

Multi-variate Empirical Mode Decomposition (MEMD) for ambient modal identification of RC road bridge

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Abstract. In this paper, an adaptive MEMD based modal identification technique for linear time-invariant systems is proposed employing multiple vibration measurements. Traditional empirical mode decomposition (EMD) suffers from mode-mixing during sifting operations to identify intrinsic mode functions (IMF). MEMD performs better in this context as it considers multi-channel data and projects them into a n -dimensional hypercube to evaluate the IMFs. Using this technique, modal parameters of the structural system are identified. It is observed that MEMD has superior performance compared to its traditional counterpart. However, it still suffers from mild mode-mixing in higher modes where the energy contents are low. To avoid this problem, an adaptive filtering scheme is proposed to decompose the interfering modes. The Proposed modified scheme is then applied to vibrations of a reinforced concrete road bridge. Results presented in this study show that the proposed MEMD based approach coupled with the filtering technique can effectively identify the parameters of the dominant modes present in the structural response with a significant level of accuracy.

Keywords: hilbert transform; Multi-variate Empirical Mode Decomposition; intrinsic mode function; operational modal analysis; modal parameters of bridge

1. Introduction

The output-only vibrations based modal identification techniques have shown excellent capabilities in the health monitoring of civil infrastructure. Researchers use different time, frequency, and time-frequency domain algorithms, and many literature can be found on this topic (Maria and Silva 2001, Zhao *et al.* 2018, Li *et al.* 2014, Mahato and Chakraborty 2016, Mahato *et al.* 2020). Recently, robust signal processing methods including blind source separation (BSS) principles (Antoni *et al.* 2004, Hazra *et al.* 2010, Huang and Nagarajaiah 2014, Yang *et al.* 2020), wavelet transformation (Staszewski and Robertson 2007, Mahato and Chakraborty 2019), and Hilbert-Huang transform (Huang *et al.* 1998, Yang *et al.* 2003, 2004, Chen *et al.* 2017, Mahato *et al.* 2017) have been investigated in numerous structural health monitoring applications. Unlike any other time-frequency decomposition tools, EMD can deal with the nonlinear and non-stationary

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signals using an adaptive transformation and therefore has gained significant popularity in structural condition assessment. In practical applications, the EMD technique uses only a single sensor measurement to obtain a subset of the modal information (Yang *et al.* 2003). Because of its self-adaptive nature without requiring a basis function, the EMD technique has become popular among the researchers in the time-frequency domain. Originally developed by Huang *et al.* (1998), the EMD decomposes a signal into several oscillatory waveforms called IMF. These IMFs are obtained by multiple averaging and interpolating the raw signal, which is known as sifting.

However, this operation introduces a significant amount of mode-mixing in the resulting IMFs (Huang *et al.* 2003). Recently, EMD has been successfully employed for modal identification of structural systems in conjunction with Hilbert Transform (e.g., Hilbert-Huang Transform) (Peng *et al.* 2005a, b, Mahato *et al.* 2015), where a set of band-pass filters are used to alleviate the mode-mixing. Researchers often use EMD with BSS (Hazra *et al.* 2012a, b) and random decrement technique (RDT) (He *et al.* 2011) to extract the modal information from limited measurements. However, these techniques require significant user intervention in designing the band-pass filters, which are a prerequisite for further analysis. A noise assisted technique commonly called the ensemble EMD (EEMD) method (Wu and Huang 2009) has been developed to evade the mode-mixing. In EEMD, a synthetic noise is introduced repeatedly, followed by an ensemble-averaging of the resulting IMFs, making it computationally prohibitive. The standard EMD method primarily works only for a particular signal, which is obtained from one sensor only.

While dealing with multiple sensors, it faces two problems (Rehman and Mandic 2010a). First, it is not secured that the number of IMFs, obtained from different sensors is the same i.e., it often varies from sensor to sensor. Secondly, because the signals from multiple sensors are treated individually, EMD cannot provide the combined information. With this in view, a multivariate version of EMD is recently introduced to exploit the advantage of data fusion (Rehman and Mandic 2010b). In this technique (i.e., MEMD), multi-dimensional envelopes are created using the projections of the raw signal along the different directions. Then the average of these envelopes is utilized to obtain the local mean. The MEMD technique has been successfully used to separate the source signals of multi-channel measurements in biomedical applications (Fleureau *et al.* 2011) and mechanical systems (Zhao *et al.* 2012). However, the mode mixing is still an issue in higher modes having low energy contributions (Zhao *et al.* 2012) as in the standard EMD, although in a much lesser proportion.

In recent, MEMD is used for modal parameters identification in the laboratory environment (Sadhu 2017). Similar to the previous cases, in this study, the mode-mixing phenomena is reported as a major hinder point on the applicability of MEMD towards structural health monitoring (SHM). Here, MEMD and EEMD are combinedly used to circumvent this issue. In the available literature, the MEMD technique's effectiveness has not been explored towards condition assessment of a field structure where there is very little control over the measurement noise, vibration magnitudes, and other parameters.

In this paper, the MEMD technique is implemented towards the modal identification of structures with adaptive filtering. The proposed methodology combines the merits of MEMD and filtering schemes to circumvent mode-mixing while dealing with multi-channel vibration measurements. Thus, two main contributions of this work are – (1) implementation of MEMD for modal identification of structures and (2) development of a new adaptive version tailored towards modal identification of real-life structures in a practical scenario.

2. EMD & its variants for modal identification

Empirical mode decomposition is a powerful signal decomposition tool for nonlinear and non-stationary signals. An IMF generated for this purpose must satisfy the following two conditions – (1) the number of extrema and the number of zero-crossings must be either equal or differ at most by one, and (2) at any point, the mean value of the envelope defined by local maxima and minima is zero. The sifting process is implemented by identifying local extrema in a data ($y(t)$) between successive pairs of zero crossings and connecting them by a cubic spline line to create an envelope. If the envelope mean is e_1 , the difference $y(t) - e_1 = I_1$ is the first IMF, provided it satisfies the necessary conditions. If not, the sifting process is repeated by treating I_1 as the original data until an IMF is obtained. The IMF is then subtracted from the original signal, and the sifting is continued to decompose the data into multiple IMFs (Huang *et al.* 1998)

$$y(t) = \sum_{j=1}^n I_j(t) + \epsilon_n \quad (1)$$

where, I_j represents j^{th} IMF and ϵ_n the residue left after EMD. Due to successive interpolation operation in the sifting process, IMFs obtained by the EMD are sensitive to noise and results in mode-mixing (Huang *et al.* 2003).

To alleviate this problem, different adaptive filtering schemes are developed by the researchers (Peng *et al.* 2005a, b, Ong *et al.* 2008) to improve the performance of the traditional EMD scheme. In this context, Chen and Wang (2012) have proposed analytical mode decomposition (AMD) where the original signal is multiplied by a sinusoid with known frequency to identify low pass and high pass components using the Bedrosian theorem. The process is continued by varying the frequency of the masking signal to identify the modal frequencies present in the response.

2.1 Multi-variate EMD

Traditionally, EMD is based on the computation of local mean by averaging the envelopes. However, for multivariate signals, the local extrema may not be well defined. Moreover, the concept of modes in IMFs is not evident for multivariate signals. To address these issues, multiple envelopes are constructed by projecting the signal in a n -dimensional space, as proposed by Rehman and Mandic (2010b, 2011), which are averaged to obtain the local mean. It is a generalization of the bivariate and trivariate EMD (Rehman and Mandic 2010a).

The estimation of local mean entails finding a suitable set of direction vectors to perform integration of all envelopes in the n -dimensional space. The procedure involves uniform angular sampling along a n -hypersphere, a generalization of an ordinary sphere's surface to an arbitrary dimension. For any natural number n , a n -hypersphere of radius is defined as the set of points in $(n + 1)$ -dimensional Euclidean space. Let $\{\theta_i: i = 1 - (n - 1)\}$ be $(n - 1)$ angular coordinates, then a n -dimensional coordinate system having $\{x_i\}_{i=1}^n$ as the n coordinates on a unit $(n - 1)$ sphere are given by

$$x_{n-1} = \left(\prod_{i=1}^{n-2} \sin \theta_i \right) \cos(\theta_{n-1}) \quad (2)$$

For example, a 2-dimensional case in the 3 coordinates can be written as

$$\begin{aligned}x_1 &= \cos \theta_1 \\x_2 &= \sin \theta_1 \times \cos \theta_2 \\x_3 &= \sin \theta_1 \times \sin \theta_2 \times \cos \theta_3\end{aligned}\quad (3)$$

The uniform angular sampling is adequate for bivariate signals, as it produces non-uniformly distributed samples. For trivariate signals, it generates the points with greater concentration at poles of the sphere (Rehman and Mandic 2010a). To address this problem, a low-discrepancy Hammersley (Rehman and Mandic 2010a, b) sampling scheme is used to generate the direction vectors in 4–dimension space. The discrepancy estimate for Hammersley sampling is better compared to other sampling methods (like importance sampling, uniform angular sampling). Thus, it provides more uniformly distributed sampling on a sphere (Rehman and Mandic 2010b). In turn, it gives a suitable set of direction vectors for generating signal projections and the corresponding signal envelopes, ensuring enhanced local mean estimates.

Consider a sequence of n -dimensional vectors $y(t) = \{y_1(t), y_2(t), \dots, y_n(t)\}$ that represents a multivariate signal with n -components, and $D_k = \{d_1^k, d_2^k, \dots, d_n^k\}$ denotes a set of direction vectors along the k^{th} directions on a $(n - 1)$ sphere. Then MEMD is performed using the following steps (Rehman and Mandic 2011)

- Choose a suitable set of direction vectors, D , using Hammersley sequence
- Calculate the k^{th} projection, $p^k(t)$ of the input signal $y(t)$ along the k^{th} direction vector, X^k , for all k (i.e., $k = 1, 2, \dots, L$ where L is the total number of direction vectors in D)
- Find the time instants, t_i^k corresponding to the maxima of the projected signal, $p^k(t)$ for all k
- Interpolate $[t_i^k, y(t_i^k)]$ to obtain multivariate envelopes, $e^k(t)$, for all k
- For a set of L direction vectors, the mean $E(t)$ of the envelope curves is obtained as

$$E(t) = \frac{1}{L} \sum_{k=1}^L e^k(t) \quad (4)$$

- Extract the residual $r(t)$ using $r(t) = y(t) - E(t)$. If $r(t)$ satisfies the stoppage criterion for a multivariate IMF, apply the above steps to $(y(t) - r(t))$ to extract the IMF ($\tilde{i}(t)$), otherwise apply it on $r(t)$

3. Proposed adaptive MEMD approach

MEMD based signal processing has shown promising results in time-frequency analysis (Rehman and Mandic 2011). Using these futures, an adaptive MEMD based modal identification of the vibrating system is proposed next. For this purpose, a multi-degrees-of-freedom (MDOF) system is considered, which can be described as follows

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{F}(t) \quad (5)$$

Here, the structural properties are represented by \mathbf{M} , \mathbf{K} , and \mathbf{C} (i.e., mass, stiffness, and damping matrices respectively), where \mathbf{X} is the state vector representing displacement while differentiation with respect to time is represented by the over-dot. External force time-histories are represented by

vector $\mathbf{F}(t)$. The coupled differential equations are solved by transforming them into modal coordinates using the following orthogonal transformation

$$\mathbf{X} = \sum_{j=1}^n \phi_j \mathbf{z}_j(t) \quad (6)$$

Using Eq. (6) in Eq. (5), one obtains

$$\ddot{\mathbf{z}}_j(t) + 2\eta_j \omega_{n_j} \dot{\mathbf{z}}_j(t) + \omega_{n_j}^2 \mathbf{z}_j(t) = \mathbf{f}_j(t) \quad (7)$$

Where η_j, ω_{n_j} and \mathbf{f}_j are the modal damping, natural frequency and modal force, respectively.

To extract the modal response from the recorded acceleration data, the proposed adaptive MEMD method is used here. Using this technique on the multi-channel acceleration records, a set of IMFs with a residual error are obtained as follows

$$MEMD[\ddot{\mathbf{X}}(t)] = MEMD \left[\sum_{j=1}^n \phi_j \mathbf{z}_j(t) \right] = \sum_{j=1}^n \mathbf{c}_j(t) + \mathbf{r}(t) \quad (8)$$

Ideally, if MEMD can extract the modal response, the above equation indicates that the j^{th} IMF would correspond to j^{th} mode i.e., $\ddot{\mathbf{z}}_j(t)$. However, due to mode-mixing j^{th} IMF may contain effects from the surrounding modes. The contiguous modes result in mode-mixing as can be observed from the amplitude spectra of the respective IMFs. To separate these modes, adaptive band-pass filtering based on a user-defined bandwidth level is proposed in this study. Using the user-defined bandwidth parameters, the responses in different DOFs are filtered and MEMD is invoked to identify the modal frequencies present in the respective band. This filtering and successive multivariate decomposition process are repeated to identify the dominant modes present in the structural response within a particular frequency band where the mode mixing has occurred. The IMFs obtained from the above scheme have two distinct constituents – (a) the synchronous motions in modal coordinates and (b) the effects of the excitation frequencies, if any. These two constituents can be delineated using the instantaneous phase of the filtered IMFs corresponding to each channel. Once the modal frequencies are identified from filtered IMFs, the mode shapes can be estimated from the respective IMFs using finite element model updating (Mahato *et al.* 2015).

Once the natural frequencies and mode shapes are identified, the remaining task is to evaluate the modal damping ratios. Considering stationary Gaussian white noise forcing function with zero mean, the filtered IMFs obtained from the previous step contain both free and forced modal responses. Estimating modal damping ratios directly from the filtered IMFs can be erroneous due to the non-decaying nature of the forced modal responses. This can be addressed using RDT as proposed in the literature (Cheng *et al.* 1982). Using this technique, the free modal response $\ddot{\mathbf{z}}^{fr}(\tau)$ can be obtained i.e.

$$\ddot{\mathbf{z}}^{fr}(\tau) = \frac{1}{R} \sum_{i=1}^R \ddot{\mathbf{z}}(t_i + \tau) \quad (9)$$

Here, R represents the triggering value corresponding to a time-point t_i and τ represents the lag parameter. Using Eq. (9), the free-response corresponding to j^{th} mode is given by

$$\ddot{\mathbf{z}}^{fr}(t) = e^{-\eta_j \omega_{n_j} t} A_j \cos(\omega_{d_j} t - \phi_j) \quad (10)$$

where the constant terms are as follows

$$\begin{aligned} A_j &= \sqrt{C_{1j}^2 + C_{2j}^2} & \phi_j &= \tan^{-1} \left(\frac{C_{2j}}{C_{1j}} \right) \\ C_{1j} &= \eta_j^2 \omega_{n_j}^2 a_j + 2\eta_j \omega_{n_j} b_j + \omega_{d_j} a_j \\ C_{2j} &= \frac{\omega_{d_j}^2 b_j - \eta_j^2 \omega_{n_j}^2 b_j - \eta_j^3 \omega_{n_j}^3 a_j - \eta_j \omega_{n_j} \omega_{d_j}^2 a_j}{\omega_{d_j}} \end{aligned} \quad (11)$$

In the above equation, a_j and b_j are the two constants that depend on the initial conditions and ω_{d_j} is the damped natural frequency (i.e., $\omega_{d_j} = \omega_{n_j} \sqrt{1 - \eta_j^2}$). Instantaneous amplitude and phase can be obtained by applying HT on the free-response given in Eq. (10) i.e.

$$S(t) = A_j e^{-\eta_j \omega_{n_j} t} \quad (12)$$

and

$$\theta(t) = \omega_{d_j}(t) - \phi_j \quad (13)$$

The damped natural frequency ω_{d_j} can be estimated using the average slope of the instantaneous phase in Eq. (13). Finally, to estimate the j^{th} modal damping ratio, the logarithm of Eq. (12) is taken which leads to

$$\ln\{S(t)\} = -\eta_j \omega_{n_j} t + \ln(A_j) \quad (14)$$

Thus, the modal frequency (ω_{n_j}) and modal damping (η_j) are evaluated from the slope of the Eqs. (13) and (14). Once, the modal frequencies are extracted, the mode shapes can be obtained by updating the finite element model. Here, an optimization technique is used to update the model

Algorithm 1: Iterative filtering based MEMD

- 1: Measure $\ddot{\mathbf{X}}$ (i.e., acceleration response) at multiple DOFs (at least three)
 - 2: Apply MEMD to the multichannel data
 - 3: Observe all the IMFs to locate mode-mixing if any
 - 4: Select a frequency range encompassing the mode-mixed regions only. Design a set of band-pass filters by equally dividing the mode-mixed frequency range
 - 5: Generate a band-passed signal for all the measurements
 - 6: Apply MEMD on each band-passed signal from the measured DOF and extract a single IMF. In case multiple IMFs are observed, reduce the bandwidth of the band-pass filters
 - 7: Identify natural frequency and damping as described in Eqs. (14)-(13)
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and it is given by (Mahato *et al.* 2017)

$$J(\rho, f_{ck}) = \|\omega_{FE} - \omega_{n_j}\|^2 \quad (15)$$

In this study, the optimization is performed using two variables for a concrete road bridge – concrete density (ρ) and compressive strength of concrete (f_{ck}), as they directly influence the bridge dynamics. The steps of the proposed algorithm are shown in Algorithm 1.

4. Numerical results

The adaptive MEMD technique proposed in the previous section is used here for numerical analysis to demonstrate its efficiency for structural system identification. It is to extract the modal parameters from the field experiment. A reinforced concrete road bridge is used for the experimental verification where a heavy truck is passed over the bridge with a constant velocity.

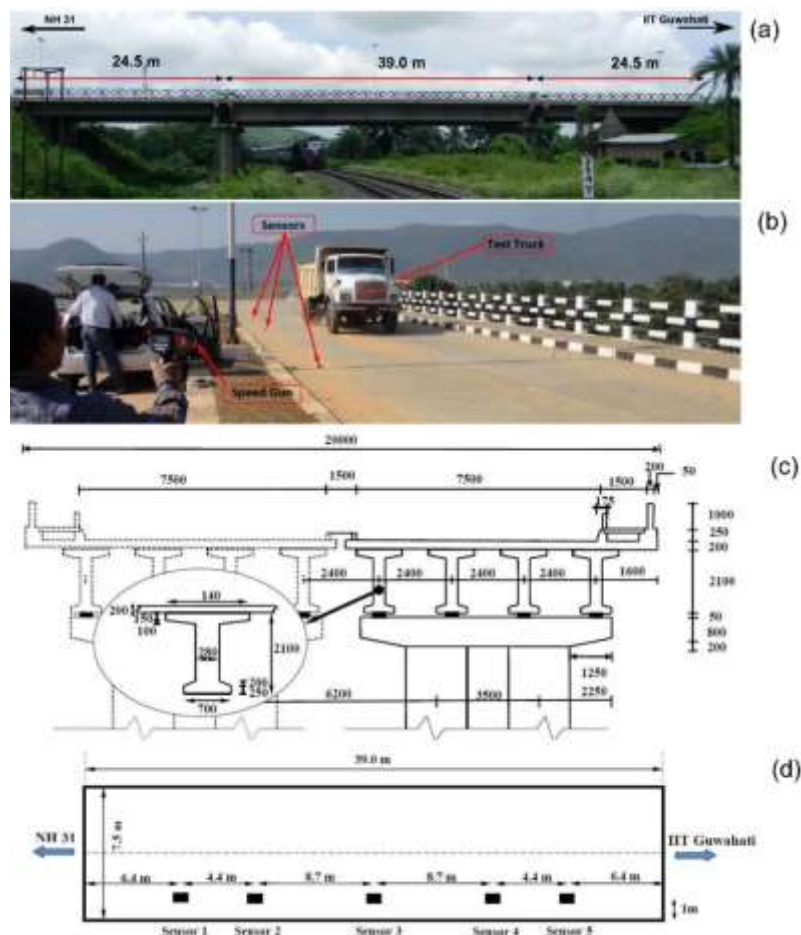


Fig. 1 (a) Longitudinal view; (b) experimental arrangement; (c) cross section view; (d) sensor locations for the field study

4.1 Experimental Implementation

The bridge near IIT Guwahati is considered in this study as shown in Fig. 1. It connects IIT Guwahati and National Highway–31. It has three spans, which are simply supported at both ends and are separated from each other by expansion joints. The length of the central span is 39 m which is instrumented for the experimental verification. Fig. 1 also shows the bridge cross-section along with other details (e.g., girder dimension and spacing etc.). From this figure, it may be noticed that the bridge is symmetric in cross-section and a small gap in the middle of the deck that separates the two sides of the bridge along the centerline. It is excited by a heavy truck along the outer lane as shown in Fig. 1 and the vibrations are recorded using 5 accelerometers placed symmetrically over the deck (Fig. 1(d)).

Fig. 2 shows the five vibration responses of the bridge and their respective Fourier amplitude spectra, which clearly show only one peak corresponding to the first natural frequency of the bridge. The recorded acceleration time histories from different channels are analyzed by EMD and the IMFs corresponding to each channel are considered here for modal identification. Fig. 3 shows two IMFs obtained from EMD analysis of the measured data. From this figure, one can identify the first mode of the bridge (i.e., 3.113 Hz) while the second IMF fails to suggest the presence of any mode owing to poor signal to noise ratio (SNR). In this context, it may be mentioned that the two IMFs, shown in Fig. 3 have significant energy as compared to other spurious modes.

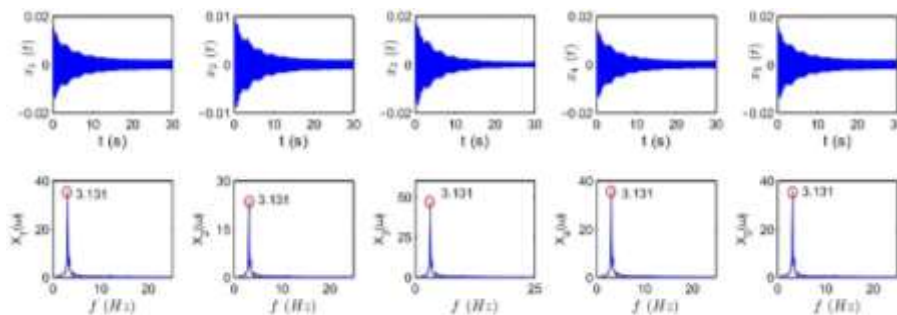


Fig. 2 Recorded accelerations and their spectra

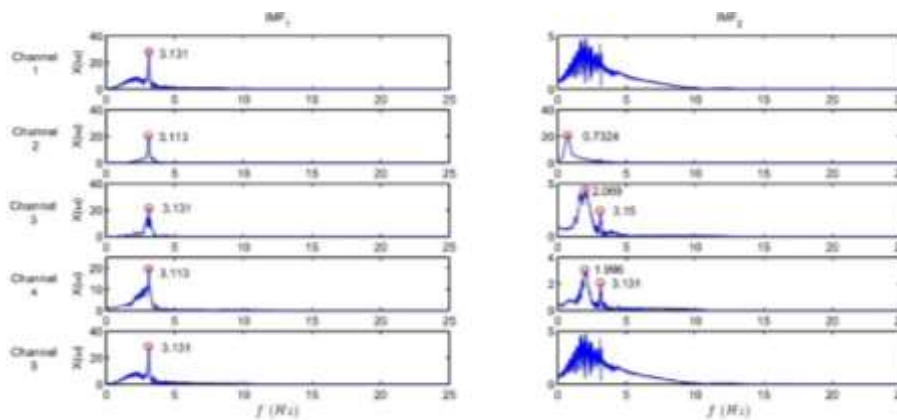


Fig. 3 Extracted IMFs using EMD

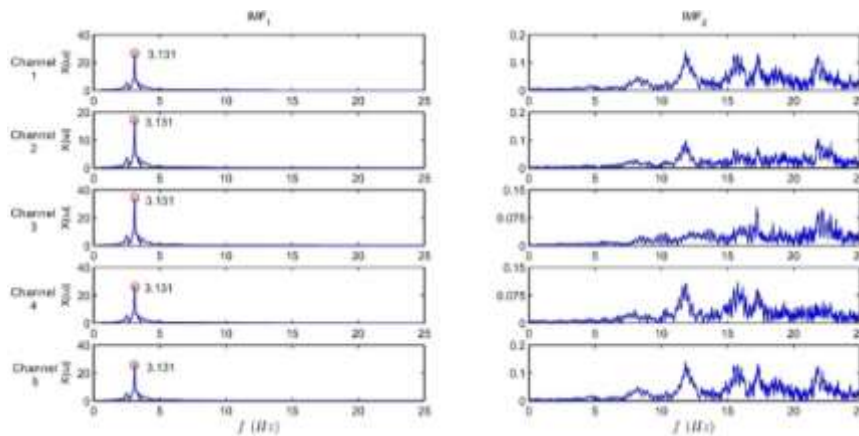


Fig. 4 Extracted IMFs using MEMD (mode-mixing in 2nd IMF)

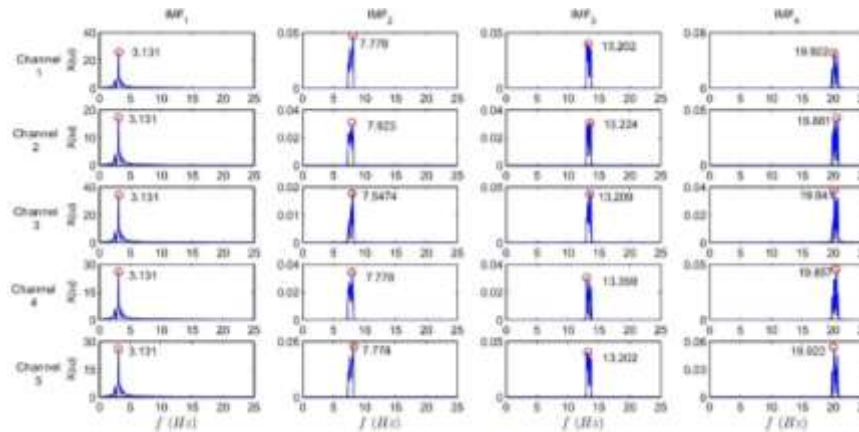


Fig. 5 IMFs obtained using the proposed algorithm

MEMD is then applied to the five recorded data set and the two dominant IMFs are shown in Fig. 4. From this figure, one can distinctly identify the first mode of the structure in IMF_1 while the second IMF shows mode-mixing with some distinct peaks and their respective frequency values. Once the mode-mixing zone is identified as shown in Fig. 4, signals from different channels are passed through a set of band-pass filters iteratively, as explained in the proposed algorithm. MEMD is then performed using the filtered signals from all five channels and the modified IMFs are shown in Fig. 5. All four IMFs in Fig. 5 distinctly show the modal frequencies with high SNR. This, in turn, highlights the efficiency of the proposed adaptive identification scheme. Table 1 shows the identified frequencies obtained from the analysis. The results in this table show that identified frequencies are well within 5% of the values obtained from the modal analysis using the finite element model. Once the dominant modal frequencies are identified, mode shapes corresponding to these natural frequencies are obtained from the updated finite element model as shown in Fig. 6. Finally, the modal damping ratios are evaluated from the filtered IMFs using Eqs. (13) and (14). Table 1 shows the modal damping ratios that could not be verified as the original damping ratios are unknown in this case. However, damping values obtained from different channels are found to be consistent with each other. Together, the results in Fig. 5 and

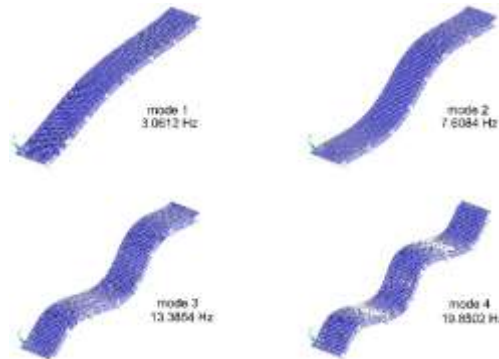


Fig. 6 Identified mode-shapes

Table 1 Identified parameters from bridge response

Finite Element Method	Identified values														
	Channel 1				Channel 2			Channel 3			Channel 4			Channel 5	
ω_n (Hz)	ω_n (Hz)	ε (%)	η (%)	ω_n (Hz)	ε (%)	η (%)	ω_n (Hz)	ε (%)	η (%)	ω_n (Hz)	ε (%)	η (%)	ω_n (Hz)	ε (%)	η (%)
3.1187	3.1310	-0.39	5.48	3.1310	-0.39	5.21	3.1310	-0.39	5.84	3.1310	-0.39	5.35	3.1310	-0.39	5.48
7.7353	7.7779	-0.55	0.55	7.9230	-2.43	0.82	7.5474	2.43	0.51	7.7780	-0.55	0.94	7.7779	-0.55	0.55
13.587	13.2017	2.84	0.20	13.2236	2.67	0.40	13.2097	2.78	0.27	13.359	1.68	0.22	13.202	2.84	0.20
20.123	19.9224	1.00	0.18	19.8808	1.20	0.30	19.8415	1.40	0.20	19.857	1.32	0.21	19.922	1.00	0.18

Table 1 indicate the superiority and efficiency of the proposed identification strategy's compared to conventional EMD for operational modal analysis.

5. Conclusions

An adaptive multi-variate empirical mode decomposition is proposed in this study to identify the modal parameters of linear structural systems. Although the superiority of MEMD based time-frequency analysis had already been established in the literature, it still suffers mode-mixing in higher modes. To avoid this difficulty, an adaptive filtering scheme of the recorded response is proposed in this study to improve its performance for modal identification. The proposed methodology has the followings edge compare to the previous versions of MEMD –

- The response of an existing structure is considered to show the efficiency of the proposed technique and the complete numerical study clearly indicates the superiority of the proposed adaptive MEMD scheme in terms of its performance to identify the modal parameters (i.e., natural frequency, modal damping, and mode shape) offering significant level of accuracy.
- It offers flexibility to the user to select different bandwidth depending upon the problem to solve. Thus, it helps the user in the signal decomposition based on the degree and extent of mode-mixing observed in the decomposed IMFs.
- The proposed identification algorithm using adaptive multivariate empirical mode decomposition is equally applicable for free and forced vibration response and does not demand for any pre-processing of the data. Hence, it can be readily adopted for any type of

loading for operational modal analysis.

With these in view, it may be concluded that the proposed identification algorithm can be adopted for efficient signal processing related to operational modal analysis of civil infrastructures for structural health monitoring.

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