

Declutching control of a point absorber with direct linear electric PTO systems

Xian-tao Zhang^{*}, Jian-min Yang^a and Long-fei Xiao^b

State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai, 200240, China

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Abstract. Declutching control is applied to a hemispherical wave energy converter with direct linear electric Power-Take-Off systems oscillating in heave direction in both regular and irregular waves. The direct linear Power-Take-Off system can be simplified as a mechanical spring and damper system. Time domain model is applied to dynamics of the hemispherical wave energy converter in both regular and irregular waves. And state space model is used to replace the convolution term in time domain equation of the heave oscillation of the converter due to its inconvenience in analyzing the controlled motion of the converters. The declutching control strategy is conducted by optimal command theory based on Pontryagin's maximum principle to gain the controlled optimum sequence of Power-Take-Off forces. The results show that the wave energy converter with declutching control captures more energy than that without control and the former's amplitude and velocity is relatively larger. However, the amplification ratio of the absorbed power by declutching control is only slightly larger than 1. This may indicate that declutching control method may be inapplicable for oscillating wave energy converters with direct linear Power-Take-Off systems in real random sea state, considering the error of prediction of the wave excitation force.

Keywords: wave energy; direct linear electric PTO; declutching control; state space; optimal command

1. Introduction

Floating oscillating bodies constitute a large class of wave energy converters, especially for offshore deployment (Falcao 2010). The oscillating motion of a floating body or the relative motion between two moving bodies is converted to electricity by the Power-Take-Off (PTO) system. Among various power conversion systems, two typical PTO systems are hydraulic cylinders driving a hydraulic motor and direct linear electric PTO (Lopez *et al.* 2013). For the hydraulic cylinders PTO system, the body motion is converted into hydraulic energy. And then the hydraulic energy is converted into electrical energy by electrical generator that is driven by a fast hydraulic motor. For the direct linear electric PTO system, the oscillating motion of the body is converted into electricity directly by linear permanent magnet generators. Like mechanical oscillators, the oscillating wave energy converter is characteristic of frequency-dependent response showing the phenomenon of resonance. The maximum power is obtained when the wave period

*Corresponding author, Master, E-mail: zhxter@outlook.com

^a Professor

^b Professor

agrees with the natural period. However, the conversion is less powerful with wave periods off resonance, in particular so if the resonance bandwidth is narrow.

Thus control strategies have been applied to the oscillating wave energy converter in order to enhance the power capture efficiency. Early work focused on the use of mechanical impedance matching schemes to maximize the velocity and hence the captured power from regular waves (Falnes, 2007). However, this method leads to unrealistically large oscillation amplitude, which is not applicable for physical constraint handling or nonlinear PTO systems (Falnes 2002a). Then latching control strategies were introduced to the realm of wave energy utilization. Hoskin and Nichols (1986) studied the latching control strategy and first converted determination of latching and releasing time into an optimal problem and made use of the Pontryagin's maximum principle to solve it. Babarit and Clement (2006) studied latching control of wave energy converters in irregular waves and applied the strategy of latching control to SEAREV wave energy converter.

Falcao (2008) applied latching control to the oscillating-body converters equipped with a high-pressure hydraulic power take-off (PTO) mechanism. As an alternative to latching control strategies, Babarit *et al.* (2009) considered another strategy of declutching control or unlatching control to SEAREV wave energy converters with hydraulic cylinders driving hydraulic motors. Declutching control strategy consists in switching on and off alternatively the wave energy converters' PTO systems, which can be achieved practically using a simple by-pass valve for the hydraulic cylinder PTO system. Their results showed that declutching control strategy can lead to energy absorption performance equivalent to that of pseudo-continuous control, improving the power capture performance in both regular and irregular waves.

In present study, the declutching control strategy is applied to a hemispherical wave energy converter with direct linear electric PTO system. The linear electric PTO system can be simplified as a linear spring and a linear damper. As for the direct linear electric PTO system, declutching control can be obtained by adding a power-electronics switch. The converter is a single degree-of-freedom body oscillating in heave direction with other degrees-of-freedom restricted. Time domain model, which was first introduced to ships in wavy seas by Cummins (1962), is applied to analyze the dynamic response of the hemispherical wave energy converter in both regular and irregular waves. Due to the complexity and inconvenience of convolution term for calculation in time domain model, state space model is used to replace the convolution one. The optimal command theory based on Pontryagin's maximum principle is used to determine the controlled sequences of PTO forces. Comparisons are made between power capture by the hemispherical wave energy converter with and without declutching control. And the response of the converter's displacement and velocity is also presented.

2. Description of the model

A hemisphere floating under the water surface is adopted as the geometry for the converter (Fig. 1). The floater is rigidly connected to the direct linear electric Power-Take-Off (PTO) system, which is simplified as a linear damper and a linear spring. Due to the symmetry of the hemisphere, only one-directional wave is adopted to analyze the interaction with the converter. Only heave oscillation of hemisphere is considered with other degrees of freedom ideally constrained. It should be mentioned neither the viscous effect of the fluid nor the frictions in the PTO system are considered.

In this paper, the water depth is assumed to be infinite. The following values of parameters of

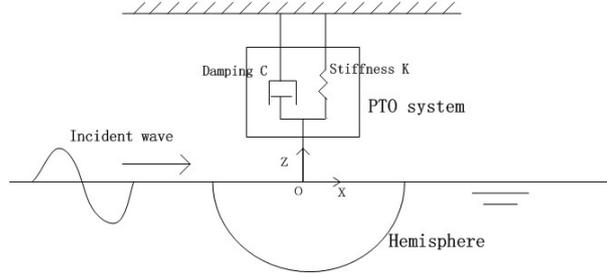


Fig. 1 The sketch of a hemispherical oscillating wave energy converter

the converter are adopted for analysis of oscillating wave energy converters with latching control. The radius of the hemisphere is $R = 5$ m. The density of water is $\rho = 1025$ Kg/m³. The gravity acceleration is $g = 9.81$ m/s². The mass of the hemispherical converter is the same as the mass of displaced water in calm surface, $m = 2\pi R^3 \rho / 3$. The area of the cross section of the hemisphere is $S = \pi R^2$. As indicated by Vicente *et al.* (2013), the equivalent stiffness of the PTO system is $K = 0.1\rho g S$, with a 25% variation.

The equivalent damping coefficient of the PTO system is chosen as the same value of radiation damping of the converter in resonance condition (Evans 1980), which satisfies the following conditions

$$\omega = \left\{ \frac{\rho g S + K}{m + A_z(\omega)} \right\}^{1/2} \quad (1)$$

$$C = B_z(\omega) \quad (2)$$

here, ω is the circular frequency of the incident waves. $A_z(\omega)$ and $B_z(\omega)$ is the hydrodynamic coefficients of added mass and radiation damping of the hemisphere for heave oscillation. C is the equivalent damping coefficient of the PTO system. According to Eq. (1), the natural (resonance) frequency of the hemispherical wave energy converter studied is about 1.4 rad/s. And in our calculation, the value of C is 91000 kg/s with a 25% variation.

3. Analysis in regular waves

In regular waves, wave absorption by the oscillating wave energy converter can be analyzed using the time-domain model, which was first introduced to ship motion in wavy seas by Cummins (1962). The equation of motion in heave direction of the hemispherical wave energy converter (shown in Fig. 1) can be expressed as

$$(m + A_z(\infty))\ddot{z}(t) + \int_0^t k_z(t - \tau)\dot{z}(\tau)d\tau + C\dot{z}(t) + (\rho g S + K)z(t) = f_{dz,regular} \quad (3)$$

here, $A_z(\infty)$ are the limiting value of the added mass for $\omega = \infty$. $k_z(t)$ is the retardation function.

$f_{dz,regular}$ is the wave excitation force of the hemispherical converter in regular waves.

The moduli of wave excitation force $|F_{dz}|$, which includes Froude-Kriloff force and diffraction force (Faltisen 1990), is proportional to the amplitude of the incident waves A_ω . This can be written as $|F_{dz}| = \Gamma_z(\omega)A_\omega$, where $\Gamma_z(\omega)$ is the excitation force coefficient, F_{dz} is the complex amplitude of the wave excitation force f_{dz} . Falnes (2002b) studied the excitation force coefficients of heave oscillation for an axisymmetric body in deep water and related the excitation force coefficients with radiation damping, known as Haskind's relation

$$\Gamma_z(\omega) = \left(\frac{2g^3 \rho B_z(\omega)}{\omega^3} \right)^{1/2} \quad (4)$$

From mathematical view, declutching control is conducted by introducing a control variable u , which is a sequence of binary value of 0 or 1, to the damping force of the PTO systems

$$F_{d,PTO} = -uC\dot{z} \quad (5)$$

here, $F_{d,PTO}$ is the damping force of PTO system. $u = 0$ means declutching control is applied, and the PTO system is disengaged from the oscillating body. Then the controlled equation of motion in heave direction of the converter can be written as

$$(m + A_z(\infty))\ddot{z}(t) + \int_0^t k_z(t-\tau)\dot{z}(\tau)d\tau + uC\dot{z}(t) + (\rho gS + K)z(t) = f_{dz,regular} \quad (6)$$

3.1 State space model

The convolution terms in Eqs. (3) and (6) indicate the memory effect of fluid in the radiation forces. The kernel (also called retardation function) can be written as

$$k_z(t) = \frac{2}{\pi} \int_0^\infty B_z(\omega) \cos \omega t d\omega \quad (7)$$

Hulme (1982) gave the tabulated values, as well as asymptotic expressions, for the coefficients of added mass and radiation damping of a floating hemisphere oscillating in deep water in dimensionless form. These theoretical values of radiation damping are taken for our calculation.

The convolution terms in the time domain equation makes it inconvenient for analysis of motion control of the converters. Thus the state-space model (Alves 2012) is used to replace the convolution term. A general state-space model has the form

$$\begin{aligned} \dot{X}(t) &= A'X(t) + B'u(t) \\ y(t) &= C'X(t) \end{aligned} \quad (8)$$

where $u(t)$ and $y(t)$ are called the input and output respectively of the state-space. $X(t)$ is the state vector. A' , B' and C' are the coefficients of the state space. The convolution term in Eq. (3) is replaced by state-space model as

$$I_z(t) = \int_0^t k_z(t-\tau)\dot{z}(\tau)d\tau \approx \begin{cases} \dot{X}(t) = A'X(t) + B'\dot{z}(t) \\ I_z(t) = C'X(t) \end{cases} \quad (9)$$

The problem now is to solve the matrix A' , B' and C' to approximate the convolution model, which is also called system identification (Taghipour *et al.* 2008). In frequency domain, the kernel has the relationship with the coefficients of added mass and damping, expressed as follows

$$k_z(j\omega) = \int_0^{\infty} k_z(\tau)e^{-j\omega\tau}d\tau = B_z(\omega) + j\omega(A_z(\omega) - A_z(\infty)) \quad (10)$$

here $k_z(j\omega)$ is the frequency response of the convolutions using Fourier transform. j is the imaginary unit. The least square technique is then used to find a rational function \hat{k}_z that approximates $k_z(j\omega)$ for the given set of circular frequencies following the restrictions given by Perez and Fossen (2011). This approximation is restricted to imaginary values, the rational function is defined, using $s = j\omega$, as

$$\hat{k}_z(s, \theta) = \frac{p_{n-1}s^{n-1} + p_{n-2}s^{n-2} + \dots + p_1s}{s^n + q_{n-1}s^{n-1} + \dots + q_0} \quad (11)$$

here, n is the order of the system. $p_i (i=1,2,\dots,n-1)$ and $q_i (i=0,1,\dots,n-1)$ are coefficients of the numerator and denominator respectively. Eq. (11) is also called transfer function. As long as the coefficients $\theta = [p_{n-1}, p_{n-2}, \dots, p_1, q_{n-1}, q_{n-2}, \dots, q_0]$ are found using the least square method, the matrix and vectors of state space model which approximate the convolution integral can be written as

$$A' = \begin{bmatrix} -q_{n-1} & -q_{n-2} & -q_{n-3} & \dots & -q_1 & -q_0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix}; \quad B' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}; \quad C' = [p_{n-1}, p_{n-2}, \dots, p_1, 0] \quad (12)$$

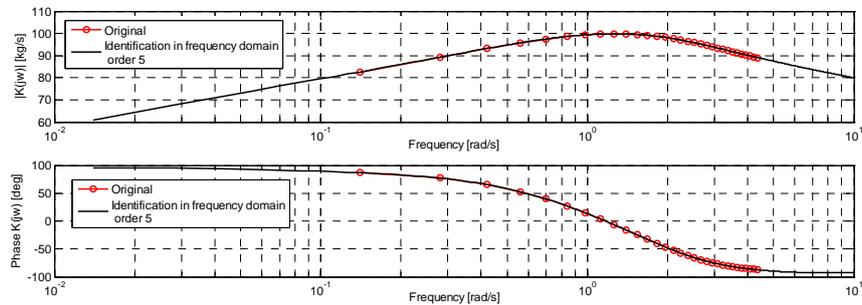


Fig. 2 The identification of the retardation function in frequency domain: the upper one is the amplitude of $k_z(j\omega)$; the lower one is the angle of $k_z(j\omega)$

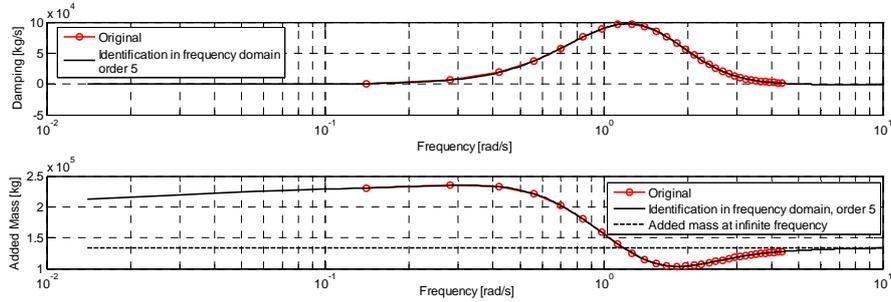


Fig. 3 The identification results of the added mass and radiation damping in frequency domain: the upper one is the radiation damping; the lower one is the added mass

For the hemispherical wave energy converter in this study, by the method of iteration, the order of the system is chosen $n=5$, providing a rather good identification of the retardation function in frequency domain. The identification results of the order $n=5$ are shown in Figs. 2 and 3.

As shown in these two figures, the identification results agree well with the original data. The results of coefficients for state space model in Eq. (12) are

$$A' = \begin{bmatrix} -3.8400 & -7.1237 & -5.8309 & -1.9262 & -0.0538 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}; B' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}; C' = [92160 \ 426370 \ 176590 \ 4070 \ 0] \quad (13)$$

As a result, the Eq. (3) can be replaced by

$$(m + A_z(\infty))\ddot{z}(t) + C'X(t) + C\dot{z}(t) + (\rho gS + K)z(t) = f_{dz\text{regular}} \quad (14)$$

$$\dot{X}(t) = A'X(t) + B'\dot{z}(t)$$

Denote a vector λ with the dimension of 7×1 , and $\lambda = [z \ \dot{z} \ X]'$. Then

$$\dot{\lambda} = P\lambda + Q \quad (15)$$

$$\text{with } P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{\rho gS + K}{m + A_z(\infty)} & \frac{C}{m + A_z(\infty)} & \frac{C'}{m + A_z(\infty)} \\ 0 & B' & A' \end{bmatrix} \text{ and } Q = \begin{bmatrix} 0 \\ \frac{f_{dz\text{regular}}}{m + A_z(\infty)} \\ 0 \end{bmatrix}.$$

Then Eq. (15) can be calculated using the 4th order Runge-Kutta method.

3.2 The optimal command theory

The optimal command theory based on Pontryagin's maximum principle, which is proposed by Hoskind and Nichols (1986), is used to conduct declutching control on the hemispherical wave energy converter in regular waves. Consider wave excitation forces on the hemispherical converter over a long duration $[0, T]$, the objective is to gain the maximum energy in this duration

$$\max_u E = \int_0^T W(t, \lambda, u) dt \quad (16)$$

here, E is the absorbed energy. $W(t, \lambda, u)$ is the instantaneous power. For the hemispherical wave energy converter, $W(t, \lambda, u) = u C z^2 = u C \lambda_2^2$. And $u = 1$ indicates declutching control is applied and $u = 0$ means no control is applied. The state vector λ satisfies Eq. (14), which can be rewritten as

$$\dot{\lambda} = f(t, \lambda, u) \quad (17)$$

A Hamiltonian function is defined as

$$H = W(t, \lambda, u) + \delta^T f(t, \lambda, u) \quad (18)$$

δ is the adjoint state vector with the same dimension as λ , which can be obtained by

$$\dot{\delta}_i = -\frac{\partial H}{\partial \lambda_i} \quad (19)$$

The state vector λ and the adjoint state vector δ have the initial and final condition respectively

$$\lambda(0) = 0; \quad \delta(T) = 0 \quad (20)$$

The optimal command theory based on Pontryagin's maximum principle, transform the objective of gaining the maximum energy E to maximizing the value of the Hamiltonian function H based on choosing the optimum value of u for every time step. The calculation is an iterative process: the device motion (without control) is numerically calculated by Eq. (15). Then the adjoint state vector can be computed by Eq. (19). After both λ and δ are obtained, the optimum value of control variable u at every time step can be calculated by maximizing the value of H . And then iterate the process until the results converge to a steady state.

The declutching control strategy needs the knowledge of the future of the excitation force, which requires prediction of incoming waves. Some promising results of short-term wave forecasting have been obtained by Fusco and Ringwood (2010) using linear AR (auto-regressive) models. By modelling the wave height at a single point using AR systems of varying orders, predictions are made up to ten seconds in the future. However, these considerations are beyond the scope of this study, in which the time sequences of wave excitation force is supposed to be already known.

3.3 Comparison of motion with and without declutching control

For the declutching control based on optimal command theory, the averaged power captured by the hemispherical converter is calculated for regular waves of several circular frequencies. And the wave amplitude is chosen 1m with a 50% variation.

The controlled motion of the hemispherical wave energy converter approximated by state space

model in regular waves is expressed as

$$(m + A_z(\infty))\ddot{z}(t) + C'X(t) + (C + Gu)\dot{z}(t) + (\rho gS + K)z(t) = f_{dzregular}$$

$$\dot{X}(t) = A'X(t) + B'\dot{z}(t) \quad (21)$$

here, G is a coefficient with constant value. And the Hamiltonian function is

$$H = uC\lambda_2^2 + \delta_1\lambda_2 + \delta_2\left(\frac{f_{dz}}{m+A_z(\infty)} - \frac{\rho gS+K}{m+A_z(\infty)}\lambda_1 - \frac{uC}{m+A_z(\infty)}\lambda_2 - \frac{1}{m+A_z(\infty)}\sum_{j=1}^5 C'_j\lambda_{j+2}\right) \quad (22)$$

$$+ [\delta_3, \delta_4, \delta_5, \delta_6, \delta_7] \cdot \left[B'_1\lambda_2 + \sum_{j=1}^5 A'_{1j}\lambda_{j+2}, B'_2\lambda_2 + \sum_{j=1}^5 A'_{2j}\lambda_{j+2}, B'_3\lambda_2 + \sum_{j=1}^5 A'_{3j}\lambda_{j+2}, B'_4\lambda_2 + \sum_{j=1}^5 A'_{4j}\lambda_{j+2}, B'_5\lambda_2 + \sum_{j=1}^5 A'_{5j}\lambda_{j+2} \right]^T$$

here, C'_j is the j^{th} element of the vector C' . B'_j is the j^{th} element of the vector B' . A'_{ij} is the element at row i , column j of the matrix A' .

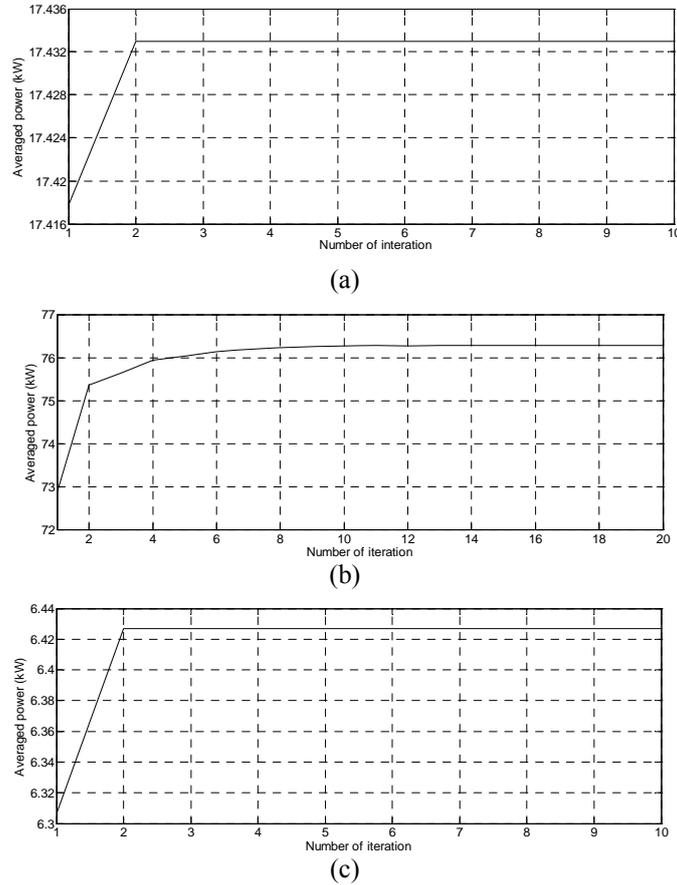


Fig. 4 Absorbed power as a function of the number of iteration with $A_\omega=1 \text{ m}$ $K = 0.1\rho gS$, $C = 91000 \text{ kg/s}$
 (a) $\omega = 0.7 \text{ rad/s}$; (b) $\omega = 1.4 \text{ rad/s}$ and (c) $\omega = 1.96 \text{ rad/s}$

By Eq. (19), the adjoint vector δ satisfies

$$\dot{\delta} = \begin{bmatrix} 0 & \frac{\rho g S + K}{m + A_z(\infty)} & 0 \\ -1 & \frac{uC}{m + A_z(\infty)} & -B^{tr} \\ 0 & \frac{C^{tr}}{m + A_z(\infty)} & -A^{tr} \end{bmatrix} \cdot \delta + \begin{bmatrix} 0 \\ -2uC\lambda_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (23)$$

From Eq. (22) we find the Hamiltonian function H is a linear function of the control variable u . So the optimum value of u can be only the extreme ones 0 or 1, which follows the principle

$$u = \begin{cases} 0 & \text{if } \lambda_2^2 - \frac{\lambda_2 \delta_2}{m + A_z} > 0 \\ 1 & \text{otherwise} \end{cases} \quad (24)$$

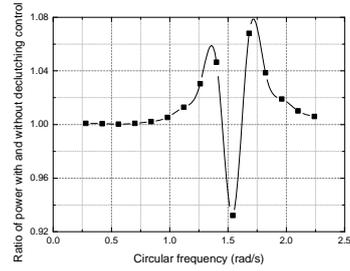
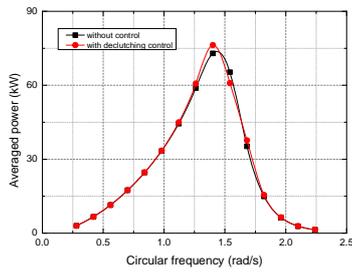
As determination of control variable is an iterative process, the averaged power is chosen as the criteria for convergence. The duration for calculation is chosen as $T = 4000s$. Figs. 4(a)-4(c) are some illustrated examples of convergence studies with regular waves of several frequencies with $A_\omega=1$ m, $K=0.1 \rho g S$, $C=91000$ kg/s. It can be found that the calculation reaches to a steady state within 10 iteration steps. Generally, almost all the iterations for the selected circular frequencies converged within 10 iteration steps.

Figs. 5(a)-5(g) show the comparison of absorbed power by the hemispherical wave energy converter with and without declutching control. From the left figure, it can be seen that declutching control can slightly increase the power capture by the oscillating wave energy converter with direct linear Power-Take-Off systems, majorly in the frequency region near the natural period of the converter. An interesting phenomenon can be found that declutching control becomes destructive at the incident frequency equaling 1.54 rad/s, as shown in the right figure of Fig. 5. For a given time interval, declutching control amplifies the oscillation amplitude of the converters, which is constructive to power capture performance. Whereas the control strategy reduces the time during which the PTO system is working as $u=0$ indicates no energy is converted by the PTO system. Thus the destructive effects near 1.54rad/s may indicate that the benefit of amplification of the oscillation amplitude of the converter by declutching control is counteracted by reduction of the time of work by the PTO systems of the converter. The amplification ratio of the averaged power by declutching control is slightly larger than 1, which indicates that declutching control may be inapplicable for the wave energy converter with direct linear electric Power-Take-Off systems if the error of prediction of wave excitation force is considered. Variation of values of stiffness (Figs.5 (a), 5(d) and 5(e)) and damping coefficient (Figs. 5(a), 5(b) and 5(c)) of the PTO system changes the absorbed power with and without declutching control, as well as the amplification ratio of power. As the system is linear, it can be found that variation of wave amplitudes does not influence the amplification ratio of power, as shown in Figs. 5(a), 5(f) and 5(g).

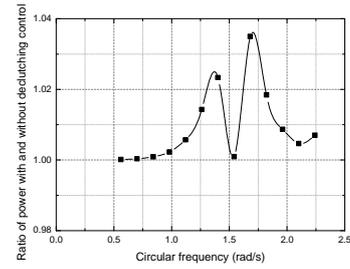
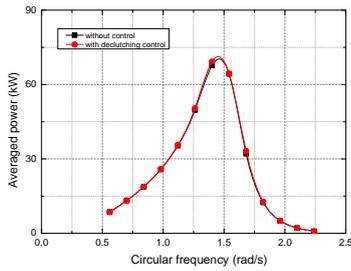
Figs. 6(a)-6(i) are some examples of time series of wave elevation and controlled heave motion of the oscillating wave energy converter, together with time sequences of controlled variables, based on optimal command method. Obviously the motion is slightly magnified by the control

strategy. Compare

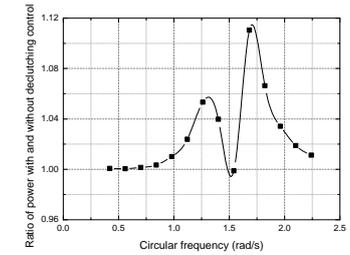
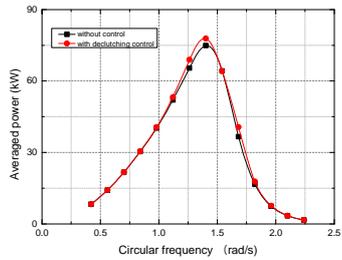
the controlled variable u in Figs. 6(c), 6(f) and 6(i), it can be observed that a larger portion of declutching time indicates a larger value of amplification ratio of the power with and without declutching control. Generally, the declutching time of the oscillating wave energy converter only takes a small portion of the whole duration, which implies that the optimal case works very closely to the original uncontrolled condition. Thus declutching control seems to slightly affect power capture performance of the converter with direct linear electric PTO systems.



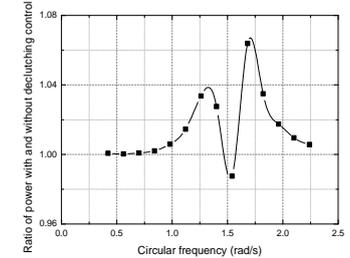
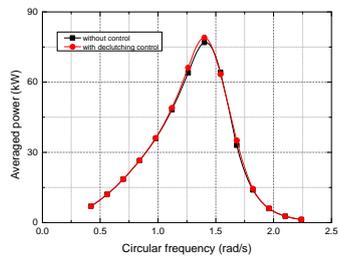
(a)



(b)



(c)



(d)

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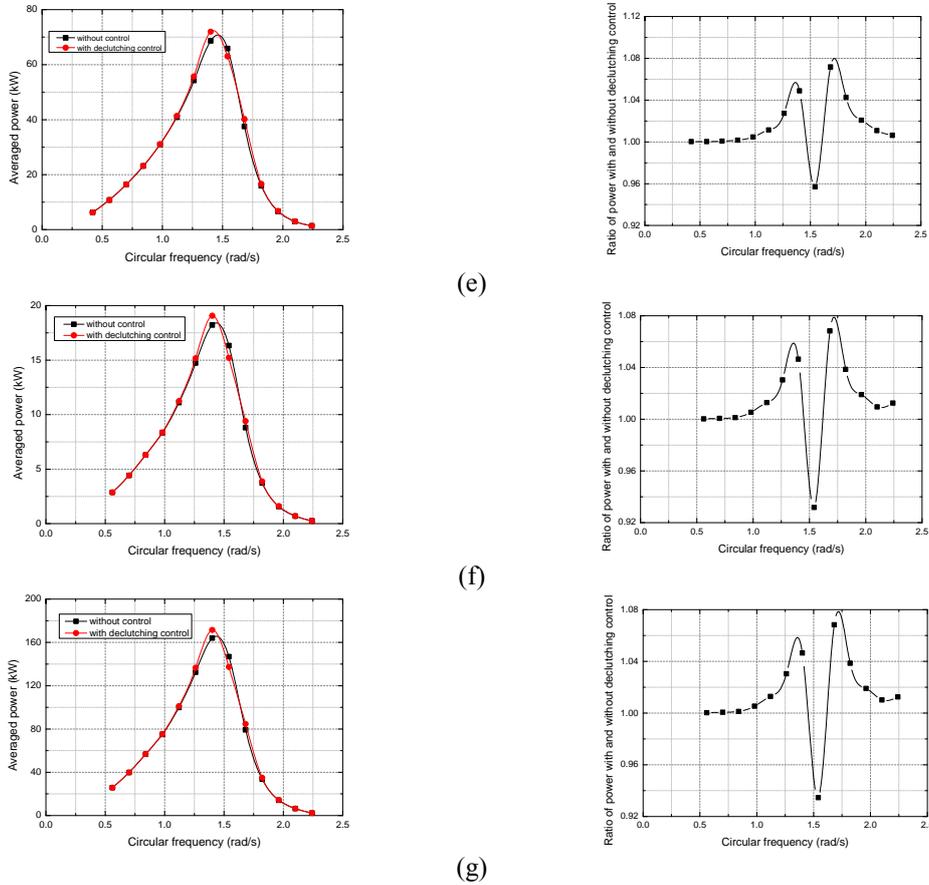
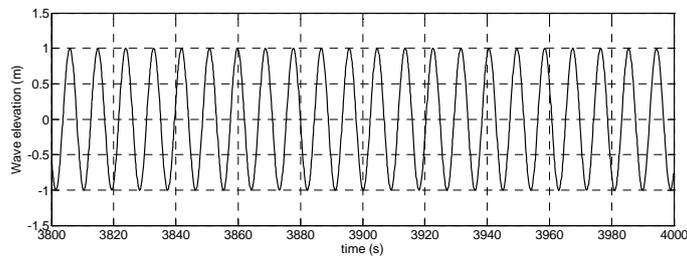
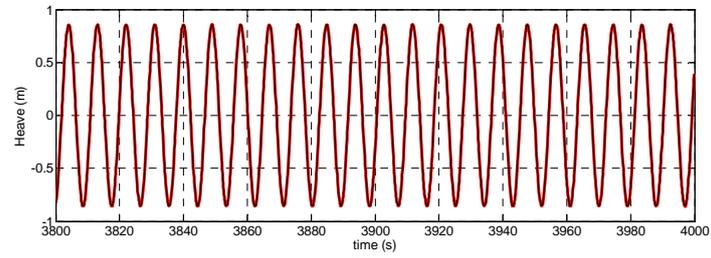


Fig. 5 Comparison of the power obtained by the hemispherical wave energy converter with and without control , the left one is the averaging absorbed power and the right one is the ratio: (a) $A_w=1m$, $K = 0.1\rho gS$, $C = 91000kg / s$; (b) $A_w=1m$, $K = 0.1\rho gS$, $C = 68250kg / s$; (c) $A_w= 1 m$, $K = 0.1\rho gS$, $C = 113750kg / s$; (d) $A_w=1m$, $K = 0.075\rho gS$, $C = 91000kg / s$; (e) $A_w=1m$, $K = 0.125\rho gS$, $C = 91000kg / s$; (f) $A_w=0.5m$, $K = 0.1\rho gS$, $C = 91000kg / s$ and (g) $A_w=1.5m$, $K = 0.1\rho gS$, $C = 91000kg / s$;

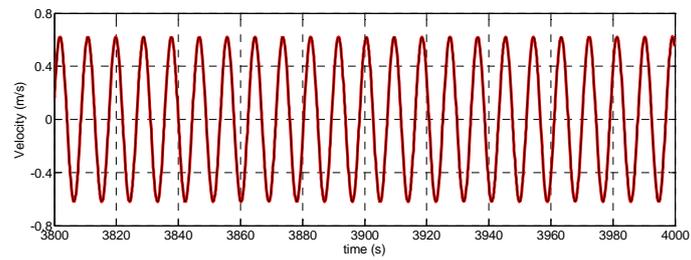


(a)

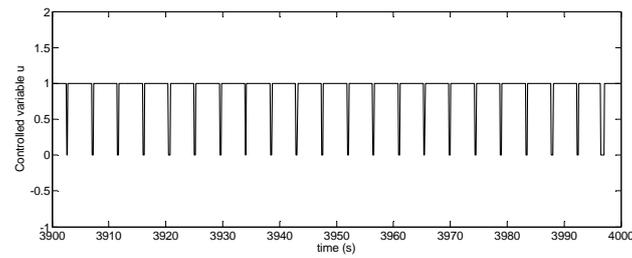
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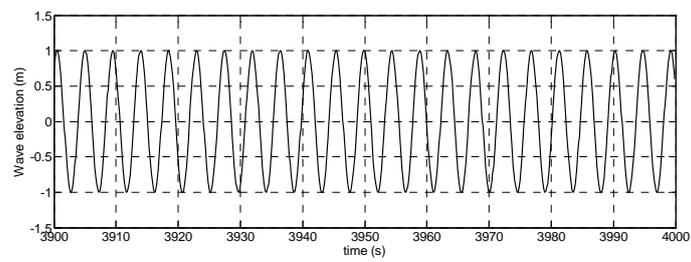
(b)



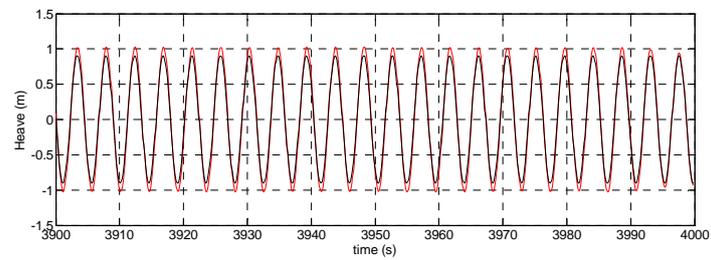
(c)



(d)

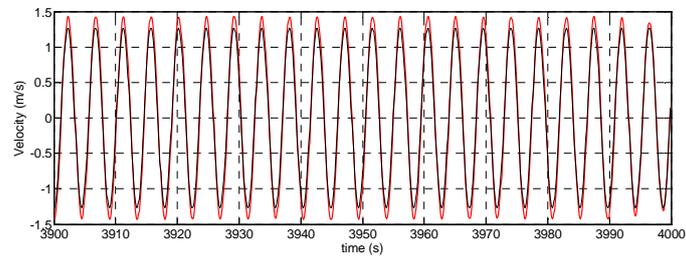


(e)

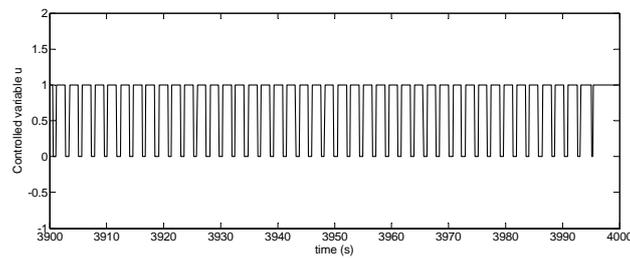


(f)

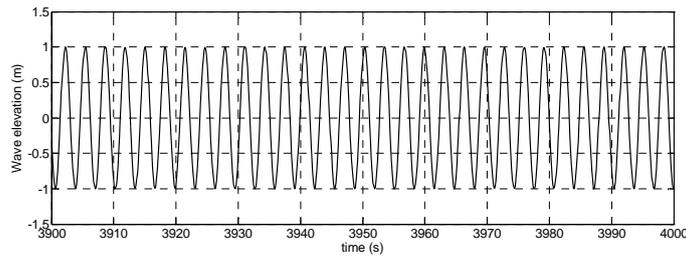
Continued-



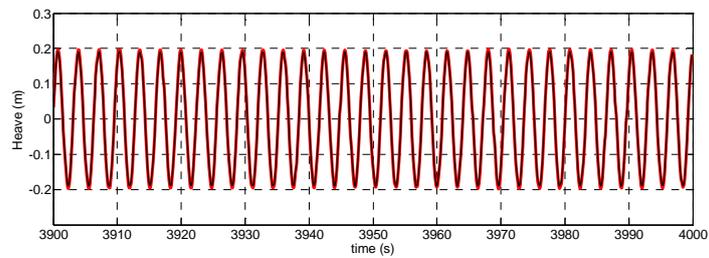
(g)



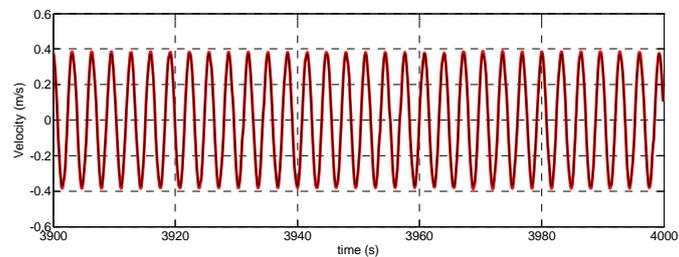
(h)



(i)



(j)



(k)

Continued-

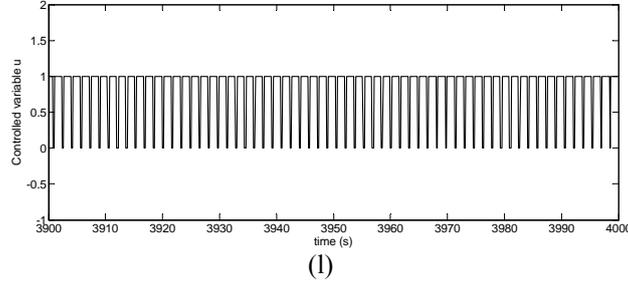


Fig. 6 Time series of wave elevation and response of amplitude and velocity (red line represent that with declutching control, and black line means no control), together with time sequences of controlled variable u , $A_\omega=1$ m, $K=0.1\rho gS$, $C=91000\text{kg/s}$: (a), (b), (c) and (d): $\omega=0.7$ rad/s; (e), (f), (g) and (h): $\omega=1.4$ rad/s; (i), (j), (k) and (l): $\omega=1.96$ rad/s

4. Analysis in irregular waves

Real irregular waves may be represented, in a fairly good approximation, as a superposition of regular waves, by defining a wave spectrum. As the semi-sphere oscillating body is axisymmetric and insensitive to wave direction, the wave spectrum is assumed to be one-dimensional. JONSWAP spectrum distribution is adopted in the calculation, which is defined by

$$S(\omega) = 319.34 \frac{\zeta_{w/3}^2}{T_p^4 \omega^5} \exp\left(-\frac{1948}{(T_p \omega)^4}\right) \chi^{\exp\left[-\frac{(0.159\omega T_p - 1)^2}{2\sigma^2}\right]} \quad (25)$$

where $\zeta_{w/3}$ is the significant wave amplitude and T_p is the peak period of the spectrum. χ is the peak lifting factor, the value of which is 3 in our calculation. σ is the peak shape parameter, when $\omega \leq \omega_p$, $\sigma=0.07$; when $\omega > \omega_p$, $\sigma=0.09$. ω_p is the peak frequency, which is related to peak period by $\omega_p = 2\pi / T_p$. The spectral distribution has the unit of m^2/s .

For time-series calculations in irregular waves, the spectral distribution in Eq. (25) should be discretized as the sum of a large number N of regular waves with frequency $\omega_n = \omega_0 + n\Delta\omega$, where ω_0 is the lowest frequency considered, $\Delta\omega$ is a small frequency interval. In our calculation, $\omega_0 = 0.14\text{rad/s}$, $\Delta\omega = 0.00042$. The amplitude of the n th wave component is $A_{oi} = \sqrt{2S(\omega_i)\Delta\omega}$. The excitation force in irregular waves can be written as

$$f_{dzirregular}(t) = \sum_i \Gamma_z(\omega_i) A_{oi} \cos(\omega_i t + \varphi_i + \theta_i) \quad (26)$$

here, φ_i is the phase response of the excitation force and θ_i is chosen as a random real number in the interval $(0, 2\pi)$. In the calculation, the significant wave amplitude is chosen as $\zeta_{w/3} = 1\text{m}$, together with $K=0.1\rho gS$, $C=91000\text{kg/s}$.

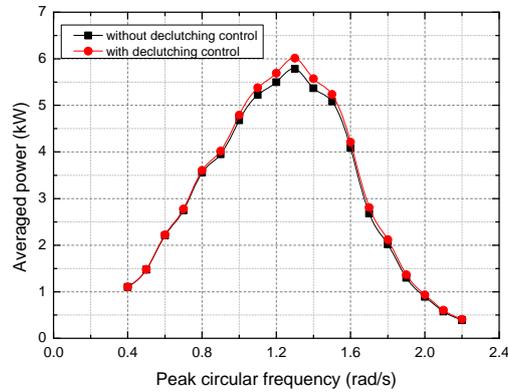


Fig. 7 Power obtained by the hemispherical wave energy converter without and with declutching control in irregular waves of various peak frequencies

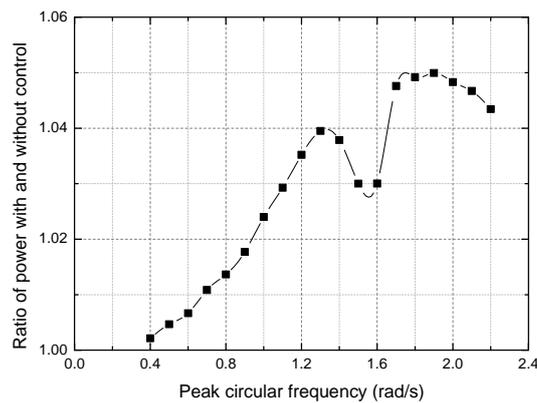


Fig. 8 Ratio of power obtained by the hemispherical wave energy converter without and with declutching control in irregular waves of various peak frequencies

Fig. 7 is the averaged power absorbed by the hemispherical wave energy converter with and without declutching control in irregular waves. It can be seen that declutching control can slightly enhance the performance of power capture for the peak circular frequencies considered. The ratio of power with and without declutching control is shown in Fig. 8. For the peak frequency less than the natural frequency of the converter, the ratio becomes larger with the value of peak frequency of irregular waves increasing. Near the natural frequency region, the ratio slightly decreases. And in the high peak frequency region, the ratio increases with the value of peak frequency again. Like the results in regular waves, the ratio is less than 1.1 for all peak frequency considered.

Figs. 9-11 are some illustrated examples of the responses of displacement and velocity, together with time sequences of controlled variable u . The amplitude of displacement and velocity of the hemispherical wave energy converter with declutching control is slightly larger than that without control.

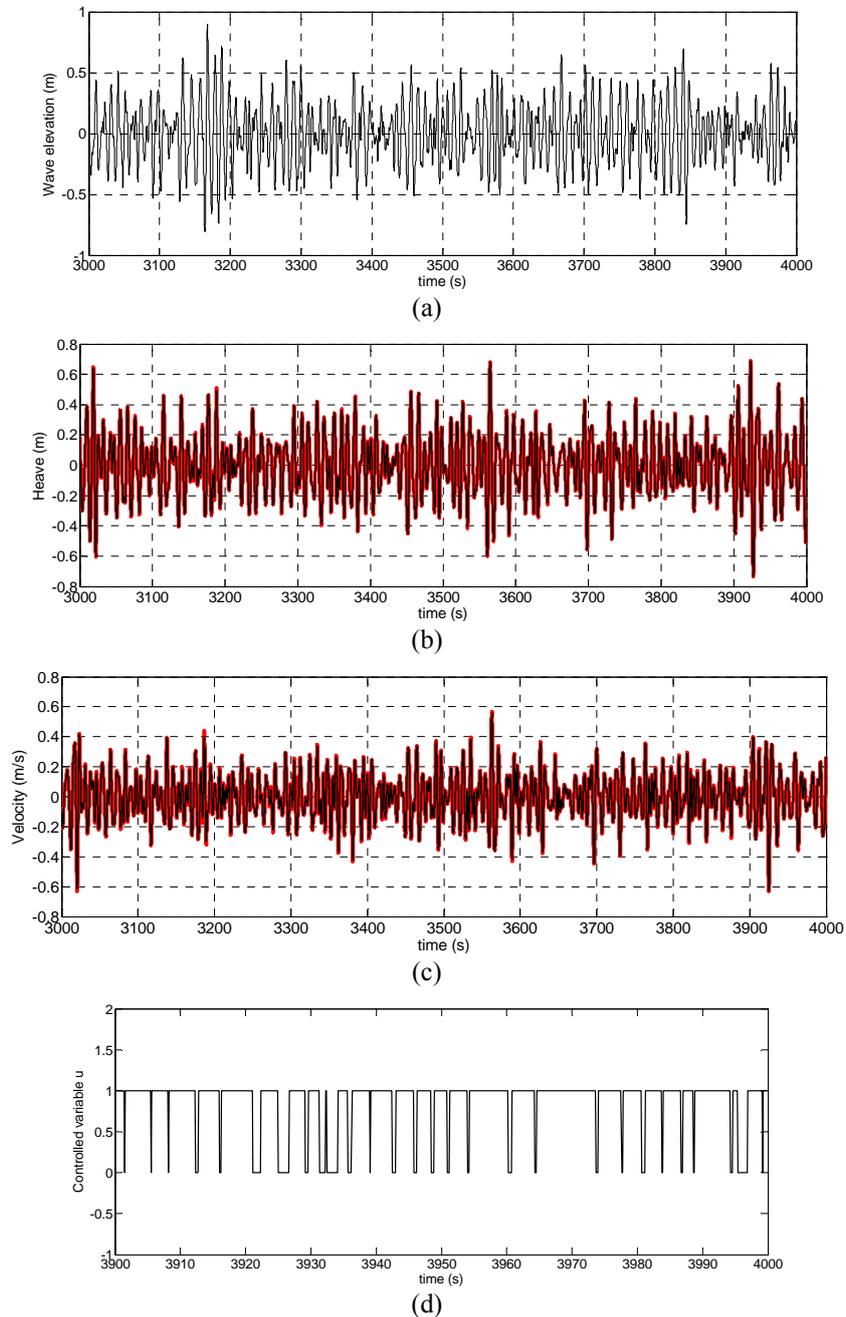


Fig. 9 Time series of wave elevation and motion of the oscillating wave energy converter without and with declutching control based on optimal command method for $\omega_p=0.6$ rad/s (red line: motion with declutching control; black line: motion without control): (a) wave elevation (b) displacement of the converter; (c) velocity of the converter and (d) time sequences of controlled variable u

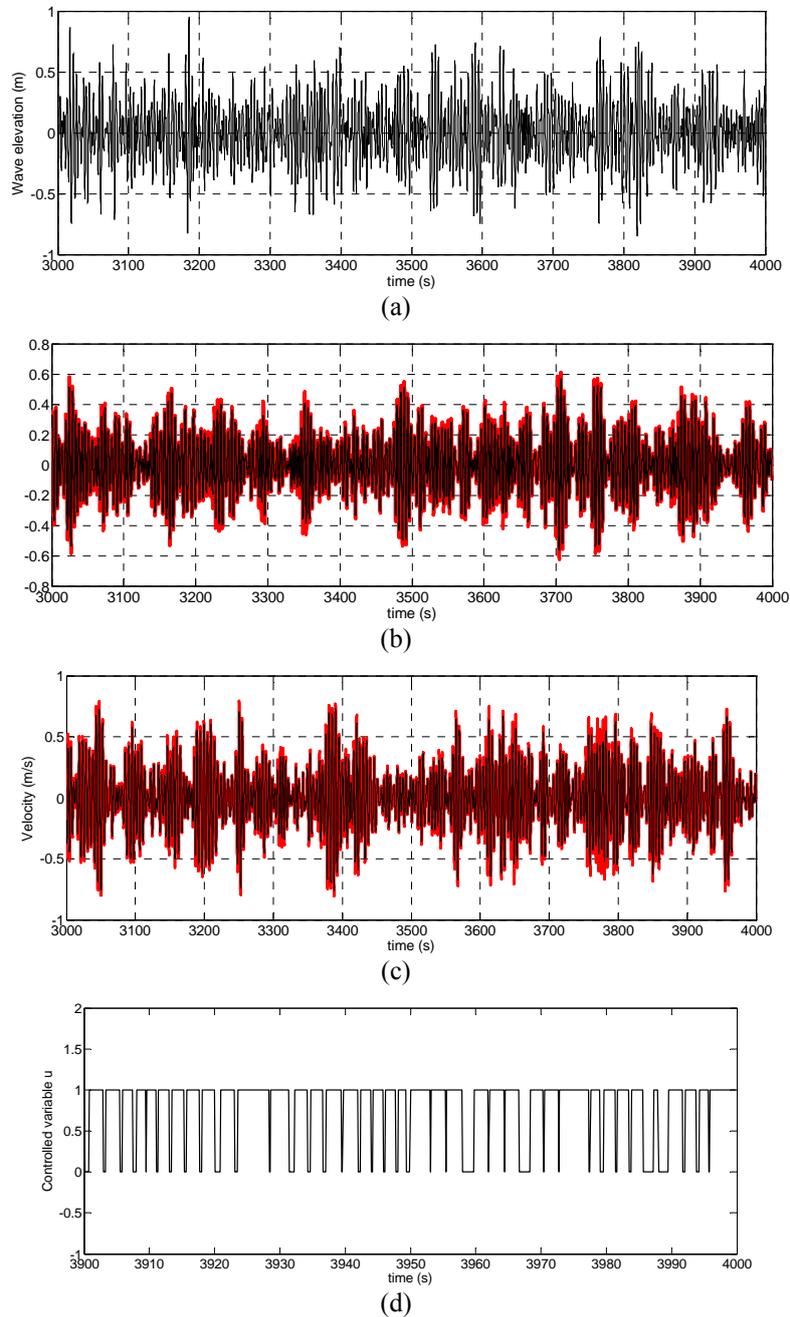


Fig. 10 Time series of wave elevation and motion of the oscillating wave energy converter without and with declutching control based on optimal command method for $\omega_p=1.3$ rad/s (red line: motion with declutching control; black line: motion without control): (a) wave elevation; (b) displacement of the converter; (c) velocity of the converter and (d) time sequences of controlled variable u

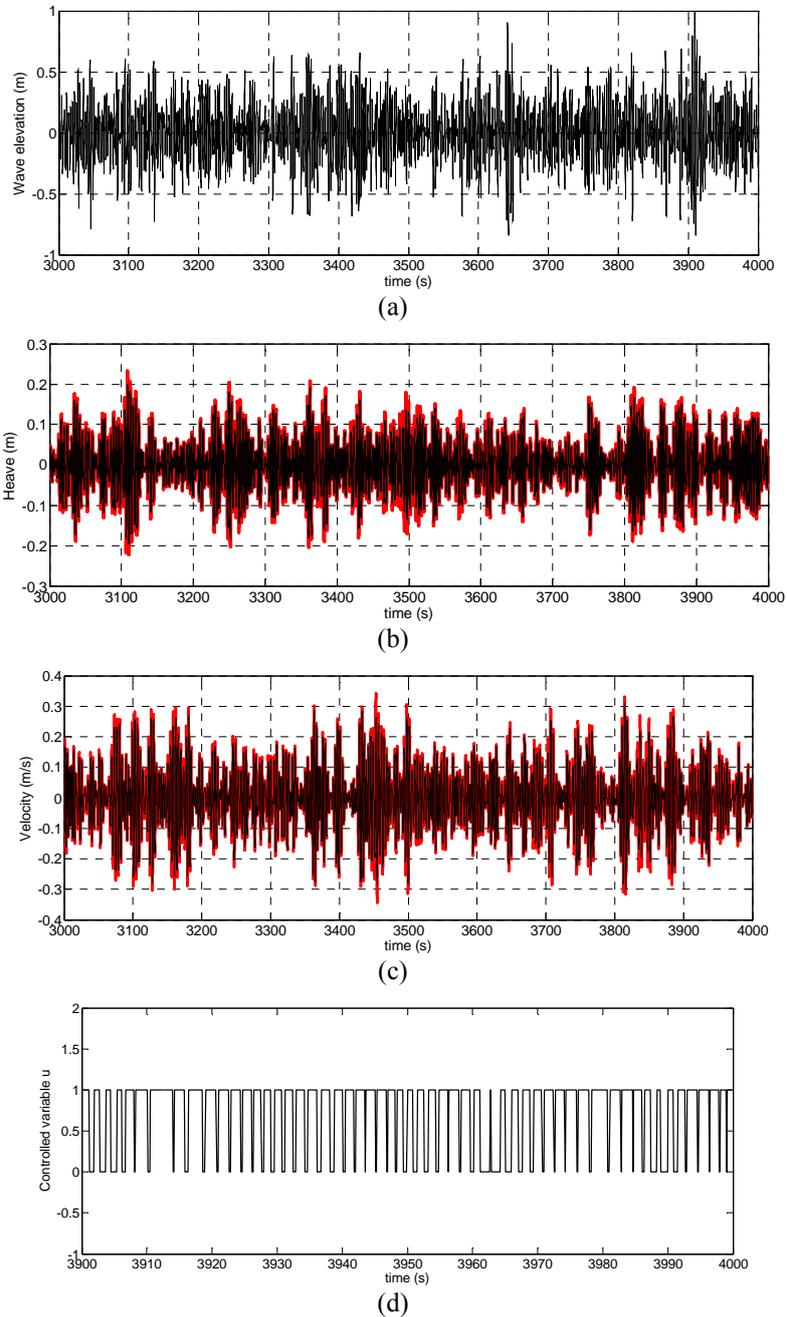


Fig. 11 Time series of wave elevation and motion of the oscillating wave energy converter without and with declutching control based on optimal command method for $\omega_p = 2.0$ rad/s (red line: motion with declutching control; black line: motion without control): (a) wave elevation; (b) displacement of the converter; (c) velocity of the converter and (d) time sequences of controlled variable u

5. Conclusions

In the present work, declutching control in both regular waves and random sea state has been investigated for a hemispherical wave energy converter with direct linear electric Power-Take-Off systems oscillating in heave direction. For the direct linear electric Power-Take-Off system, declutching control can be simply applied by adding a power-electronics switch to control Power-Take-off forces.

By means of the optimal command method, it is shown theoretically that declutching control strategy can slightly increase power capture by the oscillating wave energy converter in regular waves for all circular frequencies considered. The amplification ratio of power with and without declutching control is less than 1.1, although larger than 1. For irregular waves of various peak frequencies, this strategy leads to the amplification ratio of averaged power absorbed by the converter locating in the interval between 1 and 1.1.

In real random sea state, wave excitation force needs to be predicted for determining the time sequences of the controlled variable. Therefore, this declutching control strategy may be ineffective for oscillating wave energy converters with direct linear electric Power-Take-Off systems in real wavy seas considering the error of prediction, although the strategy has been theoretically proved to be effective for an oscillating converter with hydraulic cylinders with adequate prediction of the excitation force.

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