On the bending and stability of nanowire using various HSDTs

Djamel Ould Youcef1, Abdelhakim Kaci1, Mohammed Sid Ahmed Houari2, Abdelouahed Tounsi∗1,2,3, Abdelnour Benzair1 and Houari Heireche1

1Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbès, Algeria
2Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbés, Faculté de Technologie, Département de Génie Civil, Algeria
3Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria

(Received December 7, 2015, Revised December 27, 2015, Accepted December 28, 2015)

Abstract. In this article, various higher-order shear deformation theories (HSDTs) are developed for bending and buckling behaviors of nanowires including surface stress effects. The most important assumption used in different proposed beam theories is that the deflection consists of bending and shear components and thus the theories have the potential to be utilized for modeling of the surface stress influences on nanowires problems. Numerical results are illustrated to prove the difference between the response of the nanowires predicted by the classical and non-classical solutions which depends on the magnitudes of the surface elastic constants.

Keywords: surface effects; nanowires; bending; buckling

1. Introduction

The nanowires (NW)-based devices have found considerable range of applications in physics, engineering, and several other fields (Craighead 2000, Ekinci and Roukes 2005, He and Lilley 2008a, Jiang and Yan 2010, Liu et al. 2012, Wang and Feng 2009, Li et al. 2011, Chiu and Chen 2011a, Wang and Yang 2011, Eltaher et al. 2014). In physical applications, nanowires are often employed in advanced technological devices such as sensors, actuators, transistors, and resonators in nanoelectromechanical systems (NEMSs) (Craighead 2000, Ekinci and Roukes 2005). As is well known, conventional beam models failed to explain the size dependent mechanical response of nanostructures. In the past few years, beam theories have been developed based on non-conventional continuum theories, such as the surface elasticity theory, strain gradient theory, and coupled stress theory to account for the size effect of 1D nanoscale structures (Al-Basyouni et al. 2015). Among these efforts, beam models based on the surface elasticity theory are attracting more and more attention due to their solid physical background (Wang and Feng 2009, Song and Huang 2009, Chiu and Chen 2011b, Ansari and Sahmani 2011, Mahmoud et al. 2012, Hosseini-Hashemi et al. 2013).

Copyright © 2015 Techno-Press, Ltd.
http://www.techno-press.com/?journal=anr&subpage=7 ISSN: 2287-237X (Print), 2287-2388 (Online)
Atoms near the surface and interface of a solid experience different local environment comparatively to those away from the surface because of the reduced coordination. Thus, the surface and interface of solids present different mechanical characteristics compared with the bulk material (Gurtin and Murdoch 1975, 1978, Dingreville et al. 2005). Gurtin and Murdoch (1975, 1978) and Gurtin et al. (1998) proposed a theoretical formulation to consider this surface/interface stress impact. This approach has been largely employed to investigate the mechanical response of nano defects, nano composites, and nanostructures (Sharma et al. 2003, Duan et al. 2005, He and Lim 2001). Recently, by employing the surface Cauchy-Born model, Park and Klein (2008) and Park (2008, 2009) examined the influences of the surface stress on the resonant frequencies of metallic/silicon NWs. He and Lilley (2008a, b) have considered the surface stress on all surfaces of the NWs and the effective Young’s modulus of the NW was redefined. Yan and Jiang (2011) employed the Euler beam theory to investigate the buckling response of piezoelectric nanobeams with surface stress effect. Ansari and Sahmani (2011) adopted different beam theories for the buckling analysis of nanobeams with surface effect. Wang and Yang (2011) studied the buckling of nanobeams by considering the geometric nonlinearity.

In this work, various non-classical higher-order shear deformation beam theories are proposed to investigate the bending and axial buckling of a simply supported NWs including surface stress effect. Numerical results are presented to prove the significant effect of surface stress effects on the bending and buckling responses of NWs.

### 2. Formulation of the problem

Consider a beam of length $L$ and rectangular cross-section of thickness $h$ and width $b$. A coordinate system $x, y, z$ is employed on the central axis of the beam, whereas the $x$ axis is
On the bending and stability of nanowire using various HSDTs

Table 1 Shape functions

<table>
<thead>
<tr>
<th>Model</th>
<th>$f(z)$</th>
<th>$g(z)=1-f'(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBT based on Reddy (1984)</td>
<td>$\frac{4z^4}{3h^2}$</td>
<td>$1-\frac{4z^3}{h^2}$</td>
</tr>
<tr>
<td>SBT based on Touratier (1991)</td>
<td>$z-h\sin\left(\frac{\pi z}{h}\right)$</td>
<td>$\cos\left(\frac{\pi z}{h}\right)$</td>
</tr>
<tr>
<td>HBT based on Soldatos (1992)</td>
<td>$z-h\sinh\left(\frac{z}{h}\right)+z\cosh\left(\frac{1}{2}\right)$</td>
<td>$\cosh\left(\frac{z}{h}\right)-\cosh\left(\frac{1}{2}\right)$</td>
</tr>
</tbody>
</table>

considered along the length of the beam, the $y$ axis in the width direction and the $z$ axis is considered along the depth (height) direction. Also, the origin of the coordinate system is adopted at the left end of the beam (Fig. 1). The NW is subjected to transverse load $q$ (point load or uniform load) and axial forces $N_0$ at both ends.

2.1 Kinematics

Based on the same formulation proposed by Berrabah et al (2013) and Bourada et al (2015) where the transverse displacement is partitioned with two components (the bending part $w_b$ and the shear part $w_s$), the axial displacement, $u$, and the transverse displacement of any point of the beam, $w$, are given as

$$u(x, z) = -z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \quad (1a)$$

$$w(x, z) = w_b(x) + w_s(x) \quad (1b)$$

The shape functions $f(z)$ are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam, thus a shear correction factor is not needed. The displacement fields of the third-order beam theory (TBT) based on Reddy (1984), sinusoidal beam theory (SBT) based on Touratier (1991) and hyperbolic beam theory (HBT) based on Soldatos (1992) can be determined from Eq. (1) by employing different shape functions $f(z)$ given in Table 1.

The non-zero strains associated with the displacements in Eq. (1) are

$$\varepsilon_x = -z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \quad \text{and} \quad \gamma_{xz} = g(z)\frac{\partial w_s}{\partial x} \quad (2)$$

where

$$g(z) = 1 - f'(z) \quad \text{and} \quad f'(z) = \frac{df(z)}{dz} \quad (3)$$

2.2 Surface elasticity model for nanowires and constitutive relations

Surface impacts on the mechanical response of nanostructures can be investigated by
considering surface energy and/or surface stresses. The resulting in-plane forces lead to surface stresses which can be derived by employing surface constitutive equations as

\[ \sigma_s^i = (2\mu^s + \lambda^s)\varepsilon_s + \tau^s \quad \text{and} \quad \tau_{sz}^s = \tau^s \frac{\partial w}{\partial x} \]  

(4)

The superscript \( s \) is employed to represent the quantities corresponding to the surface.

The stress component \( \sigma_z \) is small comparatively to the \( \tau_{xz} \) for the classical beam theories and consequently it is supposed that \( \sigma_z = 0 \). However, this assumption does not respect the surface conditions considered in the Gurtin-Murdoch model. To solve this problem, it is supposed that the stress component \( \sigma_z \) changes linearly within the beam thickness and satisfies the balance conditions on the top and bottom surfaces (Lu et al. 2006). According to this assumption, \( \sigma_z \) can be determined as

\[ \sigma_z = \frac{1}{2h} \left( \frac{\partial \tau_{xz}}{\partial x} \right)_{\text{at top}} + \frac{1}{2h} \left( \frac{\partial \tau_{xz}}{\partial x} \right)_{\text{at bottom}} \]  

(5)

Based on equations (2) together with equations (4), the components of surface stress for the present beam theories can be obtained in the following form

\[ \sigma_{ss}^i = (2\mu^s + \lambda^s)\varepsilon_{ss} + \tau_{sz}^s \]

(6a)

\[ \tau_{sz}^s = \tau^s \left( \frac{\partial w_b}{\partial x} + \frac{\partial w_t}{\partial x} \right) \]

(6b)

The non-zero components of stress for the bulk (\( \sigma_{ss}^b \) and \( \tau_{sz}^b \)) of the beam can be determined as

\[ \sigma_{ss}^b = E\varepsilon_{ss} + \nu \sigma_z = E \left( -z \frac{\partial^2 w_b}{\partial x^2} - f(z) \frac{\partial^2 w_t}{\partial x^2} \right) + \frac{2z}{h} \nu \tau^s \left( \frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_t}{\partial x^2} \right) \]

(7a)

\[ \tau_{sz}^b = G\gamma_{sz} = Gg(z) \frac{\partial w_t}{\partial x} \]

(7b)

The superscript \( b \) is employed to represent the quantities corresponding to the bulk.

In this work, we consider a superposition between the quantities corresponding to the surface and the bulk and this summation is considered to facilitate only the mathematical formulation

\[ \sigma_s = \sigma_{ss}^s + \sigma_{ss}^b \quad \text{and} \quad \tau_{sz} = \tau_{sz}^s + \tau_{sz}^b \]  

(8)

### 2.3 Governing equations

The minimum total potential energy principle, is employed here to obtain the governing equations (Reddy 2002, Draiche et al. 2014).
On the bending and stability of nanowire using various HSDTs

\[ \delta \Pi = \delta (U_{\text{int}} - W_{\text{ext}}) = 0 \]  

(9)

where \(|\Pi|\) is the total potential energy. \(\delta U_{\text{int}}\) is the virtual variation of the strain energy; and \(\delta W_{\text{ext}}\) is the variation of work done by external forces. The first variation of the strain energy is given as:

\[ \delta U_{\text{int}} = \frac{\delta}{\delta z} \int_{0}^{h} \left( \sigma_{x} \delta e_{x} + \tau_{\text{sc}} \delta \gamma_{\text{sc}} \right) dz dx \]

(10)

where \(M_{b}, M_{s}\), and \(Q\) are the stress resultants defined as

\[ (M_{b}, M_{s}) = \int_{A}(z, f) \sigma_{x} dA \text{ and } Q = \int_{A} g \tau_{\text{sc}} dA \]  

(11)

The first variation of the work done by the axial compressive force is given by

\[ \delta V = \int_{0}^{l} q \delta w dz + \int_{0}^{l} N_{0} \frac{d}{dx} \frac{d}{dx} w dx \]

(12)

where \(q\) and \(N_{0}\) are the transverse and axial loads, respectively.

Substituting Eqs. (10) and (12) into Eq. (9) and carrying out the integration by parts, the equations of motion of the proposed beam theory are determined as follows

\[ \delta w_{b} : \frac{d^{2} M_{b}}{dx^{2}} - N_{0} \frac{d^{2}}{dx^{2}} (w_{b} + w_{s}) + q = 0 \]  

(13a)

\[ \delta w_{s} : \frac{d^{2} M_{s}}{dx^{2}} + Q - N_{0} \frac{d^{2}}{dx^{2}} (w_{b} + w_{s}) + q = 0 \]  

(13b)

By substituting Eqs. (6) and (7) into Eq. (8), and the subsequent results into Eq. (11), the constitutive equations for the stress resultants are obtained as

\[ M_{b} = \left[ \frac{2I_{1}v\tau^{i}}{h} - D_{11} - (2\mu + \lambda) \left( \frac{h^{i}}{6} + \frac{Ah}{2} \right) \right] \frac{\partial^{2} w_{b}}{\partial x^{2}} + \left[ \frac{2I_{1}v\tau^{i}}{h} - D_{11} - (2\mu + \lambda) \right] \frac{\partial^{2} w_{s}}{\partial x^{2}} \]  

(14a)

\[ M_{s} = \left( \frac{2I_{1}v\tau^{i}}{h} - D_{11} - (2\mu + \lambda) \right) \frac{\partial^{2} w_{b}}{\partial x^{2}} + \left[ \frac{2I_{1}v\tau^{i}}{h} - D_{11} - (2\mu + \lambda) \right] \frac{\partial^{2} w_{s}}{\partial x^{2}} + I_{p3} \tau^{i} \]  

(14b)

\[ Q = \left[ A_{55} + \mu \mu J_{p1} + \frac{1}{2} \tau^{i} \left( J_{p3} - J_{p2} \right) \right] \frac{\partial w_{s}}{\partial x} \]  

(14c)

where
\begin{align}
(D_{11}, D_{11}', H_{11}') &= \int_A E(z^2, z f(z), f^2(z)) dA \quad \text{and} \quad A_{33}' = \int_A G[g(z)] dA \\
I &= \int_A z^2 dA = \frac{bh^3}{12}; \quad I_1 = \int_A zf(z) dA;
\end{align}

\begin{align}
J_{p1} &= 2 \int_{-h/2}^{h/2} [g(z)]^2 dz; \quad J_{p2} = \int_{-h/2}^{h/2} g(z) dz; \quad J_{p3} = \int_{-h/2}^{h/2} f'(z) g(z) dz; \\
J_{p4} &= \int_{-h/2}^{h/2} f(z) dz; \quad J_{p5} = \int_{-h/2}^{h/2} [f(z)]^2 dz
\end{align}

By substituting Eq. (14) into Eq. (13), the governing equations can be expressed in terms of
displacements \((w_b, w_s)\) as

\begin{align}
&\left[ \frac{21 \nu \tau'}{h} - D_{11} - (2\mu' + \lambda') \left( \frac{h^3}{6} + \frac{Ah}{2} \right) \right] \frac{\partial^4 w_b}{\partial x^4} + \left[ \frac{21 \nu \tau'}{h} - D_{11}' - (2\mu' + \lambda') \right] \frac{\partial^4 w_s}{\partial x^4} \\
&\quad + q + H - N_0 \left( \frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2} \right) = 0
\end{align}

\begin{align}
&\left[ \frac{21 \nu \tau'}{h} - D_{11} - (2\mu' + \lambda') \right] \frac{\partial^4 w_b}{\partial x^4} + \left[ \frac{21 \nu \tau'}{h} - H_{11} - (2\mu' + \lambda') \right] \frac{\partial^4 w_s}{\partial x^4} \\
&\quad + \left[ A_{33}' + \mu' J_{p1} + \frac{1}{2} \tau' (J_{p3} - J_{p2}) \right] \frac{\partial^2 w_s}{\partial x^2} + q + H - N_0 \left( \frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2} \right) = 0
\end{align}

where \(H\) is the constant parameter which is determined by the residual surface tension \(\tau'\) (generally assumed as a positive number) and the shape of cross section. For rectangular beam cross sections, the surface elasticity tension is expressed by

\begin{equation}
H = 2b \tau'
\end{equation}

3. Closed-form solution for simply supported nanowires

A simply supported beam with length \(L\) subjected to transverse load \(q\) and axial load \(N_0\) is considered here. The following expansions of displacements \((w_b, w_s)\) are chosen to satisfy the simply supported boundary conditions of beam

\begin{align}
w_b &= \sum_{n=1}^\infty W_{bn} \sin(\alpha x) \\
w_s &= \sum_{n=1}^\infty W_{sn} \sin(\alpha x)
\end{align}
On the bending and stability of nanowire using various HSDTs

where $W_{bn}$ and $W_{sn}$ are arbitrary parameters to be determined, and $\alpha = n\pi/L$. The transverse load $q$ is also expanded in the Fourier sine series as

$$ q(x) = \sum_{n=1}^{\infty} Q_n \sin \alpha x, \quad Q_n = \frac{2}{L} \int_0^L q(x) \sin(\alpha x) \, dx $$

(19)

The Fourier coefficients $Q_n$ associated with some typical loads are given

$$ Q_n = q_0, \quad n = 1 \quad \text{for sinusoidal load,} $$
$$ Q_n = \frac{4q_0}{n\pi}, \quad n = 1, 3, 5, \ldots \quad \text{for uniform load,} $$

(20a, 20b)

Substituting the expansions of $w_{bn}$, $w_{sn}$, and $q$ from Eqs. (19) and (20) into Eq. (18), the closed-form solutions can be obtained from the following equations

$$ \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} - N_0 \alpha^2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} W_{bn} \\ W_{sn} \end{bmatrix} = \begin{bmatrix} Q_n \\ Q_n \end{bmatrix} $$

(21)

where

$$ S_{11} = \alpha^4 \left( H + \frac{2Iv \tau^3}{h} - D_{11} - \frac{2\mu^s + \lambda^s}{\mu^s} \left( \frac{h^3}{6} + \frac{Ah}{2} \right) \right) $$

(22a)

$$ S_{12} = \alpha^4 \left( H + \frac{2Iv \tau^3}{h} - D_{11} - \frac{2\mu^s + \lambda^s}{\mu^s} \left( J_{p1} + \frac{1}{2} \tau^4 (J_{p3} - J_{p2}) \right) \right) $$

(22b)

$$ S_{22} = \alpha^4 \left( H + \frac{2Iv \tau^3}{h} - H_{11} - \frac{2\mu^s + \lambda^s}{\mu^s} \left( J_{p1} + \frac{1}{2} \tau^4 (J_{p3} - J_{p2}) \right) \right) $$

(22c)

### 3.1 Bending

The static deflection is obtained from Eq. (21) by setting $N_0$ to zero

$$ w(x) = \sum_{n=1}^{\infty} \frac{Q_n}{S_{11} - S_{12}^2/S_{22}} + \frac{Q_n}{S_{22} - S_{12}^2/S_{11}} - \frac{2Q_n}{S_{11}S_{22}/S_{12} - S_{12}} \sin \alpha x $$

(23)

### 3.2 Buckling

The buckling load is obtained from Eq. (21) by setting $q$ to zero

$$ N_0 = \frac{S_{11}S_{22} - S_{12}^2}{\alpha^2 \left( S_{11} + S_{22} - 2S_{12} \right)} $$

(24)
Table 2 Comparison between maximum center deflections under uniform load of nanowires obtained with classical and non-classical solutions

<table>
<thead>
<tr>
<th>(L/h)</th>
<th>EBT classical</th>
<th>EBT non-classical</th>
<th>FBT classical</th>
<th>FBT non-classical</th>
<th>Present TBT classical</th>
<th>Present TBT non-classical</th>
<th>Present SBT classical</th>
<th>Present SBT non-classical</th>
<th>Present HBT classical</th>
<th>Present HBT non-classical</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>44.6145</td>
<td>7.2189</td>
<td>45.0980</td>
<td>7.2238</td>
<td>45.0980</td>
<td>7.2240</td>
<td>45.0973</td>
<td>7.2240</td>
<td>45.0110</td>
<td>7.2229</td>
</tr>
<tr>
<td>35</td>
<td>1322.4628</td>
<td>43.9385</td>
<td>1325.0952</td>
<td>43.9386</td>
<td>1325.0952</td>
<td>43.9386</td>
<td>1325.0913</td>
<td>43.9386</td>
<td>1324.6216</td>
<td>43.9386</td>
</tr>
<tr>
<td>40</td>
<td>2256.0632</td>
<td>57.7251</td>
<td>2259.5014</td>
<td>57.7251</td>
<td>2259.5013</td>
<td>57.7251</td>
<td>2259.4964</td>
<td>57.7251</td>
<td>2258.8829</td>
<td>57.7251</td>
</tr>
<tr>
<td>45</td>
<td>3613.7770</td>
<td>73.3501</td>
<td>3618.1285</td>
<td>73.3501</td>
<td>3618.1284</td>
<td>73.3501</td>
<td>3618.1221</td>
<td>73.3501</td>
<td>3617.3456</td>
<td>73.3501</td>
</tr>
<tr>
<td>50</td>
<td>5507.9667</td>
<td>90.8133</td>
<td>5513.3390</td>
<td>90.8133</td>
<td>5513.3389</td>
<td>90.8133</td>
<td>5513.3311</td>
<td>90.8133</td>
<td>5512.3725</td>
<td>90.8133</td>
</tr>
</tbody>
</table>

4. Numerical results and discussion

In this section, numerical results are provided for analytical solutions shown in the previous sections. The following material characteristics are used in computations as follows (Gurtin and Murdoch 1978):

\[
E = 17.73 \times 10^6 \text{ N/m}^2, \quad \nu = 0.27, \quad \lambda' = -8 \text{ N/m}, \quad \mu' = 2.5 \text{ N/m}, \quad r' = 1.7 \text{ N/m}
\]

It is supposed that \(h=b=1\) nm and \(L\) varies from \(L/h=10\) to \(50\).

Tables 2 and 3 show, respectively, the maximum deflections of a simply supported nanowire subjected to uniform load and point load by using the classical and non-classical theories. The obtained results are compared with those computed independently for the first time based on the Euler-Bernoulli beam theory (EBT), and First beam theory (FBT) for a wide range of thickness ratio. It can be seen that the results of present theories are in excellent agreement with those
On the bending and stability of nanowire using various HSDTs

Fig. 2 Variation of maximum center deflections with the aspect ratio corresponding to different values \( \tau' \) of with the assumption of \( 2\mu' + \lambda' = 0 \)

Fig. 3 Variation of maximum center deflections with the aspect ratio corresponding to different magnitudes of \( 2\mu' + \lambda' \) with the assumption of \( \tau' = 0 \)

predicted by FBT for all values of thickness ratio \( L/h \). The TBT, SBT, and HBT provide solutions which are almost the same for all values of thickness ratio \( L/h \), whereas the EBT underestimates deflections. The difference between EBT and shear deformation theories (i.e., TBT, SBT, HBT and FBT) is negligible for slender nanowires and considerable for deep nanowires. It can be proved from the results that by introducing the surface stress impacts, the deflections corresponding to all values of aspect ratio decrease which shows the fact that with consideration of the surface stress effects, the stiffness of nanowire will be increased.

Fig. 2 shows the influence of value of \( \tau' \) on the transverse deflection of nanowires. The value of
(2μ'+λ') is taken zero, and the variation of the transverse deflection with the span-to-depth ratio (L/h) of nanowire is shown corresponding to various magnitude of τ by using various higher beam theories (i.e., TBT, SBT and HBT). It is seen that the overall bending stiffness of nanowire tends to increase as the value of τ increases.

Fig. 3 presents the variation of the transverse deflection of nanowires versus the span-to-depth ratio (L/h) of nanowire for three different conditions. It is seen that by taking τ zero, the positive value of 2μ'+λ' makes nanowire stiffer. However, the non-positive value of 2μ'+λ' diminishes the stiffness of the nanowire.

Fig. 4 Variation of critical buckling load with the aspect ratio corresponding to different values of magnitudes of τ with the assumption of 2μ'+λ'=0

Table 4 Critical buckling loads corresponding to the first mode obtained with classical and non-classical solutions (nN)

<table>
<thead>
<tr>
<th>L/h</th>
<th>Ref(^{(a)}) classical</th>
<th>non-classical</th>
<th>Present TBT classical</th>
<th>non-classical</th>
<th>Present SBT classical</th>
<th>non-classical</th>
<th>Present HBT classical</th>
<th>non-classical</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.6410</td>
<td>3.9518</td>
<td>0.6410</td>
<td>3.9518</td>
<td>0.6410</td>
<td>3.9518</td>
<td>0.6410</td>
<td>3.9518</td>
</tr>
<tr>
<td>20</td>
<td>0.3623</td>
<td>3.7117</td>
<td>0.3623</td>
<td>3.7117</td>
<td>0.3623</td>
<td>3.7117</td>
<td>0.3623</td>
<td>3.7117</td>
</tr>
<tr>
<td>25</td>
<td>0.2324</td>
<td>3.5998</td>
<td>0.2324</td>
<td>3.5998</td>
<td>0.2324</td>
<td>3.5998</td>
<td>0.2324</td>
<td>3.5998</td>
</tr>
<tr>
<td>30</td>
<td>0.1616</td>
<td>3.5389</td>
<td>0.1616</td>
<td>3.5389</td>
<td>0.1616</td>
<td>3.5389</td>
<td>0.1616</td>
<td>3.5389</td>
</tr>
<tr>
<td>35</td>
<td>0.1188</td>
<td>3.5021</td>
<td>0.1188</td>
<td>3.5021</td>
<td>0.1188</td>
<td>3.5021</td>
<td>0.1188</td>
<td>3.5021</td>
</tr>
<tr>
<td>40</td>
<td>0.0910</td>
<td>3.4782</td>
<td>0.0910</td>
<td>3.4782</td>
<td>0.0910</td>
<td>3.4782</td>
<td>0.0910</td>
<td>3.4782</td>
</tr>
<tr>
<td>45</td>
<td>0.0719</td>
<td>3.4618</td>
<td>0.0719</td>
<td>3.4618</td>
<td>0.0719</td>
<td>3.4618</td>
<td>0.0719</td>
<td>3.4618</td>
</tr>
<tr>
<td>50</td>
<td>0.0583</td>
<td>3.4501</td>
<td>0.0583</td>
<td>3.4501</td>
<td>0.0583</td>
<td>3.4501</td>
<td>0.0583</td>
<td>3.4501</td>
</tr>
</tbody>
</table>

\(^{(a)}\)Taken from Ref (Ansari and Sahmani 2011)
In order to demonstrate the validity of the present formulation in the case of buckling analysis of nanowires, comparative studies are presented in Tables 4-6. The obtained results based on TBT, SBT and HBT are compared with those of Ansari and Sahmani (2011). Excellent agreement can be observed for different values of span-to-depth ratio (L/h). It can be seen that the results from the classical theories due to ignoring the surface stress effect are highly underestimate when comparing with those from the non-classical theories. This effect is more pronounced for the lower mode numbers. The obtained results confirm again that this effect is important and makes nanowire stiffer.

The effect of the values of τ' on the variation of the critical buckling load of nanowires is shown in Fig. 4. It can be observed that the increase of the value of τ' induces an increase in the overall bending stiffness of nanowire.

Table 5 Critical buckling loads corresponding to the second mode obtained with classical and non-classical solutions (nN)

<table>
<thead>
<tr>
<th>L/h</th>
<th>Ref(1)</th>
<th>Present TBT</th>
<th>Present SBT</th>
<th>Present HBT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>classical</td>
<td>non-classical</td>
<td>classical</td>
<td>non-classical</td>
</tr>
<tr>
<td>10</td>
<td>5.3019</td>
<td>8.0214</td>
<td>5.3019</td>
<td>8.0204</td>
</tr>
<tr>
<td>15</td>
<td>2.4819</td>
<td>5.5471</td>
<td>2.4819</td>
<td>5.5469</td>
</tr>
<tr>
<td>25</td>
<td>0.9185</td>
<td>4.1914</td>
<td>0.9185</td>
<td>4.1913</td>
</tr>
<tr>
<td>30</td>
<td>0.6410</td>
<td>3.9518</td>
<td>0.6410</td>
<td>3.9518</td>
</tr>
<tr>
<td>35</td>
<td>0.4723</td>
<td>3.8064</td>
<td>0.4723</td>
<td>3.8064</td>
</tr>
<tr>
<td>40</td>
<td>0.3623</td>
<td>3.7117</td>
<td>0.3623</td>
<td>3.7117</td>
</tr>
<tr>
<td>45</td>
<td>0.2866</td>
<td>3.6465</td>
<td>0.2866</td>
<td>3.6465</td>
</tr>
<tr>
<td>50</td>
<td>0.2324</td>
<td>3.5998</td>
<td>0.2324</td>
<td>3.5998</td>
</tr>
</tbody>
</table>

(1) Taken from Ref (Ansari and Sahmani, 2011)

Table 6 Critical buckling loads corresponding to the third mode obtained with classical and non-classical solutions (nN)

<table>
<thead>
<tr>
<th>L/h</th>
<th>R. Ansari et Al.</th>
<th>Present RBT</th>
<th>Present SBT</th>
<th>Present HBT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>classical</td>
<td>non-classical</td>
<td>classical</td>
<td>non-classical</td>
</tr>
<tr>
<td>15</td>
<td>5.3019</td>
<td>8.0214</td>
<td>5.3019</td>
<td>8.0204</td>
</tr>
<tr>
<td>25</td>
<td>2.0267</td>
<td>5.1512</td>
<td>2.0267</td>
<td>5.1511</td>
</tr>
<tr>
<td>35</td>
<td>1.0520</td>
<td>4.3066</td>
<td>1.0520</td>
<td>4.3066</td>
</tr>
<tr>
<td>40</td>
<td>0.8089</td>
<td>4.0967</td>
<td>0.8089</td>
<td>4.0966</td>
</tr>
<tr>
<td>45</td>
<td>0.6410</td>
<td>3.9518</td>
<td>0.6410</td>
<td>3.9518</td>
</tr>
<tr>
<td>50</td>
<td>0.5203</td>
<td>3.8478</td>
<td>0.5203</td>
<td>3.8478</td>
</tr>
</tbody>
</table>

(1) Taken from Ref (Ansari and Sahmani 2011)
The variation of the critical buckling load of nanowires as function of the span-to-depth ratio ($L/h$) of nanowire for three different conditions, is illustrated in Fig. 5. It is seen that by taking $\tau'$ zero, the non-positive value of $2\mu'+\lambda'$ makes the nanowire softer, while, for the positive value of $2\mu'+\lambda'$ the nanowire becomes stiffer.

5. Conclusions

In this work, we have presented a framework of high-order surface stresses, based on various higher-order shear deformation beam theories, to investigate the bending and buckling response of NWs. Our results showed that the bending and buckling behaviors of nanowires are significantly affected by the surface stress impacts. Indeed, it is demonstrated that the inclusion of surface stress effect makes a nanowire stiffer, and hence, leads to a reduction of deflection and an increase of buckling load. The formulation lends itself particularly well to functionally graded structures (Bouderba et al. 2013, Ait Amar Meziane et al. 2014, Belabel et al. 2014, Hebali et al. 2014, Bousahla et al. 2014, Ait Yahia et al. 2015, Belkorissat et al. 2015, Hamidi et al. 2015, Larbi Chaht et al. 2015) and nanotubes (Besseghier et al. 2015, Tounsi et al. 2013), which will be considered in the near future.

Acknowledgments

This research was supported by the Algerian National Thematic Agency of Research in Science and Technology (ATRST) and university of Sidi Bel Abbes (UDL SBA) in Algeria.
References


unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates”, Steel Compos. Struct., 18(1), 235-253.


Wang, G.F. and Feng, X.Q. (2009), “Surface effects on buckling of nanowires under uniaxial compression”,

CC