Structural monitoring and maintenance by quantitative forecast model via gray models

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Abstract. This article aims to quantitatively predict the snowmelt in extreme cold regions, considering a combination of grayscale and neural models. The traditional non-equidistant GM(1,1) prediction model is optimized by adjusting the time-distance weight matrix, optimizing the background value of the differential equation and optimizing the initial value of the model, and using the BP neural network for the first. The adjusted ice forecast model has an accuracy of 0.984 and posterior variance and the average forecast error value is 1.46%. Compared with the GM(1,1) and BP network models, the accuracy of the prediction results has been significantly improved, and the quantitative prediction of the ice sheet is more accurate. The monitoring and maintenance of the structure by quantitative prediction model by gray models was clearly demonstrated in the model.

Keywords: BP network; combined prediction; gray optimization; prediction; structural monitoring and maintenance

1. Introduction

Frost heaving of railway subgrades leads to uneven longitudinal settlement or uplift of rails, which in some cases causes train safety accidents and brings serious safety hazards. Some scholars have conducted related research on the detection of frost heaving of railway subgrades. Wu et al. (2022) used the non-contact measurement method of machine vision and optical imaging to realize the real-time monitoring and measurement of the frost heave of the Harbin-Dalian subgrade, and obtained the detection data of the subgrade surface freeze-thaw displacement (hereinafter referred to as the frost heave data). This data reflects the deformation of the railway embankment elevation and has an important impact on the smoothness of the track. At the same time, many scholars at home and abroad have conducted a lot of research on the subgrade frost heave prediction model. Asaoka (2019) proposed the Asaoka method based on the vertical one-way consolidation theory and using the measured subgrade deformation data to calculate the post-construction settlement. Sun (2020) established the GM(1,1) model to predict the subgrade frost heaving data obtained by isochronous sampling of a passenger dedicated line in Northeast my country. Good data has good effect, but it is difficult to predict complex data with many factors. Qi et al. (2018) used the inertial correction method in the BP neural network, introduced dynamic learning factors and inertial

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factors, and established a prediction model for the deformation of the frozen soil subgrade of the Qinghai-Tibet Railway. The disadvantage of this method is that the training process of the neural network model is less stable and has higher randomness. The prediction model established according to the relevant influencing factors of subgrade frost heave can reflect its development and change, but some of the influencing factors are difficult to quantify, and the prediction accuracy of the model is thus restricted. Most of the subgrade frost heaving prediction methods established at present do not reflect the dynamic randomness and many influencing factors of subgrade frost heaving well, and the prediction effect and accuracy are not ideal. Therefore, this paper adopts the method based on neural network and optimized gray combination model, taking the measurement time series of frost heave data as input, and quantitatively predicts non-equidistant subgrade frost heave data. The advantage of the model in this paper is that the gray model is suitable for small sample prediction and has good stability, but the model is relatively simple and has poor effect on data sources with complex conditions and multiple factors. The model is combined with the neural network to better solve the problem of its application limitation. Based on the input of time series, from the mathematical point of view, the statistical law and potential relationship between frost heave deformation data under the comprehensive action of complex factors can be excavated, so as to achieve the purpose of quantitative prediction with higher precision, and at the same time make the model have better versatility. The significance and value of this research lies in the prediction of frost heave deformation of subgrade, which can assist line maintenance management decision-making, realize early warning of potential safety hazards, and is also of great significance for mastering subgrade frost heave deformation laws.

2. Model building ideas and process

The research object of gray system theory is a small object with known partial information. In order to expand gray The scope of application of color theory, many scholars regard the time interval as the multiplication sub, which is similar to the method of constructing the gray GM(1,1) model to construct A non-equidistant gray GM(1,1) model. The BP network is a Multi-layer forward neural network, which can realize any input to output Non-linear mapping, with strong nonlinear capabilities, general use package The 3 -layer BP network including input layer, hidden layer and output layer can realize Now for any nonlinear signal, the high-precision approximation of the system. therefore, Firstly, the non-equidistant gray GM(1,1) is optimized and improved, and the The optimized non-equidistant gray model is then combined with the BP network, Use the BP network to correct the residual error of the model, learn from each other's strengths, and construct Establish a combined prediction model and apply the model to the frost heaving data of railway subgrade In the prediction of the data, the flow chart of the established model is shown in Figure 1 .

The modeling steps for railway embankment frost heave data include: 1) Carry out a grade test on the original embankment frost heave collection data $x^0(t_i)$, for different Qualified data carry out translation transformation; 2) Calculate time-distance weighting matrix $P$ according to its time coefficient $t(i)$ of subgrade frost-heave data passing inspection ; 3) Take time distance as multiplier for $x^0(t_i)$, and accumulate, Obtain the sequence $x^1(t_i)$; 4) Obtain the constant parameters $a$ and $u$ under the condition of the weighted matrix with the least square method ; 5) Calculate the time response function to obtain the initial predicted value $\delta^0(t_i)$; 6) Make a difference between the initial predicted value and the original data to get the residual sequence $\Delta d(t_i)$; 7) Input the residual sequence into the BP network model for training, and output the predicted
residual after correction $\Delta D(t_i)$; 8) Adding the initial predicted value and the predicted residual value to get the final predicted value $Q(t_i)$.

Not all the data can be used for GM(1,1) modeling, only the data satisfying certain conditions, the established GM(1,1) model is meaningful. The $X$ order $X = (x_1, x_2, x_3, ..., x_n)$ ratio can be expressed as formula (1). Only at that time , $\phi_k \in (0.1353, 7.389)$ non-deformed GM (1,1) modeling can be done, which is called the basic condition of GM (1,1) modeling .

$$\phi_k = \frac{x_{k-1}}{x_k}$$  \hspace{1cm} (1)

To establish an effective GM(1,1) model, the practical condition should also be met, that is, the level ratio should fall in a $\phi_k$ subinterval close to 1 $(1 - \varepsilon, 1 + \varepsilon)$ Therefore, this subinterval is called the $(1 - \varepsilon, 1 + \varepsilon) \in (0.1353, 7.389)$ grade boundary area. The method of determining the boundary area of the scale is to $X$ start from the boundary area of the original sequence, find out $\phi_k$ the boundary area at last, and then obtain the practical condition of $\phi_k \in (e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}})$.

Assuming that $[t_1, t_2, ..., t_n]$ subgrade frost heave sequences are measured within a certain time interval $x^0(t_i)$, a grade comparison test is required before using the original sequence. Level ratio detection is to check whether the original data sequence calculated by formula (1) $\varphi(i)$ falls within the limited interval $(e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}})$, if it falls within the interval, the data can be used directly, otherwise translation transformation is required, as shown in formula (2), select an appropriate The
constant $c$, until the calculated $\varphi'(i)$ new sequence $x^0(t_i)$ falls within the limited interval, at this time, the new sequence can be predicted and analyzed, and the predicted analysis is completed, and then the inverse transformation is restored.

$$x^0(t_i) = x^0(t_i) + c, \quad \text{ini} = 1, 2, 3, ..., n \quad (2)$$

Calculate the time interval series $\Delta t(k)$ according to the time series $t(i)$, where $k = 2, 3, ..., n$. The time interval is used as the multiplier, and $x^0(t_i)$the sequence is accumulated once, as shown in formula (3), to obtain the sequence $x^1(t_i)$. The differential equation of whitening form can be established from the sequence $x^1(t_i)$, such as formula (4), where $\alpha$ is called the development coefficient and $\nu$ the gray action. The role of the two is to control the uncertainty relationship between the size of the development situation of the gray system and the change of the response data.

$$x^1(t_i) = \sum_{k=2}^{i} x^0(t_k) \Delta t_k, \quad i = 1, 2, 3, ..., n \quad (3)$$

Integrate the formula (4) in $[t_{i-1}, t_i]$ the interval to get the formulas (5) and (6), and the formula $z^1(t_i)$ the background value on $x^1(t_i)$ the interval $[t_{i-1}, t_i]$. In order to obtain $\alpha \nu$ and $\alpha \mu$ 2 parameter values, use the least square method to formula (3) to get formula (7) and (8) 2 formula

$$x^0(t_i)\Delta t^i + az^1(t_i) = \nu \Delta t_i, \quad i = 2, 3, ..., n \quad (5)$$

$$z^1(t_i) = \int_{t_{i-1}}^{t_i} x1(t) \, dt = \frac{1}{2}(x^1(t_{i-1}) + x^1(t_i)) \quad (6)$$

$$\begin{bmatrix}
(a, \nu)^T = (B^TB)^{-1}B^TY
\end{bmatrix} \quad (7)$$

$$B = \begin{bmatrix}
-z^1(t_2) & \Delta t_2 \\
& \vdots & \vdots \\
-z^1(t_n) & \Delta t_n
\end{bmatrix}, \quad Y = \begin{bmatrix}
x^0(t_2)\Delta t_2 \\
& \vdots \\
x^0(t_n)\Delta t_n
\end{bmatrix} \quad (8)$$

If the initial value is specified $x^1(t_i) = x^0(t_i)$, then the time response function of formula (4) can be obtained as formula (9). Restore $x^0(t_i)$ the non-isochronous GM(1,1) model sequence that fits the original sequence $\delta^0(t_i)$ as formula (10)

$$\delta^1(t_i) = \left[-x^0(t_i) - \frac{\nu}{\alpha}\right] e^{-\alpha(t_i-t_1)} \quad (9)$$

$$\delta^0(t_i) = \frac{\delta^1(t_i) - \delta^1(t_{i-1})}{\Delta t_i} = \left[\frac{1-e^{\alpha \Delta t_i}}{\Delta t_i}\right] \left[-x^0(t_i) - \frac{\nu}{\alpha}\right] e^{-\alpha(t_i-t_1)} \quad (10)$$

obtaining the initial prediction sequence $\delta^0(t_i)$, use the MATLAB neural network toolkit to establish a BP network model and correct the residual.

Since the 3-layer BP network can approximate any nonlinear signal and system with arbitrary precision, considering the computational efficiency, a 3-layer BP network with a hidden layer is established. The input layer input is the residual sequence obtained by comparing the initial prediction of the optimized gray prediction model with the original data $\Delta d(t_i)$. The number of neurons in the hidden layer of the network is generally selected according to formula (11), where
and is the number of neurons in the input and output layers, and $\alpha$ is a constant between (0,10).

\[ n = \sqrt{p + q + \alpha} \quad (11) \]

The larger the number of neurons, the higher the training accuracy, but the slower the training rate, and it is prone to overfitting. The smaller the number of neurons, the faster the training speed, but it may lead to poor learning effect. Therefore, the model $\alpha$ in this paper is set to 9, and the number of neurons calculated by formula (11) is 10. The transfer function between the input layer and the hidden layer is selected as the tangsig hyperbolic tangent S-type transfer function, and the transfer function between the hidden layer and the output layer is a purelin linear transfer function. The output layer is the residual value of the network fit. The self-adaptive variable step size BP algorithm is selected, the learning results are observed, the network training parameters are continuously adjusted, the network is trained, and the post-training residual correction sequence is obtained $\Delta D(t_i)$. Predicted value of final subgrade frost heave $Q(t_i)$.

After adjusting the parameters several times and observing the test results, the optimal learning parameter settings determined in this paper are: learning rate 0.05, model training error precision 0.0005, and training times 1000 times.

In the non-equidistant GM(1,1) model mentioned above, the background value is calculated by using the trapezoidal formula to approximate the $x^1(t_i)$ area enclosed by the cumulative sequence and the x-axis on the interval $[t_{i-1}, t_i]$. But when the accumulated sequence changes drastically within this interval, the background value calculated by formula (6) has a large error. Literature [7] proposes a background value calculation method based on integral reconstruction, and proves that the background value constructed by this method is more in line with the actual conditions. In this method, the exponential function $c_i r$ is used to approximate the cumulative sequence $x^1(t_i)$, where the sum $c$ and $r$ are both undetermined coefficients, which are substituted into formula (6) to obtain the optimized background value calculation as formula (12)

\[ z^1(t_i) = \frac{x^1(t_i) - x^1(t_{i-1})}{\ln x^1(t_i) - \ln x^1(t_{i-1})} \quad (12) \]

In the data sequence used to establish the non-equidistant GM(1,1) model, each data has different effects on the model. For the existing data, it can be considered that the detection accuracy is the same, so it can be considered that the closer the data is to the prediction time point, the greater the role it plays in the prediction model and the higher the reliability. Each item of the original data sequence is given a weight value, and the size of the weight value is related to the time interval between the item data and the predicted data. Therefore, this paper proposes a weighted matrix based on the time interval

Define the increment factor $W$ and growth rate $w(j)$, and $w(1) = 1, w(2) = 2$, where the increment factor $W$ is a constant between (1,2). The value of the increment factor $W$ can depend on time.

The correlation of factors in the model, the larger the value, the more important the time factor. The growth rate $w(j)$ represents the time interval factor, and the smaller the interval between the data and the forecast time point, the higher the reliability in the forecast model, and the greater the weight value. Therefore, the formula for defining the weighting matrix $P$ is as formula (12)

\[
P = \begin{bmatrix}
W^{w(1)} & 0 & 0 & 0 \\
0 & W^{w(2)} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & W^{w(n)}
\end{bmatrix},
\]
\[ w(j) = \frac{t(j) - t(2)}{t(2) - t(1)} + 1, j = 3,4, ..., n \] (12)

After defining the weight matrix, the least square method is used to calculate the development system, \( a \) and \( v \)the formula (7) with the gray action is changed to the formula (13)

\[(a,v)^T = (B^T PB)^{-1}B^T PY\] (13)

In the non-equidistant GM(1,1) model, Initial value optimization based on the minimum cumulative residual error is defined \( x^1(t_i) = x^0(t_i) \), and the first value of the original data sequence is used as the initial value. However, in the actual embankment frost heave data fitting, the best fitting curve does not necessarily pass through a certain point on the original data, and the initial value in [7] is simply the mean value of the measured data. These initial value selection methods lack theoretical basis and will reduce the accuracy of the model. Therefore, this paper uses the method of calculating the minimum cumulative residual to determine the initial value. Define the initial value \( x^1(t_i) = x^0(t_i) + b \), that is, optimize the parameters for the initial value. Therefore, the final prediction formula (10) is changed to

\[ \sigma^0 = \frac{1-e^{-a\Delta t_i}}{\Delta t_i} \left[ x^0(t_i) + b - \frac{v}{a} \right] e^{-a(t_i-t_1)} \] (14)

The formula for calculating the cumulative residual \( E \) is written as

\[ E = \sum_{i=2}^{n} \left[ x^0(t_i) - \delta^0(t_i) \right]^2 + b^2 \] (15)

To make \( E \) the smallest, calculate the partial derivative expression of formula (15) with respect to \( b \), and set its value to 0 , and the value formula (16) of \( b \) under the condition of minimum cumulative residual error can be obtained

\[ b = \frac{\sum_{i=2}^{n} \frac{1}{\Delta t_i} (1-e^{-a\Delta t_i}) e^{-a\Delta t_i} x^0(t_i) + \sum_{i=2}^{n} \frac{1}{\Delta t_i} (1-e^{-a\Delta t_i}) e^{-a\Delta t_i} \left[ x^0(t_i) - \frac{v}{a} \right]}{1 + \frac{1}{\Delta t_i} (1-e^{-a\Delta t_i}) e^{-a\Delta t_i}^2} \] (16)

3. A numerical example

In 2012-12, the Harbin-Dalian Passenger Dedicated Line was officially opened for operation. The average monthly temperature in winter along the line was \(-13.5\,\text{to}\,-17.5^\circ\text{C}\), the extreme low temperature reached \(-40^\circ\text{C}\), the maximum freezing depth of the soil reached 205 cm, and frost heave was common throughout the subgrade. The data from 2013-12-19 to 2014-01-26 at k 186+600 in the downlink of Bayyuquan on the Harbin-Dalian Passenger Dedicated Line is selected as the prediction test value to test the prediction effect of the model in this paper. The actual measurement data are shown in Table 1. The measured values are all the freeze-thaw displacement values of the roadbed surface.

According to the modeling method mentioned above, the data is firstly tested for grades. The number of data \( n = 19 \) can be calculated to obtain a limited interval of \((0.905, 1.105)\). The original data calculated by formula (1) \( \varphi_{\text{max}} = 1.268 \)exceeds the limited interval, so the original data is translated and changed, and the translation constant \( c \) in formula (2) is set to 5, that is, the calculated \( x^0(t_i) = x^0(t_i) + 5 \) data sequence after transformation can be obtained \( \varphi_{\text{max}} = 1.072 \), \( \varphi_{\text{min}} = 1.009 \), passed the grade ratio test. Therefore, according to the modeling steps
Table 1 Actual measurement data

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Table 2 Fitting effect comparison

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introduced above, firstly use the $x^0(t_i)$ first 16 items of the shifted data to establish an optimization model, define the weight increment factor in the model $W = 1.4$, and obtain the initial predicted value. The initial gray model prediction error is shown in Fig. 2. It can be seen from Fig. 2 that the initial residual has a high degree of nonlinearity. Then bring the initial prediction value and the residual calculated by the original data into the BP network for training, take the number of neurons in the hidden layer as 8, and obtain the residual sequence after training, add the residual sequence to the initial prediction value, The fitting sequence of the optimized model can be obtained, and finally the translation inverse transformation is performed to obtain the final prediction result, which is compared with the actual frost heave data as shown in Fig. 3.

Table 2 shows the results of comparing the measured values of the optimized model fitting with the fitting measured values of the GM (1,1) model in literature [3] and the BP neural network.
model in literature [4]. Afterwards, the remaining three time coefficients were input to the three models respectively to obtain three sets of forecast sequences, and the comparison results with the original data are shown in Table 3.

In order to verify the optimization effect of the time-distance weighted matrix optimization item proposed in this paper and the optimization item based on the minimum residual initial value, the combination model that lacks the time-distance weighted matrix optimization item proposed above and retains the remaining optimization items is defined as model A. Define The combination model that lacks the initial value optimization item proposed above and retains the rest of the optimization items is model B. Compared with the complete optimization combination model in this paper, the prediction results are shown in Table 4.
Table 3 Comparison of prediction effects (1)

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Table 4 Comparison of prediction effects (2)

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</tbody>
</table>

By observing the above charts, it can be concluded that the combined prediction model of non-equidistant gray optimization and neural network established in this paper has an average prediction error of 1.46%. It can be seen from Figure 1 that the combined prediction model has achieved more accurate fitting and forecasting. However, the GM(1,1) model method established in literature [3] has an average prediction error of 5.69%, and the BP neural network model method established in literature [4] has an average prediction error of 10.12%, both of which are much higher than the model in this paper. It can be concluded that the prediction accuracy of the subgrade frost heave combination prediction model proposed in this paper is significantly improved compared with the existing methods. However, the average prediction errors of model A and model B are 2.052% and 2.1582%, respectively, and the accuracy is slightly lower than that of the model in this paper. Optimization effect.

At the same time, in order to verify the reliability and accuracy of the optimized combination prediction model in this paper for the prediction of frost heave of railway subgrade, the posterior difference ratio in statistics is used to verify the accuracy of the model $p$

Relative residuals

$$\mu(t_i) = \frac{\delta^1(t_i) - x^0(t_i)}{x^0(t_i)} \times 100\%$$  \hspace{1cm} (17)

Average Residuals

$$\mu' = \frac{1}{n-1} \sum_{i=2}^{n} |\mu(t_i)|$$  \hspace{1cm} (18)

Residual variance

$$S_1 = \frac{1}{n} \sum_{i=1}^{n} (\mu(t_i) - \mu')^2$$  \hspace{1cm} (19)

Assuming the variance of the original data $S_2$, the posterior difference ratio is expressed as
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\[ C = \sqrt{\frac{S_1}{S_2}} \]  

Modeling accuracy

\[ p = \left(1 - \mu'\right) \times 100\% \]  

The model in this paper is obtained by calculation \( p = 98.4\% \). The larger the model accuracy value, \( p \), the higher the model accuracy, and the smaller the posterior difference ratio \( C \), the smaller the dispersion of the prediction error. Referring to the model accuracy test table [8], it can be concluded that the prediction accuracy and reliability of this model for frost heave of railway embankment are high, and the accuracy level reaches level 1.

4. Conclusions

1) The background value and initial value of the differential equation of the non-equidistant GM(1,1) model were optimized by integral reconstruction and the method based on the minimum cumulative residual error, which improved the prediction accuracy of the model.

2) Set the weight matrix for the non-equidistant GM(1,1) model, fully consider the development trend of subgrade frost heave deformation, and improve the reliability of the prediction results.

3) Use the BP neural network residual correction model to correct the initial prediction data of the optimized non-equidistant GM (1,1) model, which makes up for the shortcomings of the gray prediction model in nonlinear prediction, improves the prediction accuracy of the model, and broadens the horizon. range of use of the model.

References


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