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# A model for damage analysis of concrete

Vui V. Cao[*1a] and Hamid R. Ronagh[1b]

*School of Civil Engineering, The University of Queensland, St. Lucia, Australia*

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**Abstract.** The damage level in structures (global scale), elements (intermediate scale) and sections (local scale) can be evaluated using a single parameter called the "Damage Index". Part of the damage attributed to the local scale relates to the damage sustained by the materials of which the section is made. This study investigates the damage of concrete subjected to monotonic compressive loading using four different damage models – one proposed here for the first time and three other well-known models. The analytical results show that the proposed model is promising yet simple and effective for evaluating the damage of concrete. The proposed damage model of concrete with its promising characteristics indicated, appears to be a useful tool in the damage assessment of structures made of concrete.

**Keywords:** damage model; concrete; damage assessment

## 1. Introduction

It is common to evaluate the damage level in structures (global scale), elements (intermediate scale) and sections (local scale) (Amziane and Dubé 2008) with a single parameter called the "Damage Index" (DI). In the local scale, the damage can be addressed based on the damage of materials of which sections are made (Amziane and Dubé 2008, Paredes *et al.* 2011). Concrete is one of the most common materials used in construction due to its many useful characteristics such as durability, forming convenience, etc. However, high compressive strength of concrete is probably its most striking feature which allows the construction of buildings, bridges, even today's high-rises with ease when combined with steel as tensile reinforcing or prestressing element. In some structural members where concrete is the major player such as columns subjected to large axial loads, or beams-columns with high compressive loads and low bending moments, the load capacity mainly depends on the concrete. Hence, the damage mechanism of these members relates mostly to the concrete. Damage of concrete has been studied over the past years by many researchers such as Cao and Chung (2001), Puri and Weiss (2006) and recently Malecot *et al.* (2010), Markovich *et al.* (2011) and Poinard *et al.* (2010).

Available damage models for concrete in the literature include those of Yu *et al*. (2010), Soh and Bhalla (2005), Chen *et al*. (2011), Amziane and Dubé (2008), as reviewed in Section 3.1. The parameters employed in these damage models are stress, stiffness and modulus. Instead of using

*Corresponding author, Ph.D. Student, E-mail: v.caovan@uq.edu.au


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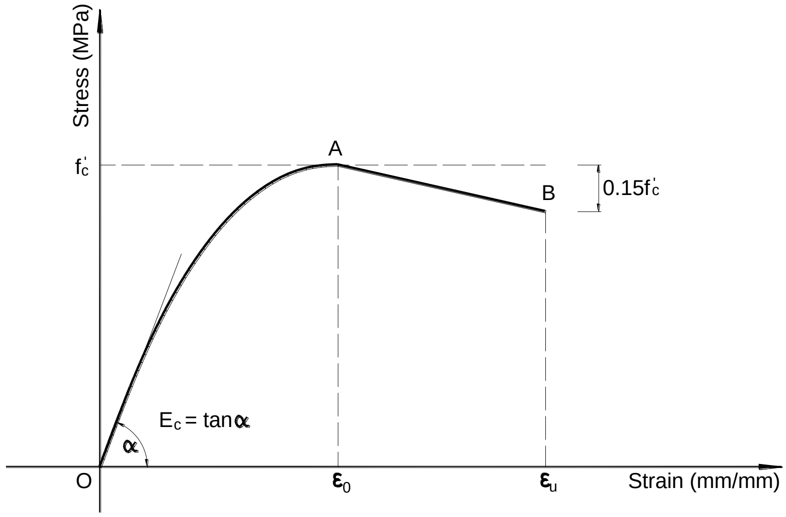



Fig. 1 Hognestad (1951) model

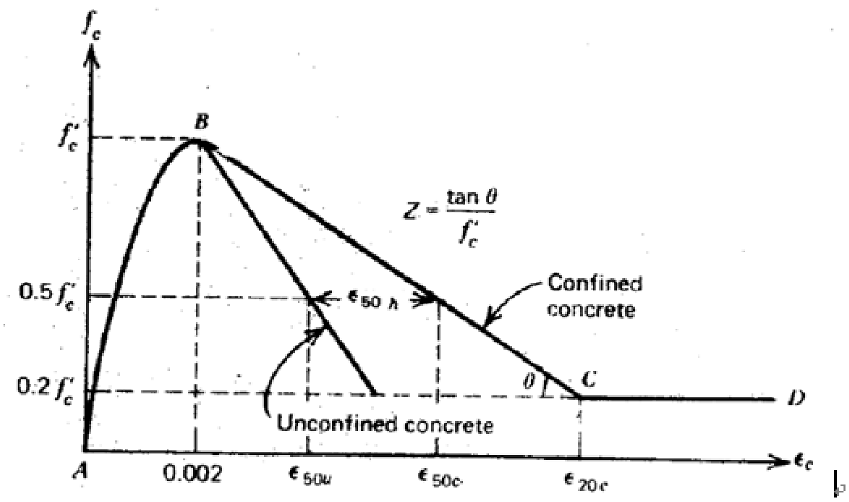



Fig. 2 Kent and Park (1971) model for concrete confined by rectangular hoops

these parameters, in this paper, the single parameter of "energy" is used. The proposed model is then used alongside other available models to evaluate the damage of concrete elements based on the compressive stress testing results on concrete specimens. It is shown that the proposed damage model can be utilised as a useful tool for the damage assessment of structures made from concrete.

## 2. Behaviour of concrete

For concrete, the stress–strain curve before maximum stress is widely approximated as a second–degree parabola (Park and Paulay 1975). Hognestad (1951) model as shown in Fig. 1 is a widely used parabola described by Eqs. (1) and (2), where, $\varepsilon_c$ is the strain; $\varepsilon_o$ is the strain at maximum stress; $f_c^{'}$ is the maximum stress reached in concrete which may differ from the cylinder strength and $E_c$ is the modulus of elasticity.

Region OA

$$f_c = f_c^{'}\left[\frac{2\varepsilon_c}{\varepsilon_o} - \left(\frac{2\varepsilon_c}{\varepsilon_o}\right)^2\right] \tag{1}$$

$$\varepsilon_o = \frac{2f_c^{'}}{E_c} \tag{2}$$

Region AB: After the maximum stress, the relation between stress and strain is linear. The stress reduces 15% comparing to $f_c^{'}$ when the strain reaches its ultimate value of 0.0038.

The transverse reinforcement may confine the concrete depending on the levels of stress and the spacing of steel spirals or hoops. The strength of concrete increases significantly when confined. However, confinement by transverse reinforcement has little effect on the stress–strain curve until the uni-axial strength of concrete is reached (Park and Paulay 1975). Richart *et al*. (1929) seems to be the pioneer studying the compressive behaviour of concrete with the effect of transverse reinforcement. Based on their tests using lateral fluid pressure, which was thought to be the same as the confining effect of transverse reinforcement, they proposed a relationship for the compression concrete strength ( $f_c^{''}$ ) with transverse pressure, the concrete strength ( $f_c^{'}$ ) without transverse pressure and the transverse pressure ( $f_l$ ) as shown in Eq. (3).

$$f_c^{''} = f_c^{'} + 4.1 f_l \tag{3}$$

For concrete confined by circular spirals, the lateral pressure shown in Eq. (4) can be obtained from equilibrium of the forces acting on the half turn of spiral (Park and Paulay 1975). This pressure depends on the diameter of the spiral ($d_s$), the area of spiral bar ($A_{sp}$), the pitch of spiral ($s$) and the yield strength of transverse reinforcement ( $f_{yh}$ ).

$$f_l = \frac{2 f_{yh} A_{sp}}{d_s s} \tag{4}$$

Concrete confined by rectangular hoops has been extensively studied by researchers (Baker and Amarakone, 1964; Blume *et al*. 1961, Chan 1955, Roy and Sozen 1964, Sargin *et al*. 1971, Soliman and Yu 1967). These stress–strain models have their own features which had been combined in the model proposed by Kent and Park (1971) as shown in Fig. 2. The stress–strain relationship up to maximum stress is the same as that of Hognestad (1951) model, however, the strain at the maximum stress is 0.002. This is also the same for unconfined and confined concrete. The difference between those types of concrete in their model is the falling branch after the maximum stress. However, Kent and Park (1971) model is conservative in most cases because it does not take into account the increase in maximum stress of confined concrete (Park and Paulay 1975).

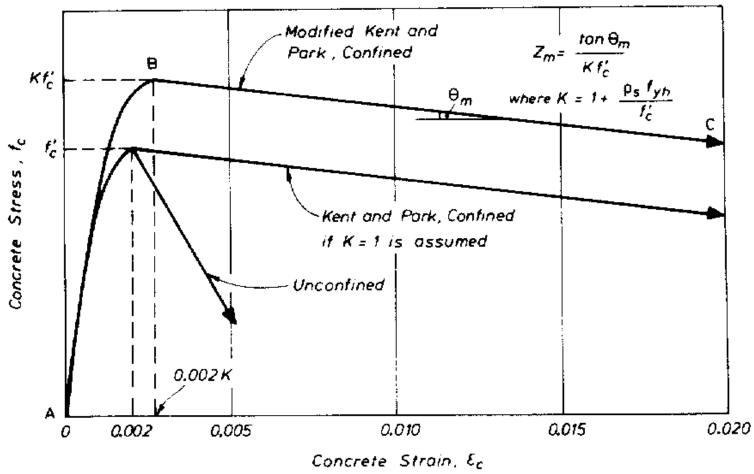



Fig. 3 Modified Kent and Park (1971) model done by Park *et al.* (1982)

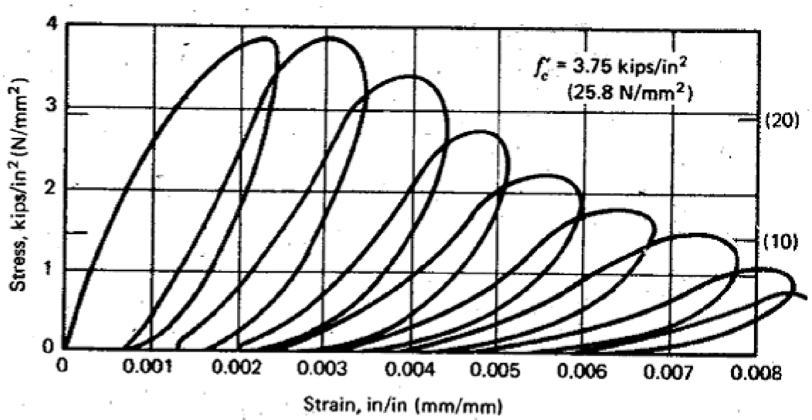



Fig. 4 Stress-strain curves for concrete cylinder with high-intensity repeated compressive loading (Sinha *et al.*, 1964)

In the same year, in recognition of the problems in the Kent and Park (1971) model, Park *et al.* (1982) modified the model by taking into account the enhancement of the concrete strength due to confinement. Fig. 3 shows the modified Kent and Park (1971) model in which the maximum stress $f_c^{'}$ and the corresponding strain of 0.002 in Kent and Park (1971) model are multiplied by the factor $K$ as shown in Eq. (5) to Eq. (10).

Region AB: $\varepsilon_c \le \varepsilon_o$

$$f_c = f_c^{"}\left[\frac{2\varepsilon_c}{\varepsilon_o} - \left(\frac{2\varepsilon_c}{\varepsilon_o}\right)^2\right] \tag{5}$$

Region BC: $\varepsilon_c \ge \varepsilon_o$

$$f_c = f_c^{"}\left[1 - Z\left(\varepsilon_c - \varepsilon_o\right)\right] \geq 0.2 f_c^{"} \tag{6}$$

In which

$$f_c^{"} = K f_c^{'} \tag{7}$$

$$\varepsilon_o = 0.002K \tag{8}$$

$$Z = \frac{0.5}{\dfrac{3 + 0.29 f_c^{'}}{145 f_c^{'} - 1000} + \dfrac{3}{4}\rho_s\sqrt{\dfrac{b^{"}}{s_h}} - 0.002K} \tag{9}$$

$$K = 1 + \frac{\rho_s f_{yh}}{f_c^{'}} \tag{10}$$

Where $\rho_s$ is the ratio of the volume of rectangular steel hoops to the volume of concrete core measured to the outside of the peripheral hoop; $f_c^{'}$ is in MPa; *b"* is the width of the concrete core measured to outside of the peripheral hoop; $s_h$ is the center-to-center spacing of hoop sets.

This modified Kent and Park (1971) model shows a good agreement with the test results of compressed concrete confined by hoop reinforcement presented by Scott *et al.* (1982). The issue were later studied by many researchers (Cusson and Paultre, 1994a, 1994b, Mander *et al.* 1988, Sheikh and Uzumeri 1982). Among those, Mander *et al.* (1988) model takes into account various types of transverse reinforcement. In addition, their model can be applied not only to monotonic loads but also to cyclic loads. However, the model has its own limitations. It is valid only within a certain range of confinement steel and the model does not include the descending portion of the confined concrete stress-strain curve (Esmaeily-Gh. and Xiao 2002).

When concrete is subjected to repeated loads, the stress–strain relation is affected by hysteresis behaviour (Park and Paulay 1975) and becomes much more complicated. Many studies of this

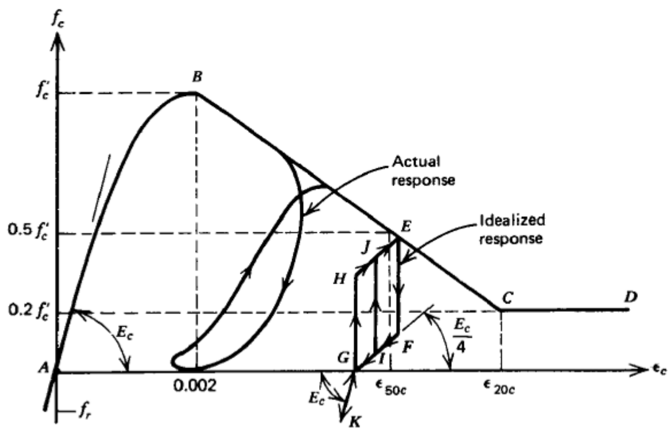



Fig. 5 Park *et al.* (1972) model

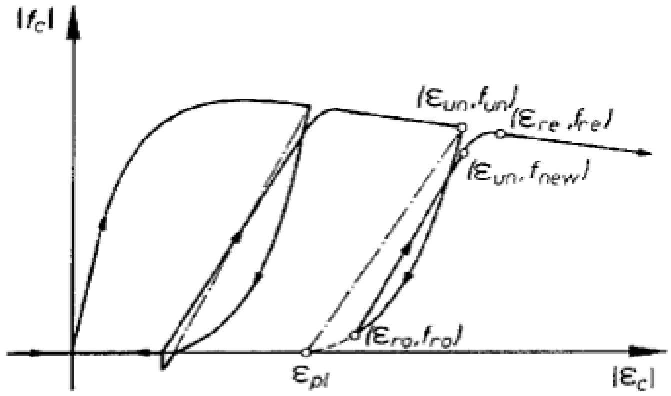



Fig. 6 Stress-strain curves for unloading and reloading branch in Mander *et al.* (1988) model

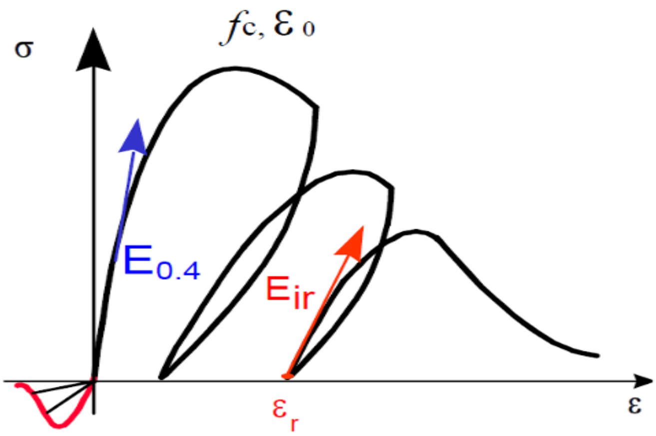



Fig. 7 Parameters for Eq. (14) (Amziane and Dubé 2008)

issue have been performed in the past. Fig. 4 shows the behaviour of concrete cylinder with high- intensity repeated axial compressive load given by Sinha *et al.* (1964) obtained from their test data.

Karsan and Jirsa (1969) also performed some tests on this behaviour. It should be noted that the results of the two above mentioned groups showed that the envelope curve was the same as the curve obtained from monotonic loading. Up to now, many models for the behaviour of concrete subjected to repeated loads have been developed based on the back bone curve obtained from monotonic loading. Park *et al.* (1972) model is shown in Fig. 5 in which the envelope curve follows the Kent and Park (1971) model for concrete confined by hoops under monotonic compression. The stress–strain curve for unloading of this model is described as bi-linear curve.

Mander *et al.* (1988) adopted Takiguchi *et al.* (1976) approach and had it modified to be suitable for both unconfined and confined concrete. The pattern of unloading curves are defined the same as the monotonic curve before the maximum stress but the unloading modulus has changed by two coefficients which relate to the stress ($f_{un}$) and strain ($\varepsilon_{un}$) of the unloading point on the back bone curve as shown in Fig. 6.

## 3. Damage models

### 3.1 Available damage models

The extent of damage occurring in concrete when subjected to loading primarily depends on two factors - the concrete itself and the applied external loading. It is now widely accepted that the magnitude of DI should, ideally, vary between 0 and 1. A concrete should not suffer any damage when it operates within its elastic limit and hence DI should be equal to 0 at this stage. On the other hand, the maximum possible magnitude for DI should be set equal to 1 referring to the event of total collapse.

There are available damage indices in literature. The parameters used for damage model commonly, are stress, deformation, stiffness and modulus. The damage of concrete subjected to uni-axial compression can be described by Eq. (11) in terms of the ratio of decaying stress to peak stress Yu *et al*. (2010), in which, $\sigma_c$ is the stress of concrete on the descending branch and $f_c^{'}$ is the peak stress.

$$DI = 1 - \frac{\sigma_c}{f_c^{'}} = \frac{f_c^{'} - \sigma_c}{f_c^{'}} \tag{11}$$

With the same pattern, Soh and Bhalla (2005) defined the damage of concrete in term of stiffness instead of stress as shown in Eq. (12), where $k_o$ is the initial stiffness of concrete and $k_d$ is the stiffness of concrete after damage.

$$DI = 1 - \frac{k_d}{k_o} = \frac{k_o - k_d}{k_o} \tag{12}$$

Chen *et al.* (2011) defined the damage of concrete as the ratio of losing modulus to the initial modulus as shown in Eq. (13), where, $E_0$ is the initial modulus of concrete and $E_d$ is the damaged modulus of concrete. Although Eqs. (12) and (13) look different, they are similar because the stiffness is the product of modulus and the cross sectional area.

$$DI = 1 - \frac{E_d}{E_o} = \frac{E_o - E_d}{E_o} \tag{13}$$

In a different way, Amziane and Dubé (2008) defined the damage of concrete using the modulus of concrete as shown in Eq. (14), in which, $E_{ir}$ is the initial reloading modulus of concrete and $E_{0,4}$ is the Young modulus define at stress of $0.4 f_c^{'}$ as shown in Fig. 7.

$$DI = 1 - \frac{E_{ir}}{E_{0,4}} \tag{14}$$

### 3.2 The proposed damage model

Fig. 8 shows the concept for the proposed damage model based on residual deformation or non-recoverable energy. Concrete experiences a total deformation $u_m$ when it is subjected to a load, a portion of which may be recovered (recoverable deformation - $u_{rec}$), whilst the rest may remain within the structure (residual deformation - $u_{res}$) when the applied forces are released. The overall behaviour of concrete may be sub-divided into two ranges: (1) Elastic range - there is no residual deformation when the load is released and hence DI = 0 and (2) Plastic range - there will be some residual deformation left within the structure when the applied load is withdrawn and in this case, DI should produce a positive magnitude between 0 and 1.

More generally, the overall behaviour of concrete is described in Fig. 8d in terms of energy. The concrete (at point A) receives a total energy ($E_{total} = E_{non\text{-}rec} + E_{rec}$) when it is subjected to the load. When the load is released (the concrete at point B), a portion of the total energy may be recovered ($E_{rec}$) and the rest is absorbed by the concrete ($E_{non\text{-}rec}$).

In simple terms, initially, DI may be defined as the ratio of the recoverable energy ($E_{rec}$) to the total energy ($E_{total}$) as shown in Eq. (15).

$$DI = \frac{E_{non-rec}}{E_{total}} = \frac{E_{non-rec}}{E_{non-rec} + E_{rec}} \tag{15}$$

Where, $E_{rec}$ is the recoverable energy.

It seems reasonable to assume that concrete suffers no damage when it is loaded up to $f_c^{'}/2$.

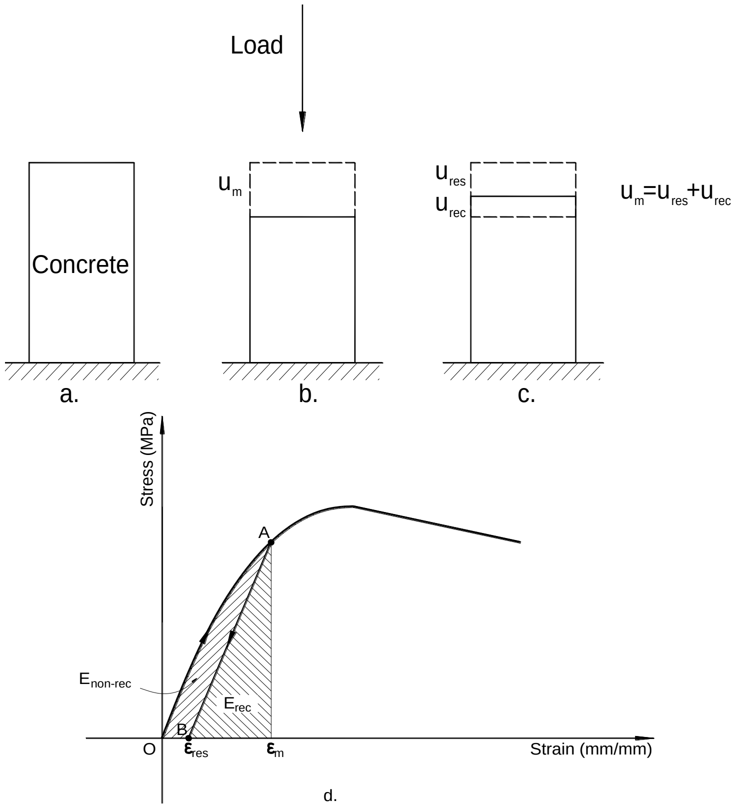



Fig. 8 Concept of the proposed DI

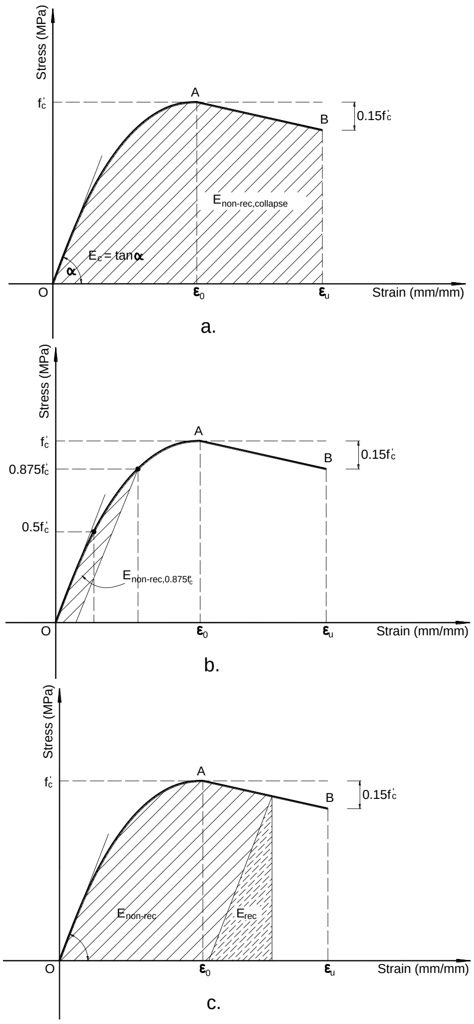



Fig. 9 Parameters for Eq. (14) (Amziane and Dubé 2008)

The reasons for this is that concrete's stress–strain curve (up to $f_c^{'}/2$) is almost linear (Park and Paulay 1975). As a result, the residual strain is 0 when unloading. Hence, it suffers no damage.

From $f_c^{'}/2$ up to $f_c^{'}$, however, the curve loses its linearity and therefore some residual strains are left after unloading. These strains are synonymous with micro cracks and therefore represent some damage. The higher is the residual strain, the higher would be the damage. The deviation from linearity becomes worse as the stress moves higher from $f_c^{'}/2$ up to $f_c^{'}$. It may speed up at around $0.75f_c^{'}$ which is mid-point of the two above.

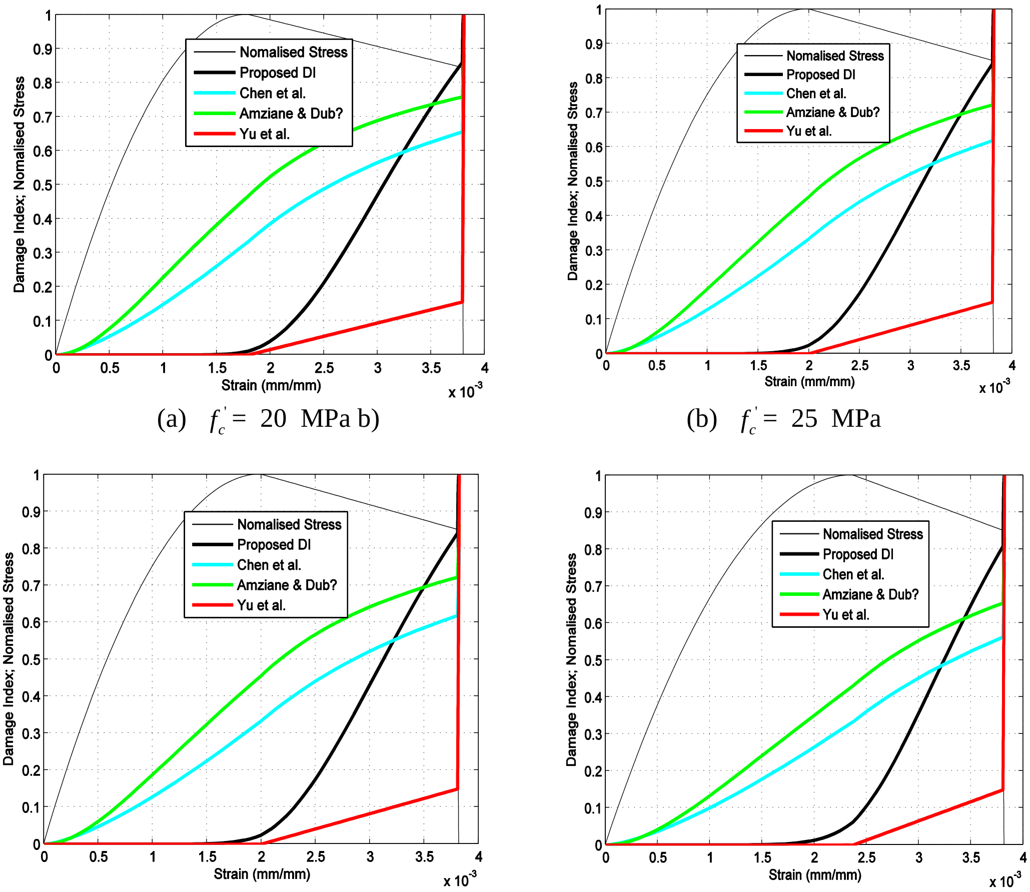
For the above reasons, the "threshold value" is proposed for the calculation of the non-recoverable energy called $E_{non\text{-}rec,0.875f'c}$ which is non-recoverable at ¾ of the second half (from $f_c^{'}/2$ up to $f_c^{'}$), as shown in Fig. 9(b). Fig. 9(a) shows the parameter non-recoverable energy at collapse ($E_{non\text{-}rec,collapse}$), while Fig. 9c represents the non-recoverable energy ($E_{non\text{-}rec}$) and recoverable energy ($E_{rec}$) at a certain loading.

It is widely known that the damage of concrete at peak stress is minimal. The damage index should increase significantly when the strain goes beyond the strain at maximum stress. Hence, the damage index for this ultimate state should be close to 1. It will reach 1 when the stress drops from $0.85 f_c^{'}$ to 0.

Eq. (15) is modified as Eq. (16) to sastify the above conditions.

$$DI = \left[ \frac{E_{non-rec}}{E_{non-rec} + E_{rec}} \right]^{(N-i)} \tag{16}$$

$$N = \frac{E_{non-rec,collapse}}{E_{non-rec,0.875 f_c^{'}}} \tag{17}$$



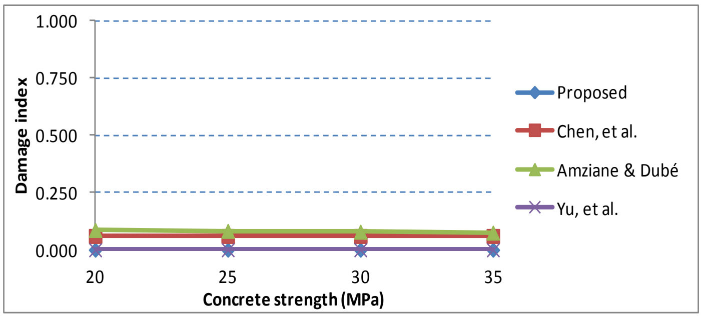

(a) $f_c^{'} =$ 20 MPa b)

(b) $f_c^{'} =$ 25 MPa

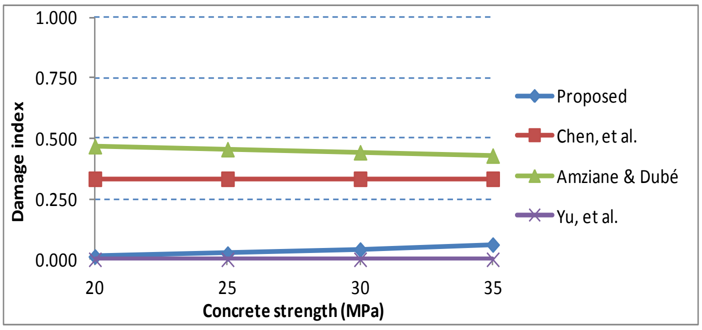
(c) $f_c^{'} =$ 30 MPa

(d) $f_c^{'} =$ 35 MPa

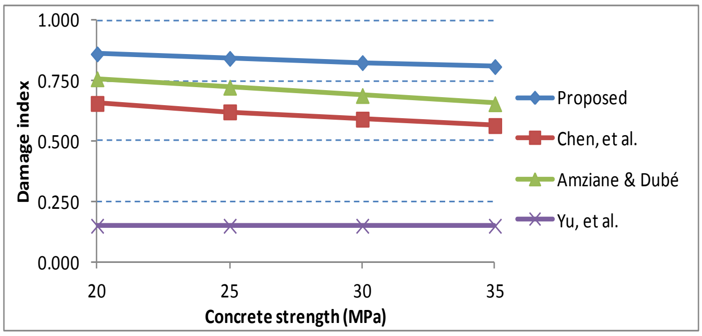
Fig. 10 Damage analysis of concrete



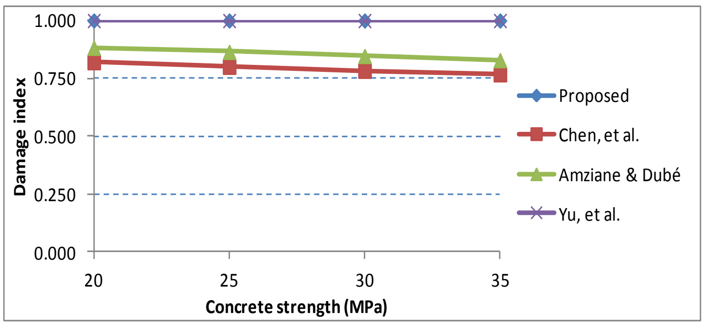

(a) at $f_c^{'}/2$




(b) at f $f_c^{'}$




(c) at $(1-0.15)f_c^{'}$




(d) at total collapse

Fig. 11 Damage of concrete at specific stresses

$$i = \frac{E_{non-rec}}{E_{non-rec,0.875f_c^{'}}} \tag{18}$$

where, $E_{non-rec,0.875f'c}$ and $E_{non-rec,collapse}$ are the non-recoverable energy at $0.875f_c^{'}$ and at the total collapse, respectively. $E_{non-rec}$ is the non-recoverable energy. Those parameters are shown in Fig. 9.

## 4. Validation of the proposed model

As shown in Fig. 4, the hysteretic stress–strain curve of concrete is complicated. The effect of unloading and reloading on the damage of concrete should be considered in case of concrete subjected to repeated loading because the energy absorbed during cyclic loading affects the damage of concrete. In this paper, only monotonic loading is considered. The total energy at any point on the stress-strain curve can be divided into 2 parts: non-recoverable energy and recoverable energy.

As mentioned above, the parabolic equations of the Hognestad (1951) and modified Kent and Park (1971) models are the same. The strain at maximum stress in Hognestad (1951) shown in Eq. (2) varies in a similar manner to that of Eq. (7). In addition, if the maximum stress $f_c^{'}$ is taken as the strength of confined concrete, those two models seem to have little difference. Furthermore, the target of this paper is to evaluate the damage of concrete, hence, the Hognestad (1951) with its simplicity will be used in this paper. The stress after the strain of 0.0038 in Hognestad (1951) is assumed to drop to 0 with a small slope to avoid difficulty in calculation without affecting the results.

The proposed damage model, together with the models proposed by Yu *et al.* (2010), Chen *et al.* (2011) and Amziane and Dubé (2008), is applied to assess the damage of different concrete with strengths varying from 20 MPa to 35 MPa. The modulus of elasticity of concrete is taken as $E_c = 5000\sqrt{f_c^{'}}$ MPa, which is also used as $E_{0.4}$ in Amziane and Dubé (2008) model because the concrete stress-strain curve is almost linear as stated by Park and Paulay (1975). The ultimate concrete strain of 0.0038 given in Hognestad (1951) is used in this paper. The line connecting the unloading point ($\varepsilon_{un}$, $f_{un}$) to the plastic strain at zero stress ($\varepsilon_{pl}$,0) in Mander *et al.* (1988) model is employed to calculate parameters for Yu *et al.* (2010), Chen *et al.* (2011) and Amziane and Dubé (2008) models. Figs. 10(a)-10(d) show the damage analyses of concrete with the strength of 20, 25, 30, 35 MPa, respectively. Overall, the damage indices produced by Chen *et al.* (2011) and Amziane and Dubé (2008) seem to be large and Yu *et al.* (2010) model seems too conservative.

Fig. 11(a) shows the damage of concrete at $f_c^{'}/2$, at which the concrete suffers no damage. The proposed and Yu *et al.* (2010) models show a good demonstration with damage indices of zero while Chen *et al.* (2011) and Amziane and Dubé (2008) models demonstrate the damage indices of 0.06 and 0.08, respectively. Fig. 11(b) represents the damage of concrete at its maximum stress. Yu *et al* (2010) model generates damage index of zero and the proposed model gives a damage index varying from 0.01 to 0.06 while the other two models produce very large damage indices (0.33 for Chen *et al.* (2011) model and around 0.45 for the Amziane and Dubé (2008) model). Fig. 11(c) represents the damage indices at the stress dropped by 15% which is also the damage index that Yu *et al.* (2010) model gives. This may be conservative. Chen *et al.* (2011) and Amziane and Dubé (2008) model produce the damage varying from 0.55 to 0.75 while the proposed model gives the index from 0.81 to 0.86. Fig. 11(d) shows the damage index at total collapse. The proposed and Yu *et al* (2010) damage models generate the damage indices of 1 while the two others produce indices of around 0.8.

## 5. Conclusions

Available damage models for concrete, in which the parameters stress, stiffness and modulus are used, have been reviewed. A new single parameter model is then proposed based on energy. The new model alongside other available models are then used to evaluate the level of damage in tested concrete specimens. The proposed model show little to no damage up to $f_c^{'}/2$ as expected simply because the behaviour to that point is almost linear and elastic. The damage index within the range $f_c^{'}/2$ up to $f_c^{'}$ increases to 6% as gradually the concrete stress-strain curve deviates from linearity. This is unlike some other models that present either no damage or a very large damage for the same range. The correct capturing of the behaviour acknowledges the proposed model as a potentially useful tool in the damage assessment of concrete.

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