An exact solution for buckling analysis of embedded piezoelectromagnetically actuated nanoscale beams

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Abstract. This paper investigates the buckling behavior of shear deformable piezoelectric (FGP) nanoscale beams made of functionally graded (FG) materials embedded in Winkler-Pasternak elastic medium and subjected to an electro-magnetic field. Magneto-electro-elastic (MEE) properties of piezoelectric nanobeam are supposed to be graded continuously in the thickness direction based on power-law model. To consider the small size effects, Eringen’s nonlocal elasticity theory is adopted. Employing Hamilton’s principle, the nonlocal governing equations of the embedded piezoelectric nanobeams are obtained. A Navier-type analytical solution is applied to anticipate the accurate buckling response of the FGP nanobeams subjected to electro-magnetic fields. To demonstrate the influences of various parameters such as, magnetic potential, external electric voltage, power-law index, nonlocal parameter, elastic foundation and slenderness ratio on the critical buckling loads of the size-dependent MEE-FG nanobeams, several numerical results are provided. Due to the shortage of same results in the literature, it is expected that the results of the present study will be instrumental for design of size-dependent MEE-FG nanobeams.

Keywords: piezoelectric nanobeam; magneto-electro-elastic FG nanobeam; buckling; nonlocal elasticity theory; higher order beam theory

1. Introduction

Magneto-electro-elastic (MEE) materials have encountered to a significant interest for their extensive potential applications, since the first report on a MEE composite including piezo-electric phase and piezo-magnetic phases in 1970s (Van Run et al. 1974). MEE materials have the potential to convert magnetic, electric and mechanical energies from one form to the others and this leads to wide application of these materials in sensing and actuating devices, control of structural vibrations and smart structure technology (Milazzo et al. 2009). Recently, analyzing the mechanical responses of MEE structural components has received a remarkable attention. A survey in literature shows that, mechanical behavior of MEE structures is studied by several researchers. Among them, Chen et al. (2005) studied free vibration of non-homogeneous transversely isotropic magneto-electro-elastic plates. Free vibration of multiphase and layered magneto-electro-elastic beam for BaTiO\textsubscript{3}-CoFe\textsubscript{2}O\textsubscript{4} composite is carried out by Annigeri et al. (2007). Kumaravel et al. (2007) researched linear buckling and free vibration behavior of layered

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and multiphase magneto-electro-elastic (MEE) beam under thermal environment. Transient dynamic response of multiphase magneto-electro-elastic cantilever beam is presented by Daga et al. (2009) using finite element method. Also, Liu and Chang (2010) presented a closed form expression for the vibration problem of a transversely isotropic magneto-electro-elastic plate. Razavi and Shoohtari (2015) studied nonlinear free vibration of symmetric magneto-electro-elastic laminated rectangular plates with simply supported boundary condition. They used the first order shear deformation theory considering the von Karman’s nonlinear strains to obtain the equations of motion, whereas Maxwell equations for electrostatics and magnetostatics are used to model the electric and magnetic behavior. Recently, Xin and Hu (2015) investigated free vibration of simply supported and multilayered magneto-electro-elastic plates.

Moreover, mechanical analysis of structures made from composition of MEEMs and FGMs have gained notable attentions in the last years. Pan and Han (2005) presented an exact solution for the multilayered rectangular plate made of FG, anisotropic, and linear magneto-electro-elastic materials. In this study, they supposed that the edges of the plate are under simply supported conditions, general mechanical, electric and magnetic boundary conditions can be applied on both the top and bottom surfaces of the plate. Also, Huang et al. (2007) studied the plane stress problem of generally anisotropic magneto-electro-elastic beams with the coefficients of elastic compliance, piezoelectricity, dielectric impermeability, piezomagnetism, magneto-electricity, and magnetic permeability being arbitrary functions of the thickness coordinate. In another study, three-dimensional (3D) static behavior of doubly curved FG MEE shells under the mechanical load, electric displacement and magnetic flux using an asymptotic approach is investigated by Wu and Tsai (2007). Li et al. (2008) investigated the problem of a functionally graded, transversely isotropic, magneto-electro-elastic circular plate acted on by a uniform load. Kattimani and Ray (2015) investigated active control of geometrically nonlinear vibrations of FG MEE plates. Sladek et al. (2015) analyzed bending of circular magneto-electro-elastic plates with functionally graded material properties using a meshless method.

The significance of size effects motivated the scientific community to explore the behaviors of the nanostructures and nanomaterials much accurately (Alizada and Sofiyev 2011a, b, Alizada et al. 2012). By minimizing the size of the structure and becoming comparable to the internal characteristic length scale, the classical continuum mechanics is unable to model such structures in which size-dependent behaviors have been experimentally observed. Due to this reason, various higher order continuum theories such as Eringen’s nonlocal elasticity theory are suggested to capture the influence of small size. Ke and Wang (2014) studied the free vibration behavior of magneto-electro-elastic (MEE) nanobeams using nonlocal theory and Timoshenko beam theory. They supposed that the MEE nanobeam is subjected to the external electric potential, magnetic potential and uniform temperature rise. In another study, Ke et al. (2014) investigated the free vibration behavior of magneto-electro-elastic (MEE) nanoplates based on the nonlocal theory and Kirchhoff plate theory. Li et al. (2014) analyzed buckling and free vibration of magneto-electro-elastic nanoplate resting on Pasternak foundation based on nonlocal Mindlin theory. Ansari et al. (2015) studied forced vibration behavior of higher order shear deformable magneto-electro-thermo elastic (METE) nanobeams based on the nonlocal elasticity theory in conjunction with the von Kármán geometric nonlinearity. Wu et al. (2015) researched surface effects on anti-plane shear waves propagating in nanoplates made from magneto-electro-elastic materials. As literature shows, there is no study investigating the small scale influence on buckling responses of MEE-FG nanobeams, so it is necessary to investigate the stability of such structures. By ignoring the effects of magnetic and electric fields only a few studies are performed to analyze mechanical behavior of

This paper presents a higher order beam model for the buckling analysis of magneto-electro-elastic FG nanobeams resting on two-parameter elastic foundation. Superiority of the present theory is that it considers the influences of shear deformation which is ignored in Euler-Bernoulli beam theory and doesn't require a shear correction factor applied in Timoshenko beam theory. The magneto-electro-elastic material properties of the beam is supposed to be variable in the thickness direction according to the power law distribution. The small size effect is captured using Eringen’s nonlocal elasticity theory. Nonlocal governing equations for the buckling of embedded MEE-FG nanobeams have been derived via Hamilton’s principle and then solved using Navier type method. Various numerical and illustrative results show the influences of elastic foundation, magnetic potential, external electric voltage, nonlocal parameter, power-law index and slenderness ratio on buckling behavior of MEE-FG nanobeams resting on elastic foundation.

2. Theoretical formulations

2.1 The material properties of MEE-FG nanobeams

Assume a magneto-electro-elastic functionally graded nanobeam composed of BaTiO$_3$ and CoFe$_2$O$_4$ materials exposed to a magnetic potential $\Upsilon(x,z,t)$ and electric potential $\Phi(x,z,t)$, with length $L$ and uniform thickness $h$, as shown in Fig. 1. The effective material properties of the MEE-FG nanobeam are supposed to change continuously in the $z$-axis direction (thickness direction) based on the power-law model. So, the effective material properties, $P$, can be stated in the following form

$$P = P_2 V_2 + P_1 V_1$$

In which $P_1$ and $P_2$ denote the material properties of the bottom and higher surfaces, respectively. Also $V_1$ and $V_2$ are the corresponding volume fractions related by

$$V_2 = \left(\frac{z}{h} + \frac{1}{2}\right)^p, \quad V_1 = 1 - V_2$$

Therefore, according to Eqs. (1) and (2), the effective magneto-electro-elastic material properties of the FG beam is defined as
$$P(z) = \left( P_2 - P_1 \right) \left( \frac{z}{h} + \frac{1}{2} \right)^p + P_1$$  (3)

where $p$ is power-law exponent which is non-negative and estimates the material distribution through the thickness of the nanobeam and $z$ is the distance from the mid-plane of the graded piezoelectric beam. It must be noted that, the top surface at $z=+h/2$ of FG nanobeam is assumed CoFe$_2$O$_4$ rich, whereas the bottom surface ($z=-h/2$) is BaTiO$_3$ rich.

### 2.2 Nonlocal elasticity theory for the magneto-electro-elastic materials

Contrary to the constitutive equation of classical elasticity theory, Eringen’s nonlocal theory (Eringen 1972a, b, Eringen 1983) notes that the stress state at a point inside a body is regarded to be function of strains of all points in the neighbor regions. For a nonlocal magneto-electro-elastic solid the basic equations with zero body force may be defined as

$$\sigma_{ij} = \int_V \alpha(|x'-x|, \tau) \left[ C_{ijkl} \varepsilon_{kl}(x') - e_{mi} E_m(x') - q_{mj} H_n(x') \right] dV(x')$$  (4a)

$$D_i = \int_V \alpha(|x'-x|, \tau) \left[ e_{iis} \varepsilon_{is}(x') + s_{im} E_m(x') + d_{im} H_n(x') \right] dV(x')$$  (4b)

$$B_i = \int_V \alpha(|x'-x|, \tau) \left[ q_{iis} \varepsilon_{is}(x') + d_{im} E_m(x') + \chi_{im} H_n(x') \right] dV(x')$$  (4c)

where $\sigma_{ij}$, $\varepsilon_{ij}$, $D_i$, $E_i$, $B_i$ and $H_i$ denote the stress, strain, electric displacement, electric field components, magnetic induction and magnetic field and displacement components, respectively; $C_{ijkl}$, $e_{mi}$, $s_{im}$, $q_{mj}$, $d_{ij}$ and $\chi_{ij}$ are the elastic, piezoelectric, dielectric constants, piezomagnetic, magnetoelectric, magnetic constants, respectively; $\alpha(|x'-x|, \tau)$ is the nonlocal kernel function and $|x'-x|$ is the Euclidean distance. $\tau = \epsilon_0 a/2l$ is defined as scale coefficient, where $\epsilon_0$ is a material constant which is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics; and $a$ and $l$ are the internal and external characteristic length of the nanostructures, respectively. Finally it is possible to represent the integral constitutive relations given by Eq. (4) in an equivalent differential form as
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\[ \sigma_{ij} - (e_{ij} a)^2 \nabla^2 \sigma_{ij} = C_{ijkl} \varepsilon_{kl} - e_{mij} E_m - q_{nij} H_n \]  \hspace{1cm} (5a)

\[ D_i - (e_{ij} a)^2 \nabla^2 D_i = e_{ijj} \varepsilon_{ij} + s_{im} E_m + d_{in} H_n \]  \hspace{1cm} (5b)

\[ B_i - (e_{ij} a)^2 \nabla^2 B_i = q_{ij} \varepsilon_{ij} + d_{in} E_m + \chi_{ij} H_n \]  \hspace{1cm} (5c)

where \( \nabla^2 \) is the Laplacian operator and \( e_{ij} a \) is the nonlocal parameter revealing the size influence on the response of nanostructures.

2.3 Nonlocal magneto-electro-elastic FG nanobeam model

Based on third order beam theory, the displacement field at any point of the beam are supposed to be in the form

\[ u_i(x, z) = u(x) + z \psi(x) - \alpha z^3 \left( \psi + \frac{\partial w}{\partial x} \right) \]  \hspace{1cm} (6a)

\[ u_j(x, z) = w(x) \]  \hspace{1cm} (6b)

in which \( \alpha = 4/3h^2 \) and \( u \) and \( w \) are displacement components in the mid-plane along the coordinates \( x \) and \( z \), respectively, while \( \psi \) denotes the total bending rotation of the cross-section.

To satisfy Maxwell’s equation in the quasi-static approximation, the distribution of electric and magnetic potential along the thickness direction is supposed to change as a combination of a cosine and linear variation as follows

\[ \Phi(x, z, t) = -\cos(\xi z) \phi(x, t) + \frac{2z}{h} V \]  \hspace{1cm} (7a)

\[ \gamma(x, z, t) = -\cos(\xi z) \gamma(x, t) + \frac{2z}{h} \Omega \]  \hspace{1cm} (7b)

where \( \xi = \pi/h \). Also, \( V \) and \( \Omega \) are the initial external electric voltage and magnetic potential applied to the MEE-FG nanobeam. Considering strain-displacement relationships on the basis of parabolic beam theory, the non-zero strains can be stated as

\[ \varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z \varepsilon_{xx}^{(1)} + z^3 \varepsilon_{xx}^{(3)} \]  \hspace{1cm} (8)

\[ \gamma_{xz} = \gamma_{xz}^{(0)} + z \gamma_{xz}^{(1)} \]  \hspace{1cm} (9)

where

\[ \varepsilon_{xx}^{(0)} = \frac{\partial u}{\partial x}, \quad \varepsilon_{xx}^{(1)} = \frac{\partial \psi}{\partial x}, \quad \varepsilon_{xx}^{(3)} = -\alpha \left( \frac{\partial \psi}{\partial x} + \frac{\partial^3 w}{\partial x^3} \right) \]  \hspace{1cm} (10)

\[ \gamma_{xz}^{(0)} = \frac{\partial w}{\partial x} + \psi, \quad \gamma_{xz}^{(2)} = -\beta \left( \frac{\partial w}{\partial x} + \psi \right) \]  \hspace{1cm} (11)
And $\beta = \frac{4}{h^2}$.

According to the Eq. (7), the non-zero components of electric and magnetic field $(E_x, E_z, H_x, H_z)$ can be obtained as

$$E_x = -\Phi_x = \cos(\xi z) \frac{\partial \phi}{\partial x}, \quad E_z = -\Phi_z = -\xi \sin(\xi z) \phi - \frac{2V}{h} \quad (12a)$$

$$H_x = -\Psi_x = \cos(\xi z) \frac{\partial \psi}{\partial x}, \quad H_z = -\Psi_z = -\xi \sin(\xi z) \gamma - \frac{2\Omega}{h} \quad (12b)$$

The Hamilton’s principle can be stated in the following form to obtain the governing equations of motion

$$\int_0^L \delta (\Pi_s + \Pi_w) \, dt = 0 \quad (13)$$

where $\Pi_s$ is strain energy and $\Pi_w$ is work done by external applied forces. The first variation of strain energy $\Pi_s$ can be calculated as

$$\delta \Pi_s = \int_0^L \left( h \frac{\partial}{\partial x} (\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{xz} \delta \varepsilon_{xz} - D_x \delta E_x - D_z \delta E_z - B_x \delta H_x - B_z \delta H_z) \right) dz \, dx \quad (14)$$

Substituting Eqs. (8) and (9) into Eq. (14) yields

$$\delta \Pi_s = \int_0^L \left( h \frac{\partial}{\partial x} (\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{xz} \delta \varepsilon_{xz} - D_x \delta E_x - D_z \delta E_z - B_x \delta H_x - B_z \delta H_z) \right) dz \, dx$$

$$+ \int_0^L \left( -D_x \cos(\xi z) \delta \left( \frac{\partial \phi}{\partial x} \right) + D_z \xi \sin(\xi z) \delta \phi - B_x \cos(\xi z) \delta \left( \frac{\partial \psi}{\partial x} \right) + B_z \xi \sin(\xi z) \delta \gamma \right) dz \, dx \quad (15)$$

in which $N, M$ and $Q$ are the axial force, bending moment and shear force resultants, respectively. Relations between the stress resultants and stress component used in Eq. (15) are defined as

$$N = \int_A \sigma_{xx} \, dA, \quad M = \int_A \sigma_{xx} z \, dA, \quad P = \int_A \sigma_{xz} \, dA$$

$$Q = \int_A \sigma_{xz} \, dA, \quad R = \int_A \sigma_{zz} \, dA \quad (16)$$

The work done due to external electric voltage, $\Pi_w$, can be written in the form

$$\Pi_w = \int_0^L \left( N_h + N_E + N_b \right) \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} + q \delta w + f \delta u - N \delta \varepsilon_{xx}^{(0)} - \hat{M} \frac{\partial \delta w}{\partial x} \frac{\partial}{\partial x}$$

$$+ \alpha P \frac{\partial^2 \delta w}{\partial x^2} - \hat{Q} \delta \varepsilon_{xz}^{(0)} - k_w \delta w + k_p \frac{\partial^2 \delta w}{\partial x^2} \right) dx \quad (17)$$

where $\hat{M} = M + \alpha P, \hat{Q} = Q - \beta R$ and $q(x)$ and $f(x)$ are the transverse and axial distributed loads and $k_w$ and $k_p$ are foundation parameters and also $N_h$, $N_b$ and $N_E$ are the buckling load and normal forces induced by magnetic potential and external electric voltage, respectively which are defined as

$$N_E = -\int_{-h/2}^{h/2} \frac{2V}{h} \, dz, \quad N_h = -\int_{-h/2}^{h/2} \frac{2\Omega}{h} \, dz \quad (18)$$
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For a magneto-electro-elastic FGM nanobeam in the one dimensional case, the nonlocal constitutive relations (5a)-(5c) may be rewritten as

\[ \sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = c_{14} e_{ux} - e_{15} E_x - q_{31} H_x \]  \hspace{1cm} (19)

\[ \sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = c_{55} \gamma_{xc} - e_{15} E_x - q_{33} H_x \]  \hspace{1cm} (20)

\[ D_x - (e_0 a)^2 \frac{\partial^2 D_x}{\partial x^2} = e_{15} \gamma_{xc} + s_{11} E_x + d_{11} H_x \]  \hspace{1cm} (21)

\[ D_z - (e_0 a)^2 \frac{\partial^2 D_z}{\partial x^2} = e_{31} e_{ax} + s_{33} E_z + d_{33} H_z \]  \hspace{1cm} (22)

\[ B_x - (e_0 a)^2 \frac{\partial^2 B_x}{\partial x^2} = q_{15} \gamma_{xc} + d_{11} E_x + \chi_{11} H_x \]  \hspace{1cm} (23)

\[ B_z - (e_0 a)^2 \frac{\partial^2 B_z}{\partial x^2} = q_{31} e_{ax} + d_{33} E_z + \chi_{33} H_z \]  \hspace{1cm} (24)

Inserting Eqs. (15) and (17) in Eq. (13) and integrating by parts, and gathering the coefficients of \( \delta u, \delta w, \delta \psi, \delta \phi \) and \( \delta \gamma \) the following governing equations are obtained

\[ \frac{\partial N}{\partial x} + f = 0 \]  \hspace{1cm} (25)

\[ \frac{\partial M}{\partial x} - \dot{Q} = 0 \]  \hspace{1cm} (26)

\[ \dot{\dot{Q}} + q - (N_h + N_{eh} + N_{e}) \frac{\partial^2 w}{\partial x^2} + \alpha \frac{\partial^2 \rho}{\partial x^2} - k_w w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \]  \hspace{1cm} (27)

\[ \int_{h/2}^{h/2} \left( \cos(\xi z) \frac{\partial D_x}{\partial x} + \xi \sin(\xi z) D_x \right) \, dz = 0 \]  \hspace{1cm} (28)

\[ \int_{h/2}^{h/2} \left( \cos(\xi z) \frac{\partial B_x}{\partial x} + \xi \sin(\xi z) B_x \right) \, dz = 0 \]  \hspace{1cm} (29)

By integrating Eqs. (19)-(24), over the beam’s cross-section area, the force-strain and the moment-strain of the nonlocal third order Reddy FG beam theory can be obtained as follows

\[ N - \mu \frac{\partial^2 N}{\partial x^2} = A_{ss} \frac{\partial u}{\partial x} + B_{ss} \frac{\partial \psi}{\partial x} - \alpha \rho_{ss} \left( \frac{\partial \rho}{\partial x} + \frac{\partial^2 \rho}{\partial x^2} \right) + A_{ss} \phi + A_{ss} \gamma - N_h - N_{eh} \]  \hspace{1cm} (30)

\[ M - \mu \frac{\partial^2 M}{\partial x^2} = B_{ss} \frac{\partial u}{\partial x} + D_{ss} \frac{\partial \psi}{\partial x} - \alpha \rho_{ss} \left( \frac{\partial \rho}{\partial x} + \frac{\partial^2 \rho}{\partial x^2} \right) + E_{ss} \phi + E_{ss} \gamma \]  \hspace{1cm} (31)
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\[ P - \mu \frac{\partial^2 P}{\partial x^2} = E_{xx} \frac{\partial u}{\partial x} + F_{xx} \frac{\partial \psi}{\partial x} - \alpha H_{xx} \left( \frac{\partial \psi}{\partial x} + \frac{\partial^2 W}{\partial x^2} \right) + F_{31}^m \phi + F_{31}^m \gamma \]  \hspace{1cm} (32)

\[ Q - \mu \frac{\partial^2 Q}{\partial x^2} = (A_{\omega} - \beta D_{\omega}) \left( \frac{\partial W}{\partial x} + \psi \right) - E_{15}^e \frac{\partial \phi}{\partial x} - E_{15}^e \frac{\partial \gamma}{\partial x} \]  \hspace{1cm} (33)

\[ R - \mu \frac{\partial^2 R}{\partial x^2} = (D_{\omega} - \beta F_{\omega}) \left( \frac{\partial W}{\partial x} + \psi \right) - F_{31}^m \frac{\partial \phi}{\partial x} - F_{31}^m \frac{\partial \gamma}{\partial x} \]  \hspace{1cm} (34)

\[ \int_{-h/2}^{h/2} \left\{ D_x - \mu \frac{\partial^2 D_x}{\partial x^2} \right\} \cos(\xi z) dz = (E_{15}^e - \beta F_{15}^e) \left( \frac{\partial W}{\partial x} + \psi \right) + F_{15}^e \frac{\partial \phi}{\partial x} + F_{15}^e \frac{\partial \gamma}{\partial x} \]  \hspace{1cm} (35)

\[ \int_{-h/2}^{h/2} \left\{ B_x - \mu \frac{\partial^2 B_x}{\partial x^2} \right\} \xi \sin(\xi z) dz = A_{31}^e \frac{\partial u}{\partial x} + (E_{31}^e - \alpha F_{31}^e) \frac{\partial \psi}{\partial x} - \alpha F_{31}^e \frac{\partial^2 W}{\partial x^2} - F_{31}^m \phi - F_{31}^m \gamma \]  \hspace{1cm} (36)

where \( \mu = (e_0 a)^2 \) and quantities used in above equations are defined as

\[ \{A_{\omega}, B_{\omega}, D_{\omega}, E_{\omega}, F_{\omega}, H_{\omega}\} = \int_{-h/2}^{h/2} c_{11} \{1, z, z^2, z^3, z^4, z^6\} dz \]  \hspace{1cm} (39)

\[ \{A_{\omega}, D_{\omega}, F_{\omega}\} = \int_{-h/2}^{h/2} c_{35} \{1, z^2, z^4\} dz \]  \hspace{1cm} (40)

\[ \{A_{31}^e, E_{31}^e, F_{31}^e\} = \int_{-h/2}^{h/2} e_{31} \{ \xi \sin(\xi z), z \xi \sin(\xi z), z^3 \xi \sin(\xi z) \} dz \]  \hspace{1cm} (41)

\[ \{E_{15}^e, F_{15}^e\} = \int_{-h/2}^{h/2} e_{15} \{ \cos(\xi z), z^2 \cos(\xi z) \} dz \]  \hspace{1cm} (42)

\[ \{F_{15}^e, F_{33}^e\} = \int_{-h/2}^{h/2} s_{15} \{ \cos^2(\xi z), s_{33} \xi^2 \sin^2(\xi z) \} dz \]  \hspace{1cm} (43)

\[ \{A_{31}^m, E_{31}^m, F_{31}^m\} = \int_{-h/2}^{h/2} q_{31} \{ \xi \sin(\xi z), z \xi \sin(\xi z), z^3 \xi \sin(\xi z) \} dz \]  \hspace{1cm} (44)

\[ \{E_{15}^m, F_{15}^m\} = \int_{-h/2}^{h/2} q_{15} \{ \cos(\xi z), z^2 \cos(\xi z) \} dz \]  \hspace{1cm} (45)
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\[ \{F_{11}^m, F_{33}^m\} = \int_{-h/2}^{h/2} \left\{ d_{11} \cos^2(\xi z), d_{33} \xi^2 \sin^2(\xi z) \right\} dz \]  

(46)

\[ \{X_{11}^m, X_{33}^m\} = \int_{-h/2}^{h/2} \left\{ \chi_{11} \cos^2(\xi z), \chi_{33} \xi^2 \sin^2(\xi z) \right\} dz \]  

(47)

The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of \( N \) from Eq.(30) into Eq.(25) as follows

\[ N_x = A_{xx} \frac{\partial u}{\partial x} + K_{xx} \frac{\partial \psi}{\partial x} - \alpha E_{xx} \frac{\partial^2 w}{\partial x^2} + A_{xx} \phi + A_{xx}^m \phi - N_E - N_H + \mu (-\frac{\partial^2 F}{\partial x^2}) \]  

(48)

Omitting \( \hat{Q} \) from Eqs. (26) and (27), we obtain the following equation

\[ \frac{\partial^2 \hat{M}}{\partial x^2} = -\alpha \frac{\partial^2 P}{\partial x^2} - q + (N_E + N_H + N_b) \frac{\partial^2 w}{\partial x^2} + k_w w - k_p \frac{\partial^2 w}{\partial x^2} \]  

(49)

Also the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of \( \hat{M} \) from Eq. (31) into Eq. (26) and using Eqs. (31) and (32) as follows

\[ \hat{M} = K_{xx} \frac{\partial u}{\partial x} + I_{xx} \frac{\partial \psi}{\partial x} - \alpha J_{xx} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} \right) + (E_{xx}^\prime - \alpha F_{xx}^\prime) \phi + (E_{xx}^\prime - \alpha F_{xx}^\prime) \gamma \\
+ \mu (-\frac{\partial^2 P}{\partial x^2}) - q + \frac{\partial}{\partial x} \left( N_E + N_H + N_b \right) \frac{\partial w}{\partial x} + k_w w - k_p \frac{\partial^2 w}{\partial x^2} \]  

(50)

where

\[ K_{xx} = B_{xx} - \alpha E_{xx}, \quad I_{xx} = D_{xx} - \alpha F_{xx}, \quad J_{xx} = F_{xx} - \alpha H_{xx} \]  

(51)

By substituting for the second derivative of \( \hat{Q} \) from Eq. (33) into Eq. (27), and using Eqs. (33) and (34) the following expression for the nonlocal shear force is derived

\[ \hat{Q} = \tilde{A}_{xx} \frac{\partial w}{\partial x} + \psi - (E_{xx}^\prime - \beta F_{xx}^\prime) \frac{\partial \phi}{\partial x} + \mu ((N_H + N_E + N_b) \frac{\partial^2 w}{\partial x^2} - \alpha \frac{\partial^2 P}{\partial x^2} - \frac{\partial q}{\partial x}) \\
+ k_w \frac{\partial w}{\partial x} - k_p \frac{\partial^2 w}{\partial x^2} - (E_{xx}^\prime - \beta F_{xx}^\prime) \frac{\partial \gamma}{\partial x} \]  

(52)

where

\[ \tilde{A}_{xx} = A_{xx}^\prime - \beta F_{xx}^\prime, \quad A_{xx}^\prime = A_{xx} - \beta D_{xx}, \quad I_{xx}^\prime = D_{xx} - \beta F_{xx} \]  

(53)

Now we use \( \hat{M} \) and \( \hat{Q} \) from Eqs. (53) and (55) and the identity

\[ \alpha \frac{\partial^2 (P - \mu \frac{\partial^2 P}{\partial x^2})}{\partial x^2} = \alpha (E_{xx} \frac{\partial u}{\partial x} + F_{xx} \frac{\partial \psi}{\partial x} - \alpha H_{xx} (\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}) + F_{xx} \frac{\partial^2 \phi}{\partial x^2} + F_{xx}^m \frac{\partial^2 \gamma}{\partial x^2}) \]  

(54)

It must be cited that inserting Eqs. (28) and (29) into Eqs. (35)-(38), does not provide an explicit expressions for \( D_x \) and \( D_z \). To overcome this problem, by using Eqs. (35)-(38), Eqs. (28)
and (29) can be re-expressed in terms of $u$, $w$, $\psi$ and $\phi$. Finally, based on third-order beam theory, the nonlocal equations of motion for a magneto-electro-elastic FG nanobeam can be obtained by substituting for $N$, $M$ and $\dot{Q}$ from Eqs. (48), (50) and (52) into Eqs. (30)-(33) as follows

$$A_{xx} \frac{\partial^2 u}{\partial x^2} + K_{xx} \frac{\partial^2 \psi}{\partial x^2} - \alpha E_{xx} \frac{\partial^3 w}{\partial x^3} + A_{31} \frac{\partial \phi}{\partial x} + A_{31} \frac{\partial \psi}{\partial x} + \mu(- \frac{\partial^2 f}{\partial x^2} + f) = 0$$

(55)

$$K_{xx} \frac{\partial^2 u}{\partial x^2} + I_{xx} \frac{\partial^2 \psi}{\partial x^2} - \alpha J_{xx} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} \right) - A_{xx} \left( \psi + \frac{\partial w}{\partial x} \right) + (E_{31} - \alpha F_{31}) \frac{\partial \phi}{\partial x} + (E_{31} - \alpha F_{31}) \frac{\partial \psi}{\partial x}$$

+ $(E_{31} - \beta F_{31}) \frac{\partial \phi}{\partial x} + (E_{31} - \beta F_{31}) \frac{\partial \gamma}{\partial x} = 0$

(56)

$$- (E_{31} - \beta F_{31}) \frac{\partial \gamma}{\partial x} + \alpha (E_{xx} \frac{\partial u}{\partial x} + J_{xx} \frac{\partial \psi}{\partial x} - \alpha H_{xx} \frac{\partial w}{\partial x}) + F_{11} \frac{\partial \phi}{\partial x} + F_{11} \frac{\partial \gamma}{\partial x} - k_e \frac{\partial w}{\partial x} + q = 0$$

(57)

$$-(E_{31} - \beta F_{31}) \frac{\partial \gamma}{\partial x} + \alpha (E_{xx} \frac{\partial u}{\partial x} + J_{xx} \frac{\partial \psi}{\partial x} - \alpha H_{xx} \frac{\partial w}{\partial x}) + F_{11} \frac{\partial \phi}{\partial x} + F_{11} \frac{\partial \gamma}{\partial x} - k_e \frac{\partial w}{\partial x} + q = 0$$

(58)

$$-(E_{31} - \beta F_{31}) \frac{\partial \gamma}{\partial x} + \alpha (E_{xx} \frac{\partial u}{\partial x} + J_{xx} \frac{\partial \psi}{\partial x} - \alpha H_{xx} \frac{\partial w}{\partial x}) + F_{11} \frac{\partial \phi}{\partial x} + F_{11} \frac{\partial \gamma}{\partial x} - k_e \frac{\partial w}{\partial x} + q = 0$$

(59)

### 3. Solution procedure

Here, on the basis the Navier method, an analytical solution of the governing equations for buckling of a simply supported magneto-electro-elastic FG nanobeam is presented. To satisfy governing equations of motion, the displacement variables are adopted to be of the form

$$u(x,t) = \sum_{n=1}^{\infty} U_n \cos \left( \frac{n \pi}{L} x \right) e^{i \omega_n t}$$

(60)

$$w(x,t) = \sum_{n=1}^{\infty} W_n \sin \left( \frac{n \pi}{L} x \right) e^{i \omega_n t}$$

(61)

$$\psi(x,t) = \sum_{n=1}^{\infty} \Psi_n \cos \left( \frac{n \pi}{L} x \right) e^{i \omega_n t}$$

(62)

$$\phi(x,t) = \sum_{n=1}^{\infty} \Phi_n \sin \left( \frac{n \pi}{L} x \right) e^{i \omega_n t}$$

(63)
An exact solution for buckling analysis of embedded piezo-electro-magnetically actuated...  

\[ \gamma(x,t) = \sum_{n=1}^{\infty} Y_n \sin \left( \frac{n\pi}{L} x \right) e^{i\omega_n t} \]  

(64)

where \( U_n, W_n, \Psi_n, \Phi_n \) and \( Y_n \) are the unknown Fourier coefficients to be determined for each \( n \) value. Using Eqs. (60)-(64) the analytical solution can be obtained from the following equations

\[
\begin{bmatrix}
  k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} \\
  k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} \\
  k_{3,1} & k_{3,2} & k_{3,3} & k_{3,4} & k_{3,5} \\
  k_{4,1} & k_{4,2} & k_{4,3} & k_{4,4} & k_{4,5} \\
  k_{5,1} & k_{5,2} & k_{5,3} & k_{5,4} & k_{5,5} \\
\end{bmatrix}
\begin{bmatrix}
  U_n \\
  \Psi_n \\
  W_n \\
  \Phi_n \\
  Y_n \\
\end{bmatrix}
= 0
\]  

(68)

where

\[ k_{1,1} = -A_m \left( \frac{n\pi}{L} \right)^2, \quad k_{1,2} = -K_m \left( \frac{n\pi}{L} \right)^2, \quad k_{1,3} = \alpha E_m \left( \frac{n\pi}{L} \right)^3, \quad k_{1,4} = -A_v \left( \frac{n\pi}{L} \right), \quad k_{1,5} = -A_g \left( \frac{n\pi}{L} \right) \]

\[ k_{2,2} = -I_m \left( \frac{n\pi}{L} \right)^2 + \alpha J_m \left( \frac{n\pi}{L} \right)^2 \]

\[ k_{3,3} = -\bar{E}_m \left( \frac{n\pi}{L} \right)^2 + J_m \left( \frac{n\pi}{L} \right)^3 \]

\[ k_{4,4} = -\left( E_{15} - \beta F_{15} \right) \]

\[ k_{5,5} = -\left( E_{31} - \alpha F_{31} \right) \left( \frac{n\pi}{L} \right)^2 \]

4. Results and discussion

This section provides some numerical examples for the buckling characteristics of MEE-FG nanobeams. To achieve this goal, the nonlocal FG beam made of BaTiO\(_3\) and CoFe\(_2\)O\(_4\), with magneto-electro-elastic material properties listed in Table 1, is assumed. The beam geometry has the following dimensions: \( L \) (length)=10 nm and \( h \) (thickness)=varied. Also, the following relation is described to calculate the non-dimensional buckling loads as well as foundation parameters

\[ N_{bcr} = N_b \left( \frac{L^2}{c_1 I} \right)_{\text{ColFe}_2O_4}, \quad K_w = k_w \left( \frac{L^4}{c_1 I} \right)_{\text{ColFe}_2O_4}, \quad K_p = k_p \left( \frac{L^2}{c_1 I} \right)_{\text{ColFe}_2O_4} \]  

(69)

In which \( I = h^3/12 \) is the moment of inertia of the cross section of the beam. To evaluate correctness of the present model, the buckling results are compared with those of nonlocal FGM Reddy beams, due to the absence of numerical results for the buckling of MEE-FG nanobeams based on the nonlocal elasticity theory, as provided in Table 2. In this paper, the material selection is carried out as follows: \( E_m = 70 \) GPa, \( v_m = 0.3 \), kgm\(^{-3}\) for Steel and \( E_c = 390 \) GPa, \( v_c = 0.3 \), for Alumina. Tables 3-5, present the influences of magnetic potential (\( \Omega \)), electric voltage (\( V \)), elastic foundation parameters (\( K_w, K_p \)), nonlocal parameter (\( \mu \)), gradient index (\( p \)) and slenderness ratio (\( L/h \)) on the non-dimensional buckling load of the S-S MEE-FG nanobeams.
Table 1 Magneto-electro-elastic coefficients of material properties (Pan and Han 2005)

<table>
<thead>
<tr>
<th>Properties</th>
<th>BaTiO$_3$</th>
<th>CoFe$_2$O$_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{11}$ (GPa)</td>
<td>166</td>
<td>286</td>
</tr>
<tr>
<td>$c_{55}$</td>
<td>43</td>
<td>45.3</td>
</tr>
<tr>
<td>$e_{31}$ (Cm$^2$)</td>
<td>-4.4</td>
<td>0</td>
</tr>
<tr>
<td>$e_{15}$</td>
<td>11.6</td>
<td>0</td>
</tr>
<tr>
<td>$q_{31}$ (N/Am)</td>
<td>0</td>
<td>580.3</td>
</tr>
<tr>
<td>$q_{15}$</td>
<td>0</td>
<td>550</td>
</tr>
<tr>
<td>$s_{11}$ ($10^9$ C$^2$m$^3$N$^{-1}$)</td>
<td>11.2</td>
<td>0.08</td>
</tr>
<tr>
<td>$s_{33}$</td>
<td>12.6</td>
<td>0.093</td>
</tr>
<tr>
<td>$\chi_{11}$ ($10^6$ Ns$^2$C$^{-2}$/2)</td>
<td>5</td>
<td>-590</td>
</tr>
<tr>
<td>$\chi_{33}$</td>
<td>10</td>
<td>157</td>
</tr>
<tr>
<td>$d_{11}$=0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2 Comparison of the non-dimensional buckling load for a S-S FG nanobeam with various power-law index ($L/h=20$)

<table>
<thead>
<tr>
<th>Nonlocal parameter</th>
<th>$\mu=1$</th>
<th>$\mu=2$</th>
<th>$\mu=3$</th>
<th>$\mu=4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RBT</td>
<td>Present</td>
<td>RBT</td>
<td>Present</td>
<td>Present</td>
</tr>
<tr>
<td>0</td>
<td>8.9258</td>
<td>8.925759</td>
<td>8.1900</td>
<td>8.190046</td>
</tr>
<tr>
<td>0.1</td>
<td>9.7778</td>
<td>9.777865</td>
<td>8.9719</td>
<td>8.971916</td>
</tr>
<tr>
<td>0.2</td>
<td>10.3898</td>
<td>10.389845</td>
<td>9.5334</td>
<td>9.533453</td>
</tr>
</tbody>
</table>

It is obvious that for all values of magnetic potential and electric voltage nonlocal parameter weakens the structure of nanobeam by showing a significant reducing influence on the non-dimensional buckling loads. Also, it is observed that elastic foundation enhances rigidity of the beam and leads to increasing the dimensionless buckling loads. Another observation is that the buckling load results are strongly dependent on the magnitude and sign of magnetic potential and electric voltage. For all values of Winkler and Pasternak foundation parameters, the negative voltages provide higher buckling loads, while negative magnetic potentials produce lower buckling loads.

The influences of magnetic potential and electric voltage on the variations of the non-dimensional buckling load of the simply supported FG nanobeams versus power-law index at $L/h=20$ are plotted in Figs. 2 and 3, respectively. As one can see the non-dimensional buckling load decreases when the gradient index rises, especially for lower values of gradient index. This
Table 3 Variation of the dimensionless buckling load of embedded S-S FG nanobeam for various nonlocal parameter, magnetic potentials and electric voltages (L/h=15)

<table>
<thead>
<tr>
<th>((K_w, K_p))</th>
<th>(\mu)</th>
<th>(\Omega=0.05)</th>
<th>(\Omega=0)</th>
<th>(\Omega=0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(p=0.2)</td>
<td>(p=1)</td>
<td>(p=5)</td>
<td>(p=0.2)</td>
</tr>
<tr>
<td>(0.0)</td>
<td>6.79306</td>
<td>6.17381</td>
<td>5.99769</td>
<td>7.47785</td>
</tr>
<tr>
<td>0</td>
<td>6.86921</td>
<td>5.86227</td>
<td>5.47846</td>
<td>7.37401</td>
</tr>
<tr>
<td>1</td>
<td>6.58536</td>
<td>5.55074</td>
<td>4.95923</td>
<td>7.27016</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(25,5)</th>
<th>(\Omega=0.05)</th>
<th>(\Omega=0)</th>
<th>(\Omega=0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p=0.2)</td>
<td>(p=1)</td>
<td>(p=5)</td>
<td>(p=0.2)</td>
</tr>
<tr>
<td>3.50854</td>
<td>5.86972</td>
<td>5.22406</td>
<td>8.68176</td>
</tr>
<tr>
<td>(0,0)</td>
<td>6.50720</td>
<td>5.92708</td>
<td>5.84582</td>
</tr>
<tr>
<td>(\Omega=0.05)</td>
<td>(\Omega=0)</td>
<td>(\Omega=0.05)</td>
<td></td>
</tr>
<tr>
<td>5.85291</td>
<td>5.35826</td>
<td>5.33722</td>
<td>7.45613</td>
</tr>
<tr>
<td>(\Omega=0)</td>
<td>(\Omega=0.05)</td>
<td>(\Omega=0.05)</td>
<td></td>
</tr>
<tr>
<td>(\Omega=0.05)</td>
<td>(\Omega=0.05)</td>
<td>(\Omega=0.05)</td>
<td></td>
</tr>
<tr>
<td>(\Omega=0)</td>
<td>(\Omega=0.05)</td>
<td>(\Omega=0.05)</td>
<td></td>
</tr>
</tbody>
</table>
Table 5 Variation of the dimensionless buckling load of embedded S-S FG nanobeam for various nonlocal parameter, magnetic potentials and electric voltages ($L/h=25$)

<table>
<thead>
<tr>
<th>($K_w,K_p$)</th>
<th>$\mu$</th>
<th>$\Omega=-0.05$</th>
<th>$\Omega=0$</th>
<th>$\Omega=+0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p=0.2$</td>
<td>$p=1$</td>
<td>$p=5$</td>
<td>$p=0.2$</td>
</tr>
<tr>
<td>(20,0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25,0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25,5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25,10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25,20)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2 Effect of external magnetic potential on the dimensionless buckling load of the S-S FG nanobeam with respect to gradient index ($L/h=20$, $V=+5$, $\mu=2$)

Reduction in buckling load is more significant with respect to the positive magnetic potentials and external electric voltages. Moreover, it is observed that influence of larger values of gradient index
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Fig. 3 Effect of external electric voltage on the dimensionless buckling load of the S-S FG nanobeam with respect to gradient index ($L/h=20$, $\Omega=+0.05$, $\mu=2$)

Fig. 4 Effect of nonlocal parameter and magnetic field on the dimensionless buckling load of the S-S FG nanobeam ($L/h=20$, $V=+5$, $\mu=0.2$)

on the magnetic potential is less than lower gradient indexes, whereas this trend is reverse for electric voltage and the impact of higher gradient indexes on electric voltage is more sensible.

The effect of nonlocal parameter on the first non-dimensional buckling load of the S-S MEE-FG nanobeams is depicted in Fig. 4 ($L/h=20$, $V=+5$, $\mu=0.2$). It is apparently seen that nonlocal parameter has a softening influence on the beam structure and reduces the buckling loads. So, nonlocal beam model produces smaller buckling loads compared to local beam model. Also, it is observed that nonlocality is independent of magnetic field.

The variations of the dimensionless buckling load of MEE-FG nanobeams versus the Winkler and Pasternak parameters for various magnetic potentials and electric voltages at $L/h=20$, $p=0.2$ and $\mu=2$ are presented in Figs. 5 and 6, respectively. One can find that, with the increase of
Fig. 5 Effect of external magnetic potential and electric voltage on the dimensionless buckling load of the S-S FG nanobeam with respect to Winkler parameter; (a) $V=+5$, (b) $\Omega=+0.05$ ($L/h=20$, $\mu=2$, $p=0.2$, $K_p=5$)

Fig. 6 Effect of external magnetic potential and electric voltage on the dimensionless buckling load of the S-S FG nanobeam with respect to Pasternak parameter; (a) $V=+5$, (b) $\Omega=+0.05$ ($L/h=20$, $\mu=2$, $p=0.2$, $K_p=25$)

Winkler and Pasternak parameters for any sign and magnitude of magnetic potential and electric voltage, the non-dimensional buckling load increases, due to the enhancement in stiffness of the MME-FG nanobeam structure. Moreover, it is clearly seen that the effect of Pasternak elastic parameter on the non-dimensional buckling load is more than Winkler parameter. Therefore, the shear layer or Pasternak parameter of elastic foundation plays an important role on the mechanical responses of FG structure and should be considered in their analysis.

Figs. 7-8 demonstrate the variations the dimensionless buckling load of nonlocal FG beams made of magneto-electro-elastic materials with respect to external electric voltage and magnetic
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Fig. 7 Effect of elastic foundation on the dimensionless buckling load of the S-S FG nanobeam with respect to electric voltage ($L/h=20$, $\Omega=+0.05$, $\mu=2$, $p=1$)

Fig. 8 Effect of elastic foundation on the dimensionless buckling load of the S-S FG nanobeam with respect to magnetic potential ($L/h=20$, $V=+5$, $\mu=2$, $p=1$)

potential, respectively at $L/h=20$ for various Winkler and Pasternak parameters. It is evident that external electric voltage has a decreasing influence on the buckling loads of MEE-FG nanobeams when it changes from negative values to positive one, whereas by varying the magnetic potential values from negative values to positive one, the non-dimensional buckling load rise. As a general consequence, it must be mentioned that the impact of magnetic field on the buckling behavior of FG nanobeams is more than electric field.

Finally, Fig. 9 depicts the variations of the non-dimensional buckling load of MEE-FG nanobeam with respect to slenderness ratio for different magnetic potentials at power-law index.
Farzad Ebrahimi and Mohammad Reza Barati

Let \( K_w = K_p = 0 \) and nonlocal parameter \( \mu = 1 \ (\text{nm})^2 \). It is shown that slenderness ratio has a significant effect on the stability of MEE-FG nanobeams. Hence, the higher values of slenderness ratio have more influence on the dimensionless buckling load. Also, it is observable that positive values of magnetic potential show an increasing influence on buckling loads of FG nanobeams, whereas the negative ones have a reducing impact. This is due to the reason that compressive and tensile in-plane forces are generated in the nanobeam when positive and negative magnetic potentials are applied, respectively.

5. Conclusions

This paper presents a nonlocal higher-order beam model for buckling analysis of magneto-electro-elastic FG nanobeams resting on two-parameter elastic foundation including linear springs and a shear layer. Governing equations obtained using Hamilton’s principle as well as nonlocal elasticity theory which captures the small size influences are solved applying Navier solution method. Magneto-electro-elastic properties of the FG nanobeams are supposed to be varied continuously through the thickness direction according to power-law model. A detailed parametric study is conducted to study the influences of the magnetic potential, electric voltage, elastic foundation, nonlocal parameter, material composition and slenderness ratio on the buckling responses of the MEE-FG nanobeams. It is deduced that nonlocality and power-law exponent yields in reduction on both rigidity of the nanobeam structure and buckling loads. But with an increment in value of Winkler or Pasternak parameters the rigidity of the MEE-FG nanobeam and buckling load growth. Also, it is observed that the magnitude and sign of magnetic potential and electric voltage have a notable influence on the buckling loads of MEE-FG nanobeams.
References


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