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Dynamic analysis of rigid roadway pavement under moving traffic loads with variable velocity

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Abstract. The study of rigid roadway pavement under dynamic traffic loads with variable velocity is investigated in this paper. Rigid roadway pavement is modeled as a rectangular damped orthotropic plate supported by elastic Pasternak foundation. The boundary supports of the plate are the steel dowels and tie bars which provide elastic vertical support and rotational restraint. The natural frequencies of the system and the mode shapes are solved using two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, known as the Modified Bolotin Method. The dynamic moving traffic load is expressed as a concentrated load of harmonically varying magnitude, moving straight along the plate with a variable velocity. The dynamic response of the plate is obtained on the basis of orthogonality properties of eigenfunctions. Numerical example results show that the velocity and the angular frequency of the loads affected the maximum dynamic deflection of the rigid roadway pavement. It is also shown that a critical speed of the load exists. If the moving traffic load travels at critical speed, the rectangular plate becomes infinite in amplitude.

Keywords: rigid roadway pavement; elastic foundation; auxiliary Levy's type problems; modified bolotin method; dynamic moving traffic load; critical speed.

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1. Introduction

Several plate elements used in civil engineering, aerospace and marine structures are supported by elastic or viscoelastic media and subjected to transverse dynamic loads. The usual approach in formulating these problems is based on the inclusion of the foundation reaction into the corresponding differential equation of the plate. The foundation is very often a complex medium, but since of interest here is the dynamic response of the rigid roadway pavement subjected to the dynamic loads, the problem reduces to finding a relatively simple mathematical expression, which could describe the response of the foundation at the contact area.

Static and free vibration analyses of plates on an elastic foundation have received considerable attention in the literature by researchers (Saha 1997, Matsunaga 2000). Models, such as beams or plates on elastic and viscoelastic foundations under moving loads are widely adopted (Gbadeyan and Oni 1992, Kim and Roesset 1998). Sun (2006) investigated an infinite plate resting on an

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elastic Winkler foundation subjected to a moving concentrated and line load of constant amplitude and speed using a Triple Fourier Transform (TFT). Kang and Zhang (2007) studied the dynamic response of rigid roadway pavement under vechicle load by using a special function. The vechicle loads are considered as impact loads and the dynamic response of rigid pavement was analyzed by mathematical method and mechanical method according to the initial condition and boundary condition. Similarly, the dynamic response of rigid pavement sitting on elastics foundation subjected to moving loads has been studied extensively (Alisjahbana and Wangsadinata 2007, 2008, Cao *et al.* 2008, Gong 2008, Beskou and Theodorakopoulos 2011).

In this paper, the dynamic responses of rigid roadway pavements subjected to dynamic traffic loads with variable velocity are discussed. The rigid roadway pavement is modeled as a rectangular damped orthotropic plate resting on a Pasternak foundation. The Pasternak foundation is a more advanced model than the Winkler foundation. The boundary supports of the plate are the steel dowels and tie bars which provide elastic vertical support and rotational restraint. For this boundary supports of the plate, the wave numbers are presented in the form $p\pi/a$ and $q\pi/b$, where p and q are real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method (Pevzner 2000). In the application of the theory of dynamic response of the orthotropic plate, the continuous elastic foundation modeled as a Pasternak foundation is representing closely the actual subsoil condition, but requiring advanced analytical treatment in solving the dynamic response problem. A Pasternak foundation model incorporates shear interaction between spring elements, mobilized through a plate placed on top of the springs, which deforms only by transverse shear. Thus, in this model compressive and shear deformation of the soil are duly simulated. The homogeneous solution of the problem is obtained by a method of separation of variables, in such a way that superposition yields a solution satisfying the boundary conditions. As the mode shapes are expressed as products of eigenfunctions, the solution of the dynamic problem is obtained on the basis of orthogonality properties of eigenfunctions. The general solution of the response of the plate to the dynamic moving load in integral form is obtained from the specific properties of the Dirac-delta function, so that it can be further integrated to obtain the various plate response equations during the time interval the load is moving within the plate boundaries, as well as after the load has left the plate.

2. The governing equations

In this research work, an orthotropic homogeneous elastic rectangular plate resting on an elastic Pasternak foundation is considered. According to the classic theory of thin plates, the transverse deflection $w(x, y, t)$ of the plate satisfies the partial differential equation

$$D_x\frac{\partial^4 w}{\partial x^4} + 2B\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 w(x,y,t)}{\partial y^4} + \rho h\frac{\partial^2 w(x,y,t)}{\partial t^2} + \gamma h\frac{\partial w(x,y,t)}{\partial t} + k_f w(x,y,t)$$
$$-G_s\nabla^2 w(x,y,t) = p(x,y,t) \tag{1}$$

in which $w(x, y, t)$ is the transverse deflection; ρ is the plate mass density per unit volume; h is plate thickness; t is the time; γ is the damping ratio; k_f is the spring stiffness and G_s is the shear

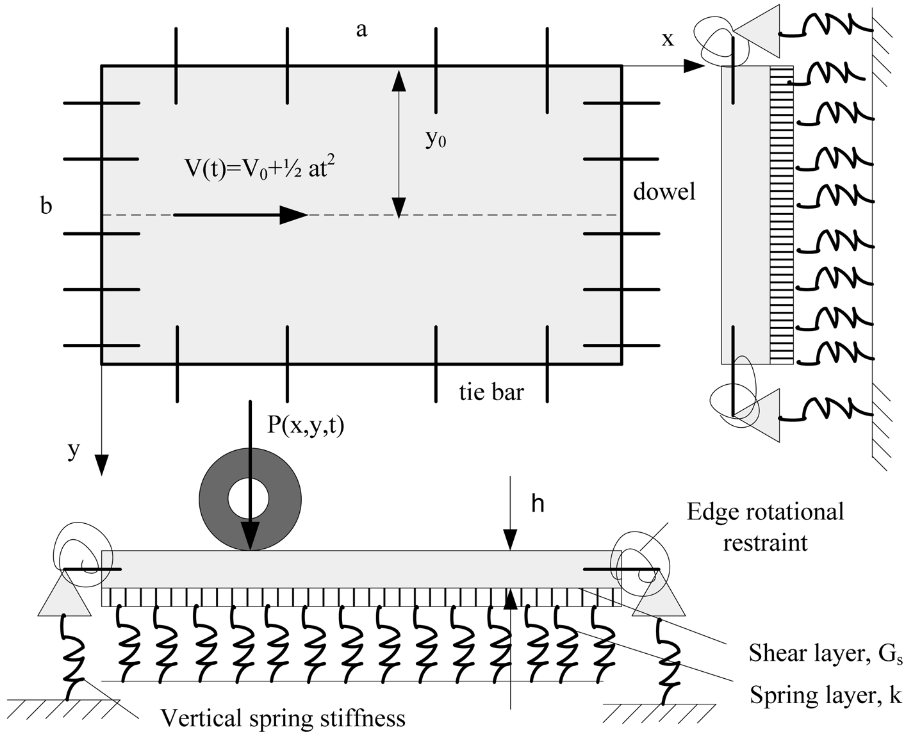

Fig. 1 A rectangular rigid concrete plate resting on a Pasternak foundation under dynamic traffic loading

modulus of the Pasternak foundation; $p\,(x, y, t)$ is the dynamic load on the plate; D_x, D_y are respectively the plate flexural rigidities in the x and y direction, B is the effective torsional rigidity.

The dynamic moving traffic load $p\,(x, y, t)$ modeled as an equivalent concentrated load of harmonically varying magnitude moving in the direction of the x axis of the plate as shown in Fig. 1 can be expressed as follows

$$p(x, y, t) = p[x(t), y(t), t] = P(t)\delta[x - x(t)]\delta[y - y(t)] = P_0\left(1 + \frac{1}{2}\cos\omega t\right)\delta[x - x(t)]\delta[y - y(t)] \tag{2}$$

$$x(t) = v_0 t + \frac{1}{2}acc(t^2); \qquad y(t) = \frac{1}{2}b \tag{3}$$

Due to the use of dowels and tie bars to join the concrete pavement plates, all four sides of the plate have elastic vertical translational support as well as elastic rotational restraint along the sides. Thus, the boundary conditions for each side of the plate are as follows

Elastic vertical support along $x = 0$

$$V_{x=0} = D_x\left[\left(\frac{\partial^3 w(x, y, t)}{\partial x^3}\right) + \left(\frac{B + 2G_{xy}}{D_x}\right)\left(\frac{\partial^3 w(x, y, t)}{\partial x \partial y^2}\right)\right] = ks_{x1}w(x, y, t) \tag{4}$$

Elastic vertical support along $x = a$

$$V_{x=a} = D_x\left[\left(\frac{\partial^3 w(x,y,t)}{\partial x^3}\right) + \left(\frac{B+2G_{xy}}{D_x}\right)\left(\frac{\partial^3 w(x,y,t)}{\partial x \partial y^2}\right)\right] = ks_{x2}w(x,y,t) \tag{5}$$

Elastic vertical support along $y = 0$

$$V_{y=0} = D_y\left[\left(\frac{\partial^3 w(x,y,t)}{\partial y^3}\right) + \left(\frac{B+2G_{xy}}{D_y}\right)\left(\frac{\partial^3 w(x,y,t)}{\partial y \partial x^2}\right)\right] = ks_{y1}w(x,y,t) \tag{6}$$

Elastic vertical support along $y = b$

$$V_{y=b} = D_y\left[\left(\frac{\partial^3 w(x,y,t)}{\partial y^3}\right) + \left(\frac{B+2G_{xy}}{D_y}\right)\left(\frac{\partial^3 w(x,y,t)}{\partial y \partial x^2}\right)\right] = ks_{y2}w(x,y,t) \tag{7}$$

3. General analysis

In order to solve the problem described above, it is assumed that the principal elastic axes of the material are parallel to the plate edges and the free vibration solution of the problem is set as

$$w(x,y,t) = W(x,y)\sin(\omega t) \tag{8}$$

where ω is the circular frequency and $W(x,y)$ is a function of the position coordinates only. Then substituting Eq. (3) into the undamped free vibration form of Eq. (1) yields

$$D_x\frac{\partial^4 W}{\partial x^4} + 2B\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 W}{\partial y^4} - \rho h\omega^2 W + k_f W - G_s\left[\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right] = 0 \tag{9}$$

The next step is to find the solution of Eq. (9) with the boundary conditions according to Eq. (4), Eq. (5), Eq. (6) and Eq. (7), to obtain the eigen frequencies and the mode shapes of the orthotropic plate with mixed support conditions at its edges. By postulating the following eigen frequency, which is analogous to the case of a plate simply supported at all edges (Alisjahbana and Wangsadinata 2007), natural frequencies of the system can be expressed as

$$\omega_{mn}^2 = \left(\frac{\pi^4}{\rho h}\right)\left[D_x\left(\frac{p}{a}\right)^4 + 2B\left(\frac{pq}{ab}\right)^2 + D_y\left(\frac{q}{b}\right)^4\right] + \frac{k_f}{\rho h} + \frac{G_s}{\rho h}\left[\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2\right] \tag{10}$$

where p and q are real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method (Pevzner 2000).

The solution of the first auxiliary problem satisfying the boundary conditions according to Eq. (4), Eq. (5), Eq. (6) and Eq. (7) can be expressed as

$$X(x) = A_1\cosh\left[\frac{\beta\pi x}{ab}\right] + A_2\sin h\left[\frac{\beta\pi x}{ab}\right] + A_3\cos\left[\frac{p\pi x}{a}\right] + A_4\sin\left[\frac{p\pi x}{a}\right] \tag{11}$$

where

$$\beta = \sqrt{\left[\frac{2q^2b^2B}{D_x} + p^2b^2 + \frac{G_s a^2 b^2}{\pi^2 D_x}\right]} \tag{12}$$

Substituting of Eq. (11) into the boundary conditions according to Eq. (4), Eq. (5), Eq. (6) and Eq. (9), the existence of a nontrivial solution yields the first characteristic determinant. The solution of the second auxiliary problem satisfying the boundary conditions according to Eq. (4), Eq. (5), Eq. (6) and Eq. (7) can be expressed as

$$Y(y) = B_1 \cosh\left[\frac{\theta\pi y}{ab}\right] + B_2 \sin h\left[\frac{\theta\pi y}{ab}\right] + B_3 \cos\left[\frac{q\pi y}{b}\right] + B_4 \sin\left[\frac{q\pi y}{b}\right] \tag{13}$$

where

$$\theta = \sqrt{\left[\frac{2p^2a^2B}{D_y} + q^2a^2 + \frac{G_s a^2 b^2}{\pi^2 D_y}\right]} \tag{14}$$

4. Dynamic response of the plate

The dynamic response of the plate can be found by using the method of variation of parameters as a general method of determining a particular solution of the corresponding non-homogeneous partial differential equation, which can be written in the following form

$$w_{mn}(x, y, t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} X_{mn}(x) Y_{mn}(y) T_{mn}(t) \tag{15}$$

where $X_{mn}(x)$, $Y_{mn}(y)$ are eigenfunctions, $T_{mn}(t)$ is a function of time which must be determined through further analysis. Having obtained the natural frequency ω_{mn} from Eq. (10) involving the spring stiffness and shear modulus of the Pasternak foundation, the differential equation for the coefficient functions $T_{mn}(t)$ can be expressed as

$$\ddot{T}_{mn}(t) + 2\zeta\omega_{mn}\ddot{T}_{mn}(t) + \omega_{mn}^2 T_{mn}(t) = \int_0^a X_{mn}(x)dx \int_0^b Y_{mn}(y)dy \frac{p(x, y, t)}{\rho h Q_{mn}} \tag{16}$$

where Q_{mn} is a normalization factor that can be expressed by

$$Q_{mn} = \int_0^a\int_0^b (X_{mn}(x))^2 (Y_{mn}(y))^2 dxdy \tag{17}$$

The particular solution of the temporal function $T_{mn}(t)$ can be represented in a form of the Duhamel convolution integral as follows (Alisjahbana and Wangsadinata 2007, Michaltsos and Raftoyiannis 2009)

$$T_{mn}^*(t) = \int_0^t \left[\frac{p(x, y, \tau)}{\rho h Q_{mn}} \int_0^a X_{mn}(x)dx \int_0^b Y_{mn}(y)dy\right]\left[\frac{e^{-\zeta\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\zeta^2)}} \sin(\omega_{mn}\sqrt{(1-\zeta^2)}(t-\tau))\right] d\tau \tag{18}$$

The general solution for the forced response deflection of the plate to an arbitrary dynamic moving load $p(x, y, t)$ is given in integral form as follows

For $0 \le t \le t_0$

$$w(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} X_{mn}(x)Y_{mn}(y)e^{-\zeta\omega_{mn}t}[a_{mn}\cos(\omega_{mn}\sqrt{1-\zeta^2}t) + b_{mn}\sin(\omega_{mn}\sqrt{1-\zeta^2}t)]$$

$$+\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} X_{mn}(x)Y_{mn}(y)\int_0^t\left[\frac{p(x,y,\tau)}{\rho h Q_{mn}}\int_0^a X_{mn}(x)dx\int_0^b Y_{mn}(y)dy\right]\left[\frac{e^{-\zeta\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{(1-\zeta^2)}}\sin(\omega_{mn}\sqrt{(1-\zeta^2)}(t-\tau))\right]d\tau \tag{19}$$

For $t > t_0$

$$w(x,y,t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} e^{-\zeta\omega_{mn}(t-t_0)}\Bigg[w_{0mn}\cos[\sqrt{(1-\zeta^2)}\omega_{mn}(t-t_0)] + \frac{v_{0mn}+\zeta\omega_{mn}w_{0mn}}{\omega_{mn}\sqrt{(1-\zeta^2)}}\sin[\sqrt{(1-\zeta^2)}\omega_{mn}(t-t_0)]\Bigg] \tag{20}$$

in which w_{0mn} and v_{0mn} are the initial deflection and velocity at $t = t_0$. Bending moments and vertical shear forces in the plate can be computed in terms of the deflection and its derivatives obtained from Eq. (20) and Eq. (21) as expressed by the following equations

Bending moments

$$M_x = -D_x\left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \nu_y\frac{\partial^2 w(x,y,t)}{\partial y^2}\right), \quad M_y = -D_y\left(\frac{\partial^2 w(x,y,t)}{\partial y^2} + \nu_x\frac{\partial^2 w(x,y,t)}{\partial x^2}\right) \tag{21a, b}$$

Shear forces

$$Q_x = -\frac{\partial}{\partial x}\left(D_x\frac{\partial^2 w(x,y,t)}{\partial x^2} + B\frac{\partial^2 w(x,y,t)}{\partial y^2}\right), \quad Q_y = -\frac{\partial}{\partial y}\left(D_y\frac{\partial^2 w(x,y,t)}{\partial y^2} + B\frac{\partial^2 w(x,y,t)}{\partial x^2}\right) \tag{22a, b}$$

5. Numerical example

Using the procedure described above, a roadway pavement subjected to a moving dynamic traffic load will be analyzed. The effect of changing the load's angular frequency ω and the damping ratio ζ will be investigated. The average wheel load is $P_0 = 80$ kN, traveling with a velocity of $v = 90$ km/hr and an acceleration of $acc = 2$ m/sec^2 along the x axis. The following numerical results have been calculated for the following case: $a = 5.0$ m, $b = 3.5$ m, $h = 0.25$ m, $E_x = 27$ GPa, $E_y = 22.5$ GPa, $\upsilon_x = 0.18$, $\upsilon_y = 0.15$, $\rho = 2{,}500$ kg/m^3, $k = 27.2$ MN/m^2, $G_s = 9.52$ MN/m, ks_{x1}, ks_{x2}, ks_{y1}, $ks_{y2} = 200$ MN/m/m, kr_{1x}, kr_{2x}, kr_{1y}, $kr_{2y} = 1.0$ m/rad/m. Table 1 shows the critical velocities of the system for three values of occurrence of the maximum absolute dynamic deflection, as also shown in Fig. 2.

Table 1 The critical velocities of the system for three values of occurrence of the maximum absolute dynamic deflection

γ (%)	Critical speed (km/hr)	Absolute dynamic deflection (m)
0	140	0.000430195
5	150	0.000203425
10	140	0.000177843

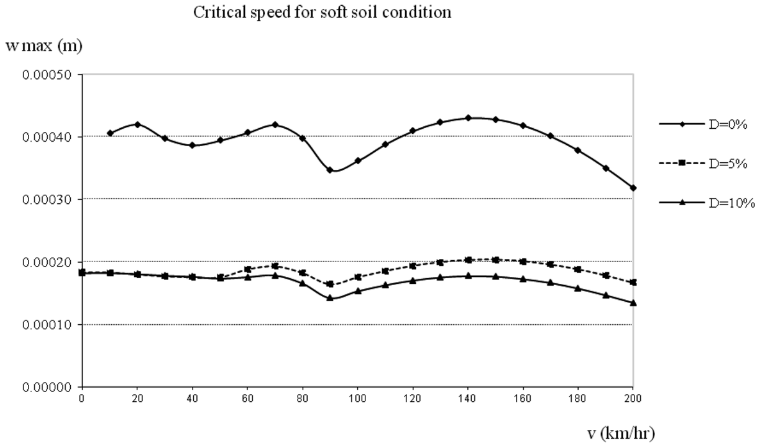

Fig. 2 Maximum dynamic deflection response spectra as a function of velocity for various values of damping ratio for $P_0 = 80$ kN, $\omega_{beban} = 100$ rad/sec, $acc = 2$ m/sec^2

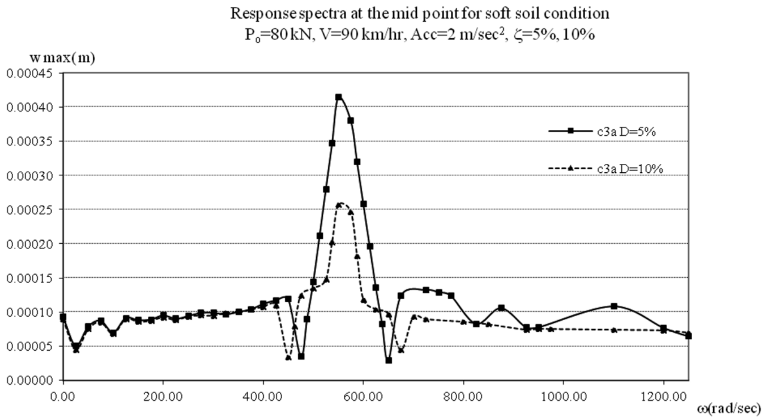

Fig. 3 Maximum dynamic deflection response spectra as a function of load's frequency for various values of damping ratio for soft soil condition

Fig. 3 shows the maximum dynamic deflection response spectra as a function of load's frequency for various values of damping ratio for soft soil condition. Fig. 4 shows various dynamics responses of the rigid runway pavement for different values of damping ratio. It can be seen that the damping ratio play a very important role in reducing the maximum dynamic deflection. Fig. 5 shows the

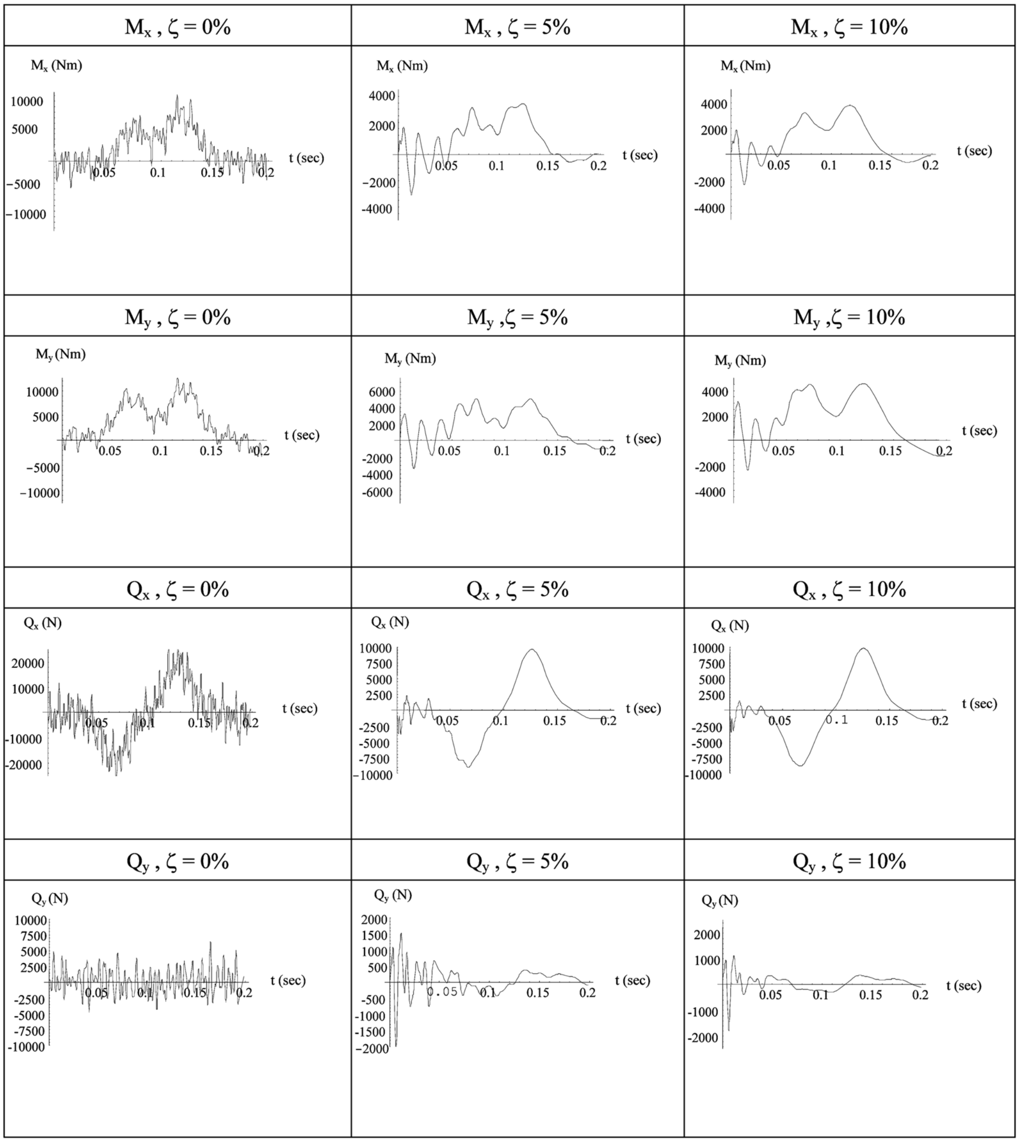

Fig. 4 Various dynamic responses of the rigid runway pavement for different values of damping ratio for soft soil condition, time interval $0 \le t \le t_0$

dynamic deflection distribution over the plate region due to the moving dynamic traffic load during the interval $0 \le t \le t_0$. It can be seen that along the edges of the plate, the distribution of the moment in the x direction is not zero due to the existence of dowels and tie bars.

6. Conclusions

In this research work, dynamic response of rigid roadway pavement subjected to dynamic traffic

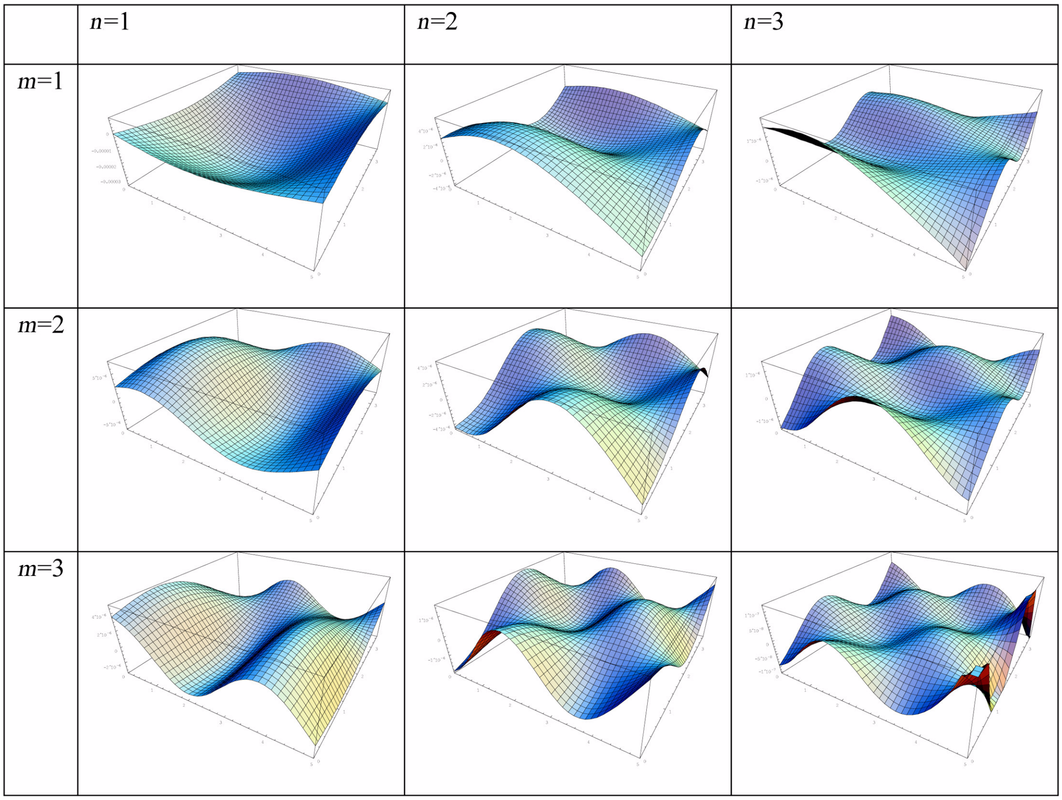

Fig. 5 The dynamic deflection distribution over the plate region to the moving dynamic traffic load calculated at $x = a/2$ and $y = b/2$ during the interval $0 \leq t \leq t_0$, $P_0 = 80$ kN, $v = 90$ km/hr, $acc = 2$ m/sec^2, $\omega_{load} = 100$ rad/sec, $t = 0.0996$ sec, $\zeta = 5\%$

loads with variable velocity are investigated. Analytical form of solution of the dynamic displacement has been obtained on the basis of the orthogonality properties of eigen functions. The whole formulation in this research is based on the assumption that the boundary supports of the orthotropic rectangular plate are steel dowels and tie bars which provide elastic vertical support and rotational restraint. This is a very realistic assumption, particularly for rigid roadway pavement joints, where one may find out the rotational and the vertical shear deformation exist along the joints.

The natural frequencies of the system and the mode numbers ($m = 1, 2, \ldots, 7$ and $n = 1, 2, \ldots, 7$) are solved using two transcendental equations to account for the effect of the non-simply supported boundary conditions.

The effect of the reasonable moving traffic load determines significantly the dynamic response of the plate, leading to the necessity to limit the aspect. Due to the existence of damping the dynamic deflection is not symmetric with respect to the angular frequency of the load (Fig. 3). At the fixed acceleration, the dynamic deflection decreases with increasing velocity of the load for all values of damping ratio considered in this research ($\zeta = 0\%$, 5% and 10%).

The present work only presents the mathematical solution which should be verified further with the results of experimental research, especially on the determination of the forces in the steel connecting devices (dowels and tie bars) along the joints.

The method presented in this paper can be applied to plates, subjected to multilane and sequential loads passing over the plate by the principle of superposition.

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