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# Local joint flexibility equations for Y-T and K-type tubular joints

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**Abstract.** It is common that analyses of offshore platforms being carried out with the assumption of rigid tubular joints. However, many researches have concluded that it is necessary that local joint flexibility (LJF) of tubular joints should be taken into account. Meanwhile, advanced analysis of old offshore platforms considering local joint flexibility leads to more accurate results. This paper presents an extensive finite-element (FE) based study on the flexibility of uni-planner multi-brace tubular Y-T and K-joints commonly found in offshore platforms. A wide range of geometric parameters of Y-T and K-joints in offshore practice is covered to generate reliable parametric equations for flexibility matrices. The formulas are obtained by non-linear regression analyses on the database. The proposed equations are verified against existing analytical and experimental formulations. The equations can be used reliably in global analyses of offshore structures to account for the LJF effects on overall behavior of the structure.

**Keywords:** fixed offshore platforms; tubular joints; local joint flexibility (LJF)

## 1. Introduction

Conventionally, in structural analysis of offshore platforms, the joints are assumed to be completely rigid. In this type of analysis local distortions of the chord circular cross sections are assumed to be negligible, and hence, no relative displacements and rotations between the chord and the brace can occur. However, at tubular joints, especially at unreinforced tubular joints, the connection is not rigid since the chord wall deforms locally as a result of loading. Significant errors can occur in estimating deflection, nominal stresses, buckling loads, natural frequencies, mode shapes and fatigue life of the platform due to the rigid connection assumption.

Numerous structures have been installed and are still in operation without reserve strength equal to conventional jacket type structures. Accounting for LJF may result in considerable redistribution of member forces, which cannot be neglected in assessment of performance and reliability of these structures. Therefore, Design codes such as API (2005) and DNV (1982 and 2010) require that the LJF effects should be engaged in global analyses of the structures.

Studies on local flexibility of tubular joints were started in the early 1980s. In 1980 Boukamp *et al*. (1980) tried to present a method for incorporating the effects of LJF into the overall response of the structure.

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Fessler *et al*. (1986) proposed parametric equations for obtaining flexibility matrices of any unreinforced single brace or multi-brace tubular joints by testing 27 Araldite tubular joints.

Later, Hu *et al*. (1993) presented an equivalent element to account for LJF of tubular joints in the structural analysis of offshore platforms.

Based on FE methods, Buitrago *et al*. (1993) published a set of equations for predicting LJF of simple tubular joints. Buitrago's parametric expressions for LJFs are simple to use in addition to having a good agreement with experimental data. As a result, they are widely used by API (2005) and DNV (1982 and 2010).

Later in 1996, Chen *et al*. (1996) proposed a semi-analytical method for estimating the LJF of tubular T/Y and symmetric K-joints.

In 1998, Morin *et al*. (1998) conducted a research with the general FE software ABAQUS and concluded that, especially for joints with axial loads in bracings it is necessary to use existing parametric formulae to account for the influence of local failure modes of tubular joints on global failure modes in reliability analyses of jacket type structures.

MSL Engineering Limited (2001) considered LJFs in spectral fatigue analyses and verified the results against the underwater inspections of MSL Services Corporation (2000) for existing structures. MSL Engineering Limited subsequently concluded that considering LJFs in spectral fatigue analyses result in a significant increase in estimating the fatigue life of offshore platforms.Consequently, fatigue analyses which account for effects of the LJF, can be performed instead of the more cost-consuming underwater inspections for estimation of the reliability of existing platforms.

Similarly, Samadani *et al*. (2009) conducted a research on two offshore platforms and showed that effects of LJF on overall behavior of jackets without joint cans is not negligible. This is particularly the case in assessment of old offshore platforms in service.

Chakrabarti *et al*. (2005) conducted a reassessment research on more than twenty platforms. They used Buitrago's (1993) formulations for considering LJF effects in fatigue analyses and showed that considering these effects can result in at least two times of increase in fatigue life of most joints they had analyzed.

Later, using FE models to account for the effect of gap size, Gho (2011) showed that the existing Y-joint formulae cannot be used reliably for predicting the LJFs of overlapped braces.

Using the proposed equations of Fessler *et al*. (1986) for LJFs of tubular joints, Alanjari *et al*. (2011) developed a two-dimensional elastic-perfectly plastic element to represent the LJF in the global analysis of offshore structures.

Due to the complexity of the problem parametric equations for LJF cannot be obtained analytically and hence, such equations are obtained by regression analyses of a given database. Thus, the reliability of these equations would be highly dependent on the size of the database. There are no equations based on a large database available yet. Therefore, this study intends to obtain more reliable equations by taking advantage of a large database generated using FE models as well as considering more effective non-dimensional parameters. Regression analyses on the database are subsequently performed to propose equations for LJFs of tubular joints.

Further studies can be carried out in order to expand the flexibility matrix proposed in this paper to a $6 \times 6$ matrix to account for out of plane bending effects on LJFs of tubular Y-T and K-joints.

## 2. Scope of the study

For the case of general multi-brace tubular joints, the LJFs depend on too many non-dimensional parameters. Therefore, it becomes too complicated to find equations that represent LJFs of such joints. Hence, the conventional approach is considering a multi-brace tubular joint as a combination of more simple joints and the interaction between these components is considered from some factors which are basically dependent on the load pattern. For instance, according to the approach defined in codes such as DNV (2010) for Multi brace joints, LJF may be extracted from a combination of joint types, i.e., from formulations such as Eq. (1).

$$\overline{LJF} = \left| \lambda_Y \, LJF_Y + \lambda_X \, LJF_X + \lambda_K \, LJF_K \right| \tag{1}$$

in which the λ values are the fractions corresponding to the joint type designated by the subscript when the joint is classified by loads.

In this study, LJF equations for K or Y-T joints ($LJF_K$ in Eq. (1)) are proposed from numerical analysis result of 814 FE models. Geometrical properties of any tubular Y-T or K-joint are functions of twelve non-dimensional parameters. However, it was investigated that only six non-dimensional parameters have considerable influence on LJFs of these joints. Hence, Six non-dimensional parameters, namely $\theta_1$, $\theta_2$,$\gamma=R_c/t_c$, $\beta_1=R_{b1}/R_c$, $\beta_2=R_{b2}/R_c$ and $\zeta=g/R_c$ were used to generate 814 FE model. On the other hand, the six non-important parameters, namely$\tau_1=t_{b1}/t_c$, $\tau_2=t_{b2}/t_c$,$\alpha=L_c/R_c$, $t_c$, $L_{b1}$ and$L_{b2}$were used with constant values in all 814 models. $R_c$, $t_c$,$R_{b1}$,$R_{b2}$,$g$, $t_{b1}$, $t_{b2}$, $L_c$, $L_{b1}$ and$L_{b2}$denote radius of the chord, wall thickness of the chord, radius of the first brace, radius of the second brace, the gap size between the two braces, wall thickness of the first brace, wall thickness of the second brace, length of the chord, length of the first brace and length of the second brace respectively. As it is shown in Fig. 1, $\theta_1$ and $\theta_2$ are the angle between the braces and the chord.

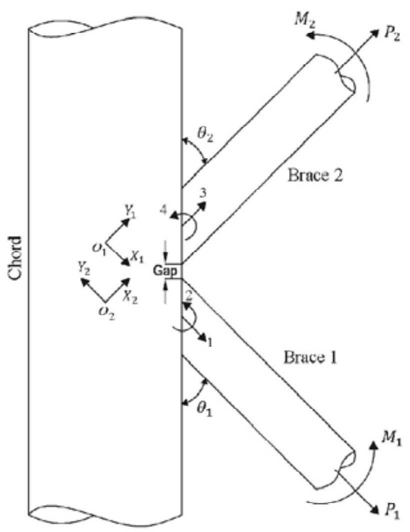



Fig. 1 Definition of local coordinate systems and positive directions for degrees of freedom

Table 1 shows the values for non-dimensional parameters of the 814 generated FE models. Constant values used to create the FE models are presented in Table 2.

American Petroleum Institute (API) recommends that the gap size should have a size of at least 5.08 cm. In this paper, an extensive range of gap sizes has been covered in order to generate a reliable database for investigating the effect of the gap on local flexibility of tubular joints.

Table 1 Ranges of non-dimensional parameters for the FE models

| $\zeta$ | $\theta_1$ | $\theta_2$ | $\gamma$ | $\beta_1$ | $\beta_2$ |
|---|---|---|---|---|---|
| 0.1, 0.2, 0.3, 0.4 | 30°, 45°, 60°, 90° | 30°, 45°, 60°, 90° | 12,15,18 | 0.25,0.5,0.75 | 0.25,0.5,0.75 |

Table 2 Constant values of non-important parameters

| $\alpha$ | $\tau_1$ | $\tau_2$ | $L_{b1}$(m) | $L_{b2}$(m) | $t_c$ (m) |
|---|---|---|---|---|---|
| 12 | 0.5 | 0.5 | 2 | 2 | 0.03175 |

## 3. Local flexibility matrix

The local coordinate systems of the planner Y-T and K-joints which describe four degrees of freedom, including two axial displacements along the braces and two rotational displacements in the plane of the joint are shown in Fig. 1. Given these degrees of freedom, the non-dimensional Eq. (2) can be used to describe the relation between the loads and the deformations:

$$[\Delta]=[F][P] \tag{2}$$

where

$$[\Delta_1]=\left[\Delta_1/D \quad \Phi_1 \quad \Delta_2/D \quad \Phi_2\right]^T \tag{3}$$

$$[F]=\begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} \tag{4}$$

$$[P]=\left[P_1/ED^2 \quad M_1/ED^3 \quad P_2/ED^2 \quad M_2/ED^3\right]^T \tag{5}$$

where $\Delta_1$ and $\Delta_2$ are the local axial displacements; $\Phi_1$ and $\Phi_2$ are the local rotational displacements at the two conjunction points between the chord and braces as shown in Fig. 1. E and D denote steel modulus of elasticity and the chord diameter, respectively. Subscripts 1 and 2 are used to refer to the loads, displacements or rotations of brace 1 and 2. According to Betti/Rayleigh reciprocal theorem which holds for elastic solids, the flexibility matrix *[F]* would be symmetric, and hence, it would have ten dependent terms, which can be obtained by analyzing the FE models. Therefore, to obtain the flexibility matrix for each FE model, the model must be analyzed in four cases as follows.

*Case 1:*

In the first attempt, only $P_1$ is applied to the model.$\Delta_1$, $\Delta_2$, $\Phi_1$ and $\Phi_2$ are to be evaluated accordingly from the analyses on the FE model. Thus, elements of the first column of the flexibility matrix *[F]* can be obtained from Eqs. (6) to (9)

$$f_{11} = (\Delta_1 / P_1)(ED) \tag{6}$$

$$f_{21} = (\Phi_1 / P_1)(ED^2) \tag{7}$$

$$f_{31} = (\Delta_2 / P_1)(ED) \tag{8}$$

$$f_{41} = (\Phi_2 / P_1)(ED^2) \tag{9}$$

*Case 2:*

In this case, only $M_1$ is appliedto the model. $\Delta_2$,$\Phi_1$ and $\Phi_2$ are to be evaluated accordingly from the FE analyses. Thus, elements of the second column of the flexibility matrix *[F]* can be obtained Eqs. (10) to (13)

$$f_{21} = f_{12} \tag{10}$$

$$f_{21} = (\Phi_1 / M_1)(ED^3) \tag{11}$$

$$f_{32} = (\Delta_2 / M_1)(ED^2) \tag{12}$$

$$f_{42} = (\Phi_2 / M_1)(ED^3) \tag{13}$$

*Case 3:*

In this case, only $P_2$ is applied to the model. $\Delta_2$ and $\Phi_2$ are to be evaluated accordingly from the FE analysis. Consequently, elements of the third column of the flexibility matrix *[F]*can be obtained from Eqs. (14) to (17)

$$f_{13} = f_{31} \tag{14}$$

$$f_{23} = f_{32} \tag{15}$$

$$f_{33} = (\Delta_2 / P_2)(ED) \tag{16}$$

$$f_{43} = (\Phi_2 / P_2)(ED^2) \tag{17}$$

*Case 4:*

In this case, only $M_2$ is applied to the model.$\Phi_2$is to be evaluated accordingly from the FE analyses. Thereby, elements of the fourth column of the flexibility matrix *[F]* can be obtained from Eqs. (18) to (21)

$$f_{14} = f_{41} \tag{18}$$

$$f_{24} = f_{42} \tag{19}$$

$$f_{24} = f_{42} \tag{20}$$

$$f_{44} = \left(\Phi_2 / M_2\right)\left(ED^3\right) \tag{21}$$

## 4. Modeling

### 4.1 Boundary conditions

Rigid plates are placed at the ends of the two braces so that the loads can be applied along the four degrees of freedom. The chord length is assumed to be equal to $12\mathrm{R_c}$ in all models in order to restrict the effects of the boundary conditions of the chord's ends on local deformations.

### 4.2 Calculations of the displacements

The braces' intersections with the chord form a spatial curve which includes two saddle points and two crest points. In experimental studies, typically these four points are used to obtain the required displacements and rotations along the given degrees of freedom. However, the average of all nodes located on the spatial curve is a more accurate indication of $\Delta_1$and$\Delta_2$; therefore, this method is used in this study. On the other hand, the rotations $\Phi_1$ and $\Phi_2$ is obtained according to Eq. (21).

$$\Phi = \frac{2\sum_{i=1}^{\frac{n-1}{2}} \left(\Delta u_x\right)_i / \left(h_i\right)}{n-1} \tag{22}$$

In this equation, n is the number of the nodes shown in Fig. 2 and $(\Delta u_x)_I$ is equal to the difference of local axial displacements of each two nodes of Fig. 2 measured in their reference coordinate system and $h_i$ is the distance between these two nodes.

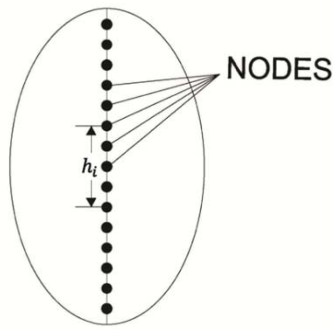



Fig. 2 Locations of the nodes on the intersection areas between the chord and the braces used to calculate the rotations

It should be noticed that the displacements and rotations obtained from the FE models yield the total values of deformations. Hence, the displacements and rotations caused by the behavior of the joint as beam-type elements were evaluated using the common slope-deflection method and were subtracted from these values to obtain the local deformations which are needed for obtaining the LJF matrices.

In order to make Eq. (22) an accurate representation of the rotations, the connecting area must be meshed perfectly in a symmetrical manner relative to the center of the connection area as in Fig. 3.

### *4.3 Material properties and meshing*

LJF effects are to be used in analysis phase of offshore platforms which is conventionally an elastic analysis. Hence, Steel material with linear elastic behavior, Young's modulus of elasticity (E) 200 GPa and Poisson ratio ($\upsilon$) of 0.3, was used to create the FE models.

The FE program ANSYS was employed to generate the models using 8-node structural shell93 elements. Meshing, with a special care about the intersection area (Fig. 3), is implemented in a good order to reach fine continuity between elements. Fig. 4 shows the deformed and undeformed shape of a model under loading conditions of case 1.

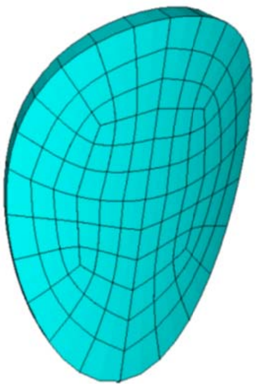


Fig. 3 Mesh generation for the intersection areas between the chord and the braces

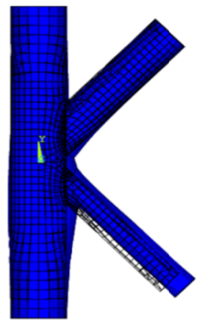


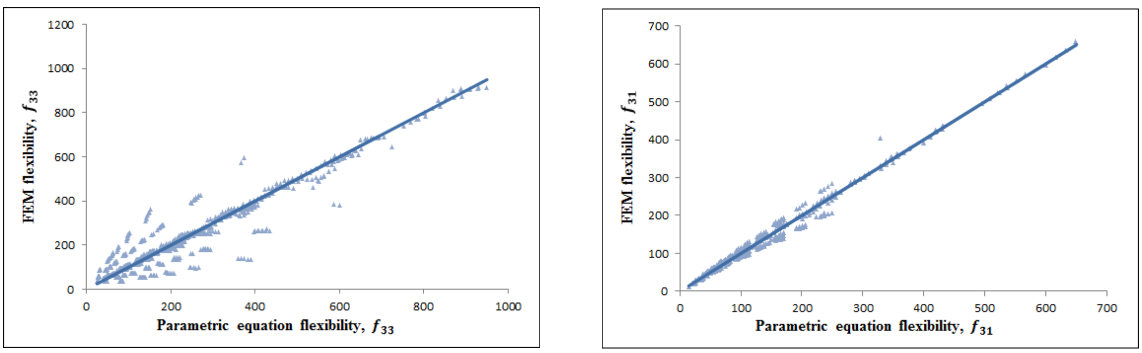
Fig. 4 Deformed and undeformed shape of a model under loading conditions of case1

## 5. Flexibility equations

A large database of flexibility matrices was obtained by analyzing the FE models. The database was subsequently used to derive parametric equations for each term of the flexibility matrix using the Levenberg-Marquardt (1944 and 1963) algorithm which is the most widely used optimization algorithm to solve a nonlinear least squares problem. By following the LM method, Eq. (23) which will be called as AMA (Asgarian-Mokarram-Alanjari) equations, are proposed for determining the flexibility matrix of uniplaner multi brace tubular Y-T and K-joints.

$$
\begin{aligned}
f_{11} &= 3.501(\sin\theta_1)^{1.898}(\sin\theta_2)^{-0.114}\gamma^{2.129}\exp(-2.302\beta_1)\exp(-0.412\beta_2)\exp(0.221\zeta)\\
f_{21} &= -10.070+0.408\left[(\sin\theta_1)^{2.457}(\sin\theta_2)^{1.375}\gamma^{2.458}\exp(-5.581\beta_1)\exp(2.761\beta_2)\exp(-2.492\zeta)\right]\\
f_{31} &= 2.789(\sin\theta_1)^{0.949}(\sin\theta_2)^{0.949}\gamma^{2.225}\exp(-1.636\beta_1)\exp(-1.636\beta_2)\exp(0.256\zeta)\\
f_{41} &= 10.116(\sin\theta_1)^{0.716}(\sin\theta_2)^{1.033}\gamma^{1.710}\exp(-3.064\beta_1)\exp(-0.863\beta_2)\exp(-0.295\zeta)\\
f_{22} &= 102.164(\sin\theta_1)^{2.411}(\sin\theta_2)^{0.042}\gamma^{2.166}\exp(-6.255\beta_1)\exp(0.003\beta_2)\exp(0.491\zeta)\\
f_{32} &= -10.116(\sin\theta_1)^{1.033}(\sin\theta_2)^{0.716}\gamma^{1.710}\exp(-0.863\beta_1)\exp(-3.064\beta_2)\exp(-0.295\zeta)\\
f_{42} &= -40.793-953.641\left[(\sin\theta_1)^{2.016}(\sin\theta_2)^{2.016}\gamma^{1.500}\exp(-6.317\beta_1)\exp(-6.317\beta_2)\exp(-3.955\zeta)\right]\\
f_{33} &= 3.501(\sin\theta_1)^{-0.114}(\sin\theta_2)^{1.898}\gamma^{2.129}\exp(-0.412\beta_1)\exp(-2.302\beta_2)\exp(0.221\zeta)\\
f_{43} &= 10.070-0.408\left[(\sin\theta_1)^{1.375}(\sin\theta_2)^{2.457}\gamma^{2.458}\exp(2.761\beta_1)\exp(-5.581\beta_2)\exp(2.492\zeta)\right]\\
f_{44} &= 102.164(\sin\theta_1)^{0.042}(\sin\theta_2)^{2.411}\gamma^{2.166}\exp(0.003\beta_1)\exp(-6.255\beta_2)\exp(0.491\zeta)
\end{aligned}
\tag{23}
$$

Figs. 5(a) and 5(b) are provided to assure the accuracy of the performed regression analyses. Figs. 5(a) and 5(b) compare the data obtained from FE analyses for $f_{33}$ and $f_{31}$ with AMA's equations for these terms. Hence, it can be concluded that equations obtained from regression analyses are well suited for the pure FE results.




(a) (b)

Fig. 5 (a) Comparison of parametric equation with the FE results for $f_{33}$ and (b) Comparison of parametric equation with the FE results for $f_{31}$

## 6. Discussion

In this section, Buitrago's (1993), Fessler's (1986) and Chen's (1996) equations for LJF of tubular joints will be compared to AMA's formulas. Chen's (1996) equations are applicable to symmetric K-joints without accounting for the effects of gap size on LJF. In Fessler's (1986) equations, on the other hand, effects of gap size on LJF are somehow taken into account only for non-diagonal terms of the local flexibility matrix. Buitrago's (1993) equations takes gap size effects into account for all terms but no equation is suggested for the interaction between rotational and translational degrees of freedom, i.e. $f_{21}$, $f_{41}$, $f_{32}$ and $f_{43}$ are assumed to be equal to zero.

Buitrago's (1993), Fessler's (1986) and Chen's (1996) equations are presented in Eqs. (24), (25) and (27), respectively. It is to be noted that in these equations, notations and parameters were redefined so as to conform to those of this paper.

- *Buitrago's (1993) equations*

$$\begin{aligned}
f_{11} &= 5.90\tau^{-0.114}\exp(-2.163\beta)\gamma^{1.869}(\zeta/2)^{0.009}(\sin\theta_1)^{1.869}(\sin\theta_2)^{-0.089} \\
f_{22} &= 52.2\tau^{-0.119}\exp(-3.835\beta)\gamma^{1.934}(\zeta/2)^{0.011}(\sin\theta_1)^{1.417}(\sin\theta_2)^{-0.108} \\
f_{33} &= 5.90\tau^{-0.114}\exp(-2.163\beta)\gamma^{1.869}(\zeta/2)^{0.009}(\sin\theta_2)^{1.869}(\sin\theta_1)^{-0.089} \\
f_{44} &= 52.2\tau^{-0.119}\exp(-3.835\beta)\gamma^{1.934}(\zeta/2)^{0.011}(\sin\theta_2)^{1.417}(\sin\theta_1)^{-0.108} \\
f_{31} &= 3.93\tau^{-0.113}\exp(-2.198\beta)\gamma^{1.847}(\zeta/2)^{-0.056}(\sin\theta_1)^{0.837}(\sin\theta_1)^{0.784} \\
f_{42} &= f_{22} - 1.83\tau^{-0.212}\beta^{-2.102}\gamma^{1.872}(\zeta/2)^{0.020}(\sin\theta_1)^{1.249}(\sin\theta_1)^{0.060} \\
f_{21} &= f_{41} = f_{32} = f_{43} = 0
\end{aligned} \tag{24}$$

- *Fessler's (1986) equations*

$$\begin{aligned}
f_{11} &= 1.95\gamma^{2.15}(\sin\theta_1)^{2.19}(1-\beta_1)^{1.3} \\
f_{31} &= 1.26\gamma^{2.3}(\sin\theta_1)^{1.58}(1-\beta_1)^{0.71}\left[\sin(\pi-\theta_1-\theta_2)\right]^{1.76}(1-\beta_2)^{0.48}\exp\left(-0.58\frac{e}{D}\right) \\
f_{41} &= 16.5\gamma^{1.2}(\sin\theta_1)^{0.71}(1-\beta_1)^{1.62}\left[\sin(\pi-\theta_1-\theta_2)\right]^{-0.36}(1-\beta_2)^{0.08}\exp\left(0.42\frac{e}{D}\right) \\
f_{22} &= 134\gamma^{1.73}(\sin\theta_1)^{1.22}\exp(-4.52\beta_1) \\
f_{32} &= -9.42\gamma^{1.84}(\sin\theta_1)^{0.79}\exp(-1.67\beta_1)\exp(-0.81\beta_2)\cos\left[0.52(\pi-\theta_1-\theta_2)\right]\exp\left(-0.52\frac{e}{D}\right) \\
f_{33} &= 1.95\gamma^{2.15}(\sin\theta_2)^{2.19}(1-\beta_2)^{1.3} \\
f_{44} &= 134\gamma^{1.73}(\sin\theta_2)^{1.22}\exp(-4.52\beta_1) \\
f_{21} &= f_{42} = f_{43} = 0
\end{aligned} \tag{25}$$

where

$$\frac{e}{D}=\frac{1}{2}\left|\zeta+\frac{\beta_1}{\sin\theta_1}+\frac{\beta_2}{\sin\theta_2}-\tan\left(\frac{\pi}{2}-\theta_1\right)-\tan\left(\frac{\pi}{2}-\theta_2\right)\right| \quad (26)$$

- *Chen's (1996) equations*

$$\begin{aligned} f_{11}&=f_{33}=4.71\gamma^{2.17}\exp(-3.25\beta)(\sin\theta)^{2.02}\\ f_{31}&=1.79\gamma^{2.39}\exp(-2.49\beta)(\sin\theta)^{3.07}\\ f_{32}&=-f_{41}=-6.69\gamma^{1.68}\exp(-2.62\beta)(\sin\theta)^{1.20}\\ f_{22}&=f_{44}=169\gamma^{1.68}\exp(-4.58\beta)(\sin\theta)^{1.25}\\ f_{42}&=-19.1\gamma^{1.43}\exp(-3.00\beta)(\sin\theta)^{0.86}\\ f_{21}&=f_{43}=0 \end{aligned} \quad (27)$$

Chen's (1996) equations are not applicable to non-symmetric joints. Thus, for the results to be compared with Chen's (1996) equations, it is necessary that symmetric joints be studied. Figs. 6-9 compare main diagonal components of the flexibility matrix obtained from AMA's equations for the case of joints with symmetric configurations with those recommended by Buitrago (1993), Fessler (1986) and Chen (1996). Figs. 10-15, on the other hand, compare main diagonal components of the flexibility matrix for the case of joints with non-symmetric configurations with those proposed by Fessler (1986) and Buitrago (1993). It is to be noted that Buitrago's (1993) equations are not applicable to non-symmetric joints in which $\beta_1\neq\beta_2$while those from AMA and Fessler (1993) are. Hence, Buitrago's (1993) equations do not suggest any curves in Figs. 10, 11, 20 and 21.

Although Fessler's (1986)equations for the main diagonal components of the LJF matrix are obtained from single-brace models, Figs. 6-15 show good agreement between the results of this paper for $f_{11}$ and $f_{33}$, and those obtained from Buitrago's (1993), Fessler's (1986) and Chen's (1996). The reason of this agreement can be understood by investigating the interaction effects between the two braces on LJFs through Figs. 10-13 and Fig. 15. Figs. 10-13 show that changing

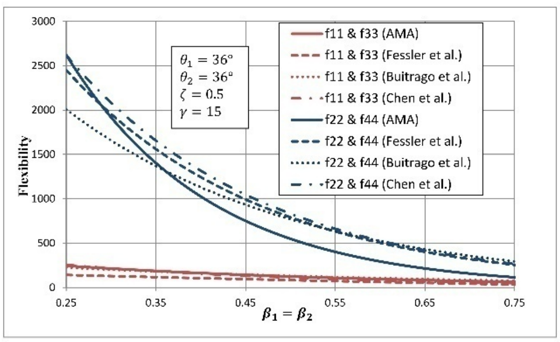



Fig. 6 Effects of $\beta$ on main diagonal terms of the LJF matrix for a symmetric K-joint

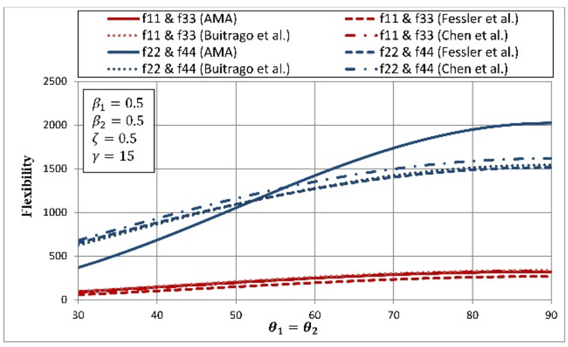



Fig. 7 Effects of $\theta$ on main diagonal terms of the LJF matrix for a symmetric K-joint

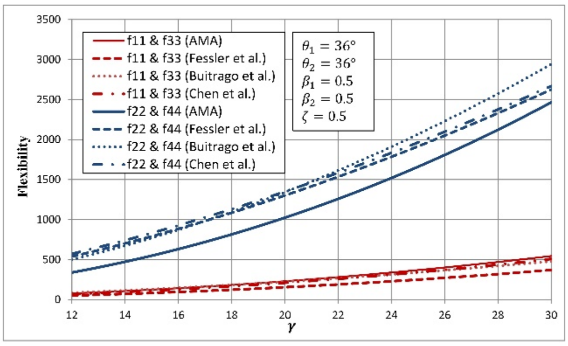



Fig. 8 Effects of $\gamma$ on main diagonal terms of the LJF matrix for a symmetric K-joint

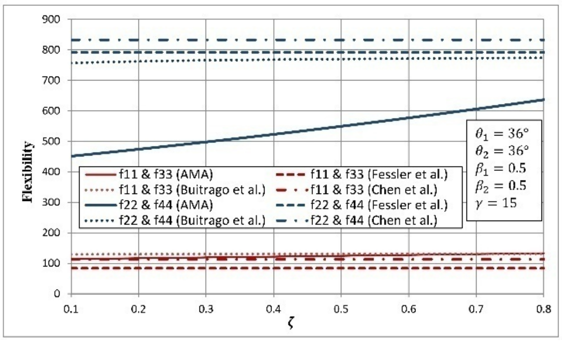



Fig. 9 Effects of $\zeta$ on main diagonal terms of the LJF matrix for a symmetric K-joint

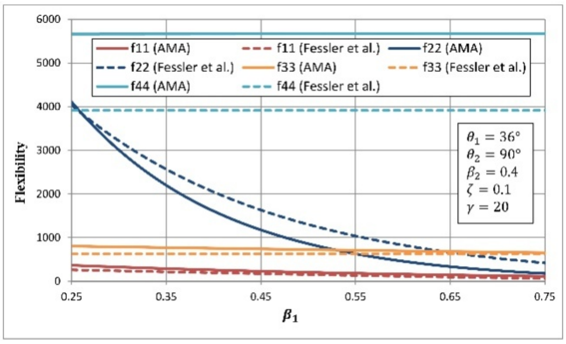



Fig. 10 Effects of $\beta_1$ on main diagonal terms of the LJF matrix for a Y-T joint

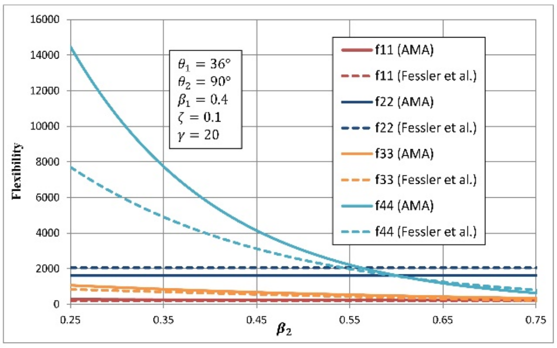



Fig. 11 Effects of $\beta_2$ on main diagonal terms of the LJF matrix for a Y-T joint

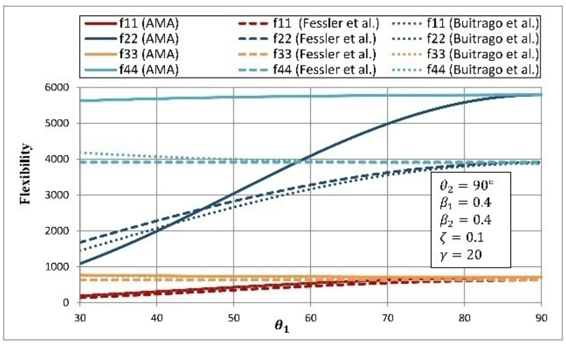



Fig. 12 Effects of $\theta_1$ on main diagonal terms of the LJF matrix for a Y-T joint

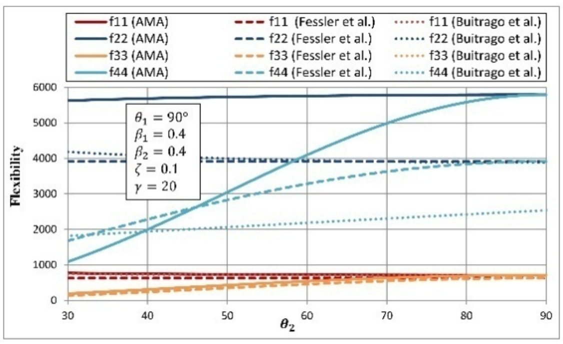



Fig. 13 Effects of $\theta_2$ on main diagonal terms of the LJF matrix for a Y-T joint

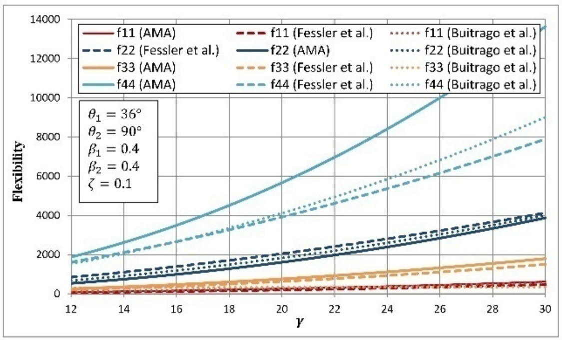



Fig. 14 Effects of $\gamma$ on main diagonal terms of the LJF matrix for a Y-T joint

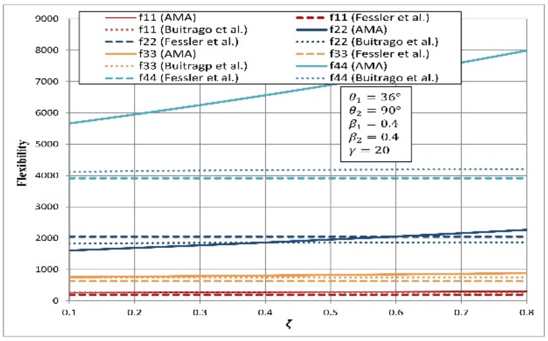



Fig. 15 Effects of $\zeta$ on main diagonal terms of the LJF matrix for a Y-T joint

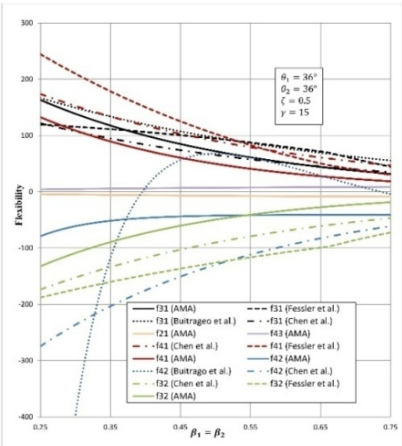



Fig. 16 Effects of β on non-diagonal terms of the LJF matrix for a Symmetric K-joint

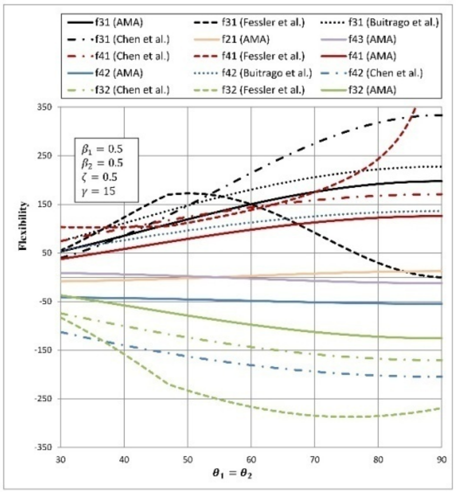



Fig. 17 Effects of $\theta$ on non-diagonal terms of the LJF matrix for a Symmetric K-joint

the values of $\beta_1$ and $\theta_1$ has no significant effect on $f_{33}$ while $f_{11}$ is not affected by changes in $\beta_2$ and $\theta_2$ values. Moreover, Fig. 15 represents that the gap size has no significant effect on $f_{11}$ and $f_{33}$.

Figs. 6-9 show good agreement between Buitrago's (1993), Fessler's (1986) and Chen's (1996) results for $f_{22}$ and $f_{44}$ while there is some difference between these results, and those obtained by the authors' formulae. The reason for this difference is that neither Fessler's (1986) nor Chen's (1996) equations account for the effects of the gap size and the local stiffening effect of the other brace on flexibilities in the brace under investigation. By investigating Figs. 10-13 it can be understood that changing the values of $\beta_1$ and $\theta_1$ has no significant effect on $f_{22}$ and $f_{44}$ is not affected by changes in $\beta_2$ and $\theta_2$ values, but Fig. 15 shows significant effects of the gap size on these two terms. Moreover, Fig. 9 shows that as the values of $\zeta$ become grater the authors' results for $f_{22}$ and $f_{44}$ get closer to those from Buitrago (1993), Fessler(1986) and Chen(1996) because the effect of the gap size becomes insignificant for large values of $\zeta$. This convergence behavior of the authors' equations confirms their accuracy. From the above discussion, it can be concluded that the authors' equations for $f_{22}$ and $f_{44}$ are more acceptable than similar equations proposed by previous researchers, since the authors' formulae account for the effects of gap size.

Fig. 17 investigates the effect of changing the angle between the brace and the chord in a symmetric joint. It can be seen that Fessler's (1986) equations for $f_{41}$ and $f_{31}$ yield inaccurate results as the angle approaches 90°. In this case, $f_{41}$ approaches infinity and $f_{31}$ approaches zero. On the other hand, in comparison with the authors' equations, Chen's (1996) equations give overestimates for these terms, since they do not account for the gap size effects while Buitrago's (1993) formulation for the case of$f_{31}$ have good agreement with the author's results. Moreover, Buitrago *et al*. (1993) have not presented any equations for $f_{41}$ while it can be observed (see Figs. 16-25) that this term cannot be neglected.

Comparing Fessler's (1986) results for $f_{31}$ and $f_{11}$ in Fig. 8 and 18 reveals that Fessler's (1986) equations do not yield reasonable results since $f_{31}$ has greater values than$f_{11}$. $f_{31}$ and $f_{11}$ are respectively equal to the axial deflections under brace 2 and brace 1 due to applying unit axial load on brace 1; therefore, $f_{11}$ should be greater than $f_{31}$ whenever the two braces have the same stiffness. The reason of this error in Fessler's (1986) equations can be recognized by regarding Fig. 21. It can be seen that $f_{31}$ in Fessler's (1986) formula is almost constant with $\beta_2$variations. It shows that Fessler's (1986) equations do not account for the effects of the other brace's stiffness on $f_{31}$ reliably, and hence it results in overestimates for$f_{31}$. On the other hand, Fig. 19 shows that Fessler's (1986) formula for $f_{31}$ gives overestimations in flexibility in addition to wrong estimations of the gap size effect since it shows an increase in flexibility with bigger gap sizes. Such deficiencies in estimation of $f_{31}$ is not present in the authors' and Buitrago's (1993) formulations and it is observed that they both yield reasonable results for this case.

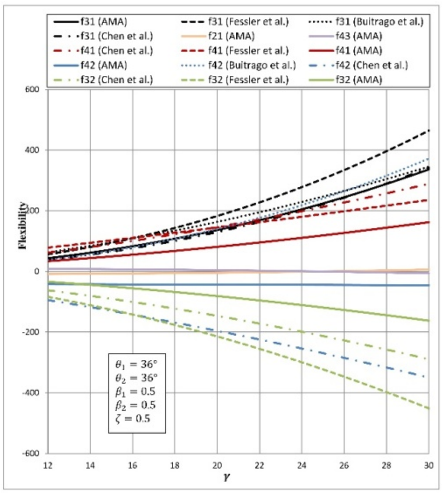



Fig. 18 Effects of $\gamma$ on non-diagonal terms of the LJF matrix for a Symmetric K-joint

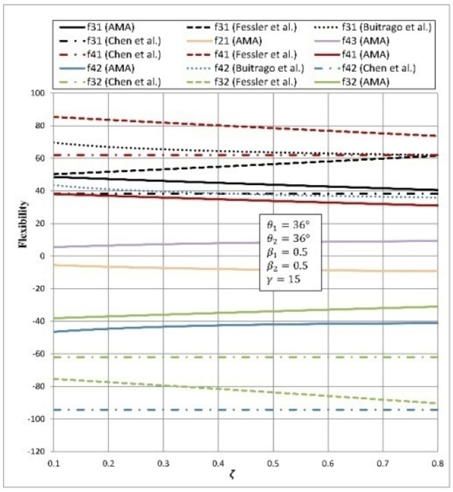



Fig. 19 Effects of $\zeta$ on non-diagonal terms of the LJF matrix for a Symmetric K-joint

$f_{21}$ and $f_{43}$ are assumed to be negligible in Buitrago's (1993), Fessler's (1986) and Chen's (1996) equations. The values predicted by the authors' formulations for these two terms in Figs. 16-19 show that this assumption is acceptable for symmetric joints. However, when the joint is not symmetric, the axial load in a brace can result in considerable rotations between the brace and the chord. Figs. 22-24 show that neglecting these two terms for large values of $\theta_1$, $\theta_2$, $\gamma$ or $\zeta$ is not acceptable. Moreover, according to Figs. 20-21, $f_{21}$ will have considerable values for small values of $\beta_1$ and large values of $\beta_1$ while $f_{43}$ will have considerable values for large values of $\beta_1$ and small values of $\beta_1$. Fig. 25 shows that neglecting these rotations is not acceptable for small values of $\zeta$ where the influence of the gap size on local joint flexibility becomes significant.

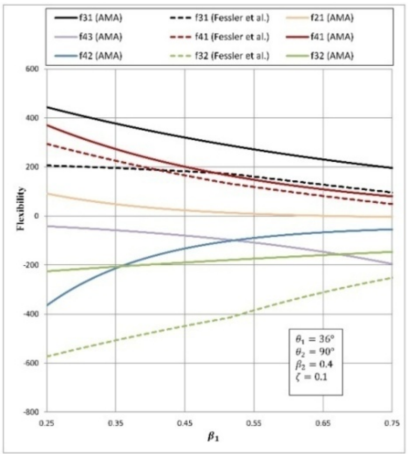



Fig. 20 Effects of $\beta_1$ on non-diagonal terms of the LJF matrix for a Y-T joint

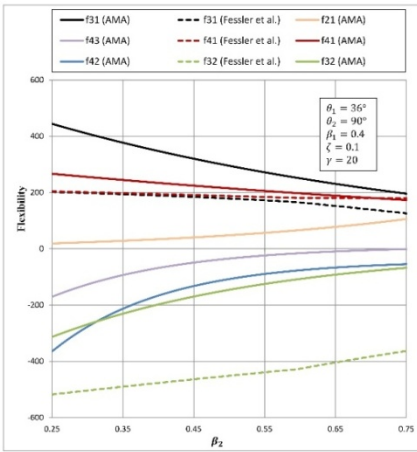



Fig. 21 Effects of $\beta_2$ on non-diagonal terms of the LJF matrix for a Y-T joint

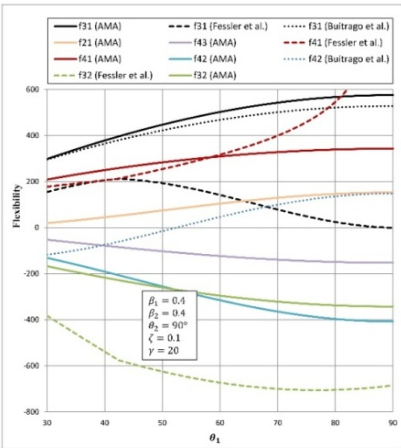



Fig. 22 Effects of $\theta_1$ on non-diagonal terms of the LJF matrix for a Y-T joint

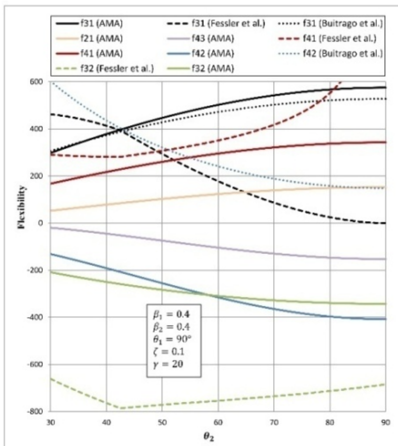



Fig. 23 Effects of $\theta_2$ on non-diagonal terms of the LJF matrix for a Y-T joint

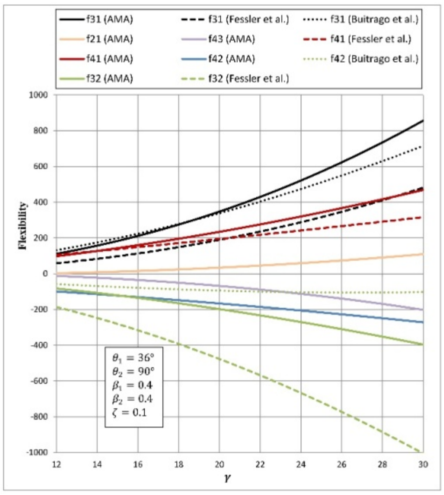



Fig. 24 Effects of $\gamma$ on non-diagonal terms of the LJF matrix for a Y-T joint

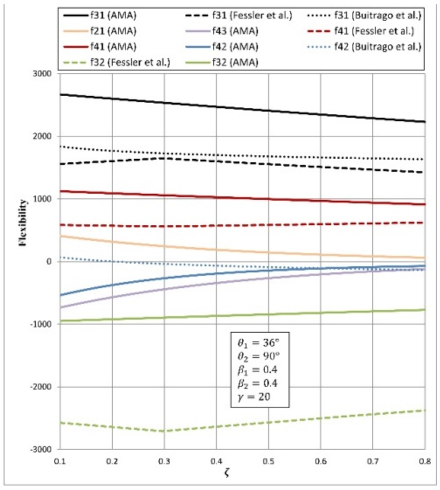



Fig. 25 Effects of $\zeta$ on non-diagonal terms of the LJF matrix for a Y-T joint

Fessler (1986) suggests taking $f_{42}$ equal to zero. Regarding Figs. 16-19 for symmetric joints, the large difference between the values of Chen's (1996) equations and those of this paper as well as the conformity between Fessler's (1986) equations and those proposed in this paper can be recognized. Hence, for symmetric joints, it can be concluded that Fessler's (1986) suggestion for neglecting the values of $f_{42}$ is almost acceptable except for joints with small values of $\beta_1$ and $\beta_2$. On the other hand, for non-symmetric joints, values of $f_{42}$ cannot be neglected. Figs. 20-25 show that the values of $f_{42}$ can be significant in non-symmetric joints. Furthermore, Buitrago's (1993) results for $f_{42}$ do not match with those from the authors or Chen (1996) in none of the Figures mentioned above. Moreover, for the case of Fig. 16 Buitrago's formulation presents inaccurate behavior since in addition to the sign change of $f_{42}$ it does not vary monotonically.

Figs. 23 and 25 show that Fessler's (1986) equation for $f_{32}$ is unreliable since it suggests a non-monotonic behavior for this term. The authors' equation, however, does not face such problem.

Figs. 16-19 show that the authors' and Chen's (1996) formulations are more close to each other while Fessler's (1986) equations seems to present overestimations in LJF of this component. Buitrago *et al*. (1993) have not suggested any formulation for this term while it is observed that this term cannot be assumed to be negligible.

## 7. Conclusions

Relative deformation of tubular joints may significantly affect analysis results of offshore platforms. This effect which is recommended to be considered by many design codes, leads to more accurate results. For the advanced analysis of old offshore platforms during their assessment process, it is recommended to consider LJF as one of the numerical model improvements.

In this paper, a large database for LJF matrix of planner tubular Y-T and K-joints was established by developing FE models in ANSYS. The models were selected such that a wide range of values for all important non-dimensional parameters of Y-T and K-joints is covered. Subsequently, regression analyses on the database were employed to provide parametric equations for obtaining LJF matrix of such types of joints.

The effect of the gap length as well as the interaction effects between the two braces on local flexibility of the joints was investigated. It was shown that neglecting these effects has been the principal reason for less accuracy in previous studies for predicting local flexibility of tubular joints. It was investigated that existing formulations for LJF matrix of tubular Y-T and K-joints have some shortcomings while formulations presented in this paper present acceptable behavior for all components of the LJF matrix. Therefore, these equations can be used reliably for considering the effects of LJFs of tubular joints on overall behavior of the structures.

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