Feasibility study of the beating cancellation during the satellite vibration test

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Abstract. The difficulties of satellite vibration testing are due to the commonly expressed qualification requirements being incompatible with the limited performance of the entire controlled system (satellite + interface + shaker + controller). Two features cause the problem: firstly, the main satellite modes (i.e., the first structural mode and the high and low tank modes) are very weakly damped; secondly, the controller is just too basic to achieve the expected performance in such cases. The combination of these two issues results in oscillations around the notching levels and high amplitude beating immediately after the mode. The beating overshoots are a major risk source because they can result in the test being aborted if the qualification upper limit is exceeded. Although the abort is, in itself, a safety measure protecting the tested satellite, it increases the risk of structural fatigue, firstly because the abort threshold has been already reached, and secondly, because the test must restart at the same close-resonance frequency and remain there until the qualification level is reached and the sweep frequency can continue.

The beat minimum relates only to small successive frequency ranges in which the qualification level is not reached. Although they are less problematic because they do not cause an inadvertent test shutdown, such situations inevitably result in waiver requests from the client.

A controlled-system analysis indicates an operating principle that cannot provide sufficient stability: the drive calculation (which controls the process) simply multiplies the frequency reference (usually called cola) and a function of the following setpoint, the ratio between the amplitude already reached and the previous setpoint, and the compression factor. This function value changes at each cola interval, but it never takes into account the sensor signal phase.

Because of these limitations, we firstly examined whether it was possible to empirically determine, using a series of tests with a very simple dummy, a controller setting process that significantly improves the results. As the attempt failed, we have performed simulations seeking an optimum adjustment by finding the Least Mean Square of the difference between the reference and response signal. The simulations showed a significant improvement during the notch beat and a small reduction in the beat amplitude. However, the small improvement in this process was not useful because it highlighted the need to change the reference at each cola interval, sometimes with instructions almost twice the qualification level. Another uncertainty regarding the consequences of such an approach involves the impact of differences between the estimated model (used in the simulation) and the actual system.

As limitations in the current controller were identified in different approaches, we considered the feasibility of a new controller that takes into account an estimated single-input multi-output (SIMO) model. Its parameters were estimated from a very low-level throughput. Against this backdrop, we analyzed the

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feasibility of an LQG control in cancelling beating, and this article highlights the relevance of such an approach.

**Keywords:** sine vibration testing; beating phenomena; control; LQG

### 1. Introduction

Satellite vibration testing is deemed critical because of some intense mechanical excitation it applies to the specimen. Acceleration must indeed be high enough in certain frequency ranges to guarantee the mechanical resistance of structures and equipment in the launch situation. However, vibration testing should be considered perfectly safe for the satellite as appropriate limits are well known and analysts ensure that compatible test strategies are defined. The root of the problem lies in the facility performance: not in hardware but in control software. To accept this position, we must simply assume that if proper electrodynamic power is available at every moment of time to produce the desired acceleration, then a particular command that achieves the strategy’s reference must also exist. This reasoning may seem convincing, but the implementation remains difficult and some considerations regarding the closed-loop system stability is sufficient to demonstrate that controller development would be specific for each satellite, requiring an accurate knowledge of the complete model (i.e., satellite + vibrator + expander-interface).

#### 1.1 Common control and performance

The state of the art in vibration control is still far from this approach: since the algorithm is based on the difference correction between reference and measuring amplitudes, it is well suited for highly-damped structures but the behavior is poor when the modes’ damping is as low as that of satellites (typically < 0.3%). The setting is not the result of a calculation taking into account the transfer function of the overall system to be controlled, and the single adjustment, called the compression factor, only slows down the drive’s dynamic to prevent the controlled system to get unstable. When the compression factor\(^1\) is small (K=1, 2 or 3, for example), the closed-loop system is highly responsive but quickly becomes unstable in the case of weakly damped structures. On the other hand, when the compression factor is high (K>10), the closed-loop system is stable but gently oscillates around the reference with a large amplitude. As explained in this paper, this type of controller does not offer a truly satisfying solution for the beating problem and it is always necessary to seek for a test strategy that minimizes the successive overshoot-undershoot from the set point.

The overshoots are a major risk source because they can lead to the test being aborted when the qualification upper limit is exceeded. Although the abort, in itself, is a safety measure for the tested satellite, it should carefully be avoided because it greatly increases the risk of structural fatigue. Firstly, because the abort threshold has been already reached and exceeded for few periods on close resonance frequencies, and then because the test must restart at the same last close-resonance frequency and remain there until the qualification level is reached to continue the sweep frequency.

The undershoots are the minimum beating at which the qualification level is not reached. They

\(^1\)See §3 for details on this parameter.
are less problematic because they do not cause an inadvertent test shutdown but, in the strictest sense of the requirements, we can consider that the satellite was not qualified on these small frequency ranges. Note that the strategy of using a negative ramp starting from a higher frequency has also never produced better results. Such a situation inevitably involves waiver requests from the client.

1.2 Investigations

In order to better understand the test strategy settings, in particular those of the compression factor, Thales Alenia Space first attempted an empirical approach (see §4). A test campaign was performed with a very simple dummy, a mere beam transversely excited by the vibrator. Given what we already knew about the controller performance and mainly the few settings it offers, we could not expect significantly better results than those obtained during the satellite tests. This approach had mostly resulted in raising the awareness of the Mechanical Environment Testing service technicians and engineers regarding the problems’ origin. Without satisfactory adjustments, we had to conclude that the current control algorithm would never be able to produce the set points where a mode is too weakly damped.

To complete the search for a vibrator setting procedure, a study was performed from a satellite SIMO model identification. Rather than empirically seeking only the most suitable compression factors, the purpose was to determine, for certain various compression factors (K=3,…,10, for example), a strategy profile that minimizes the squared difference between the desired behavior and the measurements. Section 5 presents the results and the weakness of such an approach to the problem.

1.3 New concept: LQG control and feasibility

As previous approaches proved to be poor in terms of results but dense in terms of understanding the problem, we envisaged a new control law based on an identified SIMO model from a very low-level throughput. Taking into account the estimated parameters of the process to be controlled, it becomes possible to synthesize a specific control law that could cancel the beating. Section 6 presents the results of the feasibility study.

2. Some reminders about beating

As explained in (Roy and GIRARD 2012, Nali and Bettacchioli 2014), the beating phenomenon is usually a low-frequency modulation that results from the superposition of two sources in very close frequencies. In the case of vibration testing, the causes are somewhat different since they are produced by the combination of two other parameters: a very low damped mode and a transient excitation frequency very close to the mode. The transient may be the sudden application of such a frequency but, in the particular case of vibration testing, the transient is the consequence of the permanent frequency change. In other words, at the level of a structural resonance mode, the sweep frequency must be considered as a permanent transient regime that causes beating as soon as the mode passes.

Of course, the lower the damping, the stronger the beating amplitude, but the influence of the sweep frequency rate is more difficult to describe. Indeed, the greater the sweep rate, the more
significant the transient effect is but the beating has a shorter time to settle. Conversely, if the sweep rate is slow, the analysis (Nali and Bettacchioli 2014) shows that the beating is a little smaller but persists longer. Obviously, without the risk of structural fatigue, a much slower sweep rate should ideally be used so that the system is always close to steady state, but such a solution is quite unthinkable in practice. For this reason, especially in difficult cases, it is not uncommon for us to choose the sweep rate as high as possible in order to pass the resonant mode as rapidly as acceptable. Accordingly, we derogate the specification by choosing a faster rate than initially planned, four octaves per minute instead of three for example.

Without developing any model, we can however indicate that the main structural modes are not only observable but also controllable. Hence, the beating is not inevitable and its removal only depends on the controller’s effectiveness.

3. Controller operation and limitation

The current controller (see algorithm on Fig. 1) is designed to work with any type of load and for this purpose it relies on the tuning of a single parameter (called the compression factor) per strategy channel. Its operating principle is very simple since it simply involves multiplying the reference frequency (cola, see Fig. 2) by an updated drive at the end of each period. The calculation is made from an error which is not the classical difference between the reference and the response, but as their ratio instead: for each strategy’s sensor N° i, the error at the p-cola’s period end is $e_i(p) = \frac{y_i(p)}{y_i^{ref}(p)}$.

![Drive calculation algorithm](image)

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**Fig. 1 Drive calculation algorithm**
Taking into account the error $e_i(p)$, the updated reference $y_i^{\text{ref}}(p+1)$ and the compression factors $K_i(p+1)$, a specific drive $U_i(p+1)$ is calculated as if the strategy’s sensor exists alone and the applied drive is the minimum: $U(p+1) = \min[U_i(p+1)]$. Under this condition, the control is always completed from one of the four pilots (i.e., the sensors on the vibrator table) except during the notching.

For the updated drive $U_i(p+1)$, one of the expressions (see if-condition in Fig. 1) may be available depending on whether the error is greater or less than 1. We can notice that when the compression factor is minimum ($K = 1$), the controller compensates the whole of the error. Otherwise, when $K > 1$, the controller compensates only part of the error. The compression factor, therefore works as a low-pass filter on the drive and stabilizes the system when it becomes unstable. Despite its very simple structure, the controller is robust and it is always possible to find a stable operating point independent if the compression factor is chosen high enough. However, its performance is naturally degraded when the structural modes are very weakly damped; even though the system’s response does not diverge, the set point is sometimes so poorly fulfilled that it leads to an abort after a considerable overshoot.

![Fig. 2 Cola (in green) and example of drive (in red), where $p$ is the cola’s period number and the drive’s $A_p$-amplitude modulates the cola](image)

If this control process seems apparently well-designed to quickly reset an amplitude error, it however disregards the phase response. Whatever the monitored channel phase, the drive is always in-phase with the cola. This is precisely the weakness of such a controller because the phase is one of the most important parameters that contributes to the system’s stability. We can also see that even though a 180° phase-shift drive is absolutely not a recommended solution for minimizing overshoots within the meaning of system stability, this is not permitted by the controller.

Another problem of frequency shift has already been highlighted by Nali and Bettacchioli (2014). Fig. 3 shows an example of this phenomenon. Around the second tank mode frequency, i.e., from 43.9 to 49.4 Hertz the vibrator table performs the mode’s frequency (47.8 Hertz) while the
frequency reference of the cola is not followed. From the point of view of phenomenology, as this very high modal mass is very slightly damped, the vibrator table is driven by the satellite’s inertia: the satellite behaves as an anti-resonance and, although the drive frequency is identical to that of the cola, the frequency of the table and the notching remains that of the mode. In this frequency range, the drive has just enough energy to excite the mode but not enough to impose the cola frequency on the table. It is as if, on this frequency band, there was a Phase Shift Lock (PLL) locked on the natural frequency mode.

From a qualification point of view, this is problematic because it means that the satellite can remain excited longer than expected at its natural frequency. In terms of spectra, there should be no values on this frequency band (except at the natural frequency of the mode) because the cola frequencies used as abscissa do not correspond to the excitation frequency at the base of the satellite.

4. Controller tuning attempt from an experimental approach

In an attempt to overcome the difficulties of adjusting the controller, in order to prevent set point tracking difficulties and abort risks, we have performed a test campaign with a dummy structure. The test dummy is designed to exhibit first modes and damping values in the same order of magnitude as main satellite modes and presents a transversely excited mere beam (see Fig. 4).

We gave free rein to our creativity, without any feasibility considerations. In this way, we mainly tried to change the test strategy until the system achieves the set point, by doing the following: at the end of each test run, we changed the strategy by increasing the reference (and/or the compression factor) where an undershoot occurred, and respectively, by lowering the reference (and/or the compression factor) where an overshoot occurred. Even by daring the quirkiest strategies, we have never been able to perform a test without overshoot or undershoot. The attempts failed and the conclusion was to search for a such optimal strategy by minimizing a squared error criterion.

![Image of a graph showing Cola’s frequency and frequency of the first pilot sensor on the vibrator table.](Fig. 3 Cola’s frequency (in blue) and frequency of the first pilot sensor on the vibrator table (in red) (From Bettacchioli (2014))}
5. Attempted controller tuning from a simulation approach based on a least square-beating criterion

The aim was to determine, from an identified satellite model, the specific strategy (pilot & notching references and compression factors) that reduces the overshoots and the beating amplitude with respect to a given specification. For the sake of simplicity, we limited the scope of the study to the first structural mode, which is generally less complex than the tank modes.

Fig. 5 shows a comparison between the real response of a pilot and a notch in red, and their simulation after optimization in blue. The red curves correspond to the actual answers of a pilot and a notching of the first mode during a qualification test. The blue curves are the simulations of the pilot and notching that we would have obtained with the pilot and notching strategies which are presented below, knowing that the model identified for the simulation was obtained from the actual test qualifying. In other words, the development of the qualification strategy was not carried out from a lower-level test identification to make a prediction but from the test response already known since it seemed necessary to evaluate in advance the feasibility of such an approach in the absence of dispersions on the model.

The pilot strategy and notching (on the two bottom graphs) was obtained by minimizing the quadratic energy criterion

\[ C = \frac{1}{P_2 - P_1} \sum_{p=P_1}^{P_2-1} (E_{p}^{ref} - E_{p}^{sim})^2 \]  

(1)

\( P_1 \) and \( P_2 \) denote respectively the first and last cola period of the interval over which the optimization is to be performed.

\( E_{p}^{ref} \) and \( E_{p}^{sim} \) denote respectively the energy of the reference and of the simulated response over the period \( p \) of cola.

The energies \( E_{p}^{ref} \) and \( E_{p}^{sim} \) are arbitrarily defined by

\[ E_{p}^{ref} = \frac{1}{n_p} (A_{p}^{ref})^2 \sum_{\tau=0}^{n_p-1} cola_p^2(\tau) \]  

(2)
\[ E_p^{\text{sim}} = \frac{1}{n_p} (A_p^{\text{sim}})^2 \sum_{\tau=0}^{n_p-1} \text{cola}_p^2(\tau) \]  

\( \text{cola}_p(\tau) \) is the value of \( \tau \)-th sample of the \( p \)-th cola period when \( \tau \) is expressed as the number of sampling periods. As the sampling frequency of the throughput is generally 800, 1600 or 3200 Hertz, the sampling periods may be 1.25, 0.625 or 0.3125 milliseconds.

In order to simplify the expression of criterion (1), we did not specify whether it was pilot or notching. In fact, we consider that the value to be taken into account is the value on which the control is calculated.

To obtain the result shown in Fig. 5, the optimization program was executed with a fixed step of 0.05 g, which corresponds to the order of magnitude of the Least Significant Bit (LSB) setpoint: we can clearly see the improvement provided by optimization, especially in the flat part of the notching: the overshoot around 13.5 Hertz disappeared, and after 14 Hertz the intensity of the beating of the pilot decreased.

![Fig. 5 Comparison between a real response (in red) and an optimized simulated response (in blue) of a 1st structural mode. From top to bottom: Pilot, Notching, Optimized Pilot & Notching reference strategy. In the green oval, the Notching reference strategy reaches nearly twice the nominal setpoint](image)

However, few drawbacks prevent the use of this strategy: the references are very spiked and change with each cola’s period. Under these conditions, it seems to be difficult to assess the effect of smoothing. Secondly, and this seems even more dangerous, some of these references strategy can reach nearly twice the nominal setpoint (see green oval in the Fig. 5). Because of these critical points, it appears that an optimal strategy is not directly applicable. It even seems dangerous if we take into account the fact that the identified damping and frequency of the modes can change a little when the level increases, even if the identified model is updated from the highest-level test already achieved. Indeed, by observing so closely a bristling setpoint profile, what would happen
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in the event of identification-related dispersions if the transfer function mode shifted by a few tenths of a Hertz?

In the end, we mainly had to abandon this solution, on the one hand because it was an open-loop control and on the other hand because the reduction in the number of points in the criterion (1) to achieve smoother strategy profiles could only lead to worse performance.

6. LQG approach and feasibility

In the previous sections, we saw that the performance problems of the current vibration testing are related to the controller, and tests on different control parameters settings did not result in the expected results. Therefore, we tried to study another type of controller whose setting is based on a good knowledge of the complete system transfer function: a Linear Quadratic Gaussian (LQG) synthesis. This method is not the latest but it has the advantage of being optimal (in the sense of a quadratic error criterion) while other, more recent and more robust, are suboptimal and have been thought to take account of the most important model dispersions. As identification from a Low-Level Test throughput is accurate enough (Bettacchioli 2014), the LQG synthesis seems well suited for the problem.

The LQG controller is the combination of a Kalman filter, i.e., a Linear Quadratic Estimator (LQE), with a Linear Quadratic Regulator (LQR). Often in practice, the LQR part is first processed, which consists in minimizing a cost function and then the filtering part is treated with Kalman filtering in order to attenuate the disturbances as much as possible without destabilizing the closed loop system. Without going into details, by setting the following notations:

\[ t \] is the time expressed in number of sampling periods
\[ A(t) \] is the reference magnitude at time t for the response of the channel considered
\[ X(t) \] and \[ x(t) \] are state-vector at the t-time
\[ \hat{x}(t) \] and \[ \hat{x}(t+1/t) \] state vector filtered and predicted at time t
\[ \epsilon(t) \] is an error at time t
\[ F \] and \[ f \] are discrete state transition matrix
\[ G \] and \[ g \] are command matrix

the steps in the controller development are as follows:

1) From the complete discrete state-space representation
\[
\begin{align*}
X(t + 1) &= F \cdot X(t) + G \cdot u(t) \\
Y(t) &= H \cdot X(t)
\end{align*}
\]
which allows to control the chosen mode

2) Form the augmented model that takes into account the reference
\[
\begin{align*}
\begin{bmatrix}
x(t + 1) \\
x(t + 1)
\end{bmatrix} &= \begin{bmatrix} f & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\
x(t)
\end{bmatrix} + \begin{bmatrix} g \\ 0 \end{bmatrix} u(t) \\
\epsilon(t) &= [h \quad -1]. \begin{bmatrix} x(t) \\
A(t) \cdot col\alpha(t)
\end{bmatrix}
\end{align*}
\]
3) Determining the $K^c$ control gain that minimizes the cost function

$$J = \sum_{k=1}^{N} \epsilon^T(k) Q \epsilon(k) + u^T(k) R u(k)$$

where $Q$ and $R$ are respectively $n \times n$ and $1 \times 1$ arbitrary matrix when $n$ is the state vector size.

4) From the minimum model, calculate the $K^f$ gain of the Kalman filter. This step requires knowledge of the variance of the measurement noise and the arbitrary adjustment of the variance of the states noises.

5) Implement the Kalman filtering algorithm

$$\begin{align*}
\hat{y}(t/t - 1) &= y(t) - h \hat{x}(t/t - 1) \quad \text{Innovation step} \\
\hat{x}(t/t) &= \hat{x}(t/t - 1) + K^f \hat{y}(t/t - 1) \quad \text{Filtering step} \\
\hat{x}(t + 1/t) &= f \hat{x}(t/t) + g u(t) \quad \text{Prediction step}
\end{align*}$$

6) At the beginning of each sampling period, the command is given by

$$u(t) = -K^c \hat{x}(t/t)$$

As the control must be implemented from any measurement channel of the strategy, we calculate a particular set of parameters for each channel. For each strategy’s sensor we specify only the reference requirement, i.e. neither manual notching nor specific anticipation of the references changing. On the other hand, the phase shift must be taken into account when moving a control channel from a pilot to a notching, and vice versa.

The Kalman filter is adjusted to take into account the usual magnitude orders of measurement noise. Such disturbances are also added to the simulated measurements data as the same magnitude of white noise.

The single-axis simulations can locally present some flaws, but these are always unimportant because they never lie in the notching range. Hence, by switching from a single-axis-LQG-law to another one with respect to the sensor reference in question, we obtain the global simulation in Fig. 7. All requirements are met without overshoot or undershoot, and the small beating of the nominal and redundant Earth Deck sensors about 35 Hz are unimportant because their level is negligible.

![Fig. 7 Global simulation with an LQG control. Four pilots and classical notching on Earth Deck, Upper Tank and Lower Tank](image-url)
As a result, this first approach consisting of a new specific control for each satellite looks promising but a robustness study must still be undertaken to verify the influence of the model identification dispersions and the behavior when measurement is more disturbed than standard white noise.

7. Discussion

The doubts frequently expressed regarding the performance of the vibration control algorithm are not recent and it seems useful to specify the problems and their origins. These have several origins: the drive is always in phase with the reference frequency and never takes into account the phase of the sensor used to achieve the control. Only the signal amplitude is controlled, and the filter that ensures the stability of the system introduces a delay.

Although the main cause of the problem is clearly known, namely the non-inclusion of the phase in the controller, it would be very difficult to change the phase at each cola’s period end without changing the entire algorithm, as this would result in drive discontinuities and causing shocks (see Fig. 8). It also appears impossible to gradually phase-shift the drive to ensure phase continuity because this would change the whole controller philosophy and locally change the periods, i.e., the frequencies. Naturally, empirical research has not led any results.

Under these circumstances, it is not possible to eradicate the beating, even considering theoretical profiles expected to minimize any error criterion. Their results are also unusable because they lead to local peaks which amplitudes can reach double qualification levels. Furthermore, it would be difficult to assess the impact of the identification dispersion on the results. Ultimately, as there is no other available controller, currently the best way seems to be to check the behavior of analysts’ strategies with a highly accurate simulator (Bettacchioli 2014, 2015).

Fig. 8 Cola (in green) and example of drive discontinuity (in red), where \( p \) is the cola’s period number and the drive’s \( A_p \)-amplitude modulates the cola.
Fig. 7 shows an approach based on LQG synthesis that could be effective, but a robustness study regarding the main modes dispersion is still open and needs to be performed. This point is indeed sensitive because, since the damping of the modes to be controlled is very small (generally less than 0.3%), the critical frequency band is very narrow. For this reason, the slightest error on the identification of the model used to calculate the control law can cause the controller to be effective only next to the critical frequency zone. Once more, this brings us back to the need for very good transfer function identification readjusted with a throughput. The robustness study could then be achieved, for example, by setting the controller from a very low-level test identification and then testing the controlled system from the Qualification Level identification.

If the robustness study is successful, the implementation and the validation would still be relatively fastidious. As well as providing security at the controller to ensure that the table will never go to a stop, it would be necessary to develop a real-time simulator for the initial tests as it seems unthinkable to start directly with the vibrator.

8. Conclusions

The current vibration controller does not allow good reference tracking when the mode’s damping is very low. Neither a test campaign with a dummy nor optimum setting research has provided a strategy adjustment method that cancels or mitigates the beating and/or the overshoot. It is therefore necessary to rely on a very accurate simulator to predict the behavior of the test required by analysts. In this way, Thales Alenia Space (Bettacchioli 2014, 2015) has gained a very high level of expertise in transfer function identification from very low-level test data, and test prediction from such identification have already allowed to avoid test aborts. In particular, we have understood, for a long time now, that a very accurate model of a vibrator would be interesting for analysts’ studies but that it would never entirely solve the beating problems because the misconceptions and the dispersions of the satellite’s mechanical model are at least in the order of the width of the modes’ resonance peaks (of the order of 3 Hz). Moreover, the coupling between the vibrator and the satellite will never be rigid enough to be neglected. These issues led us to abandon the commonly adopted approach and to use predictions from a very low-level test data identification.

Thanks to such an accurate simulator, we have been able to study the feasibility of canceling the beating. Amongst the possible controllers (P.I.D, H-infinity control, pole-zero placement, etc.), we considered an LQG method (i.e., an Less Quadratic control with a Kalman filter). However, this study was carried out merely to demonstrate the feasibility of canceling the beating; implementation would require not only validation tests and probably some development to improve the robustness with respect to the dispersions, but also a safety algorithm to avoid a table shock on safety stop. At this stage of the study, it is not conceivable to implement this controller to test a satellite, or even with a dummy for safety reasons of the facility. An intermediate step involves the development of a real-time simulator for initial testing.

References

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