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Numerical Bayesian updating of prior distributions for concrete strength properties considering conformity control

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Abstract. Prior concrete strength distributions can be updated by using direct information from test results as well as by taking into account indirect information due to conformity control. Due to the filtering effect of conformity control, the distribution of the material property in the accepted inspected lots will have lower fraction defectives in comparison to the distribution of the entire production (before or without inspection). A methodology is presented to quantify this influence in a Bayesian framework based on prior knowledge with respect to the hyperparameters of concrete strength distributions. An algorithm is presented in order to update prior distributions through numerical integration, taking into account the operating characteristic of the applied conformity criteria, calculated based on Monte Carlo simulations. Different examples are given to derive suitable hyperparameters for incoming strength distributions of concrete offered for conformity assessment, using updated available prior information, maximum-likelihood estimators or a bootstrap procedure. Furthermore, the updating procedure based on direct as well as indirect information obtained by conformity assessment is illustrated and used to quantify the filtering effect of conformity criteria on concrete strength distributions in case of a specific set of conformity criteria.

Keywords: Bayesian updating; concrete strength; conformity control; EN 206-1; operating characteristic; prior information

1. Introduction

Bayesian statistics can be used in order to update prior distributions of material properties taking into account additional information. Consequently, these updated distributions can be taken into account when performing structural analysis, especially in case of structural reliability calculations (see e.g. Strauss *et al*. 2008, Moser *et al*. 2011, Orton *et al*. 2012). As indicated in (Der Kiureghian 2008), the effect of parameter uncertainty on structural reliability calculations can be considerable. In case of conformity control, a first analysis with respect to the influence on structural reliability calculations was performed by the authors in (Caspeele *et al*. 2010). From these investigations it was found that conformity control may positively influence particularly reliability of lightly reinforced concrete members exposed to compression or shear. It appears that

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it has a minor effect on members with a greater reinforcement ratio or members exposed to bending. Furthermore it enables to ensure a more homogeneous safety level, which is less dependent on parameter uncertainties. Hence, prior distributions should be properly updated in case additional information is available. Not only direct test results can be used to update these distributions, but also indirect information from e.g. conformity control can be considered when updating the prior knowledge into a posterior belief. Suitable prior information with respect to mechanical properties of currently applied concrete, reinforcing steel and prestressing steel can be found in e.g (Caspeele and Taerwe 2011b, Jacinto *et al*. 2012, Wisniewsky *et al*. 2012).

The most commonly specified property of concrete is compressive strength and most control plans have been derived for this property. Most often, conformity control is used in order to investigate whether a certain inspected lot complies with a predefined (or specified) characteristic X_k of a material property (i.e., f_{ck} in case of concrete compressive strength), most often using a decision rule *d(z)* of the following type (Rackwitz 1979)

$$d(z) = \begin{cases} z > a & \text{acceptance} \\ z \leq a & \text{rejectance} \end{cases} \tag{1}$$

with z a test statistic, e.g. the sample mean $\bar{x}_n$ of n test results, and 'a' an acceptance boundary limit, e.g. in case of concrete strength $a = f_{ck} + \lambda s_n$ with f_{ck} the specific characteristic concrete compressive strength (corresponding to a certain concrete strength class), a parameter (in general depending on the chosen fractile and confidence level or other equivalent design parameter) and s_n the sample standard deviation based on n test results.

Due to the use of such conformity control to verify the specified properties, certain inspected lots are accepted and certain lots are rejected. Because of this so-called filtering effect, the original (incoming) distributions of the entire population of the material property can be updated into an outgoing distribution of the accepted inspected lots. In case of concrete strength for example, the filtering effect from conformity control leads to an increase of the mean and a decrease of the standard deviation of the outgoing predictive strength distribution in comparison to the incoming one, presented for conformity assessment (Rackwitz 1979, 1983, Taerwe 1985, Caspeele 2010). Although these favourable consequences do not form the main objective when designing conformity criteria, they reveal a significant influence on the posterior predictive distribution of material properties and thus should be taken into consideration when updating these distributions and using them in further structural calculations e.g. in structural reliability analyses.

Rackwitz (1979, 1983) describes an analytical method in order to evaluate the filter effect of some common (basic) conformity criteria, based on Bayesian statistics. However, in practice also more complex criteria are used, for example in the European Standard EN 206-1 (CEN 2000) with respect to concrete properties. In order to evaluate and compare the filtering effect of such more complex conformity criteria, a numerical algorithm is developed, based on Bayesian statistics and numerical integration, taking into account prior information. This algorithm uses the operating characteristic of the considered conformity criteria, calculated using numerical Monte Carlo simulations, which also enables to take autocorrelation between consecutive test results into account. The methodology is illustrated in case of a specific set of conformity criteria (i.e., those described in (CEN 2000)).

2. Conformity control of material properties and its filtering effect

2.1 Conformity criteria and their operating characteristics

For structural design and material production purposes a material property is characterized by a specified value X_k, which is commonly the 5%-fractile of the theoretical distribution of the material property under consideration. In case of concrete for example, the concrete strength is commonly represented by the 5% fractile of the theoretical concrete strength distribution, i.e., the specified characteristic concrete compressive strength f_{ck} (Fig. 1). In practice, the fraction below the specified value will be smaller or higher than 5%. Designating by the fraction of the population below f_{ck} in the offered strength distribution – also called the fraction defectives – it follows that

$$P\left[X \leq f_{ck}\right] = \theta \tag{2}$$

which is illustrated in Fig. 1.

For an assumed distribution function of the material property under consideration and for a given conformity criterion, one can calculate the probability that an inspected lot, characterized by a fraction defectives, is accepted. This probability is called the probability of acceptance, denoted as P_a. The function $P_a(\theta)$ is called the operating characteristic of the criterion (commonly abbreviated as OC-curve) and describes the discriminating capacity of the conformity criteria. Analytical formulas are available in order to calculate these OC-curves in case of some simple conformity criteria (Rackwitz 1979, 1983, Taerwe 1985, 1988, Taerwe and Caspeele 2006, 2011a, Caspeele 2010). However, for more complex conformity criteria or in case autocorrelation between consecutive test results is deemed important to take into account, numerical Monte Carlo simulations can be used to calculate the OC-curves, which inherently also takes into account any dependency that exists between a set of conformity criteria. More information about the numerical calculation of OC-lines, with or without taking into account autocorrelation, is available in (Taerwe 1985, 1987a, 1988, Taerwe and Caspeele 2006, 2011a, Caspeele 2010). A detailed algorithm for generating operating characteristics using Monte Carlo simulations is available in (Caspeele 2010).

As mentioned, concrete strength records from concrete plants most often reveal the presence of a significant autocorrelation between consecutive results (Soroka 1972, Rackwitz 1977, Degerman 1981, Taerwe 1985, 1987a, 1987b, 2006). A possible explanation for the existence of the correlation structure may be found in the nature of the concrete production process. A given strength value is to a certain extent dependent on the previous values due to the fact that the basic factors which contribute to the variation of concrete strength, namely cement strength, moisture

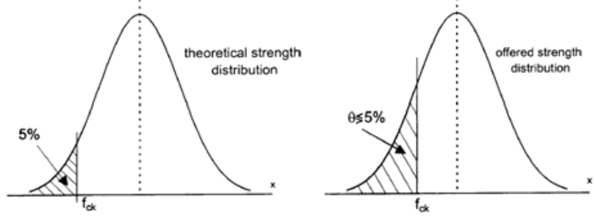


Fig. 1 Theoretical and offered strength distributions in case of concrete compressive strength

content and grading of the sand, etc. maintain a certain value during a more or less long time interval (Taerwe 1985, 1987b, 2006). In case of correlated observations the variance of the sample mean is larger than in the case of independent observations. As a result, the slope of the OC-curve – and the corresponding discriminating capacity of the conformity criteria – decreases (Taerwe 1987a). Hence, there is a significant influence of the correlation structure on an OC-curve. Autocorrelation in concrete strength records can be modelled using an autoregressive process of order 2 – an AR(2) model or Yule series – with parameters as derived in (Taerwe 1985, 1987b, 2006) based on the analysis of extensive concrete strength records of individual concrete mixes. This autoregressive process can easily be implemented into the computational method, namely in the Monte Carlo simulations for the generation of random observations from the strength distributions. More specifically, this is done by implementing Eq. (2) for the calculation of consecutive random realizations of a standard normal distribution.

$$u_i = 0.4\,u_{i-1} + 0.2\,u_{i-2} + \varepsilon_i \tag{3}$$

with u_i the i^{th} autocorrelated standard normally distributed number
ε_i a normally distributed random number with mean 0 and variance σ_ε^2.

2.2 Filtering effect of conformity control

The filter effect of conformity criteria results from the fact that – due to the conformity/non-conformity declaration – inspected lots are accepted or rejected. Because certain inspected lots with deficient quality are rejected from the accepted batches, conformity criteria inherently have a filtering effect on the distribution of the material property under consideration. The average quality of outgoing lots (after acceptance by conformity control) will be higher than the average quality of incoming lots (presented for conformity assessment), i.e., the fraction defectives decreases.

Considering the Bayesian updating principle, the posterior (filtered) joint density function of the parameters of the strength distribution after conformity has been executed is given by

$$f''_{M,\Sigma}(\mu,\sigma) = \frac{P_a(\mu,\sigma)\, f'_{M,\Sigma}(\mu,\sigma)}{\iint P_a(\mu,\sigma)\, f'_{M,\Sigma}(\mu,\sigma)\, d\mu\, d\sigma} \tag{4}$$

with $P_a(\mu,\sigma)$ the acceptance probability of a population with mean and standard deviation associated to the conformity criterion under consideration, $f'_{M,\Sigma}(\mu,\sigma)$ the prior joint density function of the mean and standard deviation of the population and $f''_{M,\Sigma}(\mu,\sigma)$ the posterior joint density function of the mean and standard deviation of the population. Eq. (3) can be evaluated using numerical integration.

For many practical situations a suitable conjugate prior for the parameters of the distribution of the material property is given by a normal-gamma distribution or a lognormal-gamma distribution (Rackwitz 1983, Vrouwenvelder 1997). Parameters that describe these latter joint density functions, are called hyperparameters (i.e., related to parameters describing the distribution of the parameters associated to a density function).

In order to reduce the computation time, it is suggested to calculate the operating characteristic (based on Monte Carlo simulations) separately for some discrete (well-chosen) fraction defectives θ_i and use linear interpolation according to

$$P_a(\mu,\sigma|\ldots) \equiv P_a(\theta) = P_a(\theta_i) + \frac{u_\theta - u_{\theta,i}}{u_{\theta,i+1} - u_{\theta,i}} P_a(\theta_{i+1}) \tag{5}$$

$$\text{with } u_\theta = \frac{X_k - \mu}{\sigma} \text{ and } u_{\theta,i} \le u_\theta \le u_{\theta,i+1} \tag{6}$$

A suitable choice of discrete values for the fraction defectives θ_i are given by the following set of 18 values: 0.1%, 0.2%, 0.3%, 0.5%, 0.7%, 1%, 2%, 3%, 5%, 7%, 10%, 15%, 20%, 25%, 30%, 35%, 40% and 50%, which adequately describe the OC-curve in a θ-P_a diagram with transformed axes according to a normal distribution.

The posterior predictive distribution of the material property (corresponding to the outgoing inspected lots) can be calculated as

$$f_o(x) = \iint f_X(x|\mu,\sigma) f''_{M,\Sigma}(\mu,\sigma) d\mu\, d\sigma \tag{7}$$

and the mean and variance of this posterior predictive distribution are calculated in the traditional way

$$\mu_o = \int x\, f_o(x)\, dx \tag{8}$$

$$\sigma_o^2 = \int (x-\mu_o)^2 f_o(x)\, dx \tag{9}$$

A brief summary of the numerical integration algorithm for quantifying the filter effect of conformity control based on prior hyperparameters for the distribution of the material property, is given in Fig. 2, considering that the material property X follows a normal or lognormal distribution.

The boundaries for μ_X (i.e., μ_{lower} and μ_{upper}) and σ_X (i.e., σ_{lower} and σ_{upper}) should be taken sufficiently wide so that the significant range of the joint density function for the mean and standard deviation is covered (and the associated error is negligible). Similarly, the boundaries for x (i.e., a and b) should be sufficiently wide to cover the significant range of the predictive density function. For practical applications (and because the numerical calculation is not at all time consuming) the boundaries can be taken rather wide (e.g. between 0 and 100 MPa in case of x) in order to avoid adjustments between different calculation sets.

3. Investigated conformity criteria for concrete strength

In order to illustrate the numerical Bayesian updating methodology with respect to the distribution of material properties, the compound conformity criteria given in Eq. (9) for the conformity evaluation of concrete strength are analyzed for the application examples in the next sections.

$$\begin{cases} \bar{x}_{15} \ge f_{ck} + 1.48\,\sigma \\ x_{\min} \ge f_{ck} - 4\ \text{MPa} \end{cases} \tag{10}$$

with $\bar{x}_{15}$ the sample mean of 15 consecutive test results, f_{ck} the specified characteristic concrete compressive strength (most often specified considering a certain concrete strength class) and x_{min}

Calculate the posterior joint density function of M and Σ

$\forall \mu_X \in [\mu_{lower}, \mu_{upper}]:$

$\quad \forall \sigma_X \in [\sigma_{lower}, \sigma_{upper}]:$

$\quad\quad u_\theta = (f_{ck} - \mu_X)/\sigma_X$

$\quad\quad \forall j:$

$\quad\quad\quad \text{if } u_{\theta,j} \leq u_\theta \leq u_{\theta,j+1}$

$$\text{then } P_a(\mu_X, \sigma_X) = P_a(\theta_j) + \frac{u_\theta - u_{\theta,j}}{u_{\theta,j+1} - u_{\theta,j}} P_a(\theta_{j+1})$$

$\quad\quad\quad \text{else if } u_\theta \leq u_{\theta,1}$

$\quad\quad\quad\quad \text{then } P_a(\mu_X, \sigma_X) = 1$

$$f''^{*}_{M,\Sigma}(\mu_X, \sigma_X) = f'^{*}_{M,\Sigma}(\mu_X, \sigma_X) P_a(\mu_X, \sigma_X)$$

Calculate an auxiliary predictive outgoing concrete strength distribution

$\forall x \in [a, b]:$

$$f_o^*(x) = \sum_{\mu_X} \sum_{\sigma_X} \frac{1}{\sigma_X} \phi\left(\frac{x - \mu_X}{\sigma_X}\right) f''^{*}_{M,\Sigma}(\mu_X, \sigma_X) \qquad \text{(X: Normal)}$$

$$\text{or } f_o^*(x) = \sum_{\mu_{\ln X}} \sum_{\sigma_{\ln X}} \frac{1}{\sigma_{\ln X} x} \phi\left(\frac{\ln x - \mu_{\ln X}}{\sigma_{\ln X}}\right) f''^{*}_{M,\Sigma}(\mu_{\ln X}, \sigma_{\ln X}) \qquad \text{(X: Lognormal)}$$

Calculate the predictive outgoing concrete strength distribution

$\forall x \in [a, b]:$

$$f_o(x) = \frac{f_o^*(x)}{\sum_{x \in [a,b]} f_o(x)}$$

Calculate characteristics of the outgoing concrete strength distribution

$$\mu_o = \sum_{x \in [a,b]} x\, f_o(x)$$

$$\sigma_o = \sqrt{\sum_{x \in [a,b]} [x - \mu_o]^2 f_o(x)}$$

Fig. 2 Flowchart and numerical integration algorithm for Bayesian updating of the predictive concrete strength distribution the minimum value of the same group of 15 test results.

Further, the standard deviation σ is estimated based on an initial assessment period consisting of 35 consecutive strength values, prior to the assessment period on which conformity has to be declared by Eq. (9). This σ value may be introduced in Eq. (9) on condition that the standard deviation of the latest 15 results (s_{15}) does not deviate significantly from σ. This is considered to be the case if Eq. (10) holds, which is the 95% acceptance interval of the test hypothesis that the real standard deviation is given by σ, based on a sample standard deviation from 15 test results, i.e.,

$$\sqrt{\frac{\chi^2_{14;0.025}}{14}}\sigma \le s_{15} \le \sqrt{\frac{\chi^2_{14;0.975}}{14}}\sigma \Rightarrow 0.63\,\sigma \le s_{15} \le 1.37\,\sigma \tag{11}$$

with σ the estimated standard deviation from the initial assessment period, s_{15} the sample standard deviation of the latest 15 test results and $\chi^2_{\nu;\alpha}$ the α fractile of the chi-square distribution with ν degrees of freedom.

If Eq. (10) is not satisfied, a new estimate of σ has to be calculated from the last available 35 test results.

The set of conformity criteria given by Eqs. (9) and (10) forms the basis of the conformity assessment for concrete compressive strength in the European Standard EN 206-1 (CEN 2000) in case continuous production is achieved.

In case a numerical Monte Carlo simulation approach (including the autocorrelation model as given in Eq. (2)) is applied to calculate the OC-curve corresponding to the compound conformity criterion considered in Eqs. (9) and (10), the results are depicted in Fig. 3 for different values of σ. The influence of the assumed value of the standard deviation σ of the strength population is found to be rather limited. A common choice of σ = 5MPa is suggested to be used for the analysis which is performed in the following sections. A more profound analysis of the conformity criteria under consideration is available in (Taerwe and Caspeele 2006, Caspeele 2010).

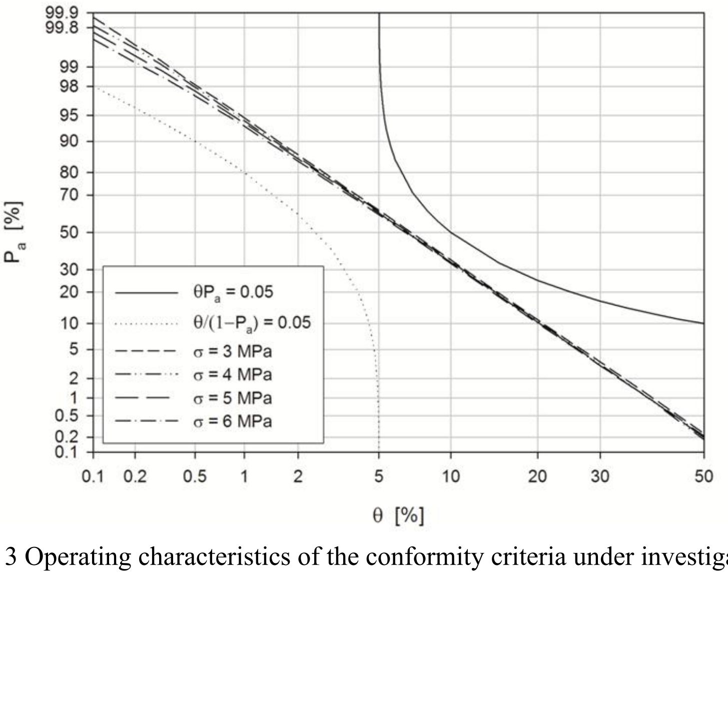


Fig. 3 Operating characteristics of the conformity criteria under investigation

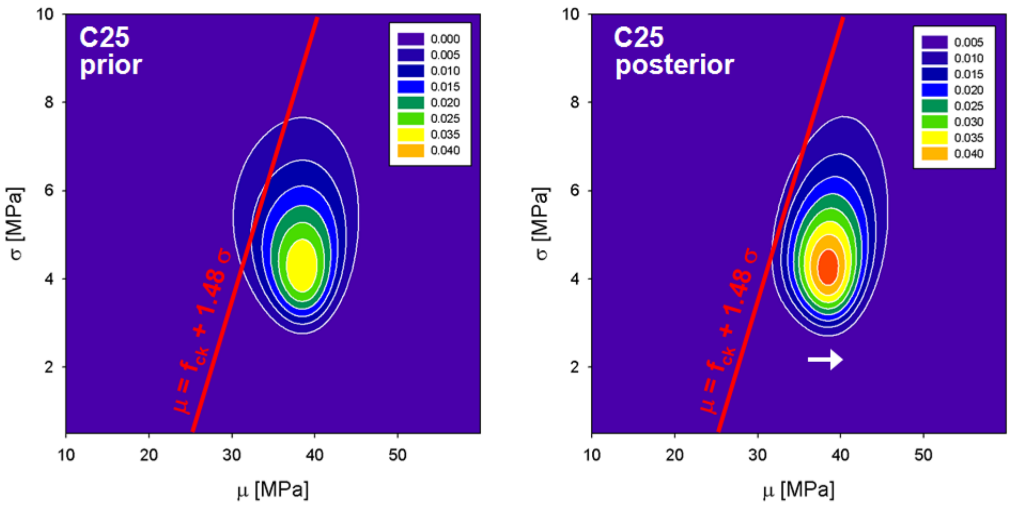


Fig. 4 Contour plot of the prior (left) and posterior (right) joint density function of the parameters for the concrete strength distribution corresponding to a concrete class C25 after conformity control based on the conformity criteria under consideration

4. Influence of conformity control on available prior distributions in literature

Suitable prior information with respect to concrete strength distributions is available in (Rackwitz 1983) in the format of hyperparameters for lognormal-gamma distributions, describing the mean M and standard deviation Σ of lognormal concrete strength distributions (considered as random variables) for different concrete grades and types. Prior estimates according to the database of more recent test results from concrete plants discussed in (Caspeele and Taerwe 2011b) showed to be comparable to the priors suggested in (Rackwitz 1983). As such, this prior information is found suitable for Bayesian updating of concrete strength distribution (Caspeele and Taerwe 2012). The filtering influence of the conformity criteria on this prior information is investigated by considering the updating algorithm as described in Section 2.2. The calculated operating characteristic corresponding to the conformity criteria described in Section 3 and illustrated in Fig. 2 is used to update the prior lognormal-gamma distributions for the parameters of concrete strength distributions in case of ready-mixed concrete.

As an example of the updating procedure with respect to the parameters of the concrete strength distribution, the prior and posterior joint density function $f'_{M,\Sigma}(\mu,\sigma)$ and $f''_{M,\Sigma}(\mu,\sigma)$ are illustrated in Fig. 4 in case of a concrete class C25 (ready-mixed concrete) with suggested prior hyperparameters $x'_{\ln X} = 3.65$, $n' = 2$, $s'_{\ln X} = 0.12$ and $\nu' = 4$ according to (Rackwitz 1983).

In comparison to the associated prior distribution, the posterior distribution is shifted slightly towards a higher mean and a smaller standard deviation, as could be expected based on the filtering effect of conformity criteria, i.e. both effects positively contribute towards a lower fraction defectives.

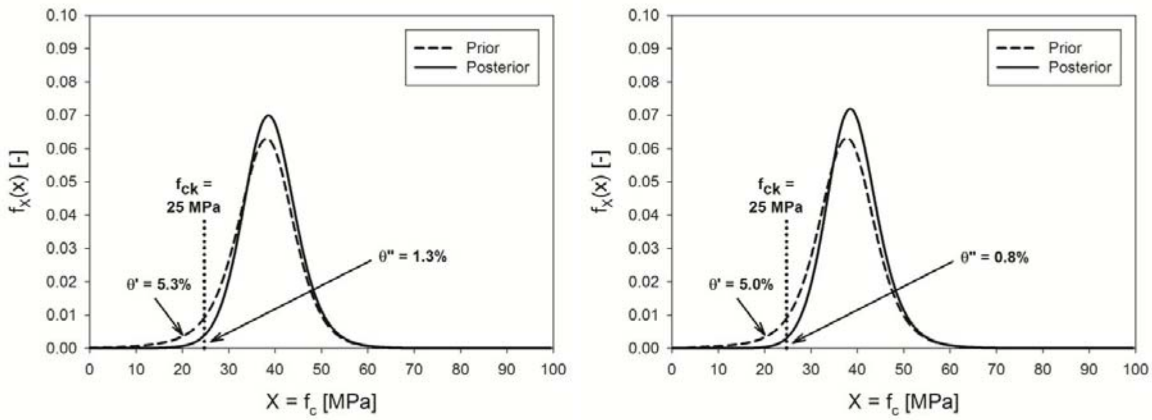
In order to quantify the filter effect of the conformity criteria on the different suggested prior distributions numerically, the mean and standard deviations of the prior and posterior predictive distributions are calculated from the joint density functions by using numerical integration.

Although the prior information in (Rackwitz 1983) is given in terms of lognormal-gamma distributions, the strength distribution of concrete can still be considered as normal or lognormal. For both assumptions, Table 1 provides the mean and standard deviation of the prior and posterior predictive distributions associated to the prior hyperparameters for different concrete grades. The application of this methodology is however not restricted to these concrete grades. It is possible to apply the procedure to a wide range of concrete classes (e.g. as those mentioned in EN 206-1), conditional on the availability of suitable prior information, e.g. based on maximum-likelihood estimations (Rackwitz 1983). Further also the fraction defectives associated to the predictive distributions are provided, based on the calculated cumulative predictive strength distributions before and after conformity control.

According to these results, the ratio of the posterior (filtered) mean to the prior mean is approximately 1.03 and the ratio of the posterior (filtered) standard deviation to the prior standard

Table 1 Mean, standard deviation and fraction defectives of prior and posterior predictive concrete strength distributions

	Normal distribution				Lognormal distribution			
	C15	C25	C35	C45	C15	C25	C35	C45
μ_i [MPa]	28.9	37.5	46.3	52.8	28.9	37.5	46.3	52.8
σ_i [MPa]	6.75	7.40	6.71	5.74	6.80	7.44	6.73	5.74
θ_i [%]	3.2	5.3	5.0	8.3	2.9	5.0	4.7	8.1
μ_o [MPa]	30.0	39.0	47.6	54.3	30.0	39.2	47.8	54.3
σ_o [MPa]	5.90	6.19	5.66	4.64	5.90	6.13	5.61	4.64
θ_o [%]	0.8	1.3	1.3	2.3	0.4	0.8	1.0	1.9
μ_o/μ_i [-]	1.035	1.040	1.027	1.027	1.035	1.046	1.031	1.027
σ_o/σ_i [-]	0.874	0.837	0.844	0.808	0.867	0.825	0.834	0.808
$X_{0.05,o}/X_{0.05,i}$ [-]	1.141	1.138	1.086	1.077	1.146	1.153	1.095	1.077



(a) (b)

Fig. 5 Prior and posterior predictive concrete strength distributions for a C25 concrete class in case of a normal distribution (a) and lognormal distribution (b)

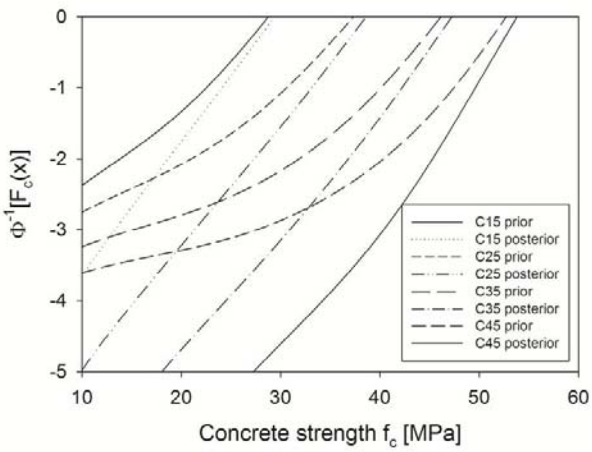


Fig. 6 Cumulative prior and posterior predictive distributions in case of a lognormal concrete strength distribution

deviation is approximately 0.84, again illustrating the increase of mean strength and the decrease in standard deviation of the strength distribution of the accepted concrete lots. In these particular cases, the influence of conformity control on the standard deviation of the predictive distribution is significantly higher compared to the effect on the mean. As a result of the filter effect on the mean and standard deviation, the fraction defectives in the predictive distribution significantly decreases when taking into account conformity control. In general, the fraction defectives in the predictive concrete strength distributions is found to decrease from about 5% to about 1%. The difference between the characteristics associated to the normal and lognormal strength distributions are rather small, which can be explained by the nature of the predictive distribution.

In case of the aforementioned C25 concrete class, Figs. 5(a) and (b) illustrate the different prior and posterior predictive distributions and their associated fraction defectives, both for a normal and lognormal assumed concrete strength distribution, again illustrating the rather small difference of the distributional assumption on the posterior predictive distributions. Therefore, in the next sections only a lognormal distribution for concrete strength will be considered.

The influence of the investigated conformity criteria on the predictive cumulative strength distributions corresponding to the different concrete classes for ready-mixed concrete as mentioned in (Rackwitz 1983) is illustrated in Fig. 6 in case of lognormal distributions, with ordinates according to

$$\Phi^{-1}\left[F_c(x)\right] \equiv \Phi^{-1}\left[\int_{t=-\infty}^{x} f_X(t)dt\right] \qquad (12)$$

with $F_c(x)$ the cumulative predictive concrete strength distribution.

5. The influence of conformity control on updated prior distributions in case a database of test results is available

5.1 Investigated concrete strength record

Data from a ready mixed concrete plant (described in (Caspeele and Taerwe 2011b)) is used to illustrate the numerical updating methodology based on more specific prior information. Concrete compressive strength results (measured on standard cubes with side 150 mm) of all concrete mixes with a specified C25/30 concrete class are obtained from a concrete plant between January till August 2006, resulting in a dataset of 240 test results. This strength record is shown in Fig. 7.

The first 225 test results of this strength record will be considered as prior information for the concrete strength distribution, while the last 15 test result will be used to update prior information (in order to simulate the influence of new available information), considering also the filtering effect of the conformity criteria described by Eqs. (9) and (10).

In order to derive suitable prior hyperparameters, 3 different approaches will be used, namely:

(1) by deriving new prior hyperparameters based on maximum-likelihood estimators (MLE),
(2) by updating available prior hyperparameters in (Rackwitz 1983) or
(3) by deriving new prior hyperparameters using a bootstrap procedure

5.2 Derivation of prior information based on maximum-likelihood estimators

The use of maximum-likelihood estimators (MLE) as explained in (Rackwitz 1981, 1983) enables to derive a prior for a C25/30 concrete class, which is more closely related to the concrete plant under consideration. The first 225 test results of the dataset given in Section 5.1 are used to derive more appropriate prior hyperparameters. Assuming that concrete strength is modelled as a lognormally distributed variable, hyperparameters for a prior lognormal-gamma distribution are derived based on 15 samples means $\bar{x}_{\ln X,i}$ and sample standard deviations $s_{\ln X,i}$, each based on 15 lognormally transformed test results and calculated according to (Rackwitz 1979). Using the provided dataset, this leads to the following hyperparameters:

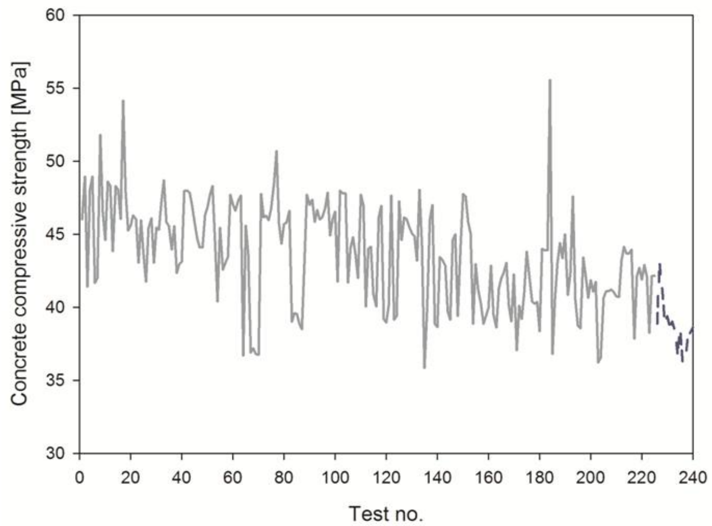


Fig. 7 Concrete strength record of a C25/30 concrete class in a ready-mixed concrete plant (January-August 2006)

$\bar{x}'_{\ln X} = 3.773$, $n' = 1.429$, $s'_{\ln X} = 0.059$ and $\nu' = 6.430$. Compared to the suggested hyperparameters by Rackwitz (1983) (see Section 4), the parameters $\bar{x}'_{\ln X}$, n' and ν' are very similar, however in case of the analyzed dataset from this specific concrete plant the standard deviation is found to be considerably smaller.

Also the filter effect of the considered conformity criteria on this prior information is quantified (Step 0) using the numerical algorithm as described in Section 2.2 in case of a lognormal concrete strength distribution. Further, as an example the last 15 test results of the given dataset are used to subsequently update the prior information using 3 times 5 successive test results (Step 1, 2 and 3) (where updating based on 5 successive results yields an acceptable updating frequency for common concrete plants) and by applying the well-known updating rules for hyperparameters as given in Eqs. (12) to (15) (Raiffa and Schlaifer 1969, Box and Tiao 1973, Rackwitz 1983, Diamantidis *et al.* 2001, Ang and Tang 2007).

$$n'' = n' + n \tag{13}$$

$$\bar{x}'' = \frac{n'\bar{x}' + n\bar{x}}{n''} \tag{14}$$

$$\nu'' = \nu' + \nu + 1 \tag{15}$$

$$s'' = \frac{1}{\nu''}\left[\left(\nu' s' + n'\bar{x}'^2\right) + \left(\nu s + n\bar{x}^2\right) - n''\bar{x}''^2\right] \tag{16}$$

where $\bar{x}$ is the mean of an equivalent sample of size n and s is the standard deviation of an equivalent sample of size $\nu + 1$. Prior hyperparameters are indicated with $'$ and posterior hyperparameters with $''$.

The filter effect on each updated distribution is also quantified. The results of these calculations are provided in Table 2.

From these results it can be seen that the filter effect of conformity control is rather limited. This is due to the rather low fraction defectives associated to the prior information (i.e., 0.5%), which reduces the filter effect considerably because the probability of rejection is low. Of course, this is due to the fact that concrete plants try to limit the probability of rejection and thus produce concrete considering a rather high safety margin with respect to the specified characteristic strength in order to avoid non-conformity declarations.

5.3 Updating available prior hyperparameters

The same prior information which was used in section 4, i.e., based on the hyperparameters suggested in (Rackwitz 1983) for a C25 concrete class in case of ready-mixed concrete, is used again in this section and updated by the additional 15 last test results of the investigated strength record, using the updating rules described by Eqs. (12)-(15).

First, the filter effect of the considered conformity criteria on this prior information is quantified (Step 0) equivalent to the method explained in section 4 in case of a lognormal concrete strength distribution. Further, the last 15 test results of the given dataset are used to subsequently update the prior information using 3 times 5 successive test results (Steps 1, 2 and 3). Again, the filter effect on each updated prior distribution is quantified. The results of these calculations are presented in Table 4.

Table 2 Results from the case study in case of specific prior information based on maximum-likelihood estimators

		Case study – MLE prior			
		Step 0	Step 1	Step 2	Step 3
Test results	$\bar{x}_{\ln X}$	-	3.695	3.644	3.624
	n	-	5	5	5
	$s_{\ln X}$	-	0.044	0.023	0.029
	v	-	4	4	4
(Updated) Prior parameters	$\bar{x}_{\ln X}$	3.773*	3.712	3.682	3.665
	n	1.429*	6.429	11.429	16.429
	$s_{\ln X}$	0.059*	0.049	0.041	0.037
	v	6.430*	11.43	16.43	21.43
Prior predictive characteristics	μ_i [MPa]	43.256	40.908	39.716	39.051
	σ_i [MPa]	4.113	2.379	1.815	1.565
	θ_i [%]	4.9×10^{-1}	1.0×10^{-2}	6.3×10^{-4}	7.9×10^{-4}
Posterior predictive characteristics	μ_o [MPa]	43.380	40.908	39.716	39.051
	σ_o [MPa]	3.912	2.377	1.815	1.565
	θ_o [%]	1.3×10^{-1}	8.7×10^{-3}	6.2×10^{-4}	7.9×10^{-4}
Filter effect	μ_o/μ_i [-]	1.003	1.000	1.000	1.000
	σ_o/σ_i [-]	0.951	0.999	1.000	1.000
	$X_{0.05,o}/X_{0.05,i}$ [-]	1.012	1.000	1.000	1.000

Table 4 Results from the case study in case of prior information based on literature data

		Case study – Rackwitz prior			
		Step 0	Step 1	Step 2	Step 3
Test results	$\bar{x}_{\ln X}$	-	3.695	3.644	3.624
	n	-	5	5	5
	$s_{\ln X}$	-	0.044	0.023	0.029
	v	-	4	4	4
(Updated) Prior parameters	$\bar{x}_{\ln X}$	3.65*	3.682	3.666	3.654
	n	2*	7	12	17
	$s_{\ln X}$	0.12*	0.073	0.054	0.046
	v	4*	9	14	19
Prior predictive characteristics	μ_i [MPa]	37.719	39.667	39.079	38.621
	σ_i [MPa]	7.078	3.529	2.379	1.936
	θ_i [%]	12.0	0.50	3.1×10^{-2}	3.7×10^{-3}
Posterior predictive characteristics	μ_o [MPa]	39.707	39.708	39.079	38.621
	σ_o [MPa]	5.690	3.461	2.377	1.936
	θ_o [%]	3.0	0.35	2.9×10^{-2}	3.7×10^{-3}
Filter effect	μ_o/μ_i [-]	1.053	1.001	1.000	1.000
	σ_o/σ_i [-]	0.804	0.981	0.999	1.000
	$X_{0.05,o}/X_{0.05,i}$ [-]	1.164	1.005	1.000	1.000

As can be seen from Table 4, the filter effect on the prior information is found to be more significant than for the comparable results in the previous section, which is due to the higher fraction defectives of the prior predictive concrete strength distribution. The filter effect reduces however strongly when the additional test results are taken into account.

5.4 Derivation of prior information based on a bootstrap technique

The bootstrap method was first introduced in 1979 by Efron (1979) and is currently commonly applied in non-parametric statistics, as it avoids distributional assumptions. Further, the method is of particular interest for the analysis of limited data. In the bootstrap procedure, the characteristics of a certain estimator are obtained by sampling from an approximate distribution which is generally the empirical distribution of the observed data (Most 2009). The method assumes independent and identically distributed observations and constructs a number of re-samples by random sampling with replacement from the observed dataset. Based on this re-sampling sets, the properties of the estimated parameters (including their distribution function, confidence interval, etc.) can be obtained. More information regarding this non-parametric bootstrap method is available in (Higgins 2004, Moore and McCabe 2006, Most 2009).

In order to derive the parameters $\bar{x}'$, n', s' and ν' of the prior distribution $f'_{M,\Sigma}(\mu,\sigma)$ in case of a lognormal-gamma distribution, the following approach can be used:

1. Generate N re-samples of the available set of n test results x_i with replacement. N should be taken sufficiently large so that the estimated parameters are independent of N.
2. For each re-sampled set of n test results, calculate the sample mean $\bar{x}_{\ln X.i}$ and standard deviation $s_{\ln X.i}$.
3. Based on these N estimated values of the mean and standard deviation, apply the MLE method as described in (Rackwitz 1981, Rackwitz 1981) in order to obtain the parameters $\bar{x}'_{\ln X}$, n', $s'_{\ln X}$ and ν'.

The test results used in Step 1 of the previous examples for updating the priors in literature or the derived more specific prior information of a C25/30 concrete class, can now be used directly to derive prior parameters for the concrete strength distribution of the concrete mixture under consideration.

Based on the sample of 5 test results (in Step 1), 1000 bootstrap re-samples are generated, each consisting of 5 test results taken with replacement from the 5 original test results. Based on these bootstrap samples, 1000 bootstrap values for the mean $\bar{x}_{\ln X}$ and standard deviation $s_{\ln X}$ are calculated. Based on these bootstrap samples $x_{\ln X.i}$ and $s_{\ln X.i}$, the parameters of a prior lognormal-gamma distribution are derived by using maximum-likelihood estimators. For the example, the following results are obtained: $\bar{x}'_{\ln X} = 3.692$, $n' = 12.136$, $s'_{\ln X} = 0.038$ and $\nu' = 17.774$. Due to the use of the bootstrap procedure, this prior information is more informative (i.e., larger n' and ν') than the previously mentioned priors (corresponding to a higher n' and ν' value). The ability of this prior information to represent the strength population properly is depending strongly on the number of test results used for the bootstrapping procedure. In case only a few test results are used for this purpose (as was the case for this example), the prior information can be biased. This could be partly overcome by reducing the regeneration sample size N, however no specific guidance can be given for such a procedure and as such this is not advised. By taking N sufficiently large, the estimation results become independent of N and the hyperparameters n and

Table 6 Results from the case study in case of specific prior information based on a bootstrap procedure

		Case study – Bootstrap prior		
		Step 1	Step 2	Step 2
Test results	$\bar{x}_{\ln X}$	3.695	3.644	3.624
	n	5	5	5
	$s_{\ln X}$	0.044	0.023	0.029
	ν	4	4	4
(Updated) prior parameters	$\bar{x}_{\ln X}$	3.692*	3.678	3.666
	n	12.136*	17.136	22.136
	$s_{\ln X}$	0.038*	0.034	0.032
	ν	17.774*	22.774	27.774
Prior predictive characteristics	μ_i [MPa]	40.117	39.563	39.092
	σ_i [MPa]	1.686	1.450	1.329
	θ_i [%]	1.0×10^{-4}	$6.8\text{x}10^{-6}$	1.1×10^{-6}
Posterior predictive characteristics	μ_o [MPa]	40.117	39.563	39.092
	σ_o [MPa]	1.686	1.450	1.329
	θ_o [%]	1.0×10^{-4}	$6.8\text{x}10^{-6}$	1.1×10^{-6}
Filter effect	μ_o/μ_i [-]	1.000	1.000	1.000
	σ_o/σ_i [-]	1.000	1.000	1.000
	$X_{0.05,o}/X_{0.05,i}$ [-]	1.000	1.000	1.000

ν correspond to the best fitting of a joint density function rather than a sample number based belief in the mean and standard deviation. As such, the bootstrap procedure also enables to make inferences, even with little information.

Again the filter effect of the considered conformity criteria on this prior bootstrap-based information (Step 1) is quantified using the same updating algorithm in case of a lognormal concrete strength distribution. The prior information is then consecutively updated using 2 sets of 5 successive test results (Steps 2 and 3) and the filter effect on each updated prior distribution is quantified. The numerical results of these calculations are given in Table 6.

As can be seen from Table 6, also in case of a bootstrap-based prior the filter effect remains negligibly small, which is again due to the low fraction defectives of the original strength population. Due to the fact that the prior information already contains much more specific information regarding the strength population of which the additional test results are taken, the influence of the filter effect decreases faster when additional test results become available than for the similar results of Table 2 in case of priors based on maximum-lihelihood estimators.

6. Conclusions

- In general, quality control has a favourable effect on material properties due to the fact that the existence of quality requirements (such as conformity criteria) compels producers to deliver high quality products in order to avoid rejection by quality assessment. This effect has an influence on the probabilistic modelling of material properties of accepted inspected lots and also influences structural reliability analyses.

• As conformity criteria are used to reject or accept concrete lots, they pose a filtering effect with respect to the predictive distribution of the material property under consideration, which can be related to the probability of acceptance associated to the applied conformity control scheme. As a result, the quality (in terms of fraction defectives) of accepted inspected lots will be higher than the quality of the incoming population which is submitted for conformity control or compared to the situation where no conformity assessment takes place. In case of a one-sided conformity criterion for concrete strength for example, the mean of the posterior predictive strength distribution of the material property will increase, while the standard deviation will decrease compared to the prior predictive strength distribution

• In order to enable Bayesian updating of the predictive distributions of material properties, prior hyperparameters for the distribution of the material property have to be derived. Available prior information in literature can be easily updated due to the fact that natural conjugate prior distributions are suggested. Further, more specific prior distributions can be derived using maximum-likelihood estimators or a bootstrap technique.

• A Bayesian updating methodology was proposed in order to update prior distributions based on indirect information by conformity and/or production control, using numerical integration and taking into account OC-curves calculated by numerical Monte Carlo simulations, enabling to consider complex conformity criteria as well as to take into account autocorrelation between consecutive test results.

• The numerical Bayesian updating methodology was used to investigate the filter effect of a specific compound conformity criterion on available prior concrete strength distributions in (Rackwitz 1983) for different concrete grades. This investigation yields the following conclusions:

- The ratio of the posterior predictive mean to the prior predictive mean of the strength distribution is approximately 1.03 for all concrete grades, the ratio of the posterior predictive standard deviation to the prior predictive standard deviation is approximately 0.84 and the ratio of the posterior predictive 5% fractile to the prior predictive 5% fractile is approximately 1.10.
- Conformity control significantly decreases the fraction defectives in the predictive strength distributions, which in general decreases from about 5% to about 1%.
- The differences between the characteristics of the predictive strength distribution associated to a normal or lognormal concrete strength distribution are rather small, yielding a negligible influence of the distributional assumption.

• A case study based on data from a concrete plant was provided in order to illustrate the derivation of more specific prior hyperparameters. The data was used to update available prior information and to derive more specific prior information based on maximum-likelihood estimators and a bootstrap procedure. Further, these prior distributions were updated based on additional strength results and the influence of the investigated conformity criteria on the predictive strength distributions was quantified. Based on these results, it is found that the maximum-likelihood estimations (MLE) are preferable compared to the use of literature-based prior information, as these MLE provide more case-specific information and as such the updated distribution converges faster. The use of a bootstrapping procedure allows even faster convergence due to the use of the latest information (and not an average over a longer period), although one should be careful in applying this procedure as the prior is more informative compared to MLE. The results of this analysis as well as these conclusions would be even more pronounced in case the concrete strength record which is subjected to compliance control has lower fraction defectives.

• Although the methodology is described for concrete compressive strength, the same methodology can be used for updating the distributions of other material properties, not necessarily related to concrete.

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