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Damage identification of structures by reduction of dynamic matrices using the modified modal strain energy method

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Abstract. Damage detection of structures is one of the most important topics in structural health monitoring. In practice, the response is not available at all structural degrees of freedom, and due to the installation of sensors at some degrees of freedom, responses exist only in limited number of degrees of freedom. This paper is investigated the damage detection of structures by applying two approaches, AllDOF and Dynamic Condensation Method (DCM), based on the Modified Modal Strain Energy Method (MMSEBI). In the AllDOF method, mode shapes in all degrees of freedom is available, but in the DCM the mode shapes only in some degrees of freedom are available. Therefore by methods like the DCM, mode shapes are obtained in slave degrees of freedom. So, in the first step, the responses at slave degrees of freedom extracted using the responses at master degrees of freedom. Then, using the reconstructed mode shape and obtaining the modified modal strain energy, the damages are detected. Two standard examples are used in different damage cases to evaluate the accuracy of the mentioned method. The results showed the capability of the DCM is acceptable for low mode shapes to detect the damage in structures. By increasing the number of modes, the AllDOF method identifies the locations of the damage more accurately.

Keywords: damage identification; modal strain energy; dynamic condensation method; Model reduction method

1. Introduction

Many structures are faced with damage in their service lifetime. One of the effective ways to determine the damage location in structures is to use structural health monitoring. To prevent the progress of the damage in structures, it needs to determine the location of the damage at the early stages. These failures vary in different structures.

Initial damage may occur in one or more elements of the structure, and then these damages may develop and may lead to general failure or collapse. The high cost of construction and the importance of some structures caused damage identification to be one of the most important issues in civil engineering. The service life of the structures has been significantly increased by identifying the damaged elements and repairing them. Hence, it is essential to detect the magnitude and location of damage in the structures. Using the measured dynamic specifications of structures for damage detection is an attractive idea, because it allows for a global investigation of the structural health and

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condition even when the damage location is inaccessible (Wang and Ni 2015).

One of the ways for identifying the damage in structures is to use the measured responses of the structures (Kourehli 2017). Structural damages occur under several factors that lead to changes in structural characteristics like mode shapes, natural frequencies, damping ratio, energy dissipation, and stiffness matrix (Xu *et al.* 2010). Therefore, the damage causes the structural stiffness to decrease while mass remained constant. Consequently, the dynamic response of the structure has been increased. In other words, changing of stiffness and ductility lead the variation in the dynamic specifications of the structures such as natural frequency and mode shapes (Sohn *et al.* 2002).

Using the structural damage identification, it is possible to identify the damaged location in the structure, prevent resultant damage in different members of the structure and increase the lifetime of the structure by taking necessary measures. The studies on large structures show that the location of sensors in a structure has a considerable effect on the success of the structural damage identification.

In previous years, several studies have been conducted to detect the damage in many structures using modal strain energy techniques. The main idea of this approach was presented by Stubbs *et al.* (1994). They stated that the amount of modal strain energy, which is defined according to the mode shape curvature increases as a result of damage. Shi *et al.* (2000) used the calculation of the modal strain energy to determine the damage location of the structure. Finally, outcomes illustrated that the use of modal strain energy had an accurate performance. Kim *et al.* (2003) evaluated the frequency parameters, mode shape, and strain energy to determine the damage to the structures. Finally, he indicated that the strain energy parameter is more sensitive to damage identification than the other parameters. Ge and Lui (2005) proposed an approach based on a finite element model. They provided the method based on the dynamic specifications of the structure, such as natural frequencies and mode shapes. The method is examined on a frame, beam and, plate, and the outcomes demonstrated that the method could be used to detect the moderate damages. Hu and Wu (2009) developed a technique based on the modal strain energy to determine the location of the damage in plates.

Pradeep *et al.* (2014) also addressed the modal strain energy in the identification of the honeycomb sandwich structure. Liu *et al.* (2014) conducted a practical study for detecting the damage in the wind turbines using the improved strain energy method. The experimental results indicated that the presented method could properly detect the damage location for different damage cases. Moradipour *et al.* (2015) used an improved MSE (modal strain energy) procedure to detect damage location in 2D structures. They mathematically developed an MSE method, and then a beam and a two-dimensional frame were used to indicate the efficiency of the method. The results illustrate that the presented method is a reliable approach to detect damage considering five modes of the structure. Li *et al.* (2016) presented a technique with modal strain energy for offshore structures. The outcomes showed the presented method has the efficiency in identifying the damage in marine structures.

Ashory *et al.* (2016) used efficient modal strain energy in determining the damage in plates with laminated composite. The obtained results illustrated the MSE method has a better performance in composite plates in comparison with other methods.

Ghannadi and Kourehli (2018) discussed some model reduction methods for structures. The used natural frequencies to evaluate each of the methods. Zare Hosseinzadeh *et al.* (2019) proposed an effective method for cross-sectional damage localization and quantification in beams. They used the Iterated Improved Reduction System (IIRS) method to reduce the model.

Lale Arefi *et al.* (2020) proposed a modified modal strain energy-based index for damage detection of structures using the improved reduction system method. They used the Improved Reduction System (IRS) method for incomplete mode shapes. The results showed the proposed index is reliable to identify the location of damage accurately. Lale Arefi and Gholizad (2020) used the modal strain energy method with the System Equivalent Reduction Expansion Process (SEREP) method in truss structures. The outcomes illustrated the proposed approach is useful for identifying the damage in truss structures.

Based on the recent studies that used the modal strain energy method, the responses at all DOFs are considered. In practice, the responses are not available in all degrees of freedom, so using the reduction method is required. One of the ways to investigate the accuracy of a monitoring system to evaluate the health of structures is by using sensors. In structures, the response is not always known at all degrees of freedom, and the sensors can only be in a certain degree of freedom. Therefore, the study of methods for reduction degrees of freedom can be greatly useful. Some model reduction methods, such as the Improved Reduction System (IRS) method and System Equivalent Reduction Expansion Process (SEREP) method have been investigated in the recent study. The Dynamic Condensation Method (DCM) is one of the reduction methods that have not been studied in locating the damage of structures by the modified modal strain energy technique, which is discussed in this article. The advantage of the proposed method is to present a method for identifying the location of the damage when responses exist only in limited degrees of freedom. The DCM method could identify damaged elements in a low number of modes. But other methods such as the Guyan method or IRS method can identify the damaged elements in the high number of modes.

In the next section, the principles of the Dynamic Condensation Method are addressed. Then the principles of damage identification in structures are presented by the modified modal strain energy method. Finally, using two standard examples, the accuracy of the mentioned technique is evaluated.

2. Dynamic condensation method

The Dynamic Condensation Method firstly proposed by Paz (1984) in order to decrease the size of the model analysis system. The Dynamic Condensation Method is an iterative method that starts by assigning an approximation value for ω_i^2 (e.g., zero), and by solving the reduced Eigen problem, the eigenvalue and eigenvector are calculated in each step.

The master and slave DOFs have been considered in this approach. In this procedure, only master DOFs are calculated in the dynamic analysis while assuming the slave DOF responses are not available. Therefore, the slave DOF responses are obtained using the responses from the master DOFs via a transmission matrix.

The equation of motion of the structure can be expressed as

$$[M]\{\ddot{u}\} + [\mathrm{C}]\{\dot{u}\} + [\mathrm{K}]\{u\} = \{\mathrm{F(t)}\} \tag{1}$$

Where K is the structural stiffness matrix, M is the structural mass matrix, C is the damping matrix, u is the response of the structure, and *F(t)* is the force vector.

Assuming proportional damping, the Eigen-solution is (Avitabile *et al.* 2014)

$$([K] - \omega_i^2 [\mathrm{M}])\{u_i\} = \{0\} \quad \mathrm{i=1,\ldots, ndf} \tag{2}$$

Where K and M are the stiffness matrix of structure and the mass matrix of structure, respectively. Besides, ω_i and u_i are the frequency of a structure and the response of the structure, respectively. Moreover, *ndf* is the number of total degrees of freedom of the structure. It is being noted that the eigenvector of the damped and undamped system is the same (Yang 2005). In this approach, the displacement vector u_i is divided into two sub vectors u_m and u_s.

The equation of motion can be expressed as (Paz 1984)

$$(\begin{bmatrix} [\mathrm{M_{mm}}] & [\mathrm{M_{ms}}] \\ [\mathrm{M_{sm}}] & [\mathrm{M_{ss}}] \end{bmatrix} \begin{Bmatrix} \ddot{u}_m \\ \ddot{u}_s \end{Bmatrix} + \begin{bmatrix} [\mathrm{K_{mm}}] & [\mathrm{K_{ms}}] \\ [\mathrm{K_{sm}}] & [\mathrm{K_{ss}}] \end{bmatrix} \begin{Bmatrix} \{u_m\} \\ \{u_s\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3}$$

Where the subscript m is the master coordinate and the subscript s is the slave coordinates. Also, *ms* and *sm* denote master/slave and slave/master coordinate.

By substituting $\{u\} = \{U\} \sin \omega_i t$ in Eq. (3) gives [15]

$$(\begin{bmatrix} [\mathrm{K_{mm}}] & [\mathrm{K_{ms}}] \\ [\mathrm{K_{sm}}] & [\mathrm{K_{ss}}] \end{bmatrix} - \omega_i^2 \begin{bmatrix} [\mathrm{M_{mm}}] & [\mathrm{M_{ms}}] \\ [\mathrm{M_{sm}}] & [\mathrm{M_{ss}}] \end{bmatrix}) \begin{Bmatrix} \{\mathrm{U_m}\} \\ \{\mathrm{U_s}\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{4}$$

Where $\{U\}$ is the eigenvector of structure.

Hence, we have the following equation

$$(\begin{bmatrix} [\mathrm{K_{mm}}] - \omega_i^2[\mathrm{M_{mm}}] & [\mathrm{K_{ms}}] - \omega_i^2[\mathrm{M_{ms}}] \\ [\mathrm{K_{sm}}] - \omega_i^2[\mathrm{M_{sm}}] & [\mathrm{K_{ss}}] - \omega_i^2[\mathrm{M_{ss}}] \end{bmatrix} \begin{Bmatrix} \{U_m\} \\ \{U_s\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5}$$

Where ω_i^2 is the approximation of the *ith* eigenvalue.

The second part of Eq. (5) can be expressed as

$$([\mathrm{K_{sm}}] - \omega_i^2[\mathrm{M_{sm}}])\{U_m\} + ([\mathrm{K_{ss}}] - \omega_i^2[\mathrm{M_{ss}}])\{U_s\} = 0 \tag{6}$$

Pre-multiplying both sides of the Eq. (6) by $([\mathrm{K_{ss}}] - \omega_i^2[\mathrm{M_{ss}}])^{-1}$gives

$$\{U_s\} = \underbrace{-\left[[K_{ss}] - \omega_i^2[M_{ss}]\right]^{-1}\left[[K_{sm}] - \omega_i^2[M_{sm}]\right]}_{\bar{T}_{iDynm}}\{U_m\} \tag{7}$$

Where ω_i^2 is the approximation of the eigenvalues. To start the process, considering zero value for the first eigenvalues ω_i^2.

Eq. (7) can be written as

$$\{\mathrm{U}_s\} = \left[\bar{\mathrm{T}}_{iDynm}\right]\{U_m\} \tag{8}$$

Therefore

$$\{\mathrm{U}_i\} = \left[T_{iDynm}\right]\{U_m\} \tag{9}$$

Where

$$\left\{T_{iDynm}\right\} = \begin{bmatrix} I \\ \bar{\mathrm{T}}_{iDynm} \end{bmatrix} \tag{10}$$

and

$$\{U_i\} = \begin{bmatrix} U_m \\ U_s \end{bmatrix} \tag{11}$$

where $\mathrm{T}_{\mathrm{iDynm}}$ and $\overline{\mathrm{T}}_{iDynm}$ demonstrate the dynamic transformation matrix between master and slave DOFs, and the dynamic transformation matrix between all DOFs and master DOFs. Also, U_i and I are the mode shapes at all DOFs and a unit matrix.

In this procedure, $\mathrm{T}_{\mathrm{iDynm}}$ matrix that considered as the dynamic transformation matrix will be as follows (Paz 1984)

$$[\mathrm{T}_{iDynm}] = \begin{bmatrix} I \\ -\left[[K_{ss}] - \omega_i^2[M_{ss}]\right]^{-1}\left[[K_{sm}] - \omega_i^2[M_{sm}]\right] \end{bmatrix} \tag{12}$$

The reduced mass $[\overline{M}_i]$ and stiffness $[\overline{K}_i]$ matrices are obtained as

$$[\overline{M}_i] = [T_{iDynm}]^T[M][T_{iDynm}] \tag{13}$$

and

$$[\overline{K}_i] = [\overline{D}_i] + \omega_i^2[\overline{M}_i] \tag{14}$$

Where

$$[\overline{D}_i] = [T_{iDynm}]^T[K][T_{iDynm}] \tag{15}$$

Where $[\overline{D}_i]$ is the reduced dynamic matrix.

Finally, the reduced Eigen problem is given by:

$$\left[[\overline{K}_i] - \omega_i^2[\overline{M}_i]\right]\{U_m\} = 0 \tag{16}$$

Eq. (15) is solved to obtain improved eigenvalues ω_i^2 and eigenvector$\{U_m\}$, and it has been repeated to achieve the suitable results (Paz 1984).

In order to identification of the damages location in structures, a modified modal strain energy approach is used, which is described in the next section.

3. The modified modal strain energy method

The products of the structural stiffness matrix and the second power of their mode shape component are modal strain energy (MSE). The modal strain energy technique is an applicable method used for detecting the location of the damage. Therefore, the modal strain energy method is applied in this paper to determine the structural damage by reducing the degrees of freedom in structures. When the modal strain energy index is applied to a structure, the local damage to members is significantly increased compared to other members.

Since the mode shape vector is proportional to the vibrational deformations of the structure, the strain energy is saved in each structural element. The strain energy of the structure derived from the vector of mode shape is called modal strain energy (MSE). The modal strain energy $\mathrm{mse}_{\mathrm{i}}^{\mathrm{e}}$ in the e^{th} element and i^{th} mode of the structure is given by (Seyedpoor 2012)

$$\mathrm{mse}_{\mathrm{i}}^{\mathrm{e}} = \frac{1}{2}\{U_{\mathrm{i}}^{\mathrm{e}}\}^{T}\mathrm{k}^{\mathrm{e}}\{\mathrm{U}_{\mathrm{i}}^{e}\} \ , \ \mathrm{i} = 1, \ \ldots, \ \mathrm{ndf} \ , \ \mathrm{e} = 1 \ , \ \ldots, \ \mathrm{nte} \tag{17}$$

Where $U_{\mathrm{i}}^{\mathrm{e}}$ is obtained from Dynamic Condensation Method (Eq. (11)), $\mathrm{K_e}$ is the stiffness matrix of element *e* of the structure, *nte* is the total number of elements and *ndf* is the number of DOFs.

By normalizing the modal strain energy of eth elements with respect to the whole modal strain energy of the structure

$$(\mathrm{nmse}_{\mathrm{i}}^{\mathrm{e}}) = \frac{mse_{e}^{i}}{\sum_{i=1}^{nte} mse_{e}^{i}} \tag{18}$$

Now, for the *nm* mode an effective parameter can be expressed as follows

$$\mathrm{mnmse}^{e} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{nm}} \mathrm{nmse}_{\mathrm{i}}^{\mathrm{e}}}{\mathrm{nm}} \ , \quad \mathrm{e} = 1 \ , \ \ldots, \ \mathrm{nte} \tag{19}$$

Where $nmse_{\mathrm{i}}^{\mathrm{e}}$ is the normalized MSE of the eth element in the ith mode.

Now, by sum squared of the normalized index of eth element for considering all modes, an efficient parameter can be defined as

$$snmse^{e} = \sum_{i=1}^{Nmode}(mnmse_{i}^{e})^{2} \quad , \qquad e = 1, \ldots, nte \tag{20}$$

Where $snmse^{e}$ is named here as the efficient parameter for e^{th} element of the structure. Finally, by determining the efficient parameter $snmse^{e}$ for healthy and damaged elements, $(snmse^{e})^{h}$ and $(snmse^{e})^{d}$, respectively, an effective index is defined here to identify damaged element. The index introduced is named as a modified modal strain energy based index (*MMSEBI*) which can be defined as the Eq. (21) (Lale Arefi *et al.* 2020)

$$MMSEBI^{e} = max\left[0, \frac{\sqrt{\frac{(snmse^{e})^{d}}{Nmode}} - \sqrt{\frac{(snmse^{e})^{h}}{Nmode}}}{\sqrt{\frac{(snmse^{e})^{h}}{Nmode}}}\right] \quad e = 1 \ , \ldots, nte \tag{21}$$

According to the above equation; the equivalent index is zero for elements that the damage had not occurred in them and the equivalent indexes are larger than zero for elements that the damage had occurred in them.

4. Numerical example

In order to evaluate and compare two methods mentioned in this study, two reference examples have been used. The first method, named the AllDOF method, considers all degrees of freedom to compute modal strain energy in all elements of structures. The Dynamic condensation method, as the second method, estimates the structural responses at all DOFs based on a limited number of DOFs and thus reduces the number of DOFs.

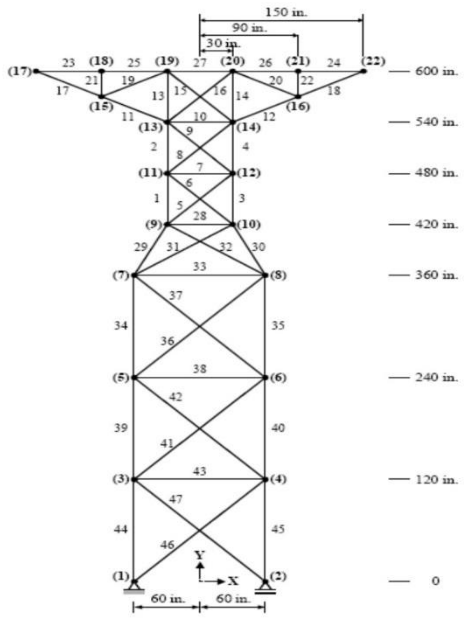


Fig. 1 A forty-seven-bar planar power line tower

The first studied example is a Forty-Seven-Bar Planar Power Line Tower, which is described in the next section. The second example is the two-dimensional 56-element frame. For both examples, three different damage cases are considered. Also, the MMSE index is utilized to obtain structural damage. It should be noted that two different modes are considered in each case to investigate the effect of the mode number in damage identification.

4.1 Forty-Seven-Bar Planar Power Line Tower

The first example used in this article is a forty-seven-bar planar power line tower to examine the efficiency of damage identification using the reduction of dynamic matrices by the modified modal strain energy method. The Forty-Seven-Bar Planar Power Line Tower with 47 members, 22 nodes, and 41 degrees of freedom is shown in Fig. 1 (Lee *et al.* 2011). The first forty degrees of freedom are used to calculate the modal strain energy. The mass density is 0.30 lb/in^3, and the elasticity modulus of the members is 30,000 psi. The elasticity modulus reduction has been used to consider the effects of damage in this structure.

Three different damaged conditions are considered to identify the damage in this truss. In damage case 1, the damage ratio in member number 27 is 30%. In damage case 2, the damage ratio of member number 9 and 29 are 30% and 25% respectively, and, in damage case 3, the damage ratio in members of number 3, 30 and 47 are 30%, %25, and 20% respectively. Table 1 presents different types of damage cases in this power line tower.

Table 1 Different damage cases for the forty-seven-bar planar power line tower

Damage cases	Element number	Damage ratio
Case 1	27	0.30
Case 2	10	0.30
	29	0.25
Case 3	3	0.30
	30	0.25
	47	0.20

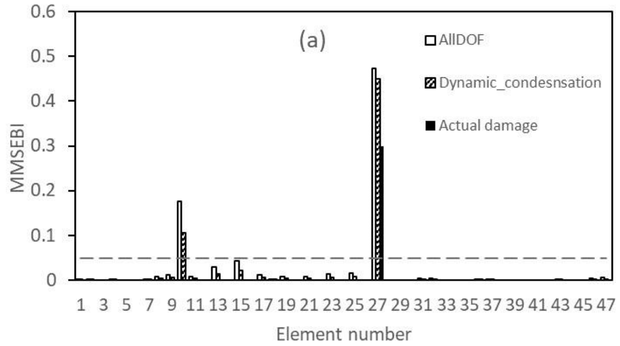
Table 2 Master DOFs for the forty-seven-bar planar power line tower

Method	Master DOFs	
	Node	Direction
AllDOF	All DOFs	
Dynamic condensation	3	1, 2
	5	2
	7	1
	10	1,
	11	1
	17	1,2
	18	1
	20	1
	22	1

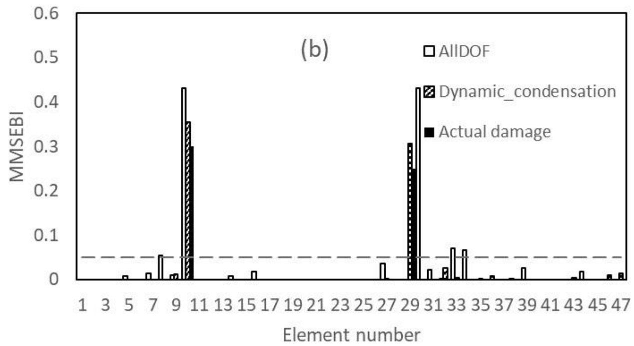
Degrees of freedom considered as the master DOFs are listed in Table 2. Accordingly, nine sensors have been utilized at nodes 3, 5, 7, 10, 11, 17, 18, 20, and 22.

Fig. 2 indicates different values of the modified modal strain energy index (MMSEBI) without noise for the three damage cases and the first four modes. The elements whose MMSEBI indices has exceeded 0.05 in the structure are considered as damaged elements. It can be seen from Fig. 2, in all cases, the damage location is correctly identified with a few false detections. Hence, in case 1, in both methods, element 27 is correctly detected with just one false identification (element 10). In damage case 2, both methods have correctly detected the location of induced damage (elements 10 and 29). The Dynamic Condensation Method has a better performance than the AllDOF method because the AllDOF method has falsely detected elements 8, 33, and 34. In damage case 3, the damaged elements 3, 30, and 47 are correctly detected, but elements 5, 7, 9, and 34 in the Dynamic

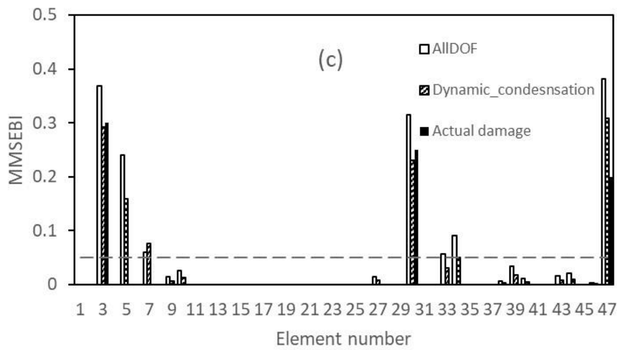
Condensation Method and the elements 5, 7, 33, and 34 in the AllDOF method are wrongly identified. It means that the Dynamic Condensation Method, like the AllDOF method, performed well in identifying the damage.



(a) Case 1

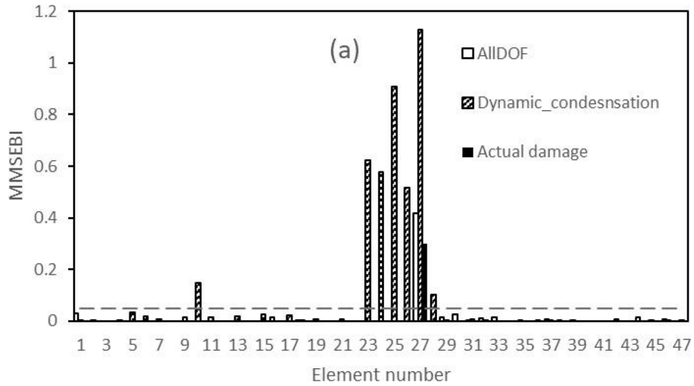


(b) Case 2

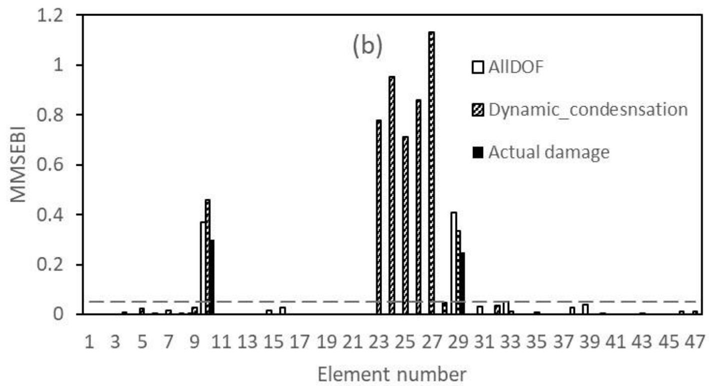


(c) Case 3

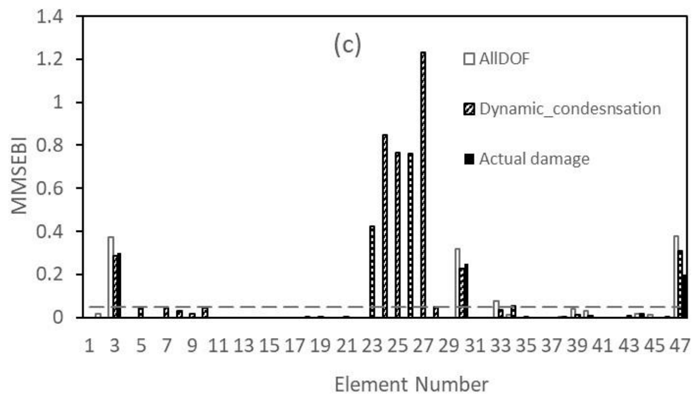
Fig. 2 Damage index values in the forty-seven-bar planar power line tower for 4 modes without noise



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 3 Damage index values in the forty-seven-bar planar power line tower for 4 modes with considering 3% noise

Furthermore, the noise level has been considered in the numerical example for the mode shapes as follows (Dinh-Cong *et al.* 2017).

$$U_i^n = U_i(1 + \mathrm{n}\beta_i) \tag{22}$$

in which U_i^n and U_i are the i^{th} mode shape with and without considering the noise, respectively. Moreover, n and β_i are the noise level and a random value between [-1 1], respectively. The noise level of 3% is used for this study.

Fig. 3 shows the MMSEBI values for the first four modes. From Fig. 3, it can be seen that the Dynamic Condensation Method is properly identified the damaged locations with a few errors. The Dynamic Condensation Method is properly identified element number 27 as damaged one and the elements 10, 23, 24, 25, 26, and 28 are falsely identified as damaged elements in case 1. In the AllDOF method, element 27 is properly identified as the damaged element without any false identification. Dynamic Condensation Method in damage case 2 properly identified damaged elements 10 and 29 while the elements 23, 24, 25, 26, 27, and 28 are incorrectly identified as damaged elements. For the AllDOF method, elements 10 and 29 are properly identified without any false identification.

Also, elements 3, 30, and 47 are properly identified, but only the element 33 in the AllDOF method and the elements 3, 23, 24, 25, 26, and 27 in the Dynamic Condensation Method are wrongly identified as damaged elements in damage case 3. Therefore, the AllDOF method provides better accuracy in damage detection.

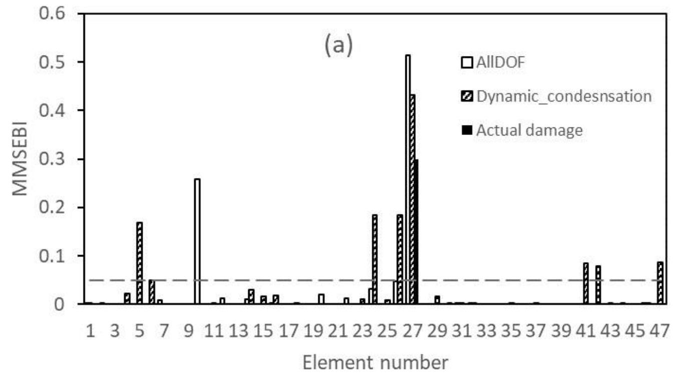
The MMSEBI values are demonstrated in Fig. 4 for the first five modes without noise. It can be seen that the damages location is properly located in all damage cases. In damage case 1, both methods have correctly identified the location of induced damage (element 27) while the Dynamic Condensation Method has falsely detected elements 5, 24, 26, 28, 41, 42, and 47.

The AllDOF method has properly identified the elements 10 and 29 with just one false detection (element 33), but the Dynamic Condensation Method has properly identified the elements 10 and 29 without any false detection in damage case 2.

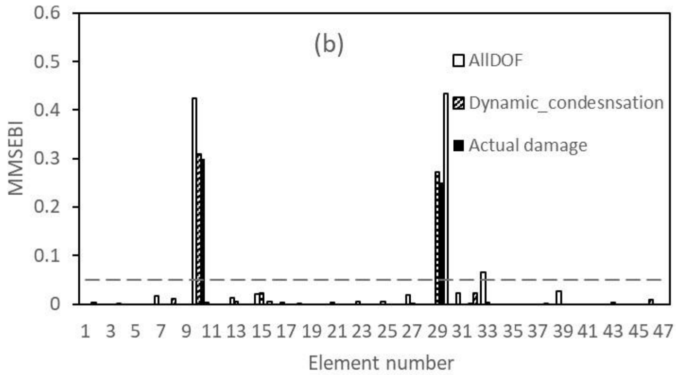
Moreover, the AllDOF method has properly determined the elements 3, 30, and 47 with two errors (elements 7 and 33), but the Dynamic Condensation Method has properly identified the elements 7, 10, and 33 without any wrong detection. Based on the obtained result, the AllDOF method produces accurate results by increasing modes number especially in case 2 and case 3.

Fig. 5 shows the MMSEBI values in three damage cases considering the first five modes with noise level 3%. From Fig. 5, it can be seen that both methods properly identified the damage element 27 in case 1. For the Dynamic Condensation Method, the elements 5, 23, 24, 25, 26, 28, 41, 42, and 47 are incorrectly identified as the damaged elements, but for the AllDOF method, element 27 has been properly identified without any false detection. Also, the AllDOF method has properly identified elements 10 and 29 without any false identification, But the Dynamic Condensation Method had led to many falsely detect elements (elements 23, 24, 25, 26, 27, and 28). By increasing modes number, it is observed that the use of the AllDOF method has led to better performance for locating damages in structures. From obtained results, the AllDOF method has a better performance than the Dynamic Condensation Method in case 3.

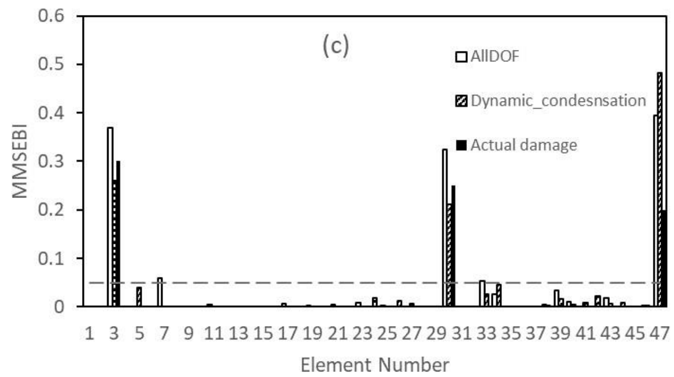
Identified damaged elements are provided in Table 3 and Table 4 for the forty-seven-bar planar power line tower. Elements with value of MMSEBI more than 0.05 are considered as damaged elements.



(a) Case 1

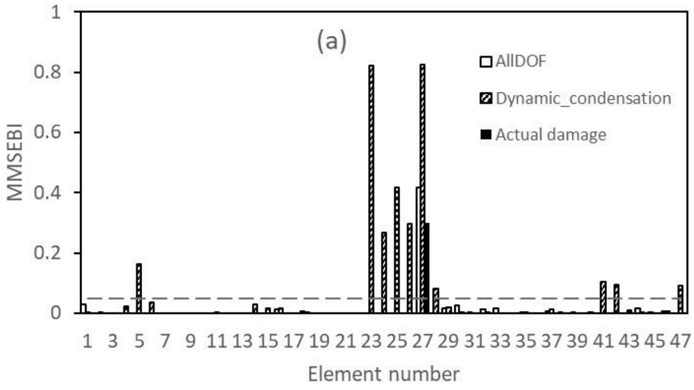


(b) Case 2

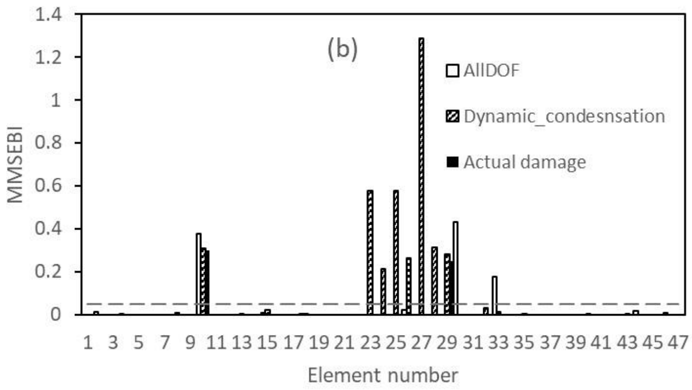


(c) Case 3

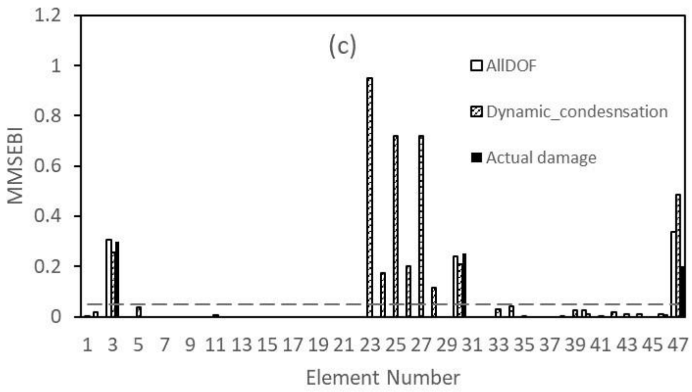
Fig. 4 Damage index values in the forty-seven-bar planar power line tower for 5 modes without noise



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 5 Damage index values in the forty-seven-bar planar power line tower for 5 modes with considering 3% noise

Table 3 Damage elements identified for the forty-seven-bar planar power line tower without considering noise

Method	Damage cases	Actual damage	4 modes	5 modes
AllDOF	Case 1	27	10, 27	10, 27
	Case 2	10, 29	8, 10, 29 33, 34	10, 29, 33
	Case 3	3, 30, 47	3, 5, 7, 30, 33, 34, 47	3, 7, 30, 33, 47
Dynamic condensation	Case 1	27	10, 27	5, 24, 26, 27, 41, 42, 47
	Case 2	10, 29	10, 29	10, 29
	Case 3	3, 30, 47	3, 5, 7, 9, 30, 34, 47	3, 30, 47

Table 4 Damage elements identified for the forty-seven-bar planar power line tower with considering 3% noise

Method	Damage cases	Actual damage	4 modes	5 modes
AllDOF	Case 1	27	27	27
	Case 2	10, 29	10, 29	10, 29, 33
	Case 3	3, 30, 47	3, 30, 33, 47	3, 30, 47
Dynamic condensation	Case 1	27	10, 23, 24, 25, 26, 27, 28	5, 23, 24, 25, 26, 27, 28, 41, 42, 47
	Case 2	10, 29	10, 23, 24, 25, 26, 27, 29	10, 23, 24, 25, 26, 27, 28, 29
	Case 3	3, 30, 47	3, 23, 24, 25, 26, 27, 30, 47	3, 23, 24, 25, 26, 27, 28, 30, 47

4.1 Two dimensional, 56-element frame

In the second example, a two-dimensional frame is used, as shown in Fig. 6 (Gomes and Silva 2008). The total number of elements in this frame is 57. Since each frame has three degrees of freedom in this frame, its total degrees of freedom is 165. The frame section of the members is rectangular and has a width and height of 0.14 and 0.24 meters, respectively. The elasticity modulus of the frame members is 25 GPa, and mass density is 2500kg/m^3.

Three different damaged cases are considered, to identify the damage in this frame. In case 1, the damage ratio of element 7 is considered to be 10%. In damage case 2, the damage ratio of element 44 is 10%. Finally, in damage case 3, the damage ratio of elements 10, 28, and 52 members is considered to be 10%. In Table 5, different damage cases are shown for this two-dimensional frame with 56 elements.

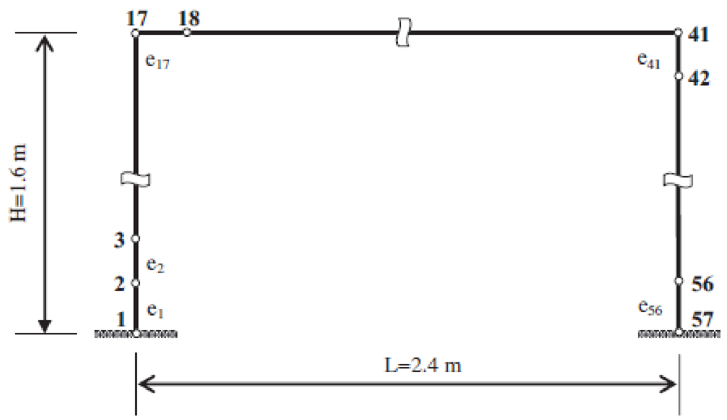


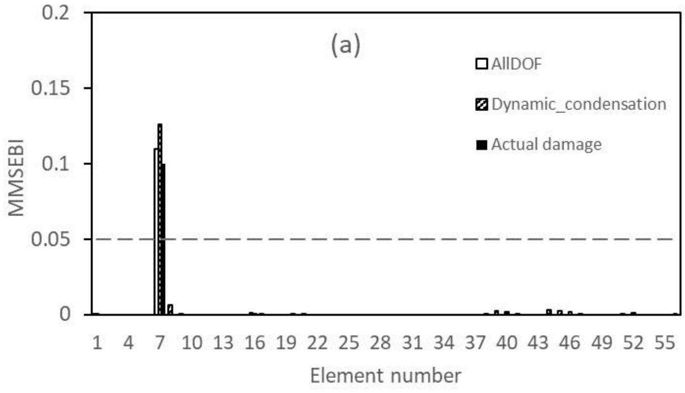
Fig. 6 A two-dimensional portal frame with 56 elements

Table 5 Different damage cases for the 56-element frame

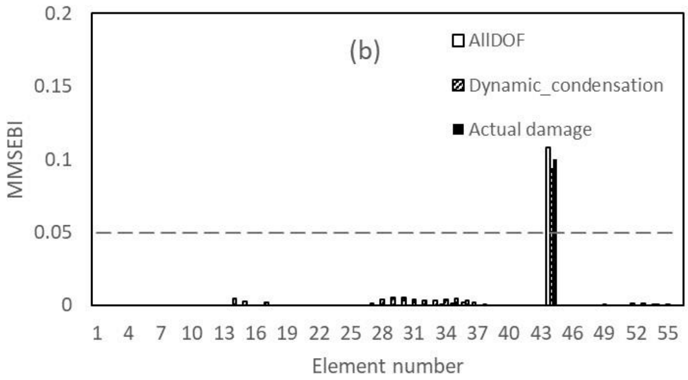
Damage cases	Element number	Damage ratio
Case 1	7	0.10
Case 2	44	0.10
	10	0.10
Case 3	28	0.10
	52	0.10

Table 6 Master DOFs for the 56-element frame

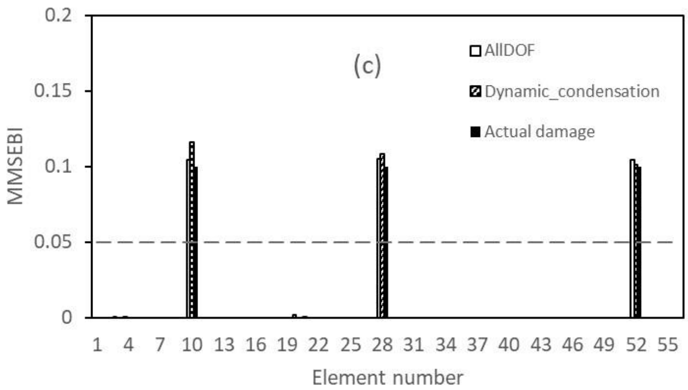
Method	Master DOFs	
	Node	Direction
AllDOF	All DOFs	
	2	1, 2
	4	1
	5	1
Dynamic Condensation	7	1, 2
	11	2
	12	2
	13	1
	14	1



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 7 Damage index values in the 56-element frame for 4 modes without noise

Both the DCM and the AllDOF methods have been used to examine the performance of detection reducing the DOFs in identifing structural damages using modal strain energy. The number of master degrees of freedom for this frame in the DCM method is 20.

Those degrees of freedom considered as the master DOFs are listed in Table 6. Accordingly, eight sensors have been utilized at nodes 2, 4, 5, 7, 11, 12, 13, and 14.

The modified modal strain energy based index (MMSEBI) values for three damage cases without noise are shown in Fig. 7. The first four modes are used in this case, to obtain MMSEBI values.

As illustrated in Fig. 7, both AllDOF and the Dynamic Condensation Method have a similar performance. As can be seen from Fig. 2, in all cases the damage location is correctly identified. Element 7 is properly identified by both methods in damage case 1. Element 44 and elements 10, 28, and 25 are properly identified without any error in damage cases 2 and 3, respectively. It means that the Dynamic Condensation Method, like the AllDOF method, performed well in identifying the damage.

Fig. 8 shows the MMSEBI values for the first four modes. As illustrated in Fig. 8, in case 1, it is observed that whereas the DCM method can successfully locate the really damaged element without any false detection, the AllDOF method has many wrong identifications (elements 7, 10, 14, 15, 16, 21, 24, 25, 26, 27, 30, 31, 34, 35, 41, 42, 43, and 44).

In damage case 2, the DCM method can properly identify the actually damaged elements with some false detections (elements 13, 4, 5, 6, 7, 11, 12, and 13). Nevertheless, the AllDOF method has many false identifications (elements 8, 9, 13, 14, 15, 16, 25, 26, 27, 31, 41, 42, 43, 45, and 49). Moreover, in damage case 3, the DCM method can successfully locate the actually damaged element with some false elements (elements 2, 3, 4, 5, 6, 7, 11, 12, and 37). On the contrary, the AllDOF method has many false elements (elements 15, 16, 23, 24, 25, 26, 27, 31, 33, 41, 42, 43, 44, and 45).

From Fig. 8, it can be seen that the Dynamic Condensation Method is properly identified as the damaged location with a few errors. The AllDOF method is properly identified as element 27 and the elements 10, 23, 24, 25, 26, and 28 are falsely identified as the damaged one in case 1. In the Dynamic Condensation Method, element 27 is properly identified as the damaged element without any false. In damage case 2 for the AllDOF methods, elements 10 and 29 are properly identified, and the elements 23, 24, 25, 26, 27, and 28 are incorrectly identified as the damaged element. For the Dynamic Condensation Method, the elements 10 and 29 are properly identified without any false.

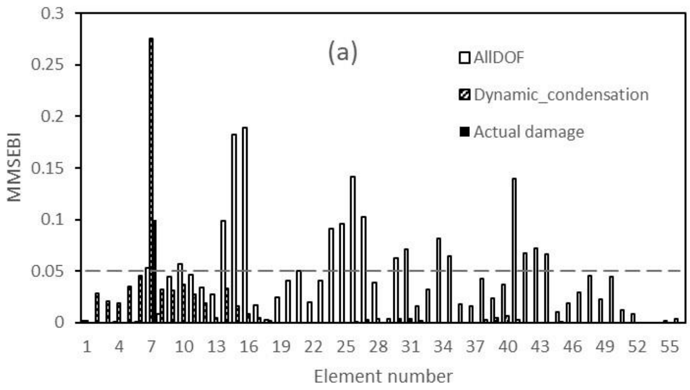
Because in the AllDOF method, all degrees of freedom are considered as the location of sensors, the responses have noise. Hence, it has a poor performance than the DCM method in identifying the damaged element, especially in a lower number of modes.

The MMSEBI values are demonstrated in Fig. 9 for the first six modes without noise. It can be seen that the damaged location has properly identified in all damage cases. In damage case 1, both methods have properly identified the location of induced damage (element 7).

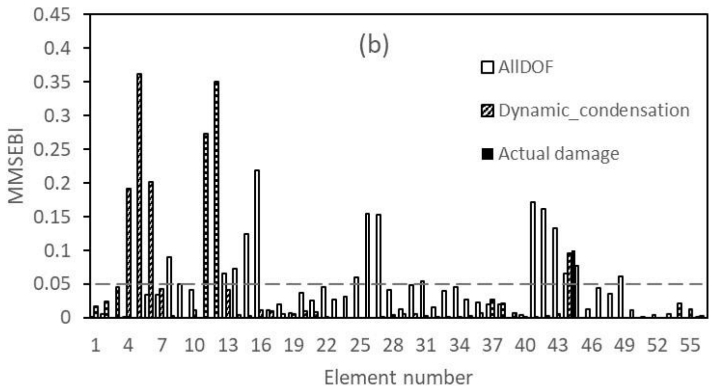
Both methods have properly identified the element 44 and elements 10, 28, and 52 in damage case 2 and 3 without any false, respectively. It means that the Dynamic Condensation Method, like the AllDOF method, performed well in identifying the damage.

Fig. 10 illustrates the result of MMSEBI for the first six modes with a 3% noise. As revealed in Fig. 10, in case 1, it is observed that whereas the DCM method can properly identify the actually damaged element with some false detections (elements 3, 4, 5, 6 and 12), the AllDOF method has many false elements (elements 13, 14, 15, 16, 24, 25, 34, 41, 43).

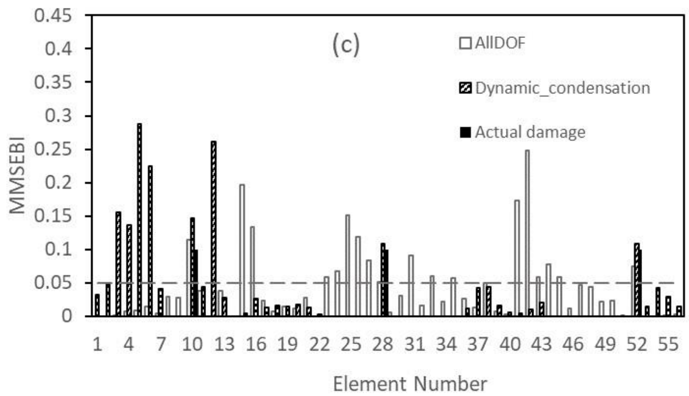
Moreover, in case 2 and 3, the DCM method can properly identify the actually damaged element with some false elements (elements 3, 4, 5, and 12 in damage case 2, and elements 4, 5, 6, and 12 in damage case 3), but the AllDOF method has many false elements (elements 11, 12, 14, 15, 23, 24, 27, 28, 31, 33, 41, 42, 47 and 48 in damage case 2, elements 14, 15, 16, 18, 24, 33, 41, 42, 48 and 49 in damage case 3).



(a) Case 1



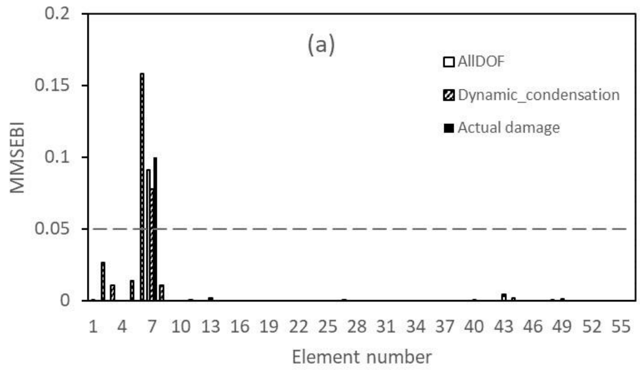
(b) Case 2



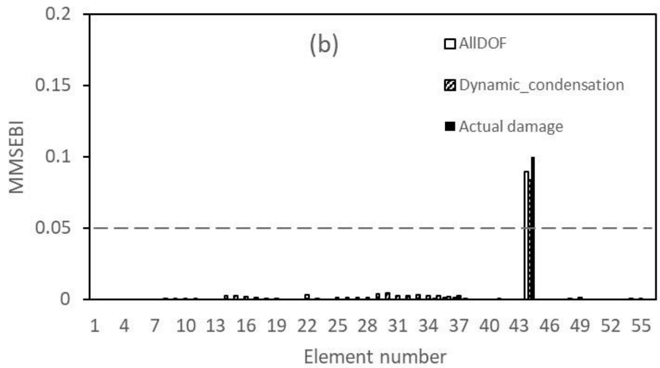
(c) Case 3

Fig. 8 Damage index values in the 56-element frame for 4 modes with considering 3% noise

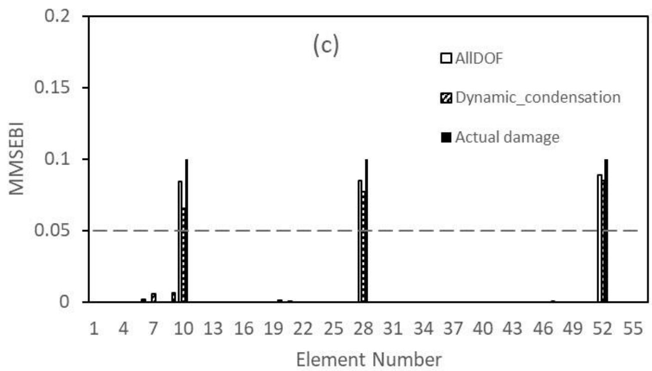
The obtained results show that the DCM method yields good performance in damage detection of the structure. Moreover, the accuracy of the AllDOF method has risen by increasing the modes number. Nevertheless, the accuracy of the DCM method has risen by decreasing the modes number.



(a) Case 1

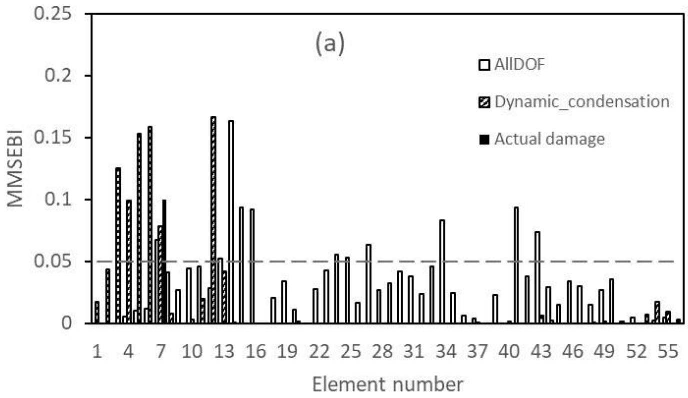


(b) Case 2

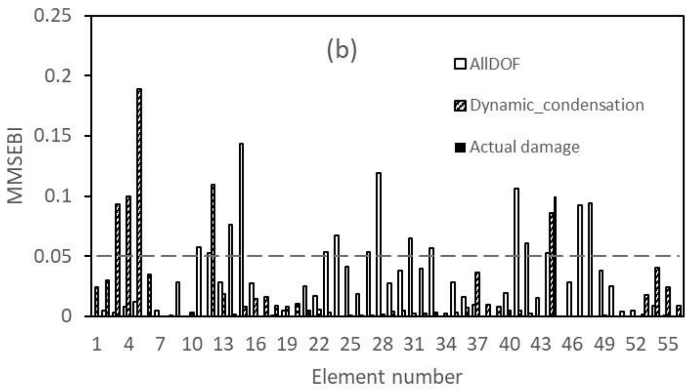


(c) Case 3

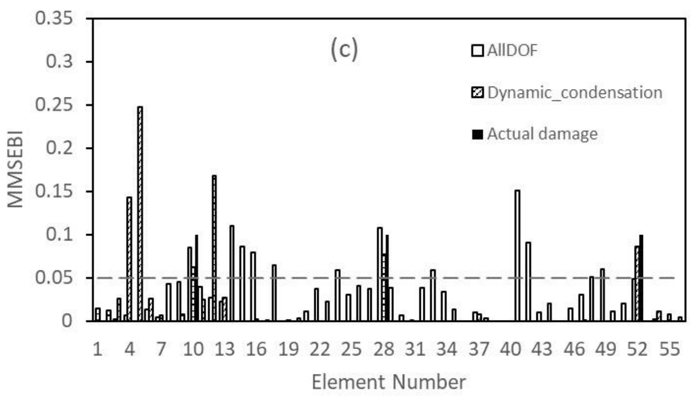
Fig. 9 Damage index values in the 56-element frame for 6 modes without noise



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 10 Damage index values in the 56-element frame for 6 modes with considering 3% noise

Table 7 Damage elements identified for the 56-element frame without considering noise

Method	Damage cases	Actual damage	4 modes	6 modes
AllDOF	Case 1	7	7	7
	Case 2	44	44	44
	Case 3	10, 28, 52	10, 28, 52	10, 28, 52
Dynamic condensation	Case 1	7	7	6, 7
	Case 2	44	44	44
	Case 3	10, 28, 52	10, 28, 52	10, 28, 52

Table 8 Damage elements identified for the 56-element frame with considering noise

Method	Damage cases	Actual damage	4 modes	6 modes
AllDOF	Case 1	7	7, 10, 14, 15, 16, 21, 24, 25, 26, 27, 30, 31, 34, 35, 41, 42, 43, 44	7, 13, 14, 15, 16, 24, 25, 34, 41, 43
	Case 2	44	8, 9, 13, 14, 15, 16, 25, 26, 27, 31, 41, 42, 43, 44, 45, 49	11, 12, 14, 15, 23, 24, 27, 28, 31, 33, 41, 42, 44, 47, 48
	Case 3	10, 28, 52	10, 15, 16, 23, 24, 25, 26, 27, 28, 31, 33, 41, 42, 43, 44, 45, 52	10, 14, 15, 16, 18, 24, 28, 33, 41, 42, 48, 49, 52
Dynamic condensation	Case 1	7	7	3, 4, 5, 6, 7, 12
	Case 2	44	4, 5, 6, 11, 12, 44	3, 4, 5, 12, 44
	Case 3	10, 28, 52	3, 4, 5, 6, 10, 12, 28, 52	4, 5, 6, 10, 12, 28, 52

The identified elements are provided in Table 7 and Table 8 for the 56-element frame. Those elements that value of MMSEBI exceeds 0.05 are considered as damaged elements. It can be observed that by increasing modes number, the DCM method with limited sensors had a better performance in identifying the damage.

5. Conclusions

Damage identification of structures is essential in structural health monitoring because it leads to reduce the costs of maintenance and increases the serviceability of structures. In practice, the sensors can be located at some specific nodes. Therefore, the response exists at those nodes of structures. In this paper, a modified modal strain energy based index is used for the identification of structural damage by using the Dynamic Condensation method and without condensation method (the AllDOF method). Therefore, two standard examples are utilized to assess the capability of the two methods described in this paper. A 3% noise is also applied for the mode shapes to attend the noise effects. Moreover, to investigate the consequence of the number of modes on the damage detection of structures, two numbers of modes have been considered.

The results showed that both methods have good efficiency without considering noise. Therefore, the DCM method, even with the reduction of degrees of freedom, has been desirable to detect the damage of the structure. Moreover, the results indicate the AllDOF method in the forty-seven-bar planar power line tower was desirable because only a few false elements were detected. Also increasing the number of modes leads to fewer false detections. Also, the results illustrated that for the 56-elements frame, the DCM method had better performance than the AllDOF method in detecting the damaged elements. By increasing the number of modes, the false detections in the AllDOF method is reduced. Finally, the numerical results of the DCM method show that the accuracy of damages identification is better than the AllDOF method when the number of mode shapes is lower. In the AllDOF method, the identification accuracy is enhanced when more mode shapes are used for analysis of the structures.

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