

Nonlinear ship rolling motion subjected to noise excitation

Arada Jamnongpipatkul, Zhiyong Su* and Jeffrey M Falzarano

Department of Civil Engineering, Texas A&M University, College Station, Texas, USA

(Received April 6, 2011, Accepted September 23, 2011)

Abstract. The stochastic nonlinear dynamic behavior and probability density function of ship rolling are studied using the nonlinear dynamical systems approach and probability theory. The probability density function of the rolling response is evaluated through solving the Fokker Planck Equation using the path integral method based on a Gauss-Legendre interpolation scheme. The time-dependent probability of ship rolling restricted to within the safe domain is provided and capsizing is investigated from the probability point of view. The random differential equation of ships' rolling motion is established considering the nonlinear damping, nonlinear restoring moment, white noise and colored noise wave excitation.

Keywords: nonlinear ship rolling; noise excitation; path integration method; Fokker Planck Equation.

1. Introduction

Safety against capsizing in heavy seas is one of the major concerns of ship operators and designers. Although the data of ship capsizing is scattered, it was often reported in the media. Existing criteria consider only the ship's static stability, which is based only on the ship's nonlinear restoring moment curve. The criteria do not correspond to the complex nature of the capsize phenomenon and the large number of possible scenarios. Dynamical behavior of ships have been of interest to many researchers and engineers, particularly in the stability of roll motion. Prior to capsizing, ships will undergo severe roll motion. The eventual motion may even be chaotic. Identifying chaotic motion and the critical conditions are important for both predicting ships' capsizing and studying the capsize mechanism.

Ship rolling even in a regular sea can exhibit complicated behavior, leading to instability and eventually capsizing. Much work has been done on the analysis of vessels subjected to a periodic excitation in a simplified sea state in order for us to understand the mechanism of ship roll motion under the influence of nonlinear stiffness and nonlinear damping. Falzarano *et al.* (1992) analyzed the global stability of a ship in regular waves by the use of the Melnikov method and lobe dynamics to define the critical parameters for the onset of chaos that might lead to capsizing and explained the unexpected capsizing in both the homoclinic and heteroclinic regions. Ship rolling in the homoclinic region is the case of ship oscillation around the loll angle while ship rolling in the heteroclinic region represents large amplitude roll motion, which results in ship rolling between

*Corresponding author, Research Scientist, E-mail: chotiros@arlut.utexas.edu

positive and negative angles of vanishing stability. The Melnikov method is a useful tool to identify the chaotic motion and its critical parameters, in order to predict and study ship capsizing. The so-called chaotic phenomena appear when the behavior of a deterministic system depends sensitively upon initial conditions and then becomes long-term unpredictable.

In order to make the models more accurate and gain a better understanding of the stability of ship rolling motion, the wave excitation was later treated as regular waves perturbed by random noise (Lin and Yim 1995). The ship may experience stochastic and chaotic motion. Some initial research about the effects of noise on chaotic behavior of nonlinear systems has been conducted in ship dynamics and also other engineering fields. Lin & Yim (Lin and Yim 1995, Yim and Lin 2001) studied the stochastic chaotic motion of ship under periodic excitation with the disturbance approximated by Gaussian white noise from a probability perspective. The joint probability density function of roll angle and roll angular velocity was calculated by applying the path integral method to solve the stochastic differential equations governing ship rolling motion. Lin and Yim found that the steady-state joint probability density functions can reflect the existing chaotic attractor on the Poincaré section and also the roll response in the heteroclinic region can be related to the capsizing through the joint probability density functions. To examine the chaotic characteristic of nonlinear roll motion in an unpredictable sea state, one cannot avoid dealing with probabilistic approaches. The shape evolution of the probability density function is another way to investigate the global system behavior. Hsieh *et al.* (1994) completed a more realistic work by analyzing a single-degree-of-freedom nonlinear rolling equation in random beam sea. Following the initial work of Falzarano (Falzarano 1990, Falzarano *et al.* 1992) *et al.* on lobe dynamics, they introduced the rate of phase space flux and studied its relation to the probability of capsizing. However, no standard method is set for identifying the chaotic motion of ship in random waves.

Gaussian white noise excitation has become an important factor in these studies. The response of a dynamical system, roll angle and roll angular velocity in ship rolling motion study, under periodic excitation and Gaussian white noise can be modeled as a Markov process whose transition probability density function is governed by a partial differential equation called the Fokker-Planck Equation (Lutes and Sarkani 2004). Er and Lu (1999) proposed the exponential-polynomial closure method used for the probability density function solution of nonlinear oscillators under Gaussian white noises. Hoon and Key-Pyo (2006) calculated a capsizing rate of a ship by solving the Fokker Planck Equation analytically. The problem of ship rolling motion in random waves has been approached several times in the past and recently (Falzarano *et al.* 2010, Ibrahim and Grace 2010). It is shown that the rolling response of a ship exhibits irregular and complicated behaviors, even in the regular sea with moderate excitation amplitude. In this paper, an analysis of large amplitude nonlinear ship rolling motion in beam seas subjected to more realistic models of excitation is presented.

Considering random sea wave excitation, one must deal with probabilistic approaches when studying stochastic stability, response, and reliability of ship roll motion. The evolution of the probability density function is another way to describe the behavior of the nonlinear roll motion in random waves. The behavior of the noisy forced ship roll motion under periodic excitation with Gaussian white noise can be modeled as a Markov process.

2. Problem description

2.1 Ship rolling equation

Ship rolling motion is a nonlinear phenomenon in nature and generally coupled with other motions, such as sway, yaw, pitch and heave. However, if it is the case of a ship at low speed in unidirectional beam waves, it is reasonable to uncouple roll motion from sway motion provided that the coordinate origin is located at an appropriate ‘roll centre’(Roberts and Vasta 2000). Then, at least for the case of beam waves, the dynamics of large amplitude ship rolling can be described by a single-degree-of-freedom equation. The nonlinear differential equation of the ship rolling motion was established considering nonlinear damping and nonlinear restoring moment. The governing equation can be expressed as follows for a ship in beam wave

$$(I_{44} + A_{44}(\omega))\ddot{\varphi} + B_{44}(\omega)\dot{\varphi} + B_{44q}(\omega)\varphi'|\dot{\varphi}'| + \Delta GZ(\varphi) = F_{sea}(t) \quad (1)$$

where I_{44} is the moment of inertia of the ships about the roll axis, $A_{44}(\omega)$ is the roll hydrodynamic added mass coefficient, $B_{44}(\omega)$ is the linear radiation coefficient, $B_{44q}(\omega)$ is the quadratic viscous damping coefficient, Δ is the vessel displacement, $GZ(\varphi)$ is the nonlinear rolling restoring moment, $F_{sea}(t)$ denotes external excitation from waves and a prime denotes a derivative with respect to time t . The method of least squares was applied to fit the nonlinear damping term $\varphi'|\dot{\varphi}'|$ into a cubic polynomial form below.

$$\varphi'|\dot{\varphi}'| = a\dot{\varphi} + b\varphi'^3 \quad (2)$$

For the restoring moment $GZ(\varphi)$ is approximated by

$$GZ(\varphi) \approx C_1\varphi - C_3\varphi^3 \quad (3)$$

where $C_1 = GM$ is the linear restoring moment coefficient and C_3 is the nonlinear coefficient. When the roll angle exceeds the angle of vanishing stability, GZ becomes negative. This means that the restoring moment becomes negative and results in the loss of stability. Eq.(1) can be re rewritten in the following non dimensional form.

$$\ddot{x}(\tau) + \mu\dot{x}(\tau) + \delta\dot{x}^3(\tau) + x(\tau) - \alpha x^3(\tau) = \varepsilon f(\tau) \quad (4)$$

where μ and δ represent, respectively, the dimensionless linear and quadratic viscous damping coefficients, α denotes the strength of the nonlinearity, $\varepsilon f(\tau)$ is the excitation and differentiation with respect to time τ is denoted by an over dot. The non-dimensional terms are defined as below

$$x = \varphi, \quad \tau = \omega_n t, \quad \omega_n = \sqrt{\frac{\Delta C_1}{I_{44} + A_{44}(\omega)}}, \quad \mu = \frac{B_{44}(\omega) + aB_{44q}(\omega)}{\Delta C_1} \omega_n$$

$$\Omega = \frac{\omega}{\omega_n}, \quad \alpha = \frac{C_3}{C_1}, \quad \delta = \frac{bB_{44q}(\omega)}{I_{44} + A_{44}(\omega)}, \quad \varepsilon f(t) = \frac{F_{sea}}{\Delta C_1}$$

2.2 Wave excitation as colored noise

In this section, the excitation term on the right-hand-side of Eqs. (1) and (4) will be defined. Random excitation in the single-degree-of-freedom roll equation is defined as a narrow banded spectrum. The simple random model is to treat the wave excitation as regular wave perturbed with the disturbance approximated by Gaussian white noise (Liqin and Yougang 2007).

$$(I_{44}(\omega) + A_{44}(\omega))\ddot{\varphi} + B_{44}(\omega)\dot{\varphi} + B_{44q}(\omega)\varphi'|\varphi'| + \Delta GZ(\varphi) = F\cos(\omega t) + \sqrt{D}N(t) \quad (5)$$

where $\sqrt{D}N(t)$ is the white noise with intensity of D and $N(t) = dW(t)/dt$, where $W(t)$ is a standard Wiener process. The behavior of the ship roll motion under combined sinusoidal and Gaussian white noise excitation constitutes a Markov vector process whose transition probability density function is governed by the Fokker-Planck Equation.

Random wave can be realistic and approximated by Pierson-Moskowitz spectrum, given in terms of wave amplitude spectrum $S_{\eta\eta}(\omega)$. The one parameter Pierson-Moskowitz spectrum is given by

$$S_{\eta\eta}(\omega) = \frac{A}{\omega^5} \exp\left(-\frac{B}{\omega^4}\right) \quad (6)$$

Where $A = 0.0081g^2$, g is the gravitational acceleration, and $B = 3.11/h_{1/3}^2$ in the formula. For the spectral density function for the linear wave excitation, $S_{ff}(\omega)$ is obtained through the expression

$$S_{ff}(\omega) = S_{\eta\eta}(\omega)|F_{rolling}(\omega)|^2 \quad (7)$$

Where $S_{ff}(\omega)$: External exciting forcing spectrum;

$F_{rolling}(\omega)$: Rolling moment amplitude per unit wave height, also defined as force RAO of rolling motion;

$S_{\eta\eta}(\omega)$: Wave spectrum.

Consider filter applied to color noise, excitation in Eq. (4) can be modeled through the filter given below.

$$\ddot{f}(t) + c_1\dot{f}(t) + k_1^2f(t) = \beta_1\dot{N}(t) \quad (8)$$

where c_1 , k_1 and β_1 are constants and $N(t)$ is Gaussian white noise. Writing Eqs. (4) and (8) in the state space format as

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= \mu x_2(t) - \delta x_2^3(t) - x_1(t) + \alpha x_1^3(t) + \varepsilon x_3(t) \\ \dot{x}_3(t) &= x_4(t) + \beta_1 N(t) \\ \dot{x}_4(t) &= -c_1 x_4(t) + k_1^2 x_3(t) \end{aligned} \quad (9)$$

Where $x_1 = x$, $x_2 = \dot{x}$, $x_3 = f$, $x_4 = \dot{f}$. Now in four-dimensional space, the response process of Eq. (9) is a Markov process. Any random process is called Markov when the probability density function of the process in the future does not depend on how the process arrived at the given state.

Hence the Markov property is a generalized causality principle and a basic assumption that is made in the study of stochastic dynamical systems. The concept of the Markov operator has been applied in the classical nonlinear random vibration analysis.

The Markov process governed by Eq. (9) is written in the stochastic Itô Eq. (10) and the transition probability density function of the response satisfies Fokker-Planck Eq. (11) governing the evolution of the probability density.

$$d\vec{X}(t) = \vec{f}(\vec{X}, t)dt + \vec{g}(\vec{X}, t)d\vec{W}(t) \tag{10}$$

Where:

$$\vec{X}(t) = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}, \vec{f}(\vec{X}, t) = \begin{Bmatrix} x_2 \\ -\mu x_2 - \delta x_2^3 - x_1 + \alpha x_1^3 + \epsilon x_3 \\ x_4 \\ -c_1 x_4 - k_1^2 x_3 \end{Bmatrix}, \vec{g}(\vec{X}, t) = \begin{Bmatrix} 0 \\ 0 \\ \beta_1 \\ 0 \end{Bmatrix} \tag{11}$$

$$\frac{\partial q(\vec{x}, t | \vec{x}^{(0)}, t_0)}{\partial t} = -\frac{\partial}{\partial x_i} [f_i(\vec{x}, t)q(\vec{x}, t | \vec{x}^{(0)}, t_0)] + \frac{1}{2} \frac{\partial^2}{\partial x_i \partial x_j} [b_{ij}(\vec{x}, t)q(\vec{x}, t | \vec{x}^{(0)}, t_0)]$$

where $\vec{X}(t)$ is an 4-dimensional state vector, \vec{f} is an 4-dimensional vector function defined as the drift coefficient, \vec{g} is an 4×1 matrix function defined as the diffusion coefficient, and $\vec{W}(t)$ is an 1-dimensional Wiener vector process, with $E[d\vec{W}(t)] = 0$ and $E[d\vec{W}(t)d\vec{W}^T(t)] = \vec{I}dt$, and where \vec{I} is an 1×1 identity matrix. f_i are components of \vec{f} , b_{ij} are elements of matrix. $\vec{b} = \vec{g}\vec{g}^T$, and the symbol $|\vec{x}^{(0)}, t_0$ denotes the given condition $\vec{X}(t_0) = \vec{x}^{(0)}$, $t \leq t_0$. The Fokker Planck Equation for system (9) becomes a four dimensional partial differential equation. Solving this equation would provide an complete probabilistic characterization of the response process.

3. Path Integration Method (PIM) procedure

The transition and steady state probability density function corresponding to the Fokker-Planck Equation can be obtained through a path integral solution procedure (Naess and Moe 2008, Yim *et al.* 2008) based on the assumption that the response vector can be approximated as jointly Gaussian. If the Fokker-Planck Equation is solved for the transition probability density, then the evolution of probability density $p(\vec{x}, t)$ of $\vec{x}(t)$ can be obtained from.

$$p(\vec{x}, t) = \int_R q(\vec{x}, t | \vec{x}^{(0)}, t_0) p(\vec{x}^{(0)}, t_0) d\vec{x}^{(0)} \tag{12}$$

where R is the range of the n -dimensional state space for \vec{x} and $p(\vec{x}^{(0)}, t_0)$ is the initial probability density of $\vec{X}(t)$ at $t = t_0$. By dividing the interval $[t_0, t]$ into N sub-intervals, a long term evolution of probability density over time can be computed in a series of shorter time steps as follows (Yu *et al.* 1997).

$$\begin{aligned}
p(\vec{x}, t) = & \int_{\mathcal{R}} q(\vec{x}, t) |(\vec{x}^{(N-1)}, t_{N-1}) d\vec{x}^{(N-1)} \int_{\mathcal{R}} q(\vec{x}^{(N-1)}, t_{N-1} | \vec{x}^{(N-2)}, t_{N-2}) d\vec{x}^{(N-2)} \\
& \dots \int_{\mathcal{R}} q(\vec{x}^{(1)}, t_1 | \vec{x}^{(0)}, t_0) p(\vec{x}^{(0)}, t_0) d\vec{x}^{(0)}
\end{aligned} \tag{13}$$

The transition probability density multiplied by the probability density function of the previous time step is integrated using a Gauss-Legendre interpolation scheme. The path integration method based on the Gauss-Legendre quadrature interpolation scheme is capable of producing an accurate description of the probability density as it evolves with time, including the tail region where the probability level is very low. However, this low probability region is important for the system reliability estimation. The basis of the Gauss-Legendre interpolation scheme is equivalent to replacing the function to be integrated directly with an interpolation polynomial of a certain order. The values of the probability are obtained at the Gauss quadrature points in sub-intervals without explicit interpolation. The desired accuracy can be achieved with enough Gaussian points.

The positions of the Gauss points and their weights can be found in any standard textbooks on numerical analysis, see e.g., (Stroud 1974). To proceed to step $i+1$, only the following probability density functions at the Gauss points are required.

$$p(x_{mm}^{(i)}, t_i) = \sum_{k=1}^K \frac{\delta_k}{2} \sum_{l=1}^{L_k} c_{kl} p(x_{kl}^{(i-1)}, t_{i-1}) q(x_{mm}^{(i)}, t_i | x_{kl}^{(i-1)}, t_{i-1}) \tag{14}$$

Eq. (14) provides a scheme to calculate the evolution of the probability density function step by step, starting from a given initial probability density function, where K is the number of sub-intervals, L_k is the number of quadrature points in sub-interval k , and δ_k is the length of sub-interval k . Each x_{kl} is the position of a Gauss quadrature point, and c_{kl} is its corresponding weight. The transition probability $q(x_{mm}^{(i)}, t_i | x_{kl}^{(i-1)}, t_{i-1})$ for each Gauss point is calculated based on the moment equations (Francescutto and Naito 2004) and the Gaussian closure assumption (Lutes and Sarkani 2004).

The transition probability density can be constructed in the Gaussian form to obtain values of $q(x_{mm}^{(i)}, t_i | x_{kl}^{(i-1)}, t_{i-1})$ at the Gauss points. This transition propagator $q(x_{mm}^{(i)}, t_i | x_{kl}^{(i-1)}, t_{i-1})$ is interpreted as the transition probability density function corresponding to the state at x_{kl} time t_{i-1} passing to another state x_{mm} at time t_i . In the one-dimensional case, assume that the closed moment equations for the mean and mean square are in the following forms (Yu *et al.* 1997).

$$\begin{aligned}
\dot{m}_1 &= h_1(m_1, m_2, t) \\
\dot{m}_2 &= h_2(m_1, m_2, t)
\end{aligned} \tag{15}$$

where $m_1 = E[X]$, $m_2 = E[X^2]$ and E denotes an ensemble average. Since the state of the system at the previous time step is assumed to be known, namely, $X = x_{kl}^{(i-1)}$, $t = t_{i-1}$, Eq. (15) are solved for the following step $t_i = t_{i-1} + \Delta t_i$ with the initial conditions

$$m_1(t_{i-1}) = x_{kl}^{(i-1)}, \quad m_2(t_{i-1}) = [x_{kl}^{(i-1)}]^2 \tag{16}$$

The short time transition probability density from $x_{kl}^{(i-1)}$ at t_{i-1} to x_{mm}^i at t_i can be approximated as

$$q(x_{mm}^i, t_i | x_{kl}^{(i-1)}, t_{i-1}) = \frac{1}{\sqrt{2\pi}\sigma(t_i)} \exp\left\{-\frac{[x_{mm}^i - m_1(t_i)]^2}{2\sigma^2(t_i)}\right\} \quad (17)$$

where $\sigma^2(t_i) = m_2(t_i) - [m_1(t_i)]^2$. Using Eq. (17), the short time transition probability density $q(x_{mm}^i, t_i | x_{kl}^{(i-1)}, t_{i-1})$ at the Gauss points can be evaluated. Within a short time interval, the transition probability density has a significant value only in the neighborhood of the starting point $x_{kl}^{(i-1)}$. Therefore, for each starting point, only a few destination Gauss points need to be taken into consideration. The transition probabilities at other Gauss points may be neglected. This can be implemented by saving only the results of those destination points for which the transition probabilities cannot be ignored.

Assuming the initial probability density function obeys the Gaussian distribution; Eq. (14) provides a scheme by the path integration method to calculate the evolution of a probability density step by step, starting from a given initial probability density. The probability densities at desired time will be calculated from Eq. (14) with transition probability given by Eq. (17) and the initial probability density.

4. PIM Application to ship rolling problem

4.1 Ship rolling system

Firstly, the ship rolling motion in the unforced ($F_{sea}(t) = 0$) and undamped ($B_{44} = B_{44q} = 0$) system is considered. This system is referred to as the unperturbed and Hamiltonian system. It will be used for evaluating the effects of damping and wave excitation later.

$$(I_{44} + A_{44}(\omega))\varphi'' + \Delta(C_1\varphi - C_3\varphi^3) = 0 \quad (18)$$

Eq. (18) can be rewritten in the following non dimensional form

$$\ddot{x}(t) + x(t) - \alpha x^3(t) = 0 \quad (19)$$

The phase portrait of Eq. (19) is shown in Fig. 1. A typical ship with or without a loll angle will have positive and negative angles of vanishing stability. These angles of vanishing stability are also shown in phase portrait. They are connected to one another by a heteroclinic connection. This special curve is called a separatrix. Initial conditions inside these connections result in bounded rolling oscillations while initial conditions outside result in unbounded oscillations or so called capsizing.

The perturbed waveforms may be modeled as regular waves with Gaussian white noise as the external disturbance as in Eqs. (5) and (20). With some noise intensity, the response appears random as shown in the Poincaré map of Fig. 2,

$$\ddot{x}(t) + \mu\dot{x}(t) + \delta x^3(t) + x(t) - \alpha x^3(t) = H/2\cos\omega t + \sqrt{D}N(t) \quad (20)$$

The associated Fokker-Planck Equation governing the evolution of the probability density function of the roll motion is derived and numerically solved by the path integral method based on Gauss-Legendre interpolation to obtain the joint probability density functions in state space with parameters $\mu = 0.1321$, $\delta = 0.02656$, $\alpha = 0.9018$.

In the presence of a small random disturbance to the external period excitation, the imprint of the Poincaré map is preserved and can be identified using the joint probability density function on the Poincaré section as shown in Figs. 4, 5 and 7. The probability density function indicates the preferred locations of the trajectories in the average sense.

The contour plots of the joint probability density function and their marginal probability density functions are shown in the Figures to follow. It is seen that the value of joint probability density decreases gradually as time progresses. For example, in Fig. 5, the maximum value of joint probability density is about 0.5 when $t = 12.95$ s, and it is about 0.0011 when $t = 74.45$ s. Moreover, for a high intensity of white noise, the value of joint probability density decreases more quickly as time progresses. For example, when $t = 29.13$ s, the maximum value of joint probability density is about 0.34 in Fig. 4, it is about 0.12 in Fig. 5, and it is only about 0.002 in Fig. 7. These conclusions are further demonstrated by the marginal probability density function in Figs. 2, 6 and 8.

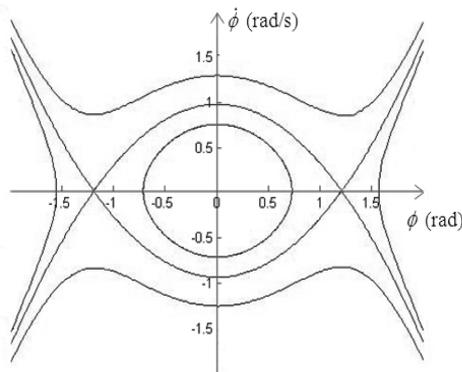


Fig. 1 Phase portrait of the unperturbed system

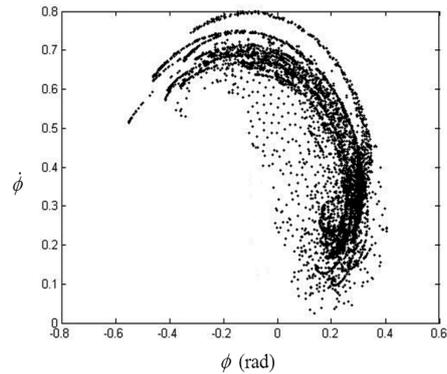


Fig. 2 The Poincaré map with noise intensity

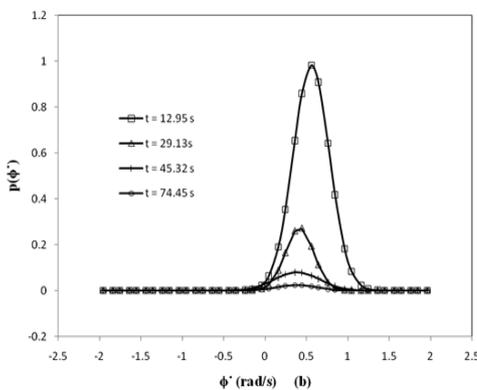
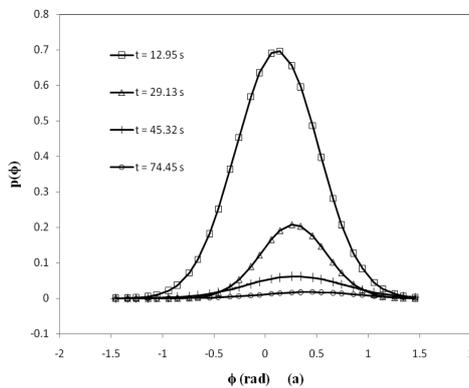


Fig. 3 Marginal probability density function with $(H, D, w) = (0.1, 0.01, 0.97)$ (a) of roll angle and (b) of roll angular velocity

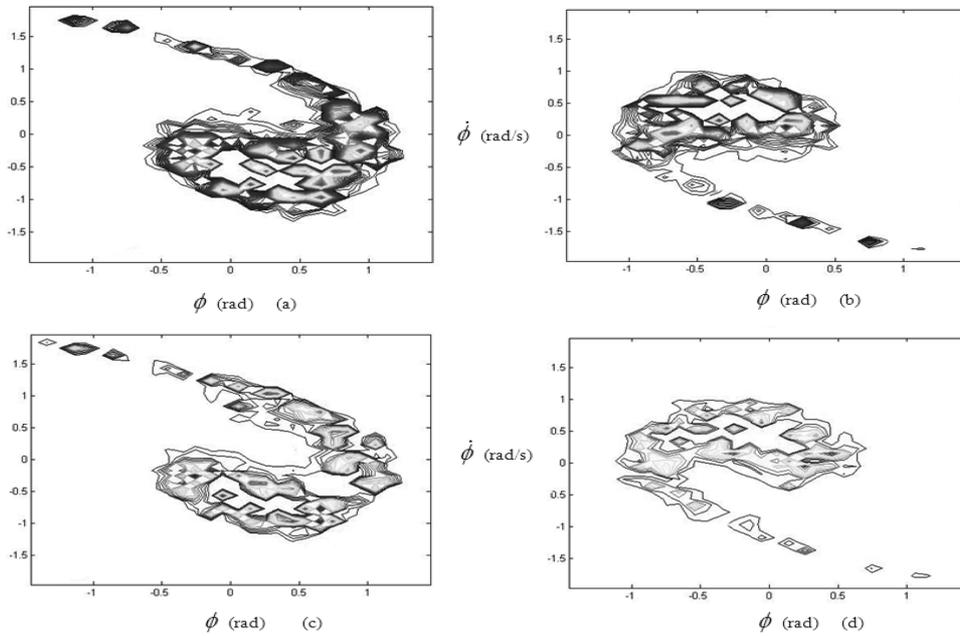


Fig. 4 Evolution of contour plot of the joint probability density function with $(H, D, \omega) = (0.3, 0.01, 0.97)$ at time (a) $t = 12.95$ s, (b) $t = 29.13$ s, (c) $t = 45.32$ s and (d) $t = 74.45$ s

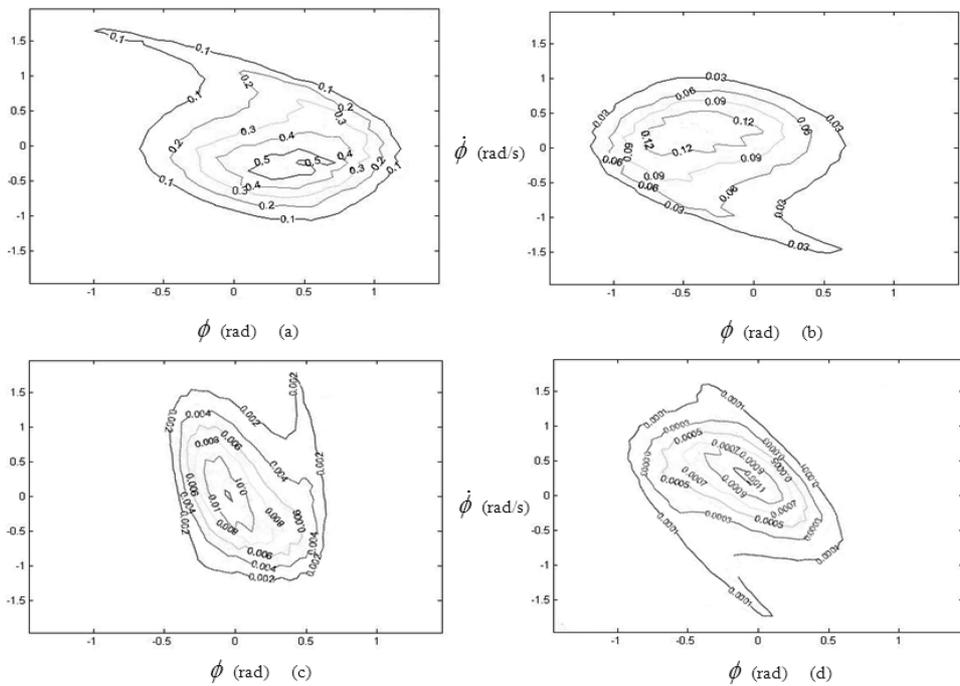


Fig. 5 Evolution of contour plot of the joint probability density function with $(H, D, \omega) = (0.3, 0.05, 0.97)$ at time (a) $t = 12.95$ s, (b) $t = 29.13$ s, (c) $t = 45.32$ s and (d) $t = 74.45$ s

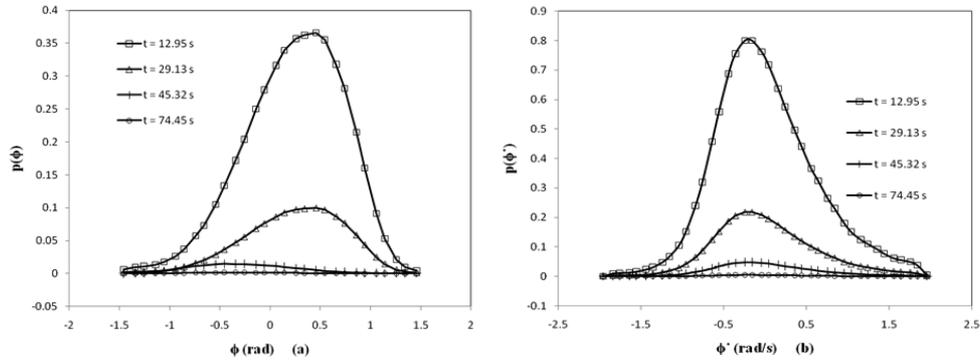


Fig. 6 Marginal probability density function with $(H, D, \omega) = (0.3, 0.05, 0.97)$ (a) of roll angle and (b) of roll angular velocity

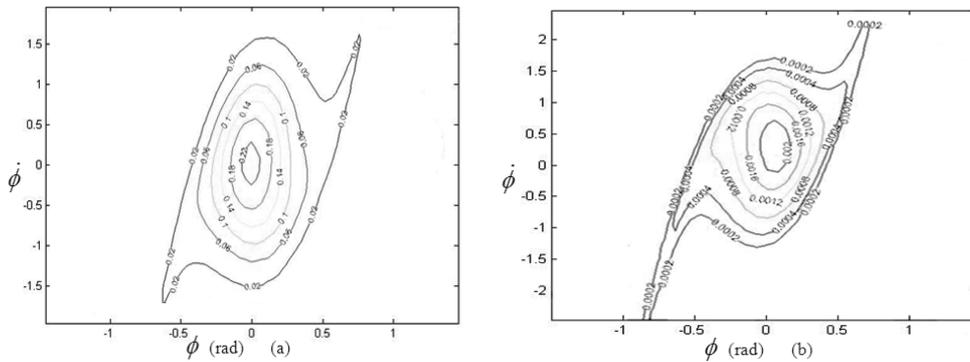


Fig. 7 Evolution of contour plot of the joint probability density function with $(H, D, \omega) = (0.3, 0.1, 0.97)$ at time (a) $t = 12.95$ s and (b) $t = 29.13$ s

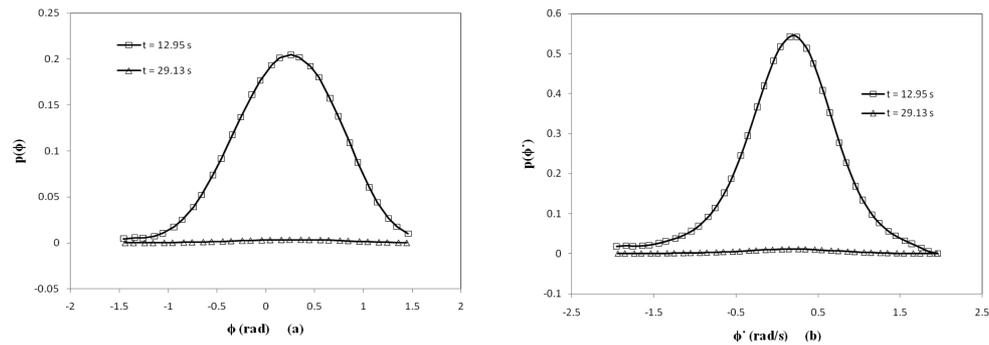


Fig. 8 Marginal probability density function with $(H, D, \omega) = (0.3, 0.1, 0.97)$ (a) of roll angle and (b) of roll angular velocity

By defining the state domain surrounded by the two heteroclinic connections of Fig. 1 as the safe domain, the probability of ship rolling restricted within the safe domain excited by above three sets of wave parameters are shown in Fig. 9.

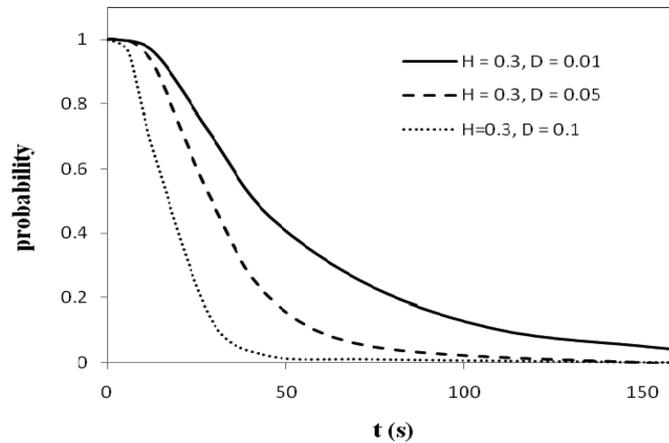


Fig. 9 Probability of the ship rolling within the safe domain when $\omega = 0.97$ rad/s

The time-dependent probability of ship rolling restricted within the safe domain is provided in Fig. 9. It is found that the probability decreases as time progresses and it decreases much more quickly for the high intensity of the white noise. At $t = 45.32$ s, the probability is about 45% when $D = 0.01$. It is about 20% when $D = 0.05$, and it is only about 2% when $D = 0.1$. The ship will finally exit the safe domain and capsize in the probability's view. To further study the qualitative behavior of the ship rolling motion and capsize in the probability space, the wave excitation is treated as colored noise. When the excitations are not white, it is quite useful to introduce the so-called filter equations. They represent the equations of motion of ideal systems excited by a white noise process input and finally give as response the particular non white excitations considered. If these equations are added to the original equations of motion, the whole system can be considered as excited by a white noise process. The output of the filter is used to drive the nonlinear system.

The primary goal is to show the possibilities of using path integration for systems where the noise is filtered, increasing the dimensionality of the problem to four. The method is based on the assumption that the input excitation can be obtained by filtering white noise processes. In this way the advantage of considering the input as a white noise process is still obtained. It is then possible to model the response of nonlinear systems subjected to white noise or filtered white noise in terms of a Markov vector process. The transitional probability density function is governed by the Fokker-Planck Equation.

Eq. (9) shows the ship rolling equation of motion with linear filtered white noise. The Poincaré map in Fig. 10 is constructed using the sum of a set of harmonic wave excitation moments as an input excitation.

As described above, the image of the Poincaré map is preserved and can be identified via the joint probability density function on the Poincaré section. The Poincaré map in Fig. 10 is used to verify the joint probability density function obtained from the colored noise method in Fig. 11.

It is shown that Fig. 11 which also follows the hypothesis concerning the image of the Poincaré map and the joint probability density function. However, the path integration method using a Gauss-Legendre interpolation scheme is not a good application for this study. The case of colored noise could not be further studied since high and low order moments' values will be of very different scales. The resulting ordinary differential equation system will be almost singular matrix. Solving

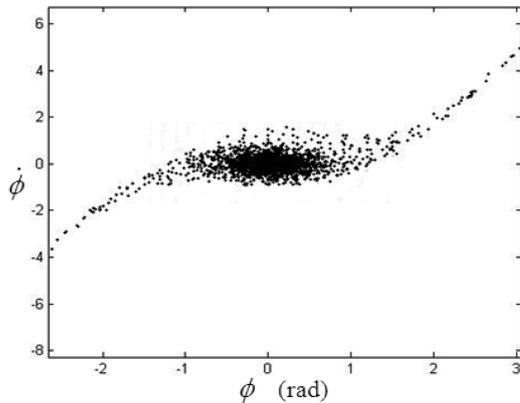


Fig. 10 The Poincaré map using the sum of a set of harmonic wave excitation

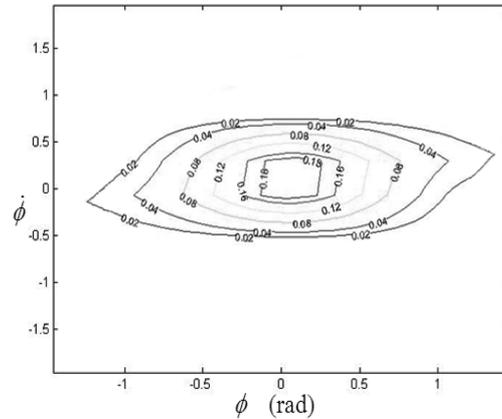


Fig. 11 Contour of joint probability density function corresponding to 6 m wave height

the ordinary differential equations numerically requires a time-stepping method that is stable over a large range of time scales. But the moment equations easily become numerically unstable as the parameter connecting lower and higher order moments are in very different scales.

5. Conclusions

This paper represents an attempt to study the qualitative behavior of the ship roll motion in the probability space. The stochastic nonlinear dynamic behaviors and the probability density function of ship rolling in the random beam waves are studied. The probability density function for the response of the system is obtained numerically for all excitation types and with varying amount of noise. We have pursued a study of the response of a nonlinear system excited by a harmonic motion with additive white noise first. The probability density function of rolling response is evaluated and the time-dependent probability of ship rolling restricted to within the safe domain is provided. It is found that the value of joint probability density decreases gradually as time progresses. And it decreases much more quickly for the high intensity of white noise. The ship will leave the safe domain for enough time and capsize in the probability's view.

A nonlinear study of ship rolling in a stochastic beam sea represented by colored noise as the Pierson-Moskowitz spectrum has been conducted. Even the path integration method based on Gauss-Legendre interpolation scheme is not a good application to ship rolling motion with high dimensionality, a preliminary result still shows the effect of colored noise to the ship rolling motion.

6. Acknowledgments

The work has been funded by the Office of Naval Research (ONR) T-Craft Tools development program ONR Grant N00014-07-1-1067 with program manager Kelly Cooper.

References

- Er, G.K. and Iu, V.P. (1999), "Probabilistic solutions to nonlinear random ship roll motion", *J. Eng. Mech.-Asce*, **125**(5), 570-574.
- Falzarano, J. (1990), *Predicting complicated dynamics leading to vessel capsizing*, Department of Naval Architecture., University of Michigan, Ph.D. dissertation. .
- Falzarano, J. and Shaw, S.W. (1992), "Application of global methods for analyzing dynamical systems to ship rolling motion and capsizing", *Int. J. Bifurcat. Chaos*, **2**(1), 101-115.
- Falzarano, J.M., Vishnubhotla, S. and Juckettan, S.E. (2010), "Combined steady state and transient analysis of a patrol vessel as affected by varying amounts of damping and periodic and random wave excitation", *J. Offshore Mech. Arct.*, **132**(1).
- Francescutto, A. and Naito, S. (2004), "Large amplitude rolling in a realistic sea", *Int. Ship. Prog.*, **51**(2), 221-235.
- Hoon, K. S. and Key-Pyo, R. (2006), "Calculation of a capsizing rate of a ship in stochastic beam seas", *Ocean Eng.*, **33**(3-4), 425-438.
- Hsieh, S.R., Troesch, A.W. and Shaw, S.W. (1994), "A nonlinear probabilistic method for predicting vessel capsizing in random beam seas", *Proceedings: Mathematical and Physical Sciences*, **446**(1926), 195-211.
- Ibrahim, R.A. and Grace, I.M. (2010), "Modeling of ship roll dynamics and its coupling with heave and pitch", *Mathematical Problems in Engineering*.
- Lin, H.A. and Yim, S.C.S. (1995), "Chaotic roll motion and capsize of ships under periodic excitation with random noise", *Appl. Ocean Res.*, **17**(3), 185-204.
- Liqin, L. and T. Yougang (2007), "Stability of ships with water on deck in random beam waves", *J. Vib. Cont.*, **13**(3), 269-280.
- Lutes, L.D. and Sarkani, S. (2004), *Random vibrations : analysis of structural and mechanical systems*, Amsterdam , Boston, Elsevier.
- Naess, A. and Moe, V. (2008), "A numerical study of the existence and stability of some chaotic attractors by path integration", *J. Vibroengineering*, **10**, 541-549.
- Roberts, J.B. and Vasta, M. (2000), "Markov modelling and stochastic identification for nonlinear ship rolling in random waves", *Phil. Trans. R. Soc. Lond. A*, **358**(1771), 1917-1941.
- Stroud, A.H. (1974), *Numerical quadrature and solution of ordinary differential equations; a textbook for a beginning course in numerical analysis*, New York, Springer-Verlag.
- Yim, S.C., Yuk, D.J., Naess, A. and Shih, I.M. (2008), "Stochastic analysis of nonlinear responses of a moored structure under narrow band excitations", *J. Offshore Mech. Arct.*, **130**(1).
- Yim, S.C.S. and Lin, H. (2001), "Unified analysis of complex nonlinear motions via densities" , *Nonlinear Dynam.*, **24**(1), 103-127.
- Yu, J.S., Cai, G.Q. and Lin, Y.K. (1997). "A new path integration procedure based on Gauss-Legendre scheme" , *Int. J. Nonlinear Mech.*, **32**(4), 759-768.