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A theoretical approach in 2d-space with applications of the periodic wave solutions in the elastic body

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Abstract. In this paper, theoretical approach with applications of the periodic wave solutions in an elastic material is applied by study the effect of initial stress, and rotation, on the radial displacement and the corresponding stresses in non-homogeneous orthotropic material. An Analytical solution for the elastodynamic equation has obtained concerning the component of displacement. The variations of stresses and displacements have shown graphically. Comparisons with previously published results in the absence of initial stress, rotation and non-homogeneity have made. Finally, numerical results have given and illustrated graphically for each case considered.

Keywords: periodic wave; initial stress; rotation; non-homogeneous; elastic body

1. Introduction

In recent years, applications in the field of the periodic wave and the employ of composite materials have experimented a great interest in aerospace, automobile, and other engineering industries (Behera and Kumari 2018, Narwariya *et al*. 2018, Fládr *et al*. 2019, Bakhshi and Taheri-Behrooz 2019, Belbachir *et al.* 2019, Abualnour *et al*. 2019, Sahla *et al.* 2019, Medani *et al.* 2019, Draoui *et al.* 2019, Ghadimi 2020, Ghannadpour and Mehrparvar 2020, Singh and Kumari 2020). Growing, attention is being devoted to elasticity due to its many engineering applications in the fields of geophysical physics, structural elements, plasma physics, and the corresponding measurement techniques of magneto-elasticity as described in the Refs. (Farhan 2017, Akbarov *et al.* 2018, Ozisik *et al.* 2018, Othman and Fekry 2018, Karami *et al*. 2019ab, Lata 2019, Karami *et al*. 2019cde, Alimirzaei *et al.* 2019, Khorasani *et al*. 2020). The interaction of electromagnetic fields with the motion of a deformable solid is receiving much attention from many researchers. Among the many essential problems considered in such studies, elastic wave propagation problems in the presence of a paper magnetic field have investigated. Considerable use was made of elastic materials, especially in aerospace industries. It is thus of considerable practical interest to investigate the elasto dynamic behavior of such materials due to the effect of suddenly applied surface pressures. Addition to aerospace industries, spherical structures may also be used in submarines, nuclear reactors, and chemical plants. The elastodynamic response of anisotropic spheres is a fundamental problem of renewed contemporary interest.

Abd-Alla *et al*. (2013) investigated an analytical solution for electrostatic potential, on wave propagation modeling in human long wet bones. They investigated effects of non-homogeneity, magnetic field and gravity field on Rayleigh waves in an initially stressed elastic half-space of orthotropic material subject to rotation, and they studied the influence of the rotation and gravity field on Stonely waves in a non-homogeneous orthotropic elastic medium. Abd-Alla and Mahmoud (2010 and 2013) investigated the magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model and the problem of radial vibrations in the non-homogeneity isotropic cylinder under the effect of initial stress and magnetic field. Mofakhami *et al*. (2006) investigated finite cylinder vibrations with different end conditions at the boundary. The hollow spheres are frequently encountered in engineering industries, and the corresponding free vibration problem has become one of the basic problems in elastodynamics. Free vibration analysis of functionally graded curved panels was carried out using a higher order formulation have been investigated by Pradyumna and Bandyopadhyay (2008). Sofiyev and Karaca (2009) investigated the vibration and buckling of laminated non-homogeneous orthotropic conical shells subjected to external pressure. Argatov (2005) investigated the approximate solution of the axisymmetric contact problem for an elastic sphere. Huang and Ho (2004) discussed the analytical solution for vibrations of a polarly orthotropic Mindlin sectorial plate with simply supported radial edges. Bahrami *et al*. (2013) investigated the wave propagation technique for free vibration analysis of annular

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circular and sectorial membranes. Towfighi and Kundu (2003) investigated elastic wave propagation in anisotropic spherical curved plates. The radially nonhomogeneous axisymmetric problem is studied by Theotokoglou and Stampouloglou (2008). Stavsky and Greenberg (2003) studied the radial vibrations of orthotropic laminated hollow spheres.

In the present paper, the equations of the elastodynamic problem for orthotropic non-homogeneous hollow sphere subject to initial stress and rotation have solved regarding displacements. An Analytical solution for the elastodynamic equations has obtained in detail for different cases. The results indicate that the periodic wave solutions in elastic material under the effect of initial stress, rotation, and non-homogeneity on radial displacement, the corresponding stresses are played a significant role in engineering, science, and pure and applied mathematics.

2. Formulation of the problem

Let us consider a system of orthogonal spherical coordinates axes and let a hollow sphere, for a spherically orthotropic elastic medium under the effect of initial stress; the spherical coordinates (r,θ,φ) are helpful with r radial θ co-latitudinal and φ meridional. The basic equations of the spherical orthotropic are given by:

$$\tau_{rr} = (c_{11}+P)\frac{\partial U_r}{\partial r} + (c_{12}+P)\frac{U_r}{r} + (c_{13}+P)\frac{U_r}{r}, \quad (1)$$

$$\tau_{\theta\theta} = (c_{12}+P)\frac{\partial U_r}{\partial r} + (c_{22}+P)\frac{U_r}{r} + (c_{23}+P)\frac{U_r}{r}, \quad (2)$$

$$\tau_{\varphi\varphi} = c_{13}\frac{\partial U_r}{\partial r} + c_{23}\frac{U_r}{r} + c_{33}\frac{U_r}{r}, \quad (3)$$

$$\tau_{r\varphi} = \tau_{r\theta} = \tau_{\theta\varphi} = 0. \quad (4)$$

where the radial displacement $U_r = U_r(r,t)$ is a function of r and t only, the circumferential displacement U_θ moreover, the longitudinal displacement U_φ, which are independent of θ *and* φ.

The dynamical equation in the r direction is given by:

$$\frac{\partial\tau_{rr}}{\partial r} + \frac{2}{r}\tau_{rr} - \frac{1}{r}\tau_{\theta\theta} - \frac{1}{r}\tau_{\varphi\varphi} + \rho\left(\overleftarrow{\Omega}\times\overleftarrow{\Omega}\times\overleftarrow{U}\right)_r = \rho\frac{\partial^2 U_r}{\partial t^2}, \quad (5)$$

where $\overleftarrow{\Omega} = (0,0,\Omega)$, $\left(\overleftarrow{\Omega}\times\overleftarrow{\Omega}\times\overleftarrow{U}\right)_r$ is a component of the centripetal acceleration in the radial direction ($\overleftarrow{r}$), due to the time-varying motion only, and ρ is the density of the material of the sphere and Ω is the rotation, and $\overleftarrow{U} = (U_r(r,t),0,0)$ is the displacement vector.

Substituting from equations (1-4) into equations (5), we get:

$$\frac{\partial\tau_{rr}}{\partial r} + \frac{2}{r}\tau_{rr} - \frac{1}{r}\tau_{\theta\theta} - \frac{1}{r}\tau_{\varphi\varphi} + \rho\Omega^2 U_r = \rho\frac{\partial^2 U_r}{\partial t^2}, \quad (6)$$

We characterize the elastic constants c_{ij} moreover, the density $\rho$of non-homogeneous material in the form

$$c_{ij} = \alpha_{ij}r^{2m}, \rho = \rho_0 r^{2m}, P = p^* r^{2m}, \quad i,j = 1,2,3, \quad (7)$$

where α_{ij} and ρ_0 are the values of c_{ij} and ρ in the homogeneous case, respectively, and m is the non-homogeneous parameter.

$$\tau_{rr} = r^{(-1+2m)}\left[(2p^* + \alpha_{12} + \alpha_{13})U_r + r(p^* + \alpha_{11})\frac{\partial U_r}{\partial r}\right] \quad (8)$$

$$\tau_{\theta\theta} = r^{(-1+2m)}\left[(2p^* + \alpha_{22} + \alpha_{23})U_r + r(p^* + \alpha_{12})\frac{\partial U_r}{\partial r}\right], \quad (9)$$

$$\tau_{\varphi\varphi} = r^{(-1+2m)}\left[(2p^* + \alpha_{23} + \alpha_{33})U_r + r(p^* + \alpha_{13})\frac{\partial U_r}{\partial r}\right]. \quad (10)$$

Substituting from equations (7-10) into equation (6), then we obtain:

$$r^{-1+m}\left[[(-2+4m)p^* + \alpha_{12} + 2m\alpha_{12} + \alpha_{13} + 2m\alpha_{13} - \alpha_{22} - 2\alpha_{23} - \alpha_{33}]U_r + r(ru_1\rho_0\Omega^2 - r\rho_0\frac{\partial^2 U_r}{\partial t^2} + 2[(1+m)(p^* + \alpha_{11}) + H_0^2\mu_0]\frac{\partial U_r}{\partial r} + r(p^* + \alpha_{11})\frac{\partial^2 U_r}{\partial r^2})\right] = 0. \quad (11)$$

In the next part, we study the analytical solution for radial vibration of an elastic spherical body of non-homogeneous orthotropic material subject to rotation

3. Solution of the problem

In this section, one obtains the analytical solution of the problem for a spherical region of inner radius $a$and outer radius b with different boundary conditions, by taking the harmonic vibrations. We assume the solution of equation (11) as the following form:

$$U_r(r,t) = u_1(r)e^{-i\omega t}, \quad (12)$$

where ω is the natural frequency of the vibrations, t is the time.

Substituting from equation (12) into equation (11), one gets:

$$e^{-it\omega}r^{-1+m}\left[[(-2+4m)p^* + \alpha_{12} + 2m\alpha_{12} + \alpha_{13} + 2m\alpha_{13} - \alpha_{22} - 2\alpha_{23} - \alpha_{33} + r^2\rho_0(\omega^2 + \Omega^2)]u_1(r) + [2r(1+m)(p^* + \alpha_{11})]\frac{du_1}{dr} + r(p^* + \alpha_{11})\frac{d^2u_1}{dr^2})\right] = 0, \quad (13)$$

where $u_1(r)$ is given in terms of m due to non-homogeneity of material. Then one has

$$r^{-1+m}\left[\alpha_1 r^2\rho_0(\omega^2+\Omega^2)u_1(r) + r\left(\alpha_2\frac{du_1}{dr}+r\alpha_3\frac{d^2u_1}{dr^2}\right)\right]=0, \tag{14}$$

where

$$\begin{aligned}\alpha_1 &= (-2+4\,\mathrm{m})p^*+\alpha_{12}+2\,\mathrm{m}\,\alpha_{12}\\ &+\alpha_{13}+2\,\mathrm{m}\,\alpha_{13}-\alpha_{22},\\ \alpha_2 &= 2(1+\mathrm{m})(p^*+\alpha_{11}),\\ \alpha_3 &= (p^*+\alpha_{11}),\end{aligned} \tag{15}$$

The equation (14) is called spherical Bessel's equation (Polyanin and Zaitsev 2003) and which its general solution is known in the form:

$$\mathrm{u}_1 = r^{\frac{1}{2}-\frac{\alpha_2}{2(p^*+\alpha_{11})}}\left[C_1\mathrm{J}_{\frac{1}{2}+\mathrm{n}}\left(\frac{\sqrt{\alpha_1}\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}}{\sqrt{p^*+\alpha_{11}}}r\right)+C_2\,\mathrm{Y}_{\frac{1}{2}+\mathrm{n}}\left(\frac{\sqrt{\alpha_1}\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}}{\sqrt{p^*+\alpha_{11}}}r\right)\right]. \tag{16}$$

Substituting from an above equation (16) into equation (12), one obtains the components of the displacements:

$$U_r = r^{\frac{1}{2}-\frac{\alpha_2}{2(p^*+\alpha_{11})}}\left[C_1\mathrm{J}_{\frac{1}{2}+\mathrm{n}}\left(\frac{\sqrt{\alpha_1}\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}}{\sqrt{p^*+\alpha_{11}}}r\right)+C_2\,\mathrm{Y}_{\frac{1}{2}+\mathrm{n}}\left(\frac{\sqrt{\alpha_1}\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}}{\sqrt{p^*+\alpha_{11}}}r\right)\right]e^{-it\omega}, \tag{17}$$

From equation (13), one gets strain components in the form:

$$e_{rr} = \sqrt{\alpha_1}r^{\frac{-1}{2}-\mathrm{n}}\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}r^{\frac{1}{2}-\mathrm{n}}\frac{\left[C_1\mathrm{J}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)+C_2\,\mathrm{Y}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)\right]e^{-it\omega}}{\sqrt{p^*+\alpha_{11}}}, \tag{18}$$

$$e_{\theta\theta} = r^{\frac{-1}{2}-\mathrm{n}}\left[C_1\mathrm{J}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)+C_2\,\mathrm{Y}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)\right]e^{-it\omega}, \tag{19}$$

$$e_{\varphi\varphi} = r^{\frac{-1}{2}-\mathrm{n}}\left[C_1\mathrm{J}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)+C_2\,\mathrm{Y}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)\right]e^{-it\omega}. \tag{20}$$

Substituting from an above equation (17) into equation (8-10), one obtains the stresses:

$$\tau_{rr} = \frac{1}{\sqrt{p^*+\alpha_{11}}}r^{-\frac{1}{2}+2m-\mathrm{n}}\left[\begin{array}{l}\sqrt{\alpha_1}r(p^*+\alpha_{11})\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}\mathrm{J}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)C_1+(2p^*+\alpha_{12}+\alpha_{13})\sqrt{p^*+\alpha_{11}}\mathrm{J}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)C_1\\ +C_2\sqrt{\alpha_1}r(p^*+\alpha_{11})\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}\mathrm{Y}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)+C_2(2p^*+\alpha_{12}+\alpha_{13})\sqrt{p^*+\alpha_{11}}\mathrm{Y}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)\end{array}\right]e^{-it\omega}, \tag{21}$$

$$\tau_{\theta\theta} = \frac{1}{\sqrt{p^*+\alpha_{11}}}r^{-\frac{1}{2}+2m-\mathrm{n}}\Big[\sqrt{\alpha_1}r(p^*+\alpha_{12})\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}\mathrm{J}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)C_1+(2p^*+\alpha_{22}+\alpha_{23})\sqrt{p^*+\alpha_{11}}\mathrm{J}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)C_1+C_2\sqrt{\alpha_1}r(p^*+\alpha_{12})\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}\mathrm{Y}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)+C_2(2p^*+\alpha_{22}+\alpha_{23})\sqrt{p^*+\alpha_{11}}\mathrm{Y}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)\Big]e^{-it\omega}, \tag{22}$$

$$\tau_{\varphi\varphi} = \frac{1}{\sqrt{p^*+\alpha_{11}}}r^{-\frac{1}{2}+2m-\mathrm{n}}\Big[\sqrt{\alpha_1}r(p^*+\alpha_{13})\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}\mathrm{J}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)C_1+(2p^*+\alpha_{23}+\alpha_{33})\sqrt{p^*+\alpha_{11}}\mathrm{J}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)C_1+C_2\sqrt{\alpha_1}r(p^*+\alpha_{13})\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}\mathrm{Y}_{-\frac{1}{2}+\mathrm{n}}(k_2\,r)+C_2(2p^*+\alpha_{23}+\alpha_{33})\sqrt{p^*+\alpha_{11}}\mathrm{Y}_{\frac{1}{2}+\mathrm{n}}(k_2\,r)\Big]e^{-it\omega}. \tag{23}$$

where

$$n = \frac{\alpha_2}{2(p^*+\alpha_{11})}, \qquad k_2 = \frac{\sqrt{\alpha_1}\sqrt{\rho_0}\sqrt{\omega^2+\Omega^2}}{\sqrt{p^*+\alpha_{11}}}, \tag{24}$$

where C_1, C_2 are arbitrary constants and $j_n(k_2r)$ and $y_n(k_2r)$ denote to spherical Bessel's function of the first and second kind of order n, respectively, which are defined in terms of Bessel's function as follows:

$$j_n(k_2r) = \sqrt{\frac{\pi}{2k_2r}}J_{n+\frac{1}{2}}(k_2r),\ y_n(k_2r) = \sqrt{\frac{\pi}{2k_2r}}Y_{n+\frac{1}{2}}(k_2r)\ ,\ k_2\ \ is\ constant.$$

It to find the necessary condition for this problem, we can determine the constants from the boundary conditions:

$$U_r(r,t) = 0, \qquad at \qquad r = a, \tag{25}$$

$$\tau_{rr}(r,t)+\sigma_{rr}(r,t) = -pe^{-i\omega t}, \qquad at \quad r = b, \tag{26}$$

where p is a constant, then from equation (16), (17) and (20) we have:

$$C_1 = \frac{pb^{-m}\sqrt{p^*+\alpha_{11}}Y_{n+\frac{1}{2}}(k_2a)}{J_{n+\frac{1}{2}}(k_2a)\left(Y_{n-\frac{1}{2}}(k_2b)+\left(\frac{c_1-n+m}{b}\right)Y_{n+\frac{1}{2}}(k_2b)\right)-Y_{n+\frac{1}{2}}(k_2b)d_9},$$

$$C_2 = \frac{-pb^{-m}(p^*+\alpha_{11})J_{n-\frac{1}{2}}(k_2a)}{J_{n+\frac{1}{2}}(k_2a)\left(Y_{n-\frac{1}{2}}(k_2b)+\left(\frac{c_1-n+m}{b}\right)Y_{n+\frac{1}{2}}(k_2b)\right)-Y_{n+\frac{1}{2}}(k_2b)d_{10}}.$$

$$d_9 = \left(J_{n-\frac{1}{2}}(k_2 b) + \left(\frac{c_1 - n - m}{b}\right) J_{n+\frac{1}{2}}(k_2 b)\right)$$

$$d_{10} = \left(J_{n-\frac{1}{2}}(k_2 b) + \left(\frac{c_1 - n - m}{b}\right) J_{n+\frac{1}{2}}(k_2 b)\right)$$

Substituting from those constants into the above displacements components and stresses then for a radial non-homogenous material, .one have the corresponding the radial displacement, radial stress

4. Discussion and numerical results

Several examples are presented to show the accuracy, and the numerical results have been obtained graphically to display the distribution of displacements, stresses through the radial direction of the inhomogeneous orthotropic hollow sphere. The elastic constants may be obtained from (Lekhnitskii 1981, Steven Chapra 2004) may be taken as an example: $\alpha_{11} = 0.134$, $\alpha_{12} = 0.101$, $\alpha_{13} = 0.099$, $\alpha_{22} = 0.674$, $\alpha_{23} = 0.151$, $\alpha_{33} = 0.297$. With the above values of the elastic constants and *b = 3 cm, a = 1 cm*.

Numerical calculations have carried out for the displacement and the stress components along the r-direction at different values of the rotation in the cases for non-homogeneous material, orthotropic material.

Figs.1-6 presents the variation of displacement, radial stress, and hoop stress along the radial direction of the non-homogeneous hollow sphere with different values of the non-homogeneity exponent m. It is seen easily from all Figures that the radial displacement satisfy the mechanical boundary conditions.

Figs. 1 and 2 show the variation of radial displacement, with increasing *r*, in case of non-homogeneity *m = 0.5*, at different values for rotation Ω *=0.3, 0.8, 1.3, 1.8* as in Fig.1, and for different values of initial stress $p^* = 0.4, 0.8, 1.2, 1.6$ as in Fig.2. The radial displacement satisfied the boundary conditions in all Figs.1-2 in the case an orthotropic nonhomogeneous hollow sphere.

Figs.3-4 present the variation of radial stress versus the radius r, in case of non-homogeneity *m = 0.5* at different values for rotation Ω *=0.3, 0.8, 1.3, 1.8* as in Fig.3, and for different values of initial stress p^*As in Fig.4.

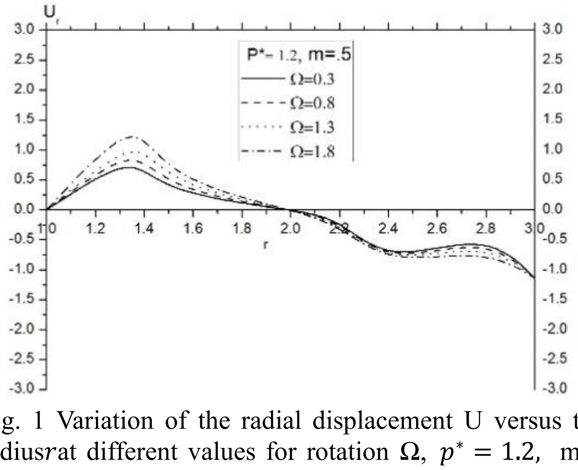

Fig. 1 Variation of the radial displacement U versus the radius$r$at different values for rotation Ω, $p^* = 1.2$, m = 0.5

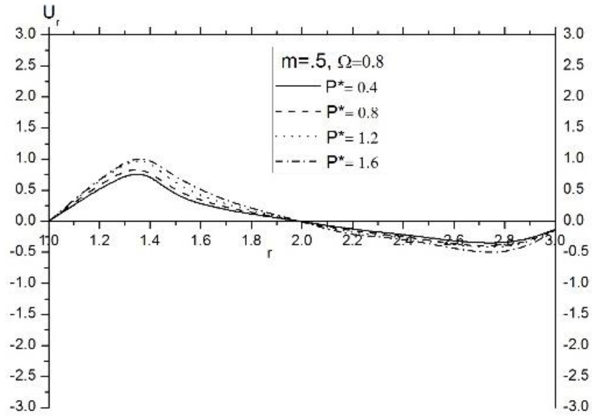

Fig. 2 Variation of the radial displacement U versus the radius r at different values for initial stress p^*, Ω =0.8, m = 0.5

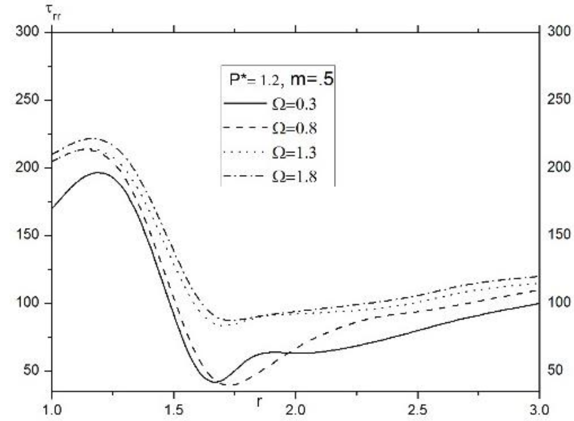

Fig. 3 Variation of the radial stressτ_{rr}versus the radius $r$at different values for rotation Ω, $p^* = 1.2$, m = 0.5

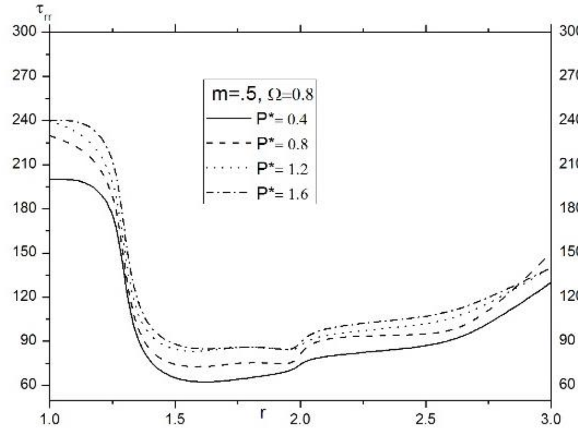

Fig. 4 Variation of the radial stressτ_{rr}versus the radius r at different values for initial stress p^* , Ω =0.8, m = 0.5

Figs. 5-6 show the variation of hoop stress versus the radius r, in case of non-homogeneity $m = 0.5$, at different values for rotation Ω *=0.3, 0.8, 1.3, 1.8* as in Fig. 5 and different values of initial stress p^* as in Fig.6. It is evident that the initial stress and rotation have a

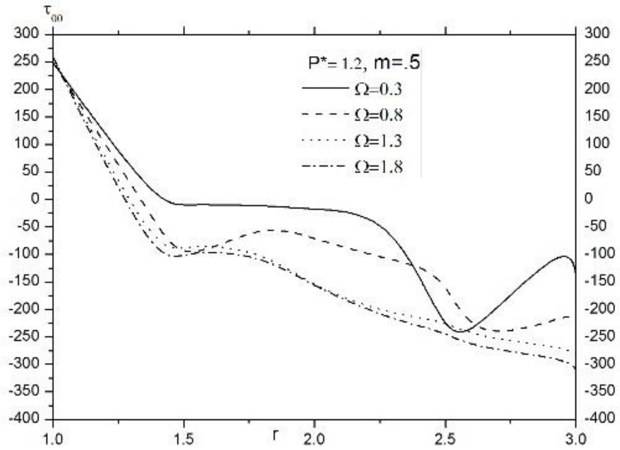

Fig. 5 Variation of the hoop stress $\tau_{\theta\theta}$ versus the radius $r$at different values for rotation $\Omega, p^* = 1.2,$ m = 0.5

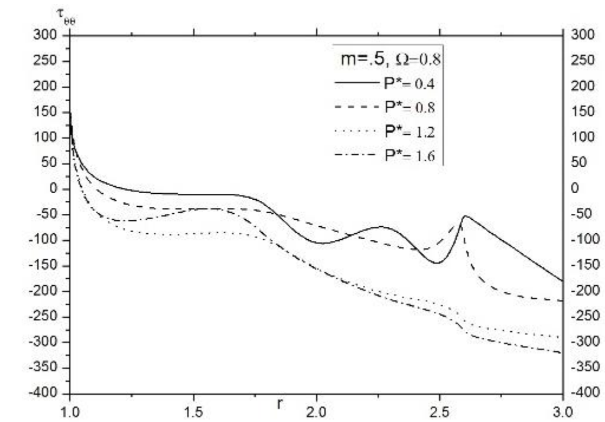

Fig. 6 Variation of the hoop stress $\tau_{\theta\theta}$versus the radius r at different values for initial stress p^*, Ω =0.8, m = 0.5

significant influence more than the influence of the initial stress on displacement, stresses. Also, the influence of initial stress, rotation, and the non-homogeneity on radial displacement, stresses is very pronounced.

These results are specific for the example considered, but other examples may have different trends because of the dependence of the results on the mechanical of the material. Also the influence of the non-homogeneity and orthotropic properties of the material is pronounced. These results are specific for the example considered; one more cases may have different trends because of the dependence of the results on the mechanical properties of the material as is displayed in Refs. (Lal *et al*. 2017, Panjehpour *et al*. 2018, Berghouti *et al*. 2019, Semmah *et al*. 2019, Ahmed *et al*. 2019, Avcar 2019, Bourada *et al*. 2019, Fenjan *et al*. 2019,Al-Maliki *et al*. 2019, Batou *et al*. 2019, Chaabane *et al*. 2019, Bedia *et al*. 2019, Barati 2019, Selmi 2019, Gupta and Anandkumar 2019, Salah *et al*. 2019, Zaoui *et al*. 2019, Nikkhoo *et al*. 2019, Kossakowski and Uzarska 2019, Hussain *et al*. 2019 and 2020ab, Taj *et al*. 2020, Timesli 2020, Kim *et al*. 2020, Boussoula *et al*. 2020, Boukhlif *et al*. 2020, Shariati *et al*. 2020ab, Tounsi *et al*. 2020, Asghar *et al*. 2020, Al-Maliki *et al*. 2020, Shokrieh and Kondori 2020) that have more applications in scientific and technical disciplines and materials science. The results in this paper compared with previous results, in the absence of initial stress, rotation, and non-homogeneity, the results coincide with the results have been obtained.

5. Conclusion

In this work, the present technique applies to applications of the periodic wave and other homogeneous material. The numerical results have obtained and represented graphically. The results indicate that the effect of initial stress, rotation, and non-homogeneity on radial displacement, stresses are pronounced. The precise solutions for non-homogeneity elastic media to get the radial displacement, stresses in orthotropic hollow sphere subjected to initial stress, rotation have obtained. The distribution of displacement, stress are drawn and discussed in detail for various effects.

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