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Analysis of a cracked bar under a tensile load in a corrosive environment

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Abstract. This brief study aims at providing a model to predict the time of service of a cracked bar in corrosive environment, in view of both the fracture mechanics and elastic failure criteria. Dolinskii's assumption on the relationship between stress and the corrosion rate is adopted. It is superimposed with fracture mechanics consideration. A comparison between the time of service of a cracked bar and that of a uniform bar is provided.

Keywords: time of service; cracked bar; corrosion; stress concentration factor

1. Introduction

The uniform corrosion models that were discussed in two recent papers by Elishakoff and Miglis (2011, 2012) deal with perfect bars and hence do not provide estimates for the time of service for a notched bar, the radius being assumed as constant along the length.

However, when considering a realistic bar, the stress concentration at the crack will occur and as a result the stress at the notch will be significantly higher than that at $x \to \infty$. It is anticipated that the stress concentration at the crack will yield an accelerated failure of the bar as compared to a non-cracked bar, because of the stress concentration at the crack tip.

The anticipated consequences are: (a) the higher stress at the crack tip would yield a higher corrosion rate, according to the models developed by Dolinskii (1969) and Gutman (2002) and (b) according to the fracture mechanics theory, if the stress intensity factor at the crack is higher than the fracture toughness, the crack will propagate, leading to the failure of the bar.

According to Pilkey (1997), the stress concentration factor f in the smallest cross section of a one sided cracked thin element is

$$f = \frac{\sigma}{\sigma_{avg}} = C_1 + C_2\left(\frac{l}{D}\right) + C_3\left(\frac{l}{D}\right)^2 + C_4\left(\frac{l}{D}\right)^3 \tag{1}$$

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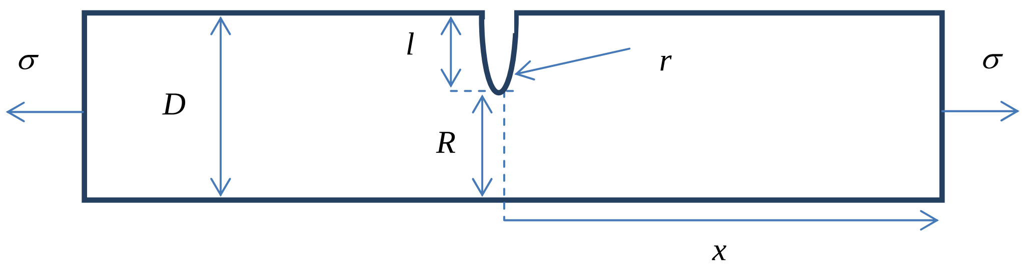

Fig. 1 Cracked bar under a tensile stress

with σ being the stress at the crack, σ_{avg} the stress at $x \to \infty$, l is the length of the crack, D is the thickness of the specimen at $x \to \infty$. The values of the parameters $C_i, i = 1, 2, 3, 4$ are listed in Table 1, where r is the radius of curvature of the crack.

Table 1 Value of the Coefficients for the Stress Concentration Factor

	If $l/r \le 2$	If $l/r > 2$
C_1	$0.907 + 2.125\sqrt{l/r} + 0.023l/r$	$0.953 + 2.136\sqrt{l/r} - 0.005l/r$
C_2	$0.710 - 11.289\sqrt{l/r} + 1.708l/r$	$-3.255 - 6.281\sqrt{l/r} + 0.068l/r$
C_3	$-0.672 + 18.754\sqrt{l/r} - 4.046l/r$	$8.203 + 6.893\sqrt{l/r} + 0.064l/r$
C_4	$0.175 - 9.759\sqrt{l/r} + 2.365l/r$	$-4.851 - 2.793\sqrt{l/r} - 0.128l/r$

On the other hand, the stress intensity factor K_I, predicting the stress state in at the crack tip in a bar is defined as

$$K_I = \sigma_{avg}\sqrt{\pi l}\,F(l/D) \tag{2}$$

Several expressions of the function $F(l/D)$ have been provided in the literature. Interested readers may consult the papers by Gross (1964), Brown (1965) and Tada (1973). According to Gross (1964)

$$F(l/D) = 1.122 - 0.231(l/D) + 10.550(l/D)^2 - 21.710(l/D)^3 + 30.382(l/D)^4 \tag{3}$$

and according to Brown (1965)

$$F(l/D) = 0.265\left(1-\frac{l}{D}\right)^4 + \left(0.857 + \left(\frac{0.265l}{D}\right)\right)\Bigg/\left(1-\frac{l}{D}\right)^{\frac{3}{2}}, (2) \tag{4}$$

Tada (1973) suggested the following function

$$\sqrt{\frac{2D}{\pi t}\tan\left(\frac{\pi l}{2D}\right)}\left(0.752+2.02\left(\frac{l}{D}\right)+0.37\left(1-\sin\left(\frac{\pi l}{2D}\right)^3\right)\right)\Big/\cos\left(\frac{\pi l}{2D}\right) \tag{5}$$

Eq. (3) provides an error of 0.5% for l/D<0.6, Eq. (4) has an error of less than 1% for l/D <0.2 and 0.5% for $l/D>0.2$.The quantity in Eq. (5) proposed Tada, leads to the error less than 0.5% for arbitrary l/D ratio. Hereinafter we adopt the latter expression for the function $F(l/D)$.

According to the fracture mechanics, Kanninen (1985) states the fracture occurs when the stress intensity factor at the crack K_I becomes larger than the fracture toughness K_{IC}. The fracture criterion reads

$$K_I \geq K_{IC} \tag{6}$$

During the corrosion process, the hybrid effect of stress and corrosion will make the crack propagate, leading to the decrease of the minimum cross section to decrease as well as the modification in the stress intensity factor to modify.

2. Mathematical Model for Predicting the Crack Propagation

As stated before, the decreasing rate of the smallest cross section at the crack can be expressed as follows

$$\frac{dR(t)}{dt} = v_1(t) \tag{7}$$

with $R(t) = D(t) - l(t)$, $v_1(t)$ being the corrosion rate at the minimum cross section. The crack propagation rate is defined as $dl(t)/dt$.

Hereinafter, for simplicity, we adopt the linear relationship between stress σ and corrosion rate $v_1(t)$ following Dolinskii (1969)

$$v_1(t) = v_0 + m\sigma(t) \tag{8}$$

On the other hand, the rate of decrease of D, the value of the diameter at $x \to \infty$ reads

$$\frac{dD}{dt} = v_0 + m\sigma_{avg}(t) \tag{9}$$

with $v(t)$ the corrosion rate of the material under a considered stress σ. Discretizing this formula between t and $t+dt$, dt being the unit of time, yields

$$R(t+dt) = \lim_{\Delta t \to 0} R(t) + \frac{dR}{\Delta t}\Delta t = R(t) + \frac{dR}{dt}dt \tag{10}$$

Similarly, the rate of decrease of D, the value of the diameter is expressed as

$$D(t+dt)=D(t)+\frac{dD}{dt}dt \tag{11}$$

Therefore, we can calculate the stress concentration factor and stress intensity factor at the time t. The initial stress concentration factor reads

$$\sigma_0=\sigma_{avg,0}\left[C_1+C_2\left(\frac{l_0}{D_0}\right)+C_3\left(\frac{l_0}{D_0}\right)^2+C_4\left(\frac{l_0}{D_0}\right)^3\right] \tag{12}$$

with $\sigma_{nom,0}$ being the initial stress at the crack, $\sigma_{avg,0}$ the initial stress at $x\to\infty$, l_0 the initial length of the crack and D_0 the initial thickness at $x\to\infty$. The coefficients C_i, $i=1,2,3,4$ have to be chosen from Table 1 depending on the value of the ratio l_0/r.

From the stress in Eq. (12), we calculate the stress concentration factor $K_{I,0}$. It reads

$$K_{I,0}=\sigma_{avg,0}\sqrt{\pi l}F(l_0/r) \tag{13}$$

with $l_0=D_0-R_0$. If the condition

$$K_I<K_{IC} \tag{14}$$

is satisfied, then we evaluate the new cross section from Eq. (10), (11), (12). Indeed, the stress being different at the crack tip and at $x\to\infty$, the attendant corrosion rates will be different, namely

$$\begin{aligned} R(t_0+\Delta t)&\approx R_0-(v_0+m\sigma_0)\Delta t\\ D(t_0+\Delta t)&\approx D_0-(v_0+m\sigma_{avg,0})\Delta t \end{aligned} \tag{15}$$

At the time $t+n\Delta t$, n being a positive integer we have

$$\begin{aligned} R(t_0+n\Delta t)&\approx R\left[t_0+(n-1)\Delta t\right]-(v_0+m\sigma_{n-1})\Delta t\\ D(t_0+n\Delta t)&\approx D\left[t_0+(n-1)\Delta t\right]-(v_0+m\sigma_{avg,n-1})\Delta t \end{aligned} \tag{16}$$

and $l(t_0+n\Delta t)=D(t_0+n\Delta t)-R(t_0+n\Delta t)$. From Eq. (16), we can calculate the ratio $l(t_0+n\Delta t)/D(t_0+n\Delta t)$ and chose the appropriate coefficients in Table 1 for Eq. (1). The stress concentration factor reads

$$\sigma_{nom,n} = \sigma_{avg,n}\left[C_1 + C_2\left(\frac{l(t_0+n\Delta t)}{D(t_0+n\Delta t)}\right) + C_3\left(\frac{l(t_0+n\Delta t)}{D(t_0+n\Delta t)}\right)^2 + C_4\left(\frac{l(t_0+n\Delta t)}{D(t_0+n\Delta t)}\right)^3\right] \tag{17}$$

Stress intensity factor becomes

$$K_{I,n} = \sigma_{avg,n}\sqrt{\pi l(t_0+n\Delta t)}\,F\left(l(t_0+n\Delta t)/r\right) \tag{18}$$

We identify the condition

$$K_{I,N} > K_{IC} \tag{19}$$

with the bar's failure, and the time of service is

$$T_s = N\Delta t \tag{20}$$

The above algorithm calculates the time of service, in a schematic form, is presented in Fig. 2. The subscript $i = n\Delta t$, $n \in \mathbb{N}$, and the dimension of the time $[\Delta t] = [s]$ is in seconds.

3. Corroboration of the model

Ibrahim et al. (2008) proposed to investigate the crack growth rate of 4340 steel in a 3.5% sodium chloride solution. The equivalent corrosion rate v_0 for this kind of steel is calculated at $1.6\cdot 10^{-9}\,m\cdot s^{-1}$, according to Chandra and Daemen (2004). Bars with a diameter of 9.5 mm were used in this study. The experiments were performed in room temperature. The bar was notched in the middle, with an indentation of 3 mm. Rusanov (2002) observed experimentally that for carbon steel the stress corrosion relationship was linear, without providing further details on the values. Due to the absence of stress corrosion relationships, we assume in this study that the coefficient m of the stress corrosion relationship equals $2\cdot 10^{-18}$, yielding the minimum mean square error. The results are depicted in Fig. 3.

It appears that the model proposed model provides a good fitting of the experimental data.

4. Conclusions

This study investigates the time of service of a *cracked bar* under a tensile load, apparently for the first time in the literature. A theoretical model predicting the time of service is proposed, using both the fracture mechanics criterion for the failure, and the stress – corrosion rate relationship to predict the service life. Additionally, the model is corroborated with available experimental data.

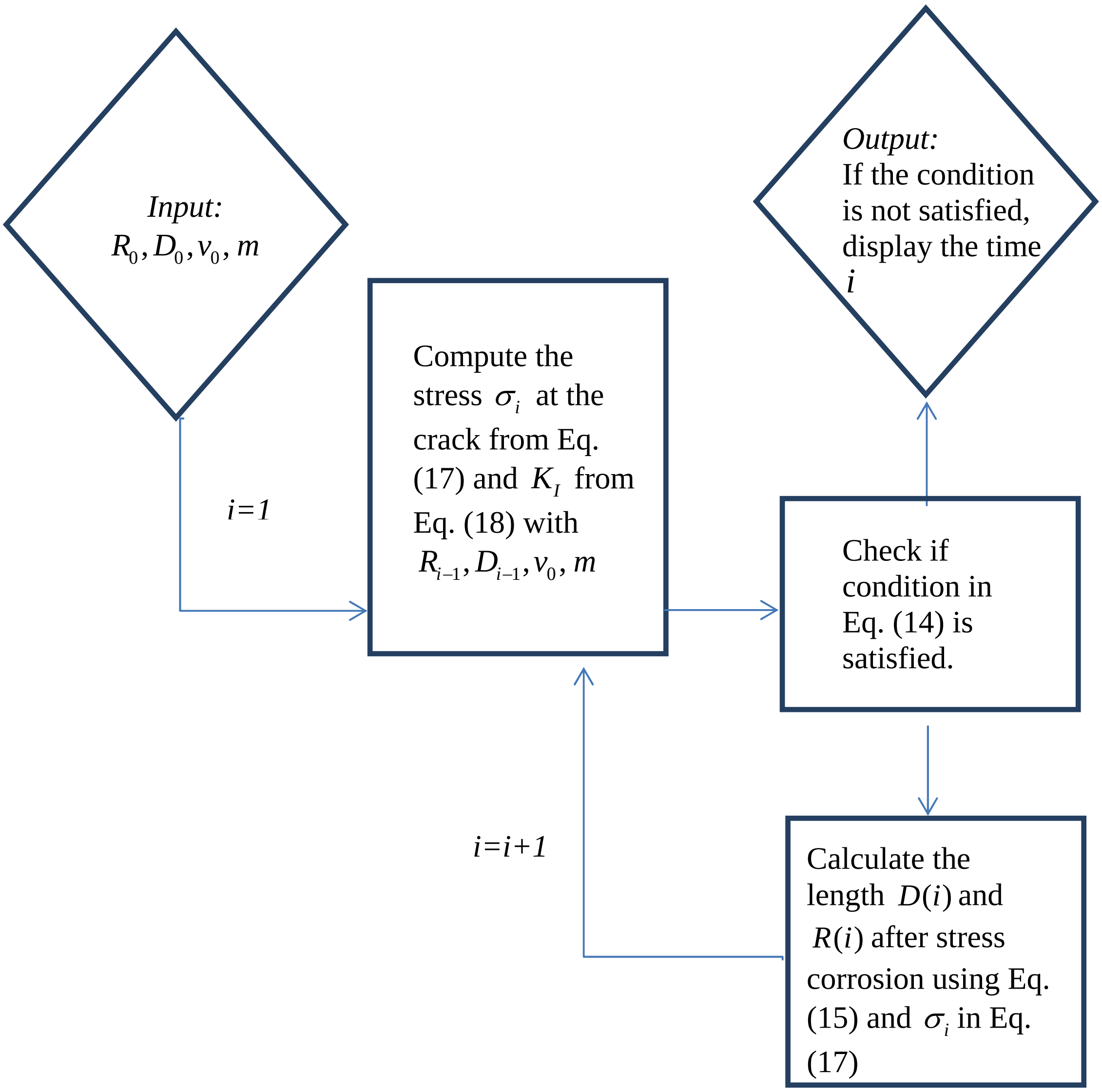

Fig. 2 Iterative algorithm for evaluation the time of service

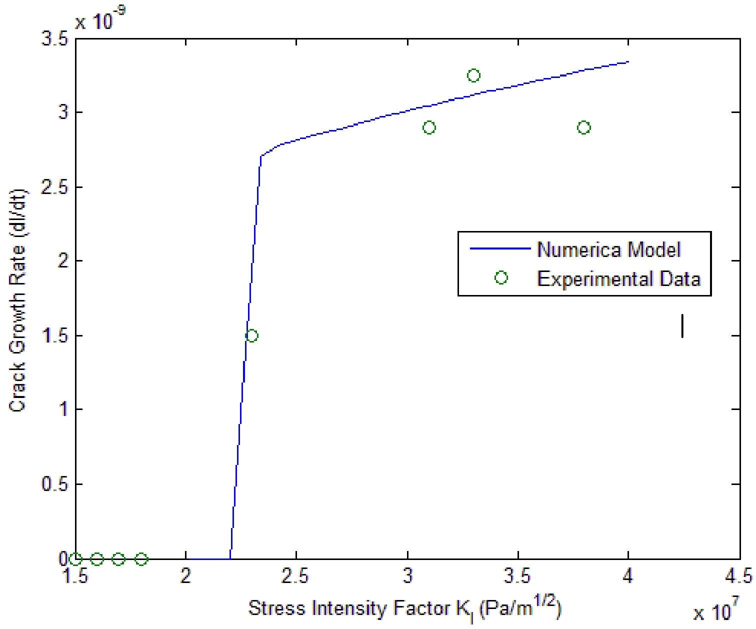

Fig. 3 Experimental corroboration of the proposed theoretical framework

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