variation of unknown input makes the modulus of bias innovation smaller. In the meantime, the larger weighting factor is obtained when the modulus of bias innovation is smaller. The estimator is unable to estimate the unknown input effectively. For this reason, the inverse estimation method with the intelligent fuzzy weighting factor, which efficiently and robustly estimates the time-varying unknown input, will be constructed in this research.

The basic configuration of the fuzzy logic system considered in this paper is illustrated here. The fuzzy logic system includes four basic components, which are the fuzzy rule base, fuzzy inference engine, fuzzifier, and defuzzifier. The value of fuzzy logic system input, $\theta(k)$, may be chosen in the interval, [0, 1].

$$\theta(k) = \frac{|\Delta\bar{Z}(k)|}{\sqrt{\Delta\bar{Z}^2(k) + \Delta t^2}} \tag{19}$$

where $\Delta\bar{Z}(k) = \bar{Z}(k) - \bar{Z}(k-1)$. Δt is the sampling interval. The proposed intelligent fuzzy weighting factor uses the input variable $\theta(k)$ to self-adjust the factor $\gamma(k)$ of the recursive least square estimator. Therefore, the fuzzy logic system consists of one input and one output variables. The value of input, $\theta(k)$, may be chosen in the interval, [0, 1], and the value of output, $\gamma(k)$, may also be in the interval, [0, 1]. The fuzzy sets for $\theta(k)$ and $\gamma(k)$ are labeled in the linguistic terms of EP (extremely large positive), VP (very large positive), LP (large positive), MP (medium positive), SP (small positive), VS (very small positive), and ZE (zero). The specific membership is defined by using the Gaussian functions shown in Fig. 2.

A fuzzy rule base is a collection of fuzzy IF-THEN rules:

IF $\theta(k)$ is zero (ZE) THEN $\gamma(k)$ is an extremely large positive (EP),

IF $\theta(k)$ is a very small positive (VS) THEN $\gamma(k)$ is a very large positive (VP),

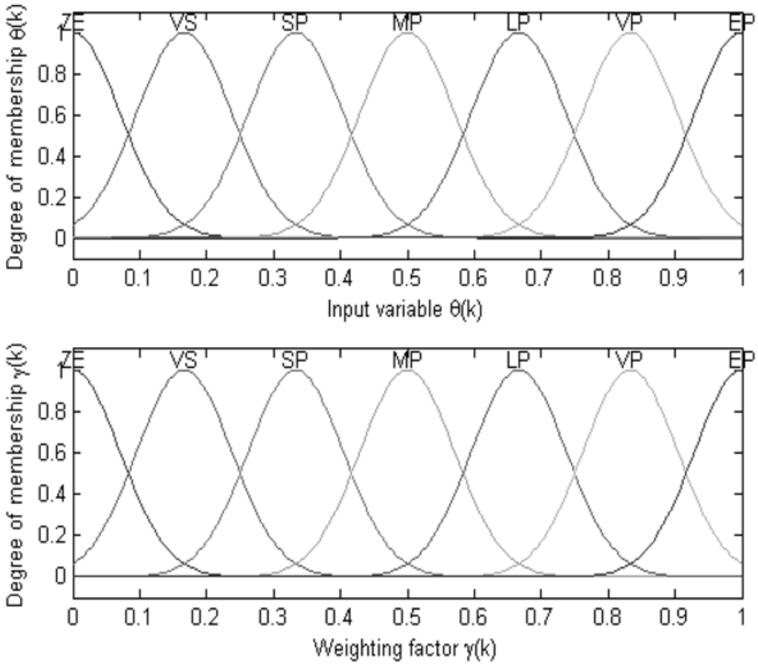

Fig. 2 Membership functions of the fuzzy sets for $\theta(k)$ and $\gamma(k)$

IF $\theta(k)$ is a small positive (SP) THEN $\gamma(k)$ is a large positive (LP),
IF $\theta(k)$ is a medium positive (MP) THEN $\gamma(k)$ is a medium positive (MP),
IF $\theta(k)$ is a large positive (LP) THEN $\gamma(k)$ is a small positive (SP),
IF $\theta(k)$ is a very large positive (VP) THEN $\gamma(k)$ is a very small positive (VS),
IF $\theta(k)$ is an extremely large positive (EP) THEN $\gamma(k)$ is zero (ZE),

where $\theta(k) \in U$ and $\gamma(k) \in V \subset R$ are the input and output of the fuzzy logic system, respectively. The fuzzier maps a crisp point $\theta(k) \in U$ into a fuzzy set A in U. Therefore, the nonsingleton fuzzier can be expressed in Wang (1994)

$$\mu_A(\theta(k)) = \exp\left(-\frac{(\theta(k)-\bar{x}_i^l)^2}{2(\sigma_i^l)^2}\right) \tag{20}$$

$\mu_A(\theta(k))$ decreases from 1 as $\theta(k)$ moves away from $\bar{x}_i^l$. $(\sigma_i^l)^2$ is a parameter characterizing the shape of $\mu_A(\theta(k))$.

The Mamdani maximum-minimum inference engine is used in this paper. The max-min-operation rule of fuzzy implication is shown in Wang (1994)

$$\mu_B(\gamma(k)) = \max_{j=1}^{c}\left\{\min_{i=1}^{d}[\mu_{A_i^j}(\theta(k)), \mu_{A_i^j \to B^j}(\theta(k), \gamma(k))]\right\} \tag{21}$$

where c is the fuzzy rule, and d is the dimension of input variables.

The defuzzifier maps a fuzzy set B in V to a crisp point $\gamma \in V$. The fuzzy logic system with the center of gravity is defined in Wang (1994)

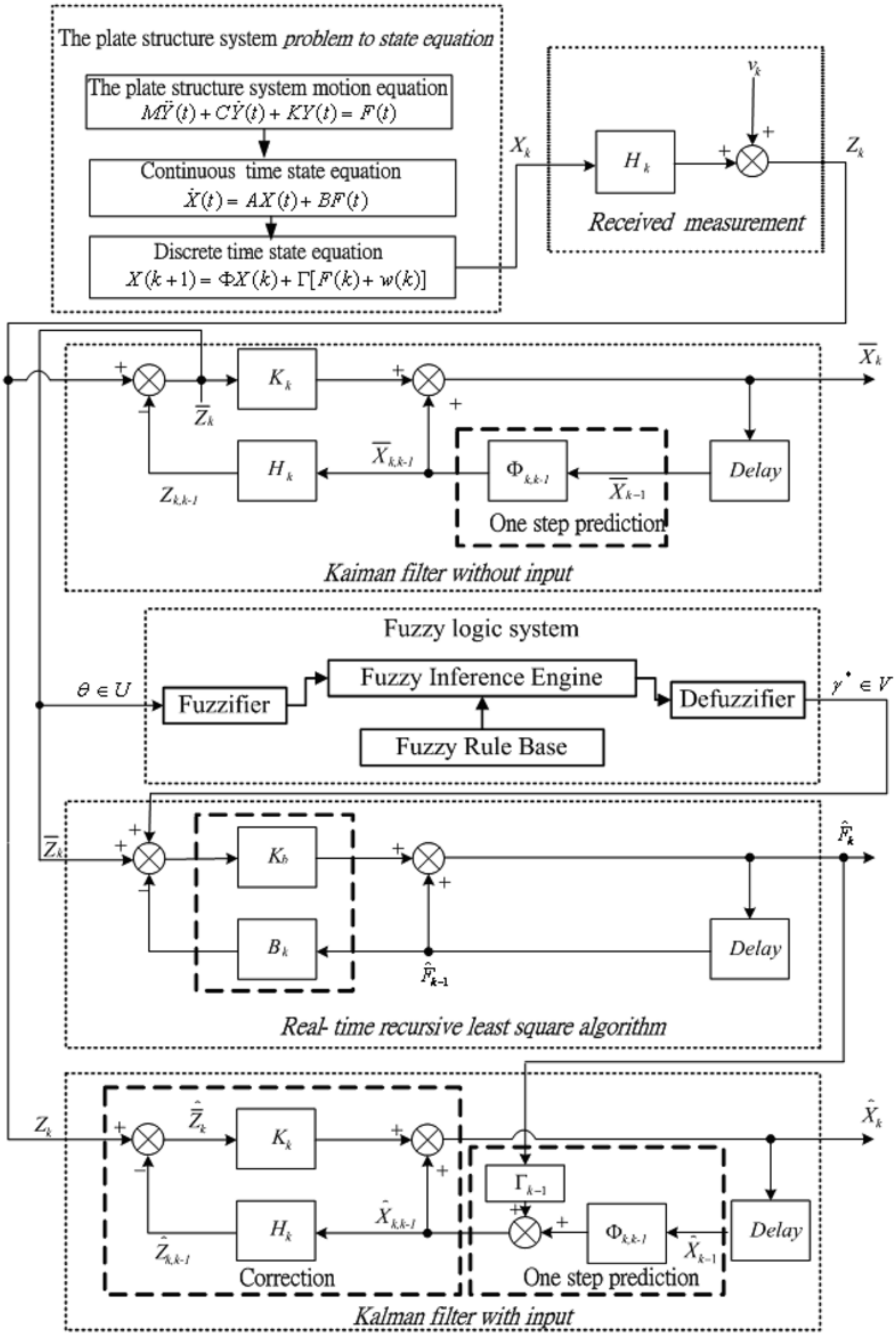

Fig. 3 Flowchart of the intelligent fuzzy weighted input estimation algorithm

$$\gamma^*(k) = \frac{\sum_{l=1}^{n} \bar{y}^l \mu_B(\gamma^l(k))}{\sum_{l=1}^{n} \mu_B(\gamma^l(k))} \tag{22}$$

n is the number of outputs. $\bar{y}^l$ is the value of the lth output. $\mu_B(\gamma^l(k))$ represents the membership of $\gamma^l(k)$ in the fuzzy set B. Substituting $\gamma^*(k)$ of Eq. (22) in Eqs. (15) and (16) allows us to configure an adaptive fuzzy weighting function of the recursive least square estimator (RLSE). A flow chart of the computation for the application of the recursive input estimation algorithm is given in Fig. 3.

4. Results and discussion

To verify the practicability and precision of the presented approach in estimating the unknown input force, a three-dimensional example is applied to the use of the input estimation method combined with the finite-element scheme. In this paper, the protective structure is modeled as a simple plate structure system. The simple plate structure is subjected to input force. The input force can be estimated by applying the dynamic responses to the proposed input estimation algorithm. The quarter modal was simulated as the symmetrical simple plate structure with the relationship of the length and wide of the plate structure, $L = 6$ m and the thickness, $h = 0.1$ m. The element mass matrix M^e and the element stiffness matrix K^e of the plate are shown as follows (Dawe 1984)

$$M^e = \frac{\rho hab}{6300} \begin{bmatrix} M_{1,1} & \cdots\cdots & M_{1,12} \\ \vdots & \cdots\cdots & \vdots \\ \vdots & \cdots\cdots & \vdots \\ M_{12,1} & \cdots\cdots & M_{12,12} \end{bmatrix}_{12\times 12}$$

and

$$K^e = \frac{Eh^3}{12(1-\nu^2)} \begin{bmatrix} K_{1,1} & \cdots\cdots & K_{1,12} \\ \vdots & \cdots\cdots & \vdots \\ \vdots & \cdots\cdots & \vdots \\ K_{12,1} & \cdots\cdots & K_{12,12} \end{bmatrix}_{12\times 12}$$

The density, $\rho = 650$ kg/m^3. The elastic modulus of all elements, $E = 3$ GPa. The Poisson ratio, $\nu = 0.3$. The proportional damping coefficient, $C = \alpha M + \beta K$, where $\alpha = 0.02$ and $\beta = 0.005$. The initial conditions of the error covariance are given as $P(0/0) = diag[10^4]$ for the KF and $P_b(0) = 10^{10}$ for the adaptive fuzzy weighted recursive least square estimator. The simulation parameters are set as follows. The sampling interval, $\Delta t = 0.01$ s and the total simulation time, $t_f = 5$s. The sensitivity matrix $M(0)$ is null. The weighting factor is an adaptive fuzzy weighting function.

Example 1: Continuous square input force

The input force is modeled by a continuous square which inputs node 16 of the plate structure. $F_{16}(t)$ is shown as the following.

$$F_{16}(t) = \begin{cases} 5\times 10^5 & 1 \le t \le 2s \\ 3\times 10^5 & 2.5 \le t \le 3.5s \quad (N) \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

First, the process noise and measurement noise are considered in the simulation process. The process noise covariance matrix, $Q = Q_w \times I_{2n\times 2n}$, where $Q_w = 10^5$. The measurement noise covariance matrix, $R = R_w \times I_{2n\times 2n}$, where $R_w = \sigma^2 = 10^{-5}$, and σ is the standard deviation of the noise. The estimates of $F_{16}(t)$ using the intelligent fuzzy weighting function, the adaptive weighting function, and the constant weighting factor, are plotted in Fig. 4. In Eqs. (15)-(17) of the recursive least square estimator, $\hat{F}(k)$ is the estimate of the unknown input. $P_b(k)$ is the error covariance of the input estimation process. $K_b(k)$ is the correction gain. $\gamma(k)$ is the weighting factor chosen in the range between 0 and 1. The two functions of $\gamma(k)$ are smoothing and forgetting. The forgetting effectiveness depends on the value of $\gamma(k)$. $K_b(k)$ gets larger as $\gamma(k)$ gets smaller according to Eq. (15). The forgetting effect therefore becomes more conspicuous according to Eq. (16). It should be noted that the faster the forgetting effect is, the lower the smoothing effect will be, that is, it introduces oscillation. The intelligent fuzzy weighting factor $\gamma(k)$ is employed to compromise between the upgrade of tracking capability and the loss of estimation precision. Fig. 5 shows the comparison between the adaptive weighting and intelligent fuzzy weighting factors in terms of the estimation results of Example 1 with $Q_w = 10^5$, $R_w = \sigma^2 = 10^{-5}$. The simulation results demonstrate that the intelligent fuzzy weighted input estimator has the property of faster convergence in the initial response. The adaptive weighted input estimator has better target tracking capability when the unknown input is larger. However, the effectiveness to reduce the effect of noise is poor. The intelligent fuzzy weighted input estimator has better target tracking capability and noise reduction effectiveness overall. Fig. 6 shows the comparison between the intelligent fuzzy weighting and constant weighting factors in terms of the estimation results of Example 1 with $Q_w = 10^5$, $R_w = \sigma^2 = 10^{-5}$. The simulation results demonstrate that the constant weighted input estimator with

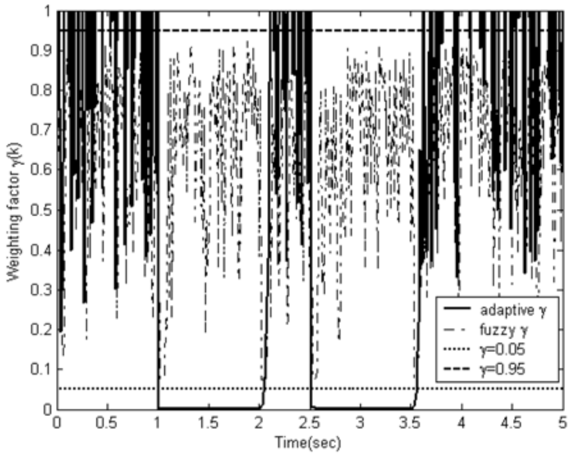

Fig. 4 Comparison of different weighting functions when the input is a continuous square input force

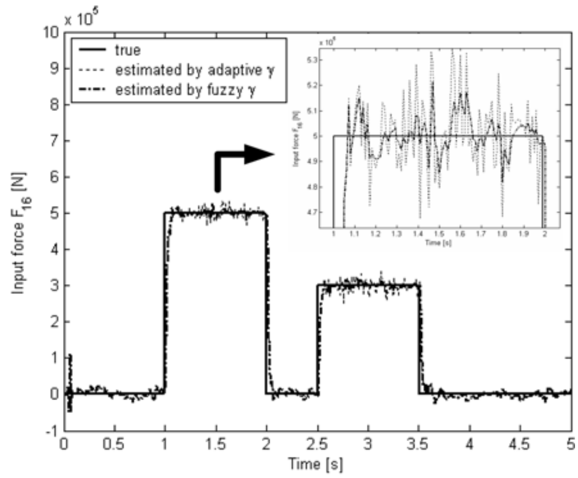

Fig. 5 Comparison between the inverse estimation using the adaptive, and fuzzy weighting functions when the input is a continuous square input force

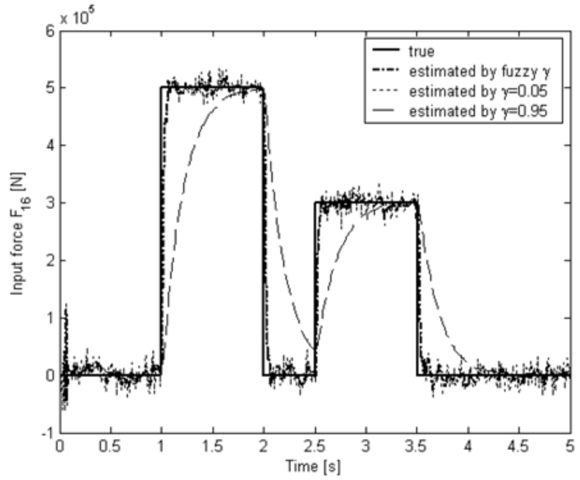

Fig. 6 Comparison of the inverse estimation using the fuzzy, and constant weighting functions when the input is a continuous square input force

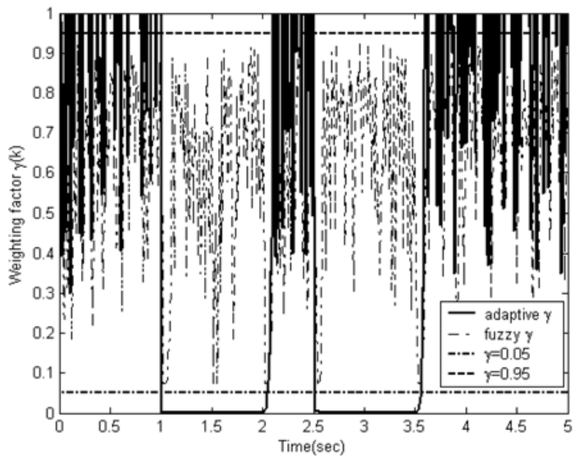

Fig. 7 Comparison of different weighting functions when the input is a continuous square input force with the transient measurement bias (15%)

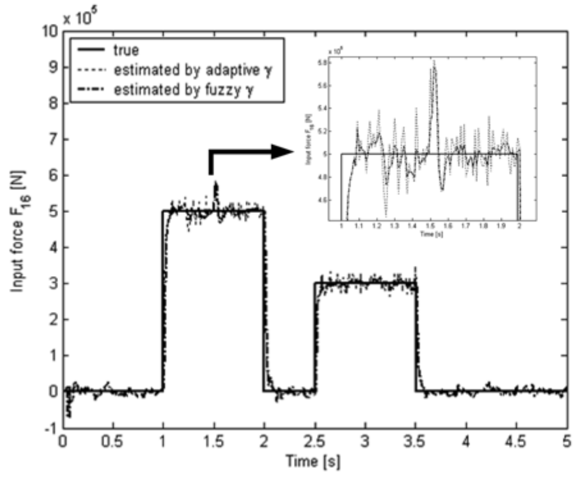

Fig. 8 Comparison between the inverse estimation using the adaptive, and fuzzy weighting functions when the input is a continuous square input force with the transient measurement bias (15%)

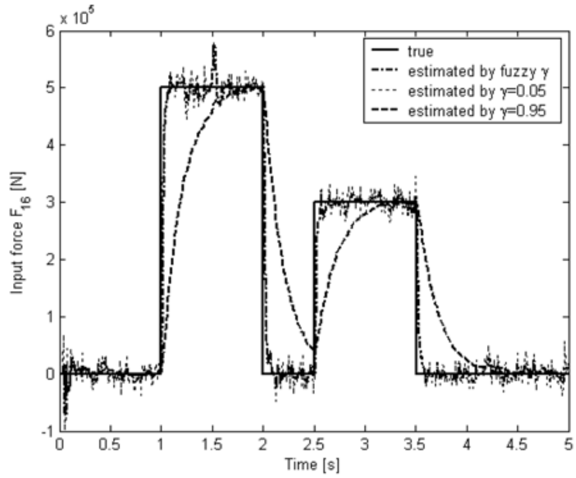

Fig. 9 Comparison of the inverse estimation using the fuzzy, and constant weighting functions when the input is a continuous square input force with the transient measurement bias (15%)

$\gamma = 0.95$ has the property of faster convergence in the initial response. The constant weighted input estimator with $\gamma = 0.05$ has better target tracking capability when the unknown input is larger. Howerer, the constant weighted input estimator with $\gamma = 0.05$ is not effecient in reducing the noise effect. Although the constant weighted input estimator with $\gamma = 0.95$ has more effective noise reduction capability, it is not effective in tracking the target. In other words, the proposed method has the property of faster convergence in the initial response, better target tracking capability and more effective noise reduction.

Moreover, the measurement bias has been taken into account during the simulation process. The measurement bias (15%) is assumed to occur at 1.5s for 3 time steps. Fig. 7 demonstrates that the smaller value of intelligent fuzzy weighting factor is transient due to the measurement bias. The value of adaptive weighting factor is smaller than 0.05 and is approximated to zero when the unknown input is large. Fig. 8 shows that the intelligent fuzzy weighted estimator has good performance as in Fig. 5. Furthermore, the effectiveness in reducing the measurement bias effect is better. By contrast, the adaptive weighted estimator has better target tracking capability; nevertheless, the capability to reduce the noise and measurement bias effect is not effective. Fig. 9 shows the comparison between the intelligent fuzzy weighting and constant weighting factors in terms of the estimation results of Example 1 with the measurement bias. The simulation results demonstrate the influence of constant weighting factor on the estimator performance as shown in Fig. 6. In short, the proposed method has the properties mentioned above. Besides, it can deal with the measurement bias more effectively.

Example 2: Decaying exponential input force

A rapid release of energy occurs when the explosive detonates. In the meantime, a tremendous input force is produced and spread out along with the vibration wave. This kind of input force has the properties of decay and transient in existence. This is the reason that the input force is often approximated in the form of decaying exponent. In this simulation, a decaying exponent input force acting on node 16 of the plate has been considered. The numerical model of the input force is shown as the following

$$F_{16}(t) = \begin{cases} 2.5\times 10^{8}\times \exp(-3t) & t \geq 2s \\ 0 & 0 \leq t < 2s \end{cases} \quad (N) \tag{24}$$

The estimates of $F_{16}(t)$ using the intelligent fuzzy weighting function, the adaptive weighting function, and the constant weighting factor with the process noise covariance matrix, $Q = Q_w \times I_{2n\times 2n}$,

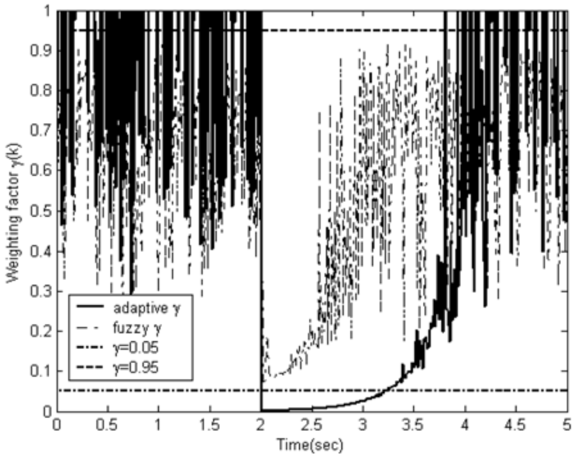

Fig. 10 Comparison of different weighting functions when the input is a decaying exponential input force

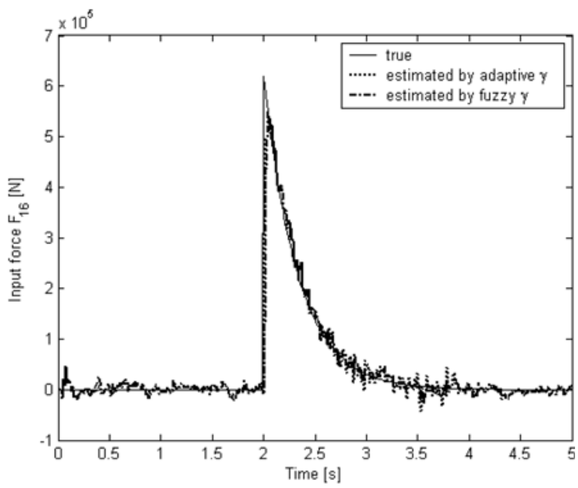

Fig. 11 Comparison between the inverse estimation using the adaptive, and fuzzy weighting functions when the input is a decaying exponential input force

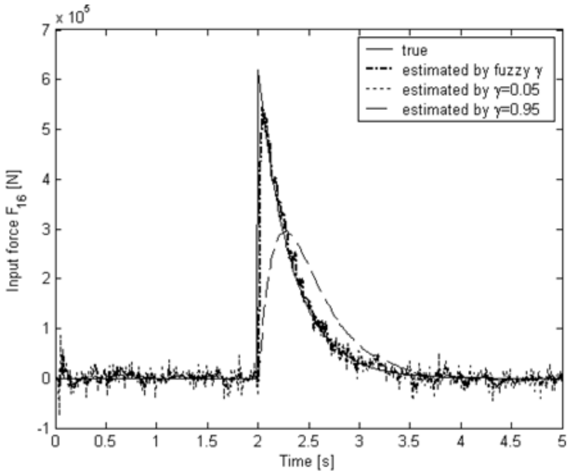

Fig. 12 Comparison of the inverse estimation using the fuzzy, and constant weighting functions when the input is a decaying exponential input force

where $Q_w = 10^5$, and the measurement noise covariance matrix, $R = R_w \times I_{2n \times 2n}$, where $R_w = \sigma^2 = 10^{-5}$ are plotted in Fig. 10. It shows that the value of intelligent fuzzy weighting factor is between 0.05 and 0.95. The weighting factor $\gamma(k)$ is employed to compromise between the upgrade of tracking capability and the loss of estimation precision. Fig. 11 shows the comparison between the adaptive weighting and intelligent fuzzy weighting factors in terms of the estimation results of Example 2. The simulation results denote that the adaptive weighted estimator has great tracking capability, but it is not capable of reducing the effect due to the measurement noise. The intelligent fuzzy weighted estimator has great tracking capability similar to the adaptive weighted estimator. Besides, it is capable of reducing the effect due to the measurement noise. The overall estimation process of the intelligent fuzzy weighted estimator is effective. Fig. 12 shows that the constant weighted input estimator with $\gamma = 0.95$ has the property of faster convergence in the initial response. The constant weighted input estimator with $\gamma = 0.05$ has better target tracking capability when the unknown input is larger. Howerer, the constant weighted input estimator with $\gamma = 0.05$ is not effecient in reducing the to noise effect. Although the constant weighted input estimator with $\gamma =$ 0.95 has more effective noise reduction capability, it is not effective in tracking the target.

The above simulation results demonstrate that the proposed method performs better than other algorithms. It is an effecient method, which shows better convergence when tracking the unknown input in the initial stage, and reduces the influence due to the measurement noise and bias.

5. Conclusions

In this paper, an intelligent fuzzy weighted input estimation method is applied to estimate the unknown input force in a plate structure system. The FEM is adopted to construct the state equation of the plate structure, and the Kalman Filter is further combined with the least square algorithm to estimate the input force. The intelligent fuzzy weighted estimator is an efficient adaptive and robust inverse estimation method for the estimation of the unknown time-varying input with the unpredicted modeling and measurement errors, and the transient measurement bias due to the instrument. Future works of this study will address the issue of the applications in the optimal control scope.

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