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Vulnerability assessment of strategic buildings based on ambient vibrations measurements

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Abstract. This paper presents a new method for seismic vulnerability assessment of buildings with reference to their operational limit state. The importance of this kind of evaluation arises from the civil protection necessity that some buildings, considered strategic for seismic emergency management, should retain their functionality also after a destructive earthquake. The method is based on the identification of experimental modal parameters from ambient vibrations measurements. The knowledge of the experimental modes allows to perform a linear spectral analysis computing the maximum structural drifts of the building caused by an assigned earthquake. Operational condition is then evaluated by comparing the maximum building drifts with the reference value assigned by the Italian Technical Code for the operational limit state. The uncertainty about the actual building seismic frequencies, typically significantly lower than the ambient ones, is explicitly taken into account through a probabilistic approach that allows to define for the building the Operational Index together with the Operational Probability Curve. The method is validated with experimental seismic data from a permanently monitored public building: by comparing the probabilistic prediction and the building experimental drifts, resulting from three weak earthquakes, the reliability of the method is confirmed. Finally an application of the method to a strategic building in Italy is presented: all the procedure, from ambient vibrations measurement, to seismic input definition, up to the computation of the Operational Probability Curve is illustrated.

Keywords: ambient vibrations; operational modal analysis; vulnerability assessment of buildings; operational condition

1. Introduction

The method presented in this paper has been developed using theories and techniques that are well known and consolidated in the scientific and technical community. They include operational modal analysis, model reduction methods and algorithm for seismic dynamic analysis. These theories and techniques, however, are assembled for producing a new and original method, aimed to the solution of a particularly vulnerability problem: the estimation of the building capability to remain operational after an earthquake.

The application of experimental modal analysis to buildings had an extensive development in the last fifteen-twenty years, mostly due to the possibility of using only the dynamic response of

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the structure to ambient noise, without any necessity to provide an external excitation. This has reduced significantly the cost and time of the tests, allowing in addition to perform measurements without interrupting the normal function of the building. This, in turn, has been made possible by the availability of sensors and measuring systems with high dynamics and relatively low cost, and by the progress of the Operational Modal Analysis, consisting of all those techniques addressed to modal parameter identification from output-only analysis, Reynders (2012), Cimellaro and De Stefano (2014). Together with the improvements mentioned above, there was a large development in using experimental modal parameters for the solution of different problems of structural engineering, as damage assessment and health monitoring, Charles *et al*. (2004), Hong-Nan Li *et al.* (2014), seismic retrofitting evaluation, see Spina and Lamonaca (1998) and in Régnier *et al*. (2013), and seismic vulnerability assessment, see Michel *et al.* (2008), Perrault *et al*. (2013), Snoj *et al.* (2013).

Seismic vulnerability evaluation is a wide and complex research field, as it is shown in Calvi *et al.* (2006) and Gueguen (2013), ranging from empirical methods operating on large geographical scales, to analytical approaches in which simplified mechanical models are developed according to structural typological classes, up to the seismic vulnerability assessment of a single existing building through nonlinear Finite Elements Models (FEMs).

Although the issue addressed in this paper may seem very particular within the general vulnerability problem, it is certainly significant for the so the called “strategic building”, i.e., buildings which, during a seismic emergency, are devoted to health care, to rescue operations management or which host the coordination centre of all the civil protection actions. It is obvious that, in order to fulfill their strategic role, these buildings must remain fully operational even for events with high return period for which an ordinary building should not collapse, but can be seriously damaged. It is important to underline that, when vulnerability assessment is performed only with reference to the operational condition, the problem is greatly simplified compared to the general case. This is because linear models can be properly used for predicting the structural response and because the results of the analysis can be expressed just in terms of inter-story drifts. As it will be shown below, in this case the informations obtained from ambient vibrations measurements are sufficient to construct a linear model for the building vulnerability assessment. The method could be applied by local government to establish a ranking of vulnerability of their buildings in order to decide which buildings to select for emergency management or how to distribute the economic resources for seismic retrofitting.

The paper is divided into five sections, plus one sixth of conclusions: in section 2 and 3 the theoretical basis of the model are illustrated, section 4 describes the probabilistic procedure for vulnerability assessment, while sections 5.1 and 5.2 are devoted respectively to the validation of the methods through some experimental seismic data and to the application of the method to a real strategic building in Italy.

2. Basic theoretical concepts

In this section some basic concepts used for the development of the method are reminded. It is observed that vertical degrees of freedom (DOFs) will not be considered, because they are in general negligible for the seismic vulnerability of most buildings. Moreover, according to the current practice, the hypothesis of proportional damping will be assumed. The equation and the symbology presented below are taken from Clough and Penzien (2003).

In structural engineering buildings are normally modeled by Finite Elements Models (FEMs) that represents the strucure as *N*-DOFs discrete mechanical systems. If the attention is limited to the linear case, the seismic behavior of a building can be properly described by the following system of N coupled linear equations

$$\mathbf{m}\ddot{\mathbf{v}}(t) + \mathbf{c}\dot{\mathbf{v}}(t) + \mathbf{k}\mathbf{v}(t) = -\mathbf{m}\mathbf{b}\ddot{\mathbf{v}}_g(t) \tag{1}$$

where t is time, the symbol $\dot{\bullet}$ states for time derivation, $\mathbf{v} \in \mathrm{R}^{\mathbf{N}}$ represents the relative displacements to the ground, **m**, **c** and **k** are the mass, the damping and the stiffness matrices, $\ddot{\mathbf{v}}_g(t) = \left[\ddot{x}_g(t), \ddot{y}_g(t)\right]^T$ (with $\bullet^T$ the transpose of $\bullet$) is the ground accelerations in two orthogonal horizontal directions x and y, and $\mathbf{b} \in \mathrm{R}^{\mathbf{N}\times\mathbf{2}}$ is the drag matrix, that assigns to each degree of freedom the inertia force corresponding to its direction. The structure of **b** is such that b_{hk}=1 if v_h represents a displacement in the direction *k* and b_{hk} =0 otherwise.

If $\mathbf{\Phi} \in \mathrm{R}^{N\times N}$ is a matrix, whose columns are the mode shapes $\mathbf{\Phi}_n$ (**n**=1,2,…,*N*) of system (1), it is possible to make a change of coordinates, through the following linear transformation

$$\mathbf{v}(t) = \mathbf{\Phi}\mathbf{V}(t) = \sum_{\mathbf{k=0}}^{\mathbf{N}} \mathbf{\Phi}_n V_n(t) \tag{2}$$

By mass-normalizing each $\mathbf{\Phi}_n$ such that

$$(\mathbf{\Phi}_h)^T \mathbf{m}\mathbf{\Phi}_k = \delta_{hk} \quad h,k = 1,...,N \tag{3}$$

where δ_{hk} is the kroneker symbol, and applying the transformation (2) to (1), *N* decoupled equations, describing the seismic response of the building, are obtained

$$V_n(t) + 2\omega_n \xi_n \dot{V}_n(t) + \omega_n^2 V_n(t) = \mathbf{\Gamma}_n(t)\ddot{\mathbf{v}}_{\mathbf{g}}(t) \tag{4}$$

In Eq. (4) $f_n = 1/2pw_n$ and ξ_n are *n*-th-th modal frequency and modal damping ratio respectively, while, $\mathbf{\Gamma}_n = \left\lfloor \Gamma_{nx}, \Gamma_{ny} \right\rfloor$ is the vector of the *Modal Participation Factor* in x and y directions. Modal Participation Factors depend on mass-normalized mode shapes and on the mass matrix through the following equation

$$\mathbf{\Gamma}_n = \mathbf{\Phi}_n^T \mathbf{m}\mathbf{b} \tag{5}$$

The set of parameters $\left\{\mathbf{\Phi}_n, f_n, \xi_n, \Gamma_{n,} n = 1,2,..., N\right\}$ defines the building *modal model.*

3. The SMAV model

SMAV is an acronym for Seismic Model from Ambient Vibrations and, in the present context, it indicates a modal model in which ω_n, ξ_n and $\boldsymbol{\Phi}_n$ are experimentally obtained from ambient noise measurements.

3.1 Participation factors

Operational modal analysis supplies only mass-unscaled mode shapes. Therefore the participation factors, unlike the modal parameters, cannot be directly derived from the measurements. However the Γ_n can be obtained using Eq. (5), provided the matrix **m** is known and the mode shapes have been identified in all the *N* degrees of freedom of the system (1).

To this end the original mechanical model (1) is condensed in a simpler one whose mass matrix can be computed on the basis of few simple informations about the geometry of the structure. Model condensation or reduction is a very complex and well studied problem, Friswell *et al.* (1995), Besselink *et al.* (2013). However for the present case the problem is much simpler, because our purpose is not to get a condensed model that preserves the key dynamic properties of the original model, such as eigenvalues and eigenvectors, but only to have a model for which all DOFs are measured and the mass matrix can be easily computed. Moreover, if the mode shapes are experimentally known, as is the present case, the partecipation factor are not very sensitive to the mass distribution. Therefore also a row estimation of the mass matrix can be sufficient for SMAV model implementation. In the following the procedure for condensing the model and computing the partecipation factor is shortly illustrated.

As mentioned above, if not all the *N* DOFs are measured, Eq. (5) can still be applied, provided that the system (1) is reduced to a condensed model characterized by N^c DOFs (where $\bullet^C$ stands for *condensed*) that are all *observable*, where the term *observable* means “directly measured” or “obtainable by the measurements”. In the following the observability property will be indicated by the symbol $\bullet^O$. For the condensed model is $N_c<N_o<N$.

In many practical cases the condensation can be performed assuming the so called “rigid floor hypothesis”, corresponding to the assumption that all the points of the building, lying on the same horizontal plan, move according to the rigid motion equations. If a rotational center is defined for each floor, all the observable DOFs lying on the *i*-th -th floor, can be expressed as a linear combination of two translations U_i, and V_i and of the rotation Θ_i of that floor. Once the translations and the rotations of all the *P* building floors are orderly collected in the vector $\mathbf{v}^C \in \mathsf{R}^{3P}$, the following linear transformation can be established between the observable DOFs $\mathbf{v}^O \in \mathsf{R}^l$ and the condensed ones

$$\mathbf{v}^O = \mathbf{D}\mathbf{v}^C \tag{6}$$

Where $\mathbf{D} \in \mathsf{R}^{\mathbf{l} \times \mathbf{N^C}}$.

The mass matrix of the condensed model can be easily derived under the hypothesis that the whole building mass is concentrated at the level of the floors, as can be admitted for framed reinforced concrete buildings, where the masses of the infill walls can be neglected. In this case the mass matrix $\mathbf{m}_C \in \mathsf{R}^{3P\times 3P}$ has the following form

$$\mathbf{m}^C = \begin{bmatrix} \mathbf{m}_1^C & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ 0 & \dots & \mathbf{m}_P^C \end{bmatrix} \tag{7}$$

in which the sub-matrix relative to the *i*-th floor is

$$\mathrm{m}_i^C = \rho \begin{bmatrix} \mathsf{A}_\mathrm{i} & 0 & 0 \\ 0 & \mathsf{A}_\mathrm{i} & 0 \\ 0 & 0 & \mathsf{I}_\mathrm{i} \end{bmatrix} \tag{8}$$

where i=1,2,…,P and $\mathbf{0} \in \mathsf{R}^{\mathbf{3\times3}}$ is a matrix of zeros. In Eq. (8) A_i e I_i are respectively the area and the polar moments of the i-th floor, defined according to the geometric property of the plan, while ρ is the mass per unit area.

By applying Eq. (6) to the observable mode shape $\mathbf{\Phi}_n^O$ and computing the pseudo-inverse of **D**, the condensed mode shape is then obtained

$$\mathbf{\Phi}_n^C = \frac{1}{\mu}(\mathbf{D}^\mathbf{T}\mathbf{D})^{-1}\mathbf{D}^\mathbf{T}\mathbf{\Phi}_n^O \tag{9}$$

where if μ must is selected such that $(\mathbf{\Phi}_n^C)^T \mathbf{m}^C \mathbf{\Phi}_n^C = 1$, also the condensed mode shapes are mass normalized.

Finally by substituting Eqs. (9) and (7) in Eq. (5) and considering the drag matrix of the condensed model $\mathbf{b}^C$, the n-th participation factor is obtained

$$\mathbf{\Gamma}_n^C = (\mathbf{\Phi}_n^C)^T \mathbf{m}^C \mathbf{b}^C \tag{10}$$

It is important to note that, for the computation of $\mathbf{\Gamma}_n^C$, it is not necessary to actually know the value of ρ, because it is eliminated when the matrix $\mathbf{m}^C$ is entered into Eq. (10) and the $\mathbf{\Phi}_n^C$ are mass normalized.

3.2 The orthogonality matrix

The condensed mode shapes $\mathbf{\Phi}_n^C$ allow defining the *ortogonality matrix* $\boldsymbol{\alpha}$, whose elements α_{hk} are expressed through the following equation

$$(\mathbf{\Phi}_h^C)^T \mathbf{m}^C \mathbf{\Phi}_k = \alpha_{hk} \tag{11}$$

In ideal condition (see Eq. (3)) is $\alpha_{hk}=\delta_{hk}$, that is every entry of $\boldsymbol{\alpha}$ outside the diagonal (*cross-term*) is equal to zero. However, in real situations some condensed mode shapes can be not orthogonal each others and therefore the cross terms can be different from zero, This is due to different causes, as errors in mode shape identification, approximations in condensed mass matrix calculation and finally by the non-fulfillment of the rigid floors hypothesis by one or more of the original identified mode shapes $\mathbf{\Phi}_n$. However, if $\alpha_{hk} \neq 0$, the ideal orthogonality conditions can be restored by correcting mode shape k in the following way

$$\mathbf{\Phi}_k^{C*} = \mathbf{\Phi}_k^C - \alpha_{hk}\mathbf{\Phi}_h^C \tag{12}$$

It is noted that the application of Eq. (12) change the mode shape k, but does not affect the mode h. This means that the mode shapes resulting from the orthogonality correction depend on which mode shape is assumed as reference and therefore left unchanged and on the order in which the corrections are applied. A simple case is when two modes are almost orthogonal while a third is not compared to the other two. In this case we proceed leaving unchanged the first two modes and correcting the third compared to the other two.

3.3 Modal mass

In general, only the first few modes of the building can be identified from ambient noise measurements. This means that the SMAV model is incomplete in the modal space.

Formally, by rewriting Eq. (2) with reference only to observable DOFs and distinguishing the N^O observable mode, i.e., the mode that can be actually identified, from the N^U non identfied modes, the following equation is obtained

$$\mathbf{v}^O(t) = \sum_{n=1}^{N^O} \mathbf{D}\mathbf{\Phi}_n^C V_n(t) + \sum_{n=N^O+1}^{N^O+N^U} \mathbf{D}\mathbf{\Phi}_n^C V_n(t) \tag{13}$$

Where $N^O+N^U=3P$, and the quantity

$$\mathbf{R}(t) = \sum_{n=N^O+1}^{N^O+N^U} \mathbf{D}\mathbf{\Phi}_n^C V_n(t) \tag{14}$$

is the error in seismic response evaluation due to the modal incompleteness. It is known [7] that the following expression holds for the total mass of the building M_T

$$\mathrm{M}_T = \sum_{n=1}^{3P} \Gamma_{nx}^2 = \sum_{n=1}^{3P} \Gamma_{ny}^2 \tag{15}$$

where Γ_{nx}^2 and Γ_{ny}^2 are the effective mass of the n-th mode in x and y directions. The influence of modal incompleteness in vulnerability assessment can be indicated by the *modal mass ratio*, which is the ratio between the observable modal mass in x and y direction and the total mass of the building

$$r_{\mathrm{M}} = \left[r_{\mathrm{M}x}, r_{\mathrm{M}y}\right] = \frac{1}{\mathrm{M}_T}\left[\sum_{n=1}^{N^O}\Gamma_{nx}^2, \sum_{n=1}^{N^O}\Gamma_{ny}^2\right] \tag{16}$$

4. Vulnerability assessment

Vulnerability assessment is performed according to the Italian Building Code. The assigned earthquake is expressed by the pseudo-acceleration response spectrum $S_{pa}(T;\xi)$, where T is the structural period and ξ is the damping ratio. The spectrum is defined considering the site seismic hazard on the bedrock and the subsoil local amplification. It is noted that modal damping is no longer a parameter of the SMAV model, because it is included in the definition of the seismic input.

In general ambient vibration frequencies are different from seismic frequencies. As a matter of a fact, from the analysis of many experimental data, it results that, for the same building, the modal frequencies identified from seismic signals may be considerably lower than the frequencies identified from ambient vibrations. In particular in the case of framed reinforced concrete buildings, with infill walls, it was observed, still well below any damage threshold, a decrease of frequency up to 40%, as reported in Gueguen (2013). On the contrary mode shapes generally show no appreciable changes, unless the building is damaged. In order to include such phenomenon in the SMAV model, a probabilistic approach will be adopted. In this approach, in place of deterministic values, a density probability function will be associate to each experimental modal frequency.

The parameter of the structural response that determines the operational or non operational condition of the building is the drift, i.e., the relative displacement (in x or y direction) of two points of contiguous floors, arranged along the same vertical, divided by the inter-story height.

4.1 Spectral analysis and drift computation

The seismic action, applied simultaneously in the two horizontal directions, is represented by the following vector of spectra

$$\mathbf{S}_a(T;\xi) = \left[aS_{pa}(T;\xi), bS_{pa}(T;\xi)\right]^T \tag{17}$$

in which is a=1=1and b=0.3 if x is the main direction of the analysis, while it is a=0.3 and b=1 if the main direction is y. The contribution of n-th mode to the building displacements is given by the following equation

$$\mathbf{v}_n^O = \left(\frac{1}{2\pi f_n}\right)^2 \mathbf{D\Phi}_n^C \mathbf{\Gamma}_n \mathbf{S}_{pa}(1/f_n;\xi) \tag{18}$$

while the drifts vector $\mathbf{\Delta}_n = \left[\Delta_{n1}, \Delta_{n2}, ..., \Delta_{nQ}\right]^T$ is

$$\mathbf{\Delta}_n = \mathbf{C}\mathbf{v}_n^O \tag{19}$$

where $\mathbf{C} \in \mathrm{R}^{Q\times N^O}$ is a sparse matrix that allows to transform the displacement vector in the vector of drifts. The j-th row of $\mathbf{C}$, associated to the j-th drift, has the following structure $\{0,...,-1/h_p,...,-1/h_p,...,0\}$, where the only non-zero elements correspond to those displacements contributing to the j-th drift and h_p is the inter-story height between the floor p and p-1.

Finally the vector of the drifts is obtained through the Complete Quadratic Combination

$$\mathbf{\Delta} = \left(\sum_{k=1}^{N^O}\sum_{h=1}^{N^O} c_{hk}\mathbf{\Delta}_k\mathbf{\Delta}_h\right)^{1/2} \tag{20}$$

where, defined $\beta_{hk} = f_h/f_k$:

$$c_{hk} = \frac{8\xi^2\beta_{hk}^{3/2}}{(1+\beta_{hk})\left[(1-\beta_{hk})^2 + 4\xi^2\beta_{hk}\right]} \tag{21}$$

4.2 Operational index

According to the Italian Building Code a building is no more operational if its maximum drift exceeds a limit value Δ^L, defined according to the structural type. Such limit is 0.003 for framed reinforced concrete buildings. The following dimensionless Operational Index is then defined:

$$I_{op} = \frac{\Delta^L}{max(|\Delta_i|)} \qquad \text{for } i = 1,2,...,Q \tag{22}$$

Where I_{op} is chosen as the minimum value obtained from the analysis in the direction x and y. The uncertainty of the ratio between seismic and ambient frequencies is considered as described below. If $\mathbf{f}^{av} = \left[f_1^{av}, f_2^{av},..., f_{N^O}^{av}\right]$ is the vector of the frequencies identified from ambient vibrations, the following set of vectors is defined

$$F = \left\{\lambda_i \mathbf{f}^{av}\right\};\ \lambda_i = 0.6 + 0.4\frac{i-1}{q-1}; i = 1,2,...,q \tag{23}$$

Where λ_i is the reduction coefficient of the ambient frequencies. If for example q=21, λ_i takes the values 0.60, 0.62,...,0.98, 1. For each element of the set F the spectral analysis described in sub-section (4.1) is performed and by repeatedly applying equations from (17) to (22), the following set of Operational Indexes is generated:

$$\mathrm{I}_{op} = \left\{I_{op}^1, I_{op}^2,..., I_{op}^{q-1}, I_{op}^q\right\} \tag{24}$$

The vulnerability of the building can than be expressed by two dimensionless quantities: the average value $\bar{I}_{op}$ and the standard deviations σ_{op} of the elements of I_{op}.

4.3 Operational probability curve

The building vulnerability can be expressed in a probabilistic form by assuming a lognormal probability distribution, Song 2004, for the random non-negative variable I_{op}. Therefore, the probability that the Operational Index of the building I_{op} is greater or equal to a given value I $(I_{op} \geq I)$, is given by the following *Operational Probability Curve*

$$P_{op}(I_{op} \geq I) = 1 - \frac{1}{\sqrt{2\pi}\sigma_{\ln I}} \int_0^I \frac{1}{\gamma} e^{-\frac{1}{2\sigma_{\ln I}^2}\left[\ln(\gamma)-\theta_I\right]^2} d\gamma \tag{25}$$

where is

$$\theta_I = \ln\left[(\bar{I}_{op})^2 \Big/ \sqrt{1 + \frac{\sigma_{I_{op}}^2}{(\bar{I}_{op})^2}}\right], \sigma_{\ln I} = \sqrt{\ln\left(1 + \frac{\sigma_{I_{op}}^2}{(\bar{I}_{op})^2}\right)} \tag{26}$$

It is worth to underline the value of $P_{op}(I_{op} \geq I)$ for I=1 gives exactly the probability that the building states operational for the assigned earthquake,

5. Experimental cases studies

Two cases studies are presented. In the first one the method is validated by comparing the experimental seismic response of a real building to the maximum drift predicted by the model. In the second one the application of the whole procedure to real strategic building, until the determination of the Operational probability Curve, is illustrated.

5.1 Case study 1: method validation

The method is validated analyzing the experimental seismic vibrations of a real building. The building belongs to the Seismic Observatory of Structures (OSS), Spina *et al*. (2009): a network of public buildings and bridges whose structural vibrations are permanently monitored. The OSS, created and managed by the Italian Department of Civil Protection, is composed of 150 structures (65% of the buildings are reinforced concrete and 35% masonry) and, since 1999 up to now, has collected more than 450 seismic recordings. Each structure of the OSS is monitored with force balance accelerometers in a number that ranges from 15 to 32, according to its size and complexity, while a tri-axial accelerometer is always positioned on the ground near the structure. The building selected for the validation is a school at Barberino di Mugello (see the photo of Fig. 1), a small town near Florence. The school is a three floors frame reinforced concrete building, with irregular plan. The seismic monitoring system is composed of three accelerometers (2 bi-axials and 1 mono-axial) per floor, according to the layout shown in Fig. 2.

The ambient vibrations of the building were recorded for 1800 s at a sampling frequency of 200 Hz. Signals were analyzed using the Test.Lab Operational Modal Analysis-LMS © software. The cross-spectra of the all recorded signals were computed respect to signals of the top floor, using *weighted correlogram method.* The PolyMax algorithm [10] was then applied to identify natural frequencies and mode shapes. Three modes has been extracted from the experimental data at frequencies 4.72 Hz, 5.35 Hz and 7.05 Hz. The corresponding mode shapes is shown in Figs. 3-5. The cross-terms of the orthogonality matrix were found: α_{12}=-0.81, α_{13}=0.10 and α_{23}==0.31, thereby indicating that the mode shapes 1 and 3 were almost orthogonal to each other, while the second mode shape was not compared with the other two. The modal mass ratios were found r_{Mx}=0.88 and r_{My}=0.80. Mode shape 2 was then corrected by applying twice Eq. (12), the first with h=1 and the second with h=3 (k=2 both times, of course). The new cross-terms were found α_{12}=0.09, α_{13}=0.10 and α_{13}=0.00, indicating the all the new mode shapes were almost orthogonal. The modal mass ratios became r_{Mx}=0.98 and r_{My}=0.97.

The three main earthquakes recorded at the building were used for the validation. The main parameters of these seismic signals are summarized in Table 1. All the events are of low intensity, with a maximum structural drift, obtained by double numerical integration of the accelerations, that is of an order of magnitude lower than the threshold for the operational limit state. The lack of experimental data from large earthquakes during which the building has gone beyond the threshold of linear elastic behavior is clearly an objective limit to the experimental validation of the method presented here. However, this limitation is partially mitigated by the fact that the purpose of

SMAV is precisely to study the behavior of the structure up to the first structural damage.

The probability density function of $\max(|\Delta_i|)$ was computed for each event. To this end the response spectra of the recorded ground accelerations in both horizontal directions were used as SMAV model input. For damping it was assumed ξ=0.05. The outcomes of the analysis are shown in Figure 6. It is observed that, for the main two earthquakes 1 and 3, the experimental drifts are characterized by a high probability of occurrence, near to the maximum of the distribution. The same is not true for earthquake 2. The difference between the expected drift and the experimental one are 0.5 %, -14.0 % and 5.5 % respectively. Is is worth to observe that the expected drift resulting from the SMAV model, is rounded down for lower intensity (earthquake 2), almost exactly for the intermediate one (earthquake 1) and rounded up for event characterized by highest drift (earthquake 3). This trend is due to the decreasing of seismic frequencies as the structural vibrations increase, while the mean frequency of the set (23) is $0.8\mathbf{f}^{av}$ for all the earthquakes.

Fig. 1 The school at Barberino di Mugello

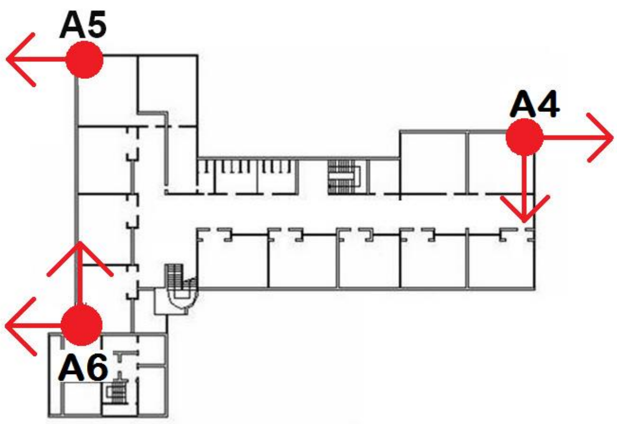

Fig. 2 Sensor layout of 3rd floor

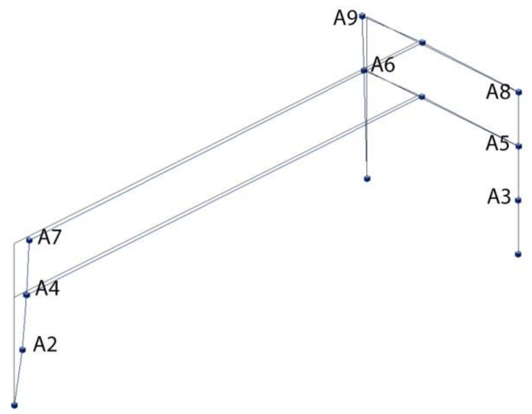

Fig. 3 Mode shape 1 – f_1=4.72 Hz

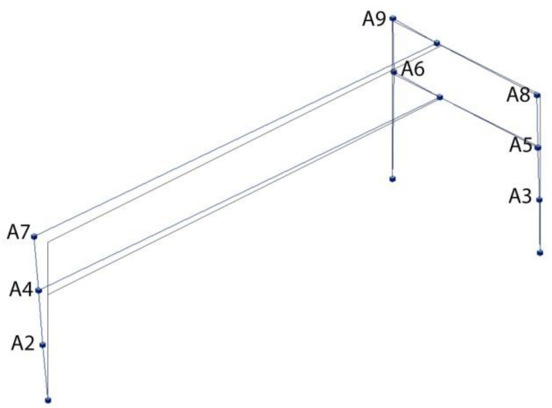

Fig. 4 Mode shape 2 – f_2=5.35 Hz

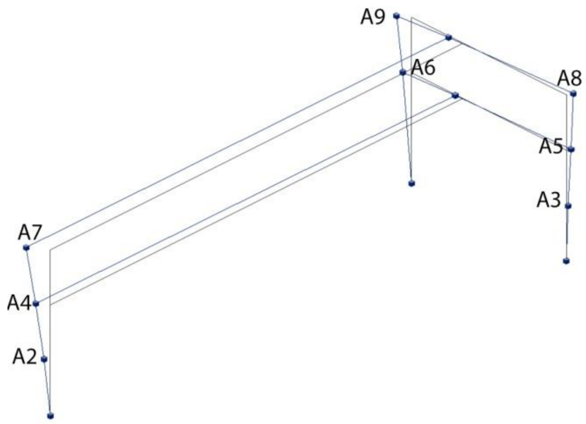

Fig. 5 Mode shape 3 – f_3=7.05 Hz

Table 1 Main earthquakes recorded at the school of Barberino di Mugello

N	date	Time UTC	M	d km	PGA m/s^2	Δ mm/m
1	1-3-2008	8:43	4.1	5	0.736	0.18
2	1-3-2008	10:43	4.1	6	0.306	0.11
3	14-9-2009	20:04	4.3	4	0.678	0.32

(M: magnitude, d: distance of the building from epicentre, PGA: Peak Ground Acceleration,Δ: maximum drift)

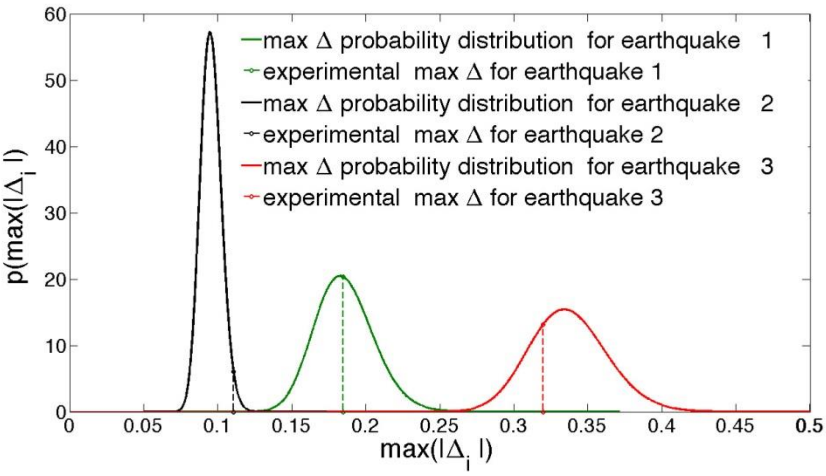

Fig. 6 Probability density functions and experimental values of maximum drifts

5.2 Case study 2: application of the method to the Faenza hospital

The method described and validated above, was applied to the hospital 'San Pier Damiano' (see Fig. 7), located in Faenza, a small italian town about 50 km from Bologna. It is a five-story frame reinforced concrete building, separated in two different dynamically independent structures by a seismic gap. Only the main structure, with a T-shaped plan, was considered in this analysis. In order to assess the bulding capability in conserving its operational condition, two different return periods were considered for the eathquake. First of all, to meet the requirements of a strategic building to be operational in the occurence of a desctructive earthquake, T_r=475 years was considered. Secondly, for comparing the actual performance of the Faenza Hospital with the request of Italian Building Code for stretegic building, the analysis was repeated for T_r=60 years. The different steps of the process leading to the final vulnerability assessment are shortly illustrated below.

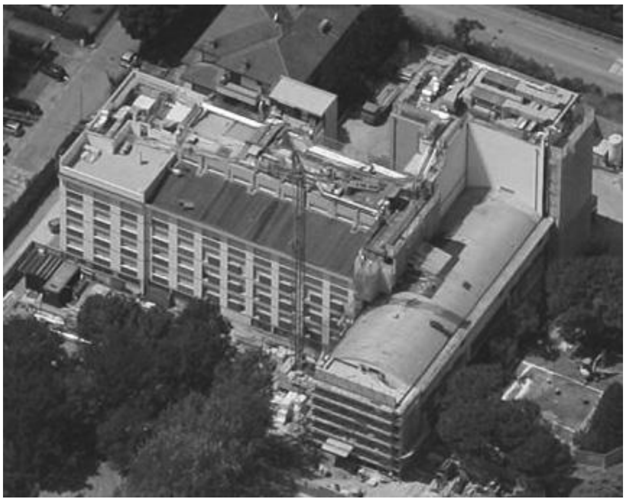

Fig. 7 The Hospital “San Pier Damiano” in Faenza

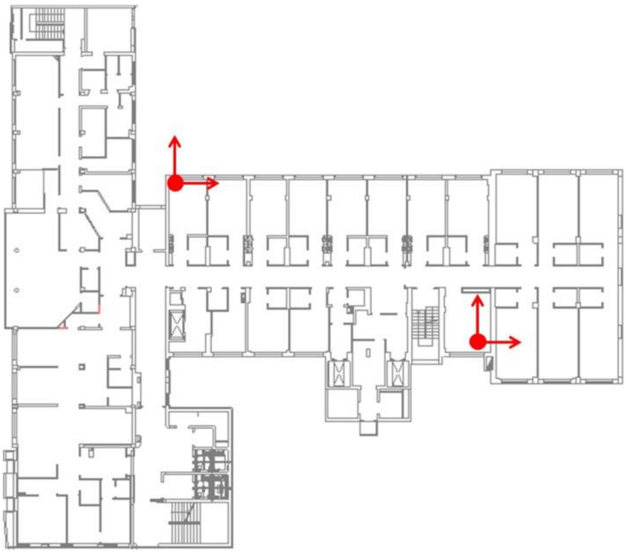

Fig. 8 Typical Sensor layout - 1st floor

Four independent units were used for vibrations measurement. Each unit integrates into a single device a force-balance triaxial accelerometer, a 24-bit analog to digital converter, a solid state disk drive for data storage, and a GPS receiver for time synchronization. Instrument are connected each other and with a laptop PC through a Wi-Fi network. The units were placed on two floors of the building at a time, simply by resting them on the floor, according to the scheme in Fig. 8. Initially the fifth and fourth floor were instrumented. Subsequently, after the recording of ambient vibrations for 1800 s at a sampling frequency of 256 Hz, the units on the fourth floor were moved downstairs. The procedure was then repeated to cover all the floors, each time leaving fixed the units at the top level. Four partial configurations were considered. It is important to observe that, even if the actual measurement points are located only on the stem of the T-shaped plan, because the SMAV model provides the seismic displacements as rigid translations and rotations, the displacement and the drifts can be computed in every points of the floor.

For each configuration modal parameters were identified according to the same method described in section 5.1. Finally the overall mode shapes were reconstructed using fixed accelerometers as reference. The three identified natural frequencies were 2.88 Hz, 3.48 Hz, and 4.00 Hz. The mode shapes are shown in Figs. 9-11.

The SMAV Model were implemented by condensing the mode shapes with Eq. (9) and by computing the matrix $\boldsymbol{m}^C$ as in Eq. (8). The resulting cross-terms of the orthogonally matrix initially were found α_{12}=0.62, α_{13}=0.71 and α_{23}=0.01. After the orthogonalization of $\boldsymbol{\Phi}_1^C$ compared to second and the third mode shapes, also first two cross-terms became close to zero, while the modal mass ratio was found r_{Mx}=0.92 and r_{My}=0.54.

The seismic hazard was defined at the geographic coordinates of the building according to Annex B of Italian Building Code. The seismic input was expressed in terms of acceleration response spectrum associated to soil category "C" (corresponding to an equivalent shear waves velocity in the upper 30 m VS,30 between 180 m/s and 360 m/s) as resulting from the available geotechnical and geophysical information for foundation soil. The corresponding pseudo-acceleration response spectra are shown in Fig. 12, where the range $[\lambda_q T_{3,} \lambda_1 T_1]$ covered by the possible building natural periods is also indicated.

The seismic inputs defined above were applied to the SMAV model. The procedure described in section 4 was performed, obtaining the outcomes shown in Table 1 and Fig. 13. It results that the building has a probability to remains operative for T_r=475 years close to zero, while for T_r=50 years such probability is about 40%. The corresponding Operational Indexes are 0.45 and 0.97. This last is near the unity but has a standard deviation of 0.25.

It results that the building is not operational nor for the demands of the civil protection (T_r=475 years) nor according to Italian Building Code (T_r=50 years). Therefore, in its present structural configuration, it does not appear suitable to perform its strategic building function.

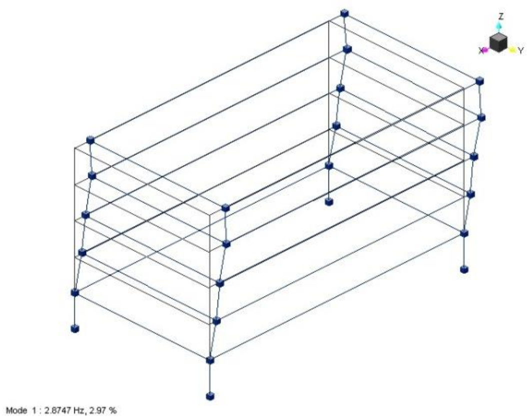

Fig. 9 Mode shapes 1 – f_1=2.88 Hz

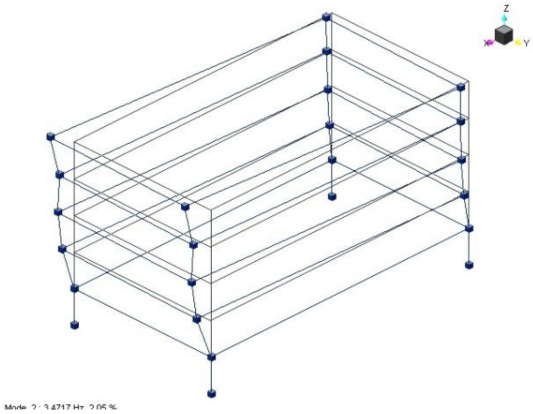

Fig. 10 Mode shapes 2 – f_2=3.48 Hz

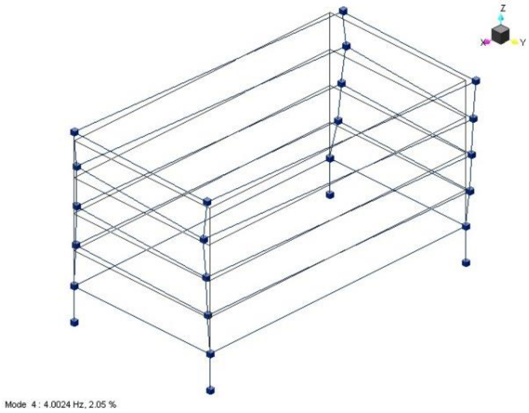

Fig. 11 Mode shapes 3 – f_3=4.00 Hz

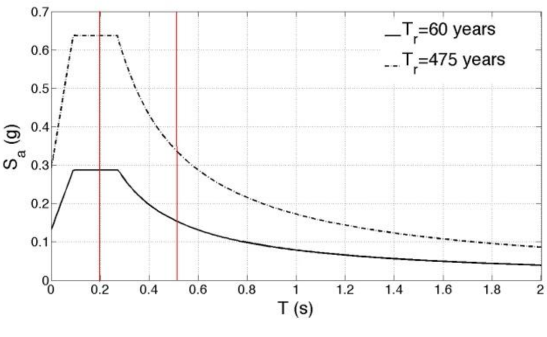

Fig. 12 Seismic input for Italian Technical Code: pseudo acceleration response spectra for soil category "C" at $T_r = 50$ years and $T_r = 475$ years - Vertical red lines defines the range in which fall the building natural periods

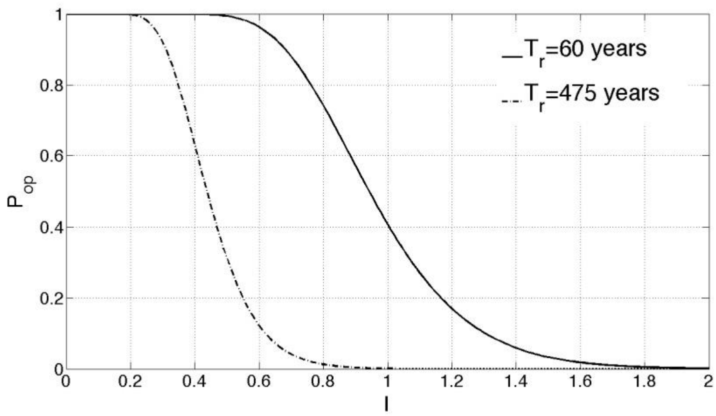

Fig. 13 Operational curves for seismic input from Italian Technical Code at 50 years and 475 years

Table 2 Dimensionless operational parameters

Seismic Input T_r (years)	$\bar{I}_{op}$	$\sigma_{I_{op}}$	$P_{op}(1)$
50	0.97	0.25	0.422
475	0.45	0.12	0.001

6. Conclusions

A new method for seismic vulnerability assessment of buildings has been presented. The method is based on the experimental measurement of ambient vibrations and is focused on the evaluation of the limit structural operational condition, that is particularly significant for strategic buildings, devoted to seismic emergency management. The theoretical mathematical basis of the method has been illustrated in detail. A probabilistic approach aimed to deal with the uncertainty of the ratio between seismic and ambient natural frequencies has been also presented. The building vulnerability depends on its maximum drift compared to a limit threshold. It is eventually expressed by an Operational Index and by an Operational Probability Curve. The validation of the method with experimental seismic data coming from a real building has been shown. The good agreement between the probabilistic prediction and the experimental drift proves the reliability of the method. Finally an application of the method for assessing the vulnerability of real building, the “San Pier Damiano Hospital” of Faenza, has been presented. All the steps composing the estimation procedure has been described. Measurement methodology, identified modal parameters and seismic input definition are also shown. The Operational Index and the Operational Probability Curve of the building has been calculated for the expected earthquake at 50 and 475 years of return period, in order to consider both seismic emergency condition and the Italian Building Code, resulting in a negative opinion about the ability of the building to carry out its strategic functions.

On the basis of the obtained results the method appears ready to be extensively applied by local authorities to classify the buildings selected for emergency management and to decide where to address possible interventions of seismic retrofit.

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