Continuously inhomogeneous beam structure with creep: a longitudinal crack study

Victor I. Rizov

Department of Technical Mechanics, University of Architecture, Civil Engineering and Geodesy, 1 Chr. Smimensky blvd., 1046 – Sofia, Bulgaria

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Abstract. The main purpose of the present paper is to evaluate the influence of increase of the coefficient of viscosity with time on the strain energy release rate for a longitudinal crack in a continuously inhomogeneous beam configuration under linear creep. The beam exhibits continuous material inhomogeneity along the width, thickness and length. The creep behavior is studied analytically by a viscoelastic model structured by one dashpot and three springs. The coefficient of viscosity of the dashpot and the modulii of elasticity of the springs are distributed continuously in the width, thickness and length directions of the beam. Besides, the coefficient of viscosity increases with time. Time-dependent solutions to the strain energy release are derived by considering the balance of the energy and by applying the compliance method. The results obtained by the two solutions are identical which proves the correctness of the analysis performed. The solutions take into account the creep behavior and the increase of the coefficient of viscosity with time. A parametric study of the strain energy release rate is carried out by using the solutions derived. It is found that the strain energy release rate decreases with increasing of the coefficient of viscosity with time.

Keywords: longitudinal crack; beam structure; viscoelastic behavior; creep; continuous material inhomogeneity

1. Introduction

Continuously inhomogeneous materials are important structural materials which are widely used in various engineering areas such as aerospace, automotive industry, astronautics, nuclear reactors, biomedicine, energy conservation and electronics. The properties of structural members made of these modern materials vary gradually along one or more coordinates. Thus, the material properties are continuous functions of the coordinates. One of the advanced continuously inhomogeneous materials is the functionally graded material. Continuous variations of microstructure and material properties of functionally graded materials are obtained by gradually changing the proportions of the constituent materials along the desired directions in the solid (Avcar and Mohammed 2018, Butcher et al. 1999, Çalışgolu et al. 2011, 2015, Demir et al. 2013, Gasik 2010, Han et al. 2001, Hedia et al. 2014, Hirai and Chen 1999, Nemat-Allal et al. 2011, Saiyathibrahim et al. 2016, Udupa et al. 2014, Sofiyev and Avcar 2010, Sofiyev et al. 2012, Uslu

*Corresponding author, Professor, E-mail: V_RIZOV_FHE@UACG.BG

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Uysal and Kremzer 2015, Uslu Uysal 2015, 2016, Wu et al. 2014). Due to the fact that the properties of functionally graded materials can be formed technologically and vary smoothly in the material, these novel composites possess remarkable advantages in comparison with traditional laminated composite. The functionally graded materials find extensive application especially in development of structural members and components subjected to non-uniform exploitation requirements.

In order to design the continuously inhomogeneous (functionally graded) structural members safely, various aspects of the fracture behavior of continuously inhomogeneous materials should be carefully analyzed (Panigrahi and Pohit 2016, Wei et al. 2012, Erdogan 1995, Uslu Uysal and Güven 2016, Yang and Chen 2008).

Various problems of linear-elastic fracture mechanics of functionally graded materials have been formulated in Erdogan (1995). The deboning fracture behaviour of functionally graded coatings has been discussed too. The effects of residual and thermal stresses on the fracture have been considered. Delamination cracks in beam structural members with functionally graded surface coatings with linear-elastic behaviour have also been studied.

Functionally graded beam structures with edge cracks have been investigated theoretically assuming linear-elastic behavior of the material (Yang and Chen 2008). The beams under consideration are functionally graded along their thickness only. Cracked beams with different end supports such as clamped-clamped, clamped-free and hinged-hinged have been analyzed.

Fracture analyses of functionally graded beam configurations with axial loading have been developed in (Wei et al. 2012). The crack has been modeled by using a rotational spring. Free vibration of cracked beams with linear-elastic mechanical behavior has been studied. The effects of crack location and end supports have been evaluated and discussed. It has been shown that the analyses developed can be used for controlling of damaged functionally graded beams structures.

Fracture behavior of continuously inhomogeneous (functionally graded) beam structures has been studied analytically in Panigrahi and Pohit (2016). Various beam configurations have been analyzed by using the neutral surface as a reference. The material is functionally graded along the beam height. The influence of crack depth on the fracture behavior has been investigated and discussed.

The present paper is focused on analysis of the effect of increase of the coefficient of viscosity with time on the strain energy release rate for a longitudinal vertical crack in a continuously inhomogeneous beam structure that exhibits linear creep behavior. It should be noted that the previous works on longitudinal fracture of continuously inhomogeneous beam configurations do not consider the influence of creep behavior (Rizov 2017, 2018a, b, 2019, 2020, Rizov and Altenbach 2020). However, inhomogeneous materials and structures under long-lasting external loads usually have creep behavior that has to be taken into account in longitudinal fracture analyses. Therefore, developing of analyses with considering of creep is an important problem of fracture mechanics of inhomogeneous structures. Such analyses will contribute for improving of safety, reliability and durability of inhomogeneous structural members subjected to long-lasting loads. It should be mentioned that analysis of longitudinal fracture is an important problem also because continuously inhomogeneous (functionally graded) materials can be built-up layer by layer (Mahamood and Akinlabi 2017, Markworth et al, 1995, Miyamoto et al. 1999) which is premise for appearance of longitudinal cracks between layers. In the present paper, the time-dependent strain energy release rate is derived by considering the balance of the energy. The results obtained are confirmed by applying the compliance method. The beam under consideration is continuously inhomogeneous along the width, thickness and length. The creep is treated by a
viscoelastic model with one dashpot and three springs (the coefficient of viscosity of the dashpot increases exponentially with time). A parametric study is performed by using the time-dependent solution to the strain energy release rate.

2. Theoretical model. Analysis of the strain energy release rate

A viscoelastic beam configuration is depicted in Fig. 1. The beam has a rectangular cross-section of width, $b$, and thickness, $h$. The length of the beam is $2(l_1 + l)$. The beam is subjected to four-point bending by two vertical forces, $F$, applied at the ends of the beam. A notch of depth, $b_1$, is cut-out in the lateral surface of the beam as shown in Fig. 1. A longitudinal vertical crack of length, $2a$, is located symmetrically with respect to the mid-span. The cross-sections of the right-hand and left-hand crack arms are rectangles of widths, $b_1$ and $b_2$, and thickness, $h$. The notch divides the right-hand crack arm in two symmetric parts of length, $a$. It is obvious that the ring-hand crack arm is free of stresses. It should be mentioned that the crack is located in beam portion, $D_2D_6$, which is loaded in pure bending (Fig. 1).

The beam exhibits linear creep behavior that is described by using the viscoelastic model shown schematically in Fig. 2. The model is structured by one dashpot with coefficient of viscosity, $\eta_0t$, and three springs with modulii of elasticity, $E_a$, $E_b$ and $E_c$. The increase of $\eta_0t$ with time is expressed as

$$\eta_0t = \eta_0e^{\rho t},$$

where $t$ is time, $\eta_0$ is the value of the coefficient of viscosity at $t = 0$, $\rho$ is a material property ($\rho > 0$).

For the viscoelastic model under constant applied stress, $\sigma \tau^*$, it can be written that

$$\sigma = \sigma_\eta + \sigma_b,$$

$$\sigma_a = E_a \varepsilon_a, \quad \sigma_b = E_b \varepsilon_b,$$

$$\sigma_\eta = \eta_0t \varepsilon_\eta \tau^*,$$

$$\varepsilon_\eta + \varepsilon_a = \varepsilon_b,$$

$$\varepsilon_b = \varepsilon,$$

where $\sigma_a$, $\sigma_b$ and $\sigma_\eta$ are the stresses in the springs with modulii of elasticity, $E_a$ and $E_b$, and in the dashpot, respectively. The strains in the springs with modulii of elasticity, $E_a$ and $E_b$, and in the dashpot are denoted by $\varepsilon_a$, $\varepsilon_b$ and $\varepsilon_\eta$, respectively.
By combining (1)-(8), one obtains

$$\sigma = \eta_0 e^{\rho t} \left[ \dot{\varepsilon} - \frac{1}{E_a} \left( \ddot{\sigma} - E_b \dot{\varepsilon} \right) \right] + E_b \varepsilon, \quad (9)$$

Since

$$\dot{\sigma} = 0, \quad (10)$$

Eq. (9) takes the form

$$\sigma = \eta_0 e^{\rho t} \left( \dot{\varepsilon} + \frac{E_b}{E_a} \dot{\varepsilon} \right) + E_b \varepsilon. \quad (11)$$
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By solving of (11) with respect to the strain, $\varepsilon$, and adding the strain in the spring with modulus of elasticity, $E_c$, one obtains the following stress-strain-time relationship for the viscoelastic model shown in Fig. 1

$$
\varepsilon = \frac{\sigma}{E_b} \left[ 1 - e^{\frac{E_b}{E_a + E_b}(e^{-\rho t} - 1)} \right] + \sigma \frac{e^{\frac{E_b}{E_a + E_b}(e^{-\rho t} - 1)}}{E_a + E_b} + \frac{\sigma}{E_c} \varepsilon
$$

where

$$Q_\varepsilon = \frac{\eta_0(E_a + E_b)}{E_a} \varepsilon
$$

The beam is made of material that is continuously inhomogeneous in width, thickness and length directions. Therefore, the modulii of elasticity and the coefficient of viscosity of the viscoelastic model vary continuously along the width, thickness and length of the beam. The distributions of the modulii of elasticity and the coefficient of viscosity in the beam cross-section are written as

$$E_a = E_{a0} e^{f_a z} e^{\frac{b+y_3}{b} + g_a z} e^{\frac{h+z_3}{h}}$$

$$E_b = E_{b0} e^{f_b z} e^{\frac{b+y_3}{b} + g_b z} e^{\frac{h+z_3}{h}}$$

$$E_c = E_{c0} e^{f_c z} e^{\frac{b+y_3}{b} + g_c z} e^{\frac{h+z_3}{h}}$$

$$\eta = \eta_{0\tau} e^{f_\eta z} e^{\frac{b+y_3}{b} + g_\eta z} e^{\frac{h+z_3}{h}}$$

where

$$-\frac{b}{2} \leq y_3 \leq \frac{b}{2}$$

$$-\frac{h}{2} \leq z_3 \leq \frac{h}{2}$$

In formulae (14)-(19), $E_{a0}$, $E_{b0}$, $E_{c0}$ and $\eta_{0\tau}$ are, respectively, the values of $E_a$, $E_b$, $E_c$ and the coefficient of viscosity, $\eta_\tau$, in the upper left-hand vertex of the beam cross-section, $f_a$, $f_b$, $f_c$, and $f_\eta$ are, respectively, material properties which control the distributions of $E_a$, $E_b$, $E_c$ and $\eta_\tau$ along the beam width, $g_a$, $g_b$, $g_c$ and $g_\eta$ are, respectively, material properties which control the distributions of $E_a$, $E_b$, $E_c$ and $\eta_\tau$ along the beam thickness, $y_3$ and $z_3$ are the centroidal axes of the beam cross-section.

Along the beam length, the distributions of $E_{a0}$, $E_{b0}$, $E_{c0}$ and $\eta_\tau$ are expressed as

$$E_{a0} = E_{a0b} e^{s_a x} e^{\frac{x_3}{X}}$$
In formulae (20)-(24), $E_{a0B}$, $E_{b0B}$, $E_{c0B}$ and $\eta_{0B}$ are, respectively, the values of $E_{a0}$, $E_{b0}$, $E_{c0}$ and $\eta_0$ at the ends of the beam, $x_3$ is the longitudinal centroidal axis of the beam, $s_a$, $s_b$, $s_c$ and $s_\eta$ are material properties controlling the distributions of $E_{a0}$, $E_{b0}$, $E_{c0}$ and $\eta_0$ along the beam length, respectively. At

$$l_1 + l \leq x_3 \leq 2(l_1 + l),$$  \hspace{1cm} (25)$$

the distributions of $E_{a0}$, $E_{b0}$, $E_{c0}$ and $\eta_0$ are written as

$$E_{a0} = E_{a0B} e^{s_a \frac{2(l_1+l)-x_3}{l_1+l}},$$  \hspace{1cm} (26)$$

$$E_{b0} = E_{b0B} e^{s_b \frac{2(l_1+l)-x_3}{l_1+l}},$$  \hspace{1cm} (27)$$

$$E_{c0} = E_{c0B} e^{s_c \frac{2(l_1+l)-x_3}{l_1+l}},$$  \hspace{1cm} (28)$$

$$\eta_0 = \eta_{0B} e^{s_\eta \frac{2(l_1+l)-x_3}{l_1+l}}.$$  \hspace{1cm} (29)$$

Formulae (20)-(29) indicate that the modulii of elasticity and the coefficient of viscosity are distributed symmetrically with respect to the mid-span.

A time-dependent solution to the strain energy release rate for the longitudinal crack in Fig. 1 is derived by considering the balance of the energy. The solution takes into account the linear creep behavior and the increase of the coefficient of viscosity with time. Due to the symmetry, only half of the beam, $D_4D_7$, is analyzed (Fig. 1). The balance of the energy is written as

$$F \delta w = \frac{\partial U}{\partial a} \delta a + G h \delta a,$$  \hspace{1cm} (30)$$

where $G$ is the strain energy release rate, $w$ is the vertical displacement of the application point, $H$, of the external force (Fig. 1), $U$ is the time-dependent strain energy cumulated in half of the beam, $\delta a$ is a small increase of the crack length.

From (30), one derives

$$G = 2 \left( \frac{F \left( \frac{\partial w}{\partial a} - \frac{1}{h} \frac{\partial U}{\partial a} \right)}{h} \right).$$  \hspace{1cm} (33)$$
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The right-hand side of (31) is doubled in view of the symmetry (Fig. 1). The time-dependent strain energy in half of the beam is written as

\[ U = U_1 + U_2, \]

where \( U_1 \) and \( U_2 \) are the strain energies in portion, \( D_4D_5 \), of the left-hand crack arm, and in portion, \( D_5D_6 \), of the beam. It should be mentioned that the strain energy cumulated in portion, \( D_6D_7 \), of the beam is not involved in (32), respectively in (31) since this strain energy does not depend on the crack length.

The time-dependent strain energy cumulated in portion, \( D_4D_5 \), of the left-hand crack arm is found as (Fig. 3)

\[ U_1 = \int_0^a \int_{-b_3}^{b_3} \int_{-h}^h u_{01} \, dx_1 \, dy_1 \, dz_1, \]

where the time-dependent strain energy density, \( u_{01} \), is written as

\[ u_{01} = \frac{1}{2} \sigma \varepsilon. \]

By applying the Bernoulli’s hypothesis for plane sections, the distribution of strains in the cross-section of the left-hand crack arm is expressed as

\[ \varepsilon = \varepsilon_{C_1} + \kappa_1 y_1 + \kappa_2 z_1, \]

where \( \varepsilon_{C_1} \) is the strain in the centre of the cross-section, \( \kappa_1 \) and \( \kappa_2 \) are the curvatures in the \( x_1y_1 \) and \( x_1z_1 \) planes, respectively. It should be mentioned that the Bernoulli’s hypothesis is applicable since beams of high length to thickness ratio are considered in the present paper.

The following time-dependent equations for equilibrium of the elementary forces in the cross-
section of the left-hand crack arm are used in order to determine $\varepsilon_{C_1}$, $\kappa_1$ and $\kappa_2$

\begin{align*}
N_1 &= \int_{-\frac{b_2}{2}}^{\frac{b_2}{2}} \frac{h}{Z} \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma d y_1 d z_1, \quad (36) \\
M_{y_1} &= \int_{-\frac{b_2}{2}}^{\frac{b_2}{2}} \frac{h}{Z} \int_{-\frac{h}{2}}^{\frac{h}{2}} \alpha z_1 d y_1 d z_1, \quad (37) \\
M_{z_1} &= \int_{-\frac{b_2}{2}}^{\frac{b_2}{2}} \frac{h}{Z} \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma y_1 d y_1 d z_1, \quad (38)
\end{align*}

where $N_1$ is the axial force, $M_{y_1}$ and $M_{z_1}$ are the bending moments with respect to $y_1$ and $z_1$. It is obvious that

\begin{align*}
N_1 &= 0, \quad (39) \\
M_{y_1} &= Fl, \quad (40) \\
M_{z_1} &= 0. \quad (41)
\end{align*}

By combining of (12) and (35), one obtains

\begin{equation}
\sigma = \left( \frac{\varepsilon_{C_1} + \kappa_1 y_1 + \kappa_2 z_1}{\varepsilon} \right) \left\{ \frac{1}{E_b} \left[ 1 - \frac{E_b}{E_a} \left( e^{-\rho t} - 1 \right) \right] + \frac{E_b}{E_a + E_b} \right\}^{-1} \cdot \varepsilon. \quad (42)
\end{equation}

After substituting of (42) in (36), (37) and (38), the equations for equilibrium are solved with respect to $\varepsilon_{C_1}$, $\kappa_1$ and $\kappa_2$ at various values of time by using the MatLab computer program.

The time-dependent strain energy in beam portion, $D_5D_6$, is written as

\begin{equation}
U_2 = \int_{-\frac{b_2}{2}}^{\frac{b_2}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} u_{02} d x_1 d y_2 d z_2, \quad (43)
\end{equation}

where $u_{02}$ is the time-dependent strain energy density in this beam portion, $y_2$ and $z_2$ are the centroidal axes of the cross-section. The strain in the centre of the cross-section and the curvatures of the beam in portion, $D_5D_6$, are determined by using the equations for equilibrium (36), (37) and (38). For this purpose, $b_2$ and $\sigma$ are replaced with $b$ and $\sigma_{D_5D_6}$, respectively. The stress, $\sigma_{D_5D_6}$, in the beam portion, $\sigma_{D_5D_6}$, is expressed by replacing $\varepsilon_{C_1}$, $\kappa_1$ and $\kappa_2$ with the strain in the centre and curvatures of the beam portion, $\sigma_{D_5D_6}$, in formula (42).

The vertical displacement, $w$, that is involved in the expression for the time-dependent strain energy release rate (31) is derived by applying the theorem of Castigliano

\begin{equation}
w = \frac{\partial U}{\partial F}. \quad (44)
\end{equation}
By substituting of (32), (33), (43) and (44) in (31), one derives

\[ G = \frac{2}{h} \frac{\partial}{\partial F} \int_{b_2}^{b_1} \left( \int_{-b_2}^{b_2} u_{01} dx_1 dy_1 dz_1 \right) + \int_{b_2}^{b_1} \left( \int_{-b_2}^{b_2} u_{02} dx_1 dy_2 dz_2 \right) \]

where the derivative, \( \frac{\partial}{\partial F} \), is obtained by the MatLab computer program. The integration in (45) is carried-out also by using the MatLab. The time-dependent solution (45) is applied to calculate the strain energy release rate at various values of time.

A time-dependent solution to the strain energy release rate is derived also by using the compliance method. For this purpose, the strain energy release rate is written as

\[ G = 2 \left( \frac{1}{2h} \int_{b_2}^{b_1} C \int_{-b_2}^{b_2} dC \right) \]

where \( C \) is the compliance of the beam. The right-hand side of (46) is doubled in view of the symmetry. The compliance is defined as

\[ C = \frac{w}{F} \]

where \( w \) is found by using (44).

By combining of (32), (33), (43), (44), (46) and (47), one obtains

\[ G = \frac{F}{h} \frac{\partial}{\partial F} \int_{b_2}^{b_1} \left( \int_{-b_2}^{b_2} u_{01} dx_1 dy_1 dz_1 \right) + \int_{b_2}^{b_1} \left( \int_{-b_2}^{b_2} u_{02} dx_1 dy_2 dz_2 \right) \]

The integration in (48) is performed by the MatLab computer program. The derivative, \( \frac{\partial}{\partial F} \), is found also by using the MatLab.

It should be noted that the strain energy release rates obtained by (48) match exactly these calculated by (45). This fact proves the correctness of the time-dependent solutions to the strain energy release rate derived in the present paper.

3. Parametric investigation

The time-dependent solution to the strain energy release rate (45) is applied here in order to carry-out a parametric investigation.

The strain energy release rate is expressed in non-dimensional form by using the formula \( G_N = G/(E_{00b}h) \). The purpose of the parametric investigation is to evaluate the influence of the increasing coefficient of viscosity with time on the strain energy release rate. The influences of the length and location of the longitudinal crack and the material inhomogeneity along the width,
Fig. 4 The strain energy release rate in non-dimensional form presented as a function of non-dimensional time (curve 1 – at \( \rho = 0.5 \), curve 2 – at \( \rho = 1.0 \) and curve 3 – at \( \rho = 1.5 \)).

thickness and length of the beam on the strain energy release rate are also evaluated. It is assumed that \( b = 0.015 \) m, \( h = 0.010 \) m, \( l_1 = 0.020 \) m, \( l = 0.040 \) m and \( F = 10 \) N.

First, the change of the strain energy release rate with time is evaluated. For this purpose, calculations of the strain energy release rate are carried-out at various values of time. In order to evaluate the effect of increase of the coefficient of viscosity with time, the strain energy release rate is obtained for three values of \( \rho \). The strain energy release rate in non-dimensional form is presented as a function of non-dimensional time in Fig. 4 for three values of \( \rho \). The time is expressed in non-dimensional form by using the formula \( tN = tE_{0AB}/\eta_{0BL} \). It can be observed in Fig. 4 that the strain energy release rate increases with time (this behavior is due to creep). The curves in Fig. 4 indicate that the strain energy release rate decreases with increasing of \( \rho \). Therefore, it can be concluded that the increase of the coefficient of viscosity with time leads to decrease of the strain energy release rate (physically, this is due to decrease of the deformability).

The influence of material inhomogeneity in the beam cross-section on the strain energy release rate is analyzed too. The findings of this analysis are shown in Fig. 5. First, the influence of the continuous variation of the value of \( E_a \) along the width and thickness of the beam cross-section on the strain energy release rate is studied by performing calculations of the strain energy release rate at various values of \( f_a \) and \( g_a \). The results of these calculations are illustrated in Fig. 5(a) where the strain energy release rate in non-dimensional form is presented as a function of \( f_a \) for three values of \( g_a \). The curves in Fig. 5(a) indicate that the strain energy release rate decreases with increasing of \( f_a \). The increase of material property, \( g_a \), also leads to decrease of the strain energy release rate (Fig. 5(a)). The reason for this behavior is increase of the beam stiffness.

An investigation of the effect of the continuous change of \( E_b \) in the width and thickness direction of the beam cross-section is carried-out by calculating the strain energy release rate at various values of \( f_b \) and \( g_b \). The results obtained are shown in Fig. 5(b). One can observe in Fig. 5(b) that the strain energy release rate decreases with increasing of \( f_b \) and \( g_b \) (this is explained by increase of the stiffness).

A similar investigation of the effect of the continuous variation of the value of modulus of elasticity, \( E_c \), along the width and thickness of the cross-section is performed. The calculated strain energy release rate is presented in non-dimensional form as a function of \( f_c \) in Fig. 5(c) at three values of \( g_c \). The curves in Fig. 5(c) show that the increase of \( f_c \) and \( g_c \) leads to decrease of the strain energy release rate.
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Fig. 5 The strain energy release rate in non-dimensional form presented as a function of (a) $f_a$ (curve 1 – at $g_a = 0.5$, curve 2 – at $g_a = 1.0$ and curve 3 – at $g_a = 2.0$); (b) $f_b$ (curve 1 – at $g_b = 0.5$, curve 2 – at $g_b = 1.0$ and curve 3 – at $g_b = 2.0$); (c) $f_c$ (curve 1 – at $g_c = 0.5$, curve 2 – at $g_c = 1.0$ and curve 3 – at $g_c = 2.0$); (d) $f_\eta$ (curve 1 – at $g_\eta = 0.5$, curve 2 – at $g_\eta = 1.0$ and curve 3 – at $g_\eta = 2.0$)

Fig. 6 The strain energy release rate in non-dimensional form presented as a function of (a) $s_a$ (curve 1 – at $s_b = 0.5$, curve 2 – at $s_b = 1.0$ and curve 3 – at $s_b = 2.0$); (b) $s_c$ (curve 1 – at $b_2/b = 0.3$, curve 2 – at $b_2/b = 0.6$ and curve 3 – at $b_2/b = 0.9$); (c) $s_\eta$ (curve 1 – at $a/l = 0.25$, curve 2 – at $a/l = 0.5$ and curve 3 – at $a/l = 0.75$)
Calculations of the strain energy release rate are carried-out also at various values of $f_\eta$ and $g_\eta$ in order to assess the effect of the continuous variation of the coefficient of viscosity along the width and thickness of the beam cross-section. The strain energy release rate in non-dimensional form is presented as a function of $f_\eta$ at three values of $g_\eta$ in Fig. 5(d). One can observe in Fig. 5(d) that the strain energy release rate decreases with increasing of $f_\eta$ and $g_\eta$.

The effect of continuous material inhomogeneity along the beam length on the strain energy release rate is also analyzed. The results obtained are shown in graphical form in Fig. 6. First, the effect of continuous variation of the modulii of elasticity, $E_a$ and $E_b$, along the beam length is studied. For this purpose, calculations of the strain energy release rate are performed at various values of $s_a$ and $s_b$. One can get an idea for the effect of continuous variation of $E_a$ and $E_b$ along the beam length from Fig. 6(a) where the strain energy release rate in non-dimensional form is presented as a function of $s_a$ at three values of $s_b$. It can be observed in Fig. 6a that the strain energy release rate decreases with increasing of $s_a$. The curves in Fig. 6(a) indicate also that increase of $s_b$ leads to decrease of the strain energy release rate. These findings are explained by increase of the stiffness.

The influence of the crack location along the beam width and the continuous variation of the modulus of elasticity, $E_c$, along the beam length on the strain energy release rate is illustrated in Fig. 6(b) where the strain energy release rate in non-dimensional form is presented as a function of $s_c$ for three values of $b_2/b$ ratio (this ratio characterizes the location of the crack along the beam width). One can observe in Fig. 6(b) that the strain energy release rate decreases with increasing of $s_c$. Concerning the influence of crack location along the beam width, the curves in Fig. 6(b) show that the strain energy release rate decreases with increasing of $b_2/b$ ratio. This behavior is due to increase of the stiffness of the left-hand crack arm with increasing of $b_2/b$ ratio.

Finally, the influences of the crack length and the continuous variation of the coefficient of viscosity along the beam length on the strain energy release rate are evaluated by performing calculations of the strain energy release rate at various values of $a/l$ ratio and $s_\eta$. The strain energy release rate in non-dimensional form is presented as a function of $s_\eta$ in Fig. 6(c) at three values of $a/l$ ratio. It can be observed that the strain energy release rate decreases with increasing of $s_\eta$ (Fig. 6(c)). The increase of $a/l$ ratio (this ratio characterizes the crack length) leads to increase of the strain energy release rate (Fig. 6(c)).

4. Conclusions

The strain energy release rate for a longitudinal crack in a continuously inhomogeneous beam configuration under linear creep is analyzed. The creep behavior is described by a linear viscoelastic model structured by one dashpot and three springs. The coefficient of viscosity of the dashpot increases with time. The beam exhibits continuous material inhomogeneity along the width, thickness and length. Therefore, the modulii of elasticity of springs and the coefficient of viscosity vary continuously in the width, thickness and length directions of the beam. Time-dependent solutions to the strain energy release rate that take into account the creep and the increase of viscosity with time are derived by using two approaches (by considering the energy balance and by applying the compliance method). The results yielded by the two approaches are identical. The solutions to the strain energy release rate are applied to perform a parametric study. It is found that the strain energy release rate increases with time (this behavior is due to the creep).
Concerning the effect of the increase of the coefficient of viscosity with time, the study reveals that the strain energy release rate decreases with increase of the coefficient of viscosity. The effects of continuous material inhomogeneity along the width, thickness and length of the beam, the crack length and the crack location along the beam width on the strain energy release rate are also assessed. The calculations indicate that the strain energy release rate decreases with increasing of material properties, $f_a$, $g_a$, $f_b$, $g_b$, $f_c$, $g_c$, $f_\eta$, $g_\eta$, $s_a$, $s_b$, $s_c$ and $s_\eta$ (these material properties control the variation of the coefficient of viscosity and the modulii of elasticity along the width, thickness and length of the beam). It is found that the strain energy release rate decreases with increasing of $b_x/b$ ratio. The increase of the crack length leads to increase of the strain energy release rate.

References


