On the use of numerical models for validation of high frequency based damage detection methodologies

Diego A. Aguirre^{1a} and Luis A. Montejo^{*2}

¹Department of Civil, Construction and Environmental Engineering, North Carolina State University, Raleigh, NC 27695, USA ²Department of Engineering Science and Materials, University of Puerto Rico at Mayaguez, Mayaguez, PR 00680, Puerto Rico

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Abstract. This article identifies and addresses current limitations on the use of numerical models for validation and/or calibration of damage detection methodologies that are based on the analysis of the high frequency response of the structure to identify the occurrence of abrupt anomalies. Distributed-plasticity non-linear fiber-based models in combination with experimental data from a full-scale reinforced concrete column test are used to point out current modeling techniques limitations. It was found that the numerical model was capable of reproducing the global and local response of the structure at a wide range of inelastic demands, including the occurrences of rebar ruptures. However, when abrupt sudden damage occurs, like rebar fracture, a high frequency pulse is detected in the accelerations recorded in the structure that the numerical model is incapable of reproducing. Since the occurrence of such pulse is fundamental on the detection of damage, it is proposed to add this effect to the simulated response before it is used for validation purposes.

Keywords: numerical models; damage detection; signal-processing; wavelets; reinforced concrete

1. Introduction

Numerical models are a useful tool which allows not only performing structural analysis for design/assessment purposes but also validating and verifying structural health monitoring (SHM) methodologies. In either case, the main goal is to reproduce the response that the structure would experience under the action of different types of loads. Given the scarce amount of data from actual instrumented structures undergoing damaging process, the high costs of large scale destructive dynamic testing and the oversimplification implied in the use of austere elastic models, developing realistic and reliable numerical models may play an important role for validation purposes of SHM methodologies. In fact, the use of numerical simulations is very common and several authors have used them in the past for validation and verification of SHM techniques (e.g., Hera and Hou 2004, Ovanesova and Suárez 2004, Law *et al.* 2005, Hou *et al.* 2006, Grabowska *et al.* 2008, Curadelli *et al.* 2008, Ren and Sun 2008, Beskhyroun *et al.* 2010, You *et al.* 2012).

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^{*}Corresponding author, Associate Professor, E-mail: luis.montejo@upr.edu

^a Ph.D. Candidate, E-mail: daaguirr@ncsu.edu

In this work focus is placed on the adequacy of advanced non-linear fiber-based models to reproduce the seismic response of reinforced concrete (RC) structures at different levels of inelastic demand and induced damage. Moreover, the feasibility of using such type of models to validate signal-processing based damage detection techniques that search for anomalies in the high frequency range of the structural response is examined. Other more common "Takeda-like" models are not evaluated as past research (Aguirre *et al.* 2013) have found them not adequate due to the unrealistic abrupt change in stiffness proper of any multi-linear hysteretic model which results in the detection of a large number of spurious anomalies. The fiber-based model is more appropriate for this purpose as this type of approach more closely reproduces the actual non-linear hysteretic response of the structure, including the effect of local damage like reinforcement rupture.

The structure to analyze is a full-scale, circular RC bridge column recently tested at the Network for Earthquake Engineering Simulation (NEES), Large High Performance Outdoor Shake Table at the University of California, San Diego (UCSD). The numerical model is first validated based on the global response of the structure (accelerations and displacements) by comparison with the actual response of the structure recorded during the tests. The adequacy for damage detection methodologies validation is evaluated by applying the methodology to both the simulated and recorded response and comparing the results obtained.

The novelty of this work relies in two key points. First, it was found that a local damage (i.e., rebar fracture) produces a small disturbance in the top acceleration response, which cannot be captured even by a simulation using a fiber-based approach. On the other hand, a parametric impulse form is proposed in order to modify the simulated acceleration response of the column so it is viable to validate/calibrate high frequency based damage detection methodologies.

2. Damage detection based on high frequency irregularities

The premise within this type of approaches is that any sudden damage suffered by the structure will be reflected as a singularity in its high frequency response. The methodology is relatively simple; it consists on extracting the high frequency component of the dynamic response of the structure and look for spikes that are then related to damage episodes. Different tools have been used to extract the high frequency component, including: the Discrete Wavelet Transform DWT (e.g., Ovanesova and Suárez 2004, Sone et al. 1995, Todorovska and Trifunac 2010, Quiñones et al. 2105), the Empirical Mode Decomposition EMD (e.g., Xu and Chen 2004, Yang et al. 2004) and high-pass filters (e.g., Bisht and Singh 2011, Montejo 2011). While promising results have been obtained, it should be noticed that most of the past research efforts have validated their results using elastic numerical models where the damage is introduced as a sudden change in stiffness, or the physical models employed are of reduced scale and complexity. The only works where more realistic scenarios were used for validation are Todorovska and Trifunac (2010), that operates on data from a six-story instrumented building severely damaged by the 1979 Imperial Valley earthquake, and Aguirre et al. (2013) that engaged data from a real scale shake table test series of a RC column. Both studies used the DWT approach and found good correlations between the results obtained and the damage observed in the actual structures. This article expands the authors' previous work (Aguirre et al. 2013) and uses the same set of experimental data to examine the limitations of using numerical models for validation of high frequency based damage detection techniques.

The methodology used in this work to extract the high frequency portion of the response is

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based on the DWT. Basically, the DWT performs a simultaneous, low-pass and high-pass filtering process which decomposes/separates a signal (e.g., structural acceleration response) into two signals: the Approximations A (i.e., the low-frequency components) and the Details D (i.e., the high-frequency components) which may contain sudden discontinuities from the analyzed response signal (Ovanesova and Suárez 2004, Beskhyroun *et al.* 2010, Velázquez and Montejo 2011). The component signals can be obtained from the convolution of the signal x(t) and the down-sampled, low-pass and high-pass filters as

$$A(t) = \left[x \otimes g \right](t) = \sum_{b=-\infty}^{\infty} x(t) g(at-b)$$
(1)

$$D(\mathbf{t}) = \left[x \otimes \mathbf{h} \right](\mathbf{t}) = \sum_{b=-\infty}^{\infty} x(\mathbf{t}) \mathbf{h}(a \, \mathbf{t} - b) \tag{2}$$

where \otimes denotes convolution; *a* represents the translation; *b* the dilation; *g* and *h* are wavelet filters which must be related to each other as a quadrature mirror filters (Beskhyroun *et al.* 2010). Numerical implementation of the DWT is accomplished by means of the fast wavelet transform (FWT) which is a fast decomposition and reconstruction algorithm developed by Mallat (1989) using a two-channel subband coder. In the FWT, a signal can be represented as

$$\chi(t) = A_j(t) + \sum_{i \le j} D_i(t)$$
(3)

where j is referred to as the level of decomposition. The signal's decomposition process can be applied to different levels as needed, so that frequency resolution can be increased, namely, each component signal (i.e., A and D) can be decomposed into another approximation and detail signals. In this work the function (mother wavelet) employed to perform the FWT is the Biorthogonal (Bior) 6.8 wavelet (Cohen *et al.* 1992). This wavelet has been successfully used in the past to detect discontinuities (e.g., Ovanesova and Suárez 2004, Todorovska and Trifunac 2010, Montejo 2011, Montejo *et al.* 2012, Montejo and Vidot 2012).

2.1 Test description, numerical model, and initial validation

A large-scale test of a reinforced concrete bridge column was carried out at the NEES shake table at UCSD. The column had a diameter of 1.22 m (4 ft) with a cantilever height of 7.32 m (24 ft). A large, reinforced concrete block with a total weight of 2245 kN (521.9 kips) was placed at its top in order to mimic the mass of a bridge and also to produce nonlinear response (Fig. 1).

Longitudinal and transverse reinforcement was provided by means of 18 No. 11 bars and butt-welded double No.5 hoops spaced at 152 mm (6 in) center to center, respectively. Ten ground motions, with increasing intensity levels, were applied at the base of the column in a sequential fashion. The different excitations imposed a large range of inelastic demand and induced damage into the column; from concrete cover cracking up to near collapse conditions including several rebar rupture episodes. Complete details regarding the test are available elsewhere (Carrea 2010, Schoettler *et al.* 2012).

The numerical model used is briefly described next for completion purposes; nonetheless, complete details regarding structural modeling are available elsewhere (Aguirre *et al.* 2013). A

detailed nonlinear finite element analysis using a distributed plasticity model was performed using OpenSees (McKenna *et al.* 2000) as shown Fig. 2. The column was modeled using 2 force-based beam-column elements with 3 Gauss-Lobatto integration points. The length for the first element (next to the fixed base) was set so that the integration weight of the fixed node matches the expected plastic hinge length, Lp, (Priestley *et al.* 2007). Confined and unconfined concrete fibers were modeled using the *ConcreteO1* material with parameters based on the Mander *et al.* model (1988) and cylinder tests. Degradation of strength and stiffness (due to cyclic loads) was taken into account for the longitudinal reinforcement by using the *ReinforcingSteel* material (Mohle and Kunnath 2006) with degradation parameters set so that rebar fracture episodes agree with the observed during the experimental tests. Considering the large weight at the top and the height of the column, second order effects were taken into account by using the OpenSees P-Delta coordinate transformation command.

Initial validation of the model was accomplished by comparison of the global responses (top acceleration (Atop) and displacement ductility (μ) as shown in Fig. 3), as well as the first natural period of vibration of the structure after each load stage (Fig. 4).



(a) Test setup Photo: https://nees.org/warehouse/project/987/



(b) cross section details

Fig. 1 Reinforced concrete bridge column



Fig. 2 Numerical model developed using OpenSees

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Fig. 3 Comparison of acceleration and displacement ductility response



Fig. 4 Comparison of the first natural period of vibration

It is seen from these figures that the numerical results are in close agreement with the response exhibited by the structure, capturing even the rupture episodes and the elongation in the natural period of vibration as the inelastic demand increased. Note from Fig. 3 that both the simulated top acceleration and displacement ductility follow not only the shape but also the peaks recorded in the experimental test. However, notice that residual displacements become important after Eq. (8) when a significant damage occurred. In addition, notice that experimental results of Eq. (10) differ from the simulated data as a consequence to the impact with the safety structure.

The natural period of the structure from the experimental data was estimated from the Fourier analysis of the structure response to low amplitude white noise time histories applied between earthquake loads. In Fig. 4 experimental observations at each load stage are included along with the maximum recorded displacement ductility (μ). Although the shifts in the natural period of vibration slightly differ between simulated and experimental results, the shape of both curves is consistent, i.e., they increase during the initial inelastic excursions up until they remain approximately constant and then they increase again when a large damage occurred. In addition, differences in the natural periods in Fig. 4 generate because simulated values were obtained from an eigenvalue analysis in the numerical model.

With this results it can be concluded that the developed numerical model is ideal for design or assessment purposes and yet for validation or calibration of SHM methodologies based on the change on the dynamic properties of the structure (e.g., Aguirre and Montejo 2014). Next section will investigate the adequateness of this type of models for validation of high-frequency based damage detection methodologies.

2.2 Detecting and analyzing rebar fracture episodes

A total of 6 rebar fracture episodes were reported to occur during the earthquake loads application: 2 of them during Eq. (8), (3) during Eq. (9), and the remaining one during Eq. (10) (Schoettler *et al.* 2012). However, results from Eq. (10) are discarded in this work since the column impacted one of the safety structures during this ground motion. Column acceleration response signals from Eqs. (8) and (9) are processed via FWT and the resulting detail functions are

analyzed. In these functions, spikes emerge indicating discontinuities which can be correlated with damage episodes (i.e., rebar ruptures). Values at each time instant (i) of the detail functions (D) are normalized according to the rule

$$Z_i = \frac{D_i - \mu}{\sigma} \tag{4}$$

where μ and σ are the mean and standard deviation of the detail values, respectively. In order to avoid the identification of spurious spikes, a threshold criterion is adopted such that any instant (*i*) where the normalized absolute value (|z|) is larger than six (6) is treated as a damage instant, i.e., where the absolute amplitude of the normalized details deviate more than 6 standard deviations from the mean value. This value was determined by inspection of results obtained from the high frequency analysis using all the earthquake load excitations. Further details about the threshold criterion adopted can be found elsewhere (Aguirre *et al.* 2013).

Figs. 5 and 6 present the column acceleration (*Atop*) and displacement ductility (μ) response along with the normalized absolute values of the detail functions for Eqs. (8) and (9), respectively, using the experimental and simulated data; for better appreciation, only the first 26 seconds from results are presented. It is seen that the results from DWT analysis significantly differ between both data sources.

Using the recorded acceleration response, two and three large spikes arise in the normalized detail functions of Eqs. (8) and (9), respectively. According to past research efforts (Aguirre *et al.* 2013), it can be inferred that those spikes are pinpointing rebar fracture episodes, which is a reasonable assumption since the number of irregularities detected coincide with the number of rupture episodes reported during the experimental tests (Schoettler *et al.* 2012). Unlike the results obtained using experimental data, notice from Figs. 5(d) and 6(d) that a large number of irregularities emerge when the simulated acceleration response is utilized to perform the DWT analyses. Moreover, notice that the emerging spikes do not correlate with the times when the rebar rupture occurred in the simulation (indicated by a dashed red line).

As reported in past research (Aguirre *et al.* 2013), irregularities detected after performing the DWT analysis are identifying the inelastic excursions rather than the rupture episodes. This clearly implies a serious limitation to the use of numerical models to validate this type of analysis.

In order to have a measure of the disturbance generated in the acceleration response signals due to the rebar fracture, a detailed analysis on each experimental rupture episode was performed using a time frame of 0.1 seconds. For this purpose, instant changes in the acceleration and displacement response are computed by removing the mean value from the signals (during the rupture episode). For a better understanding of what occurs during those episodes, the detailed analyses also include the acceleration and displacement ductility response along with results from the DWT analysis.

For the sake of brevity, only results for the two rebar rupture episodes occurring during Eq. (8) are presented in Figs. 7 and 8. In those figures the vertical dashed red line represents the instant of the rupture which is selected based upon the maximum value from the irregularities detected.

It can be seen that the rebar rupture generates a large impulse in the acceleration response (Figs. 7(d) and 8(d)), whereas the instant change is less notorious in the displacement ductility response (Figs. 7(e) and 8(e)). Notice that the impulses generated in the acceleration have some degree of resemblance (similar shape). However, differences can be seen in the impulses' shape of the 5 rupture episodes considered (Fig. 9); the variation is more noticeable for the episodes occurring during Eq. (9).



5 10 15 20 (b) Displacement ductility

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Continued-

± 0 -5



Fig. 7 Detailed analysis of first rebar rupture (t=14.57 seconds) during Eq. (8) using experimental data



Fig. 8 Detailed analysis of second rebar rupture (t=18.01 seconds) during Eq. (8) using experimental data



Fig. 9 Acceleration impulses at rupture episodes using experimental data

3. Proposed acceleration impulse approach

So far it has been shown that the developed numerical model is capable of closely reproducing the response of the structure over a large range of inelastic demands and induced damage including rebar fracture episodes. However, it failed to reproduce the additional disturbance caused in the acceleration response when a bar fractures, making it useless for validation or calibration of high frequency based damage detection methodologies. To solve this problem it is proposed to add this additional disturbance to the simulated acceleration response (i.e., the numerical model is not modified) at the time instants where the simulation indicates that rebar fracture has occurred. Instants of rupture in the simulated data were determined by analyzing the stress time history and stress-strain response of the bars. Times of rupture were identified when the stress on a given bar dropped to zero during the application of the ground motions.

Initially, acceleration impulses measured during rebar rupture episodes (experimental data) are used; for the sake of brevity only the results using data from Eq. (8) are presented. A total of 3 rebar fracture episodes occurred during the simulation of Eq. (8) at ~12.48 (2 bars) and 17.82 seconds. The impulse added to the simulated signals was obtained by averaging the two impulses presented in Figs. 7(d) and 8(d) and it is illustrated in Fig. 10. Fig. 11 presents the DWT results using the simulated signal from Eq. (8) modified by adding acceleration impulses. It can be seen the method is successful on detecting the induced rupture episodes and more importantly, the spikes emerging at these time instants surpass other spikes detected on the original simulated data (Fig. 5(d)).

Based on the impulses extracted from the experimental data, a typical impulse form is proposed to characterize the disturbance caused in the acceleration response due to fracture of the rebar. It should be noticed that the data available is quite limited since it corresponds to the same structural configuration and rebar type. Changes in the frequency and amplitude of the impulse should be expected for other structural configurations and rebar type, therefore the proposed form which is based on the real Morlet wavelet is equipped with parameters to control the frequency (f), duration (s) and peak amplitude (A) of the impulse (Eq. (5)). Fig. 12 shows the proposed acceleration impulse, its resulting velocity and displacement for A=1g and two different set of values for f and s.

$$w = A \cdot e^{-t^2/2s^2} \cdot \cos(2\pi ft)$$
(5)

Note from Fig. 12 that the higher the frequency (f) and the shorter the duration (s), the lower the influence of the impulse on the velocity and displacement response, which is desirable as the rebar rupture was no found to affect much these structural responses.

Using the proposed impulse form with parameters A=0.03 g, f=80 Hz, s=0.01, the shape of the impulse obtained is in close agreement with that obtained from the experimental data; these values are calibrated for the specific structural configuration and rebar type under consideration. The simulated results for Eqs. (8) and (9) are reanalyzed via DWT with the proposed impulse and the aforementioned parameters, added to the rebar fracture instants (Figs. 13 and 14). It is seen that the spikes in the detail functions are successfully identifying the rebar ruptures.



Fig. 10 Average acceleration impulse obtained from rupture episodes of Eq. (8)



Fig. 11 Detection of induced rupture episodes during EQ8 using modified simulated data with impulse obtain from experimental data



Fig. 12 Proposed acceleration impulse







Fig. 14 Detection of induced rupture episodes during Eq. (9) using modified simulated data with proposed impulse

4. Conclusions

An acceleration impulse approach for validation or calibration of high frequency based damage detection techniques has been proposed in this paper. The main conclusions are summarized as follows:

- Non-linear fiber-based models are a powerful tool to simulate the structural response of a structure as they are capable of closely reproducing the response of reinforced concrete members subjected to a wide range of earthquake load intensities, including the occurrence of damage like reinforcement rupture, as well as capturing the natural period elongation at increasing levels of lateral demand. However, this approach fails to reproduce the additional disturbance caused in the acceleration response when a bar fractures, making it useless for validation or calibration of high frequency based damage detection methodologies.
- A parametric impulse form was proposed to simulate this disturbance, the impulse is added to the simulated acceleration response centered at the time instants where the simulation indicates rebar fracture. This approach was found to be successful as the emerging spikes from the DWT details surpass all other spikes from the original simulated accelerations allowing the identification of the rupture episodes, just as for the experimental data.
- More experimental data is required to obtain impulse parameter values for different rebar types and structural configurations.

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