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A new 3D interface element for three dimensional finite element analysis of FRP strengthened RC beams

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Abstract. The analysis of interfacial stresses in structural component has been the subject of several investigations but it still requires more effort and studies. In this study a general three-dimensional interface element has been formulated for stress and displacement analyses in the interfacial area between two adjacent plate bending element and brick element. Interface element has 16 nodes with 5 degrees of freedom (DOF) in each node adjacent to plate bending element and 3 DOF in each node adjacent to brick element. The interface element has ability to transfer three translations from each side of interface element and two rotations in the side adjacent to the plate element. Stiffness matrix of this element was formulated and implemented in three-dimensional finite element code. Application of this element to the reinforced concrete (RC) beam strengthened with fiber reinforced polymer (FRP) including variation of deflection, slip between plate and concrete, normal and shear stresses distributions in FRP plates have been verified using experimental and numerical work of strengthened RC beams carried out by some researchers. The results show that this interface element is effective and can be used for structural component with these types of interface elements.

Keywords: interface element; bond; plate bending element; three dimensional; FRP.

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1. Introduction

The use of fiber reinforced polymers plates in the RC members are a well-established technique used to strengthen RC members in shear and flexure. These strengthening have been done by several researchers to structural components for example, piles and slabs, such as work of Reddy (2009), Harajli and Soudki (2003) and Ebead and Marzouk (2004). Application of these type of strengthening to concrete beams have been reported in numerous scientific work, as reported by Saadatmanesh and Ehsani (1991), Triantafillou and Pleveris (1992), Sharif *et al*. (1994), Arduini and Nanni (1997), Grace *et al*.(1999), El-Mihilmy and Tedesco (2000), Rahimi and Hutchinson (2001), Seim *et al*.(2001). They show, in particular, the flexural strength of beams can be significantly increased by application of FRP plates bonded to the beam tension face.

More recently, several experimental studies have indicated that bonding FRP plates to the RC members is a very effective technique to upgrade the capacity of RC members such as work

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reported by Gao *et al*. (2004), Anania *et al*. (2005), Pham and Al-Mahaidi (2006), Coronado and Lopez (2006), Guido *et al*. (2007), Barbato (2009) and Yasmeen *et al*. (2010).

Formulation and application of the interface element has been carried out by several investigators. Lu *et al*. (2006) were used interface elements to model bonding between FRP plates to concrete beams. Chore *et al*. (2009, 2010) used interface elements for modeling the soil structure interaction.

A few researchers such as Ebead and Marzouk (2005), have simulated the behavior of the concrete- FRP interface, using a very fine mesh for modeling the adhesive layer that defined as a linear elastic material. However most researchers who have studied the behavior of retrofitted structures have ignored the effect of the interfacial behavior between FRP plates and concrete at all, such as work of Hu *et al*. (2004), Lundquist *et al*. (2005) and Santhakumar and Chandrasekaran (2004).

In this study, we use the finite element method to model the structural behavior of beams strengthened with FRP plate using new interface element. Special three-dimensional interface element is developed for stress and displacement analyses in adhesively bonded FRP plates. Adhesive or interface element has 16 nodes with 5 and 3 degrees of freedom in each node for nodes adjacent to plate bending element and brick element respectively. The proposed element has ability to model and transfer translations and rotations based on the adherends materials. For justification, the study was carried out using a series of beams that had been experimentally tested for flexural behavior and reported by Rahimi and Hutchinson (2001) and later numerical study by Coronado and Lopez (2006).

Application of this element to the strengthened FRP reinforced concrete beams including variation of deflection, slip between plates and concrete, normal and shear stresses distributions in FRP plates have been verified using experimental and numerical mentioned works, that show the ability of this element in capturing the interfacial behavior between the FRP plates and concrete.

2. Proposed finite element model

The aim of the present study is to model the response of FRP strengthened RC beams which focus on interface behavior between FRP plates and concrete. The following elements have been used for modeling in this study.

(a) Plate bending element with 5 DOF is used to model the FRP plates. Detail formulation of this element has been presented somewhere else, Viladkar *et al*. (1992).
(b) 20 node isoparametric solid brick elements are used to represent the concrete.
(c) Truss-linkage element is used to model the bars and slip between bars and concrete. Detail formulation of this element has been presented somewhere else, Kohnehpooshi *et al*. (2010).
(d) Interface element is sandwiched between plate bending element and concrete brick element. Formulation of this element is presented in the following section.

2.1 Interface element between concrete and FRP

This interface element is developed to connect plate and brick elements together and has 16 nodes as shown in Fig. 1. It connects 8 nodes of the top brick element and 8 nodes of the bottom plate with identical coordinates. It’s mechanical action for nodes adjacent to the plate is represented by 5springs, including 3 orthogonal springs connected in the horizontal, vertical and lateral directions

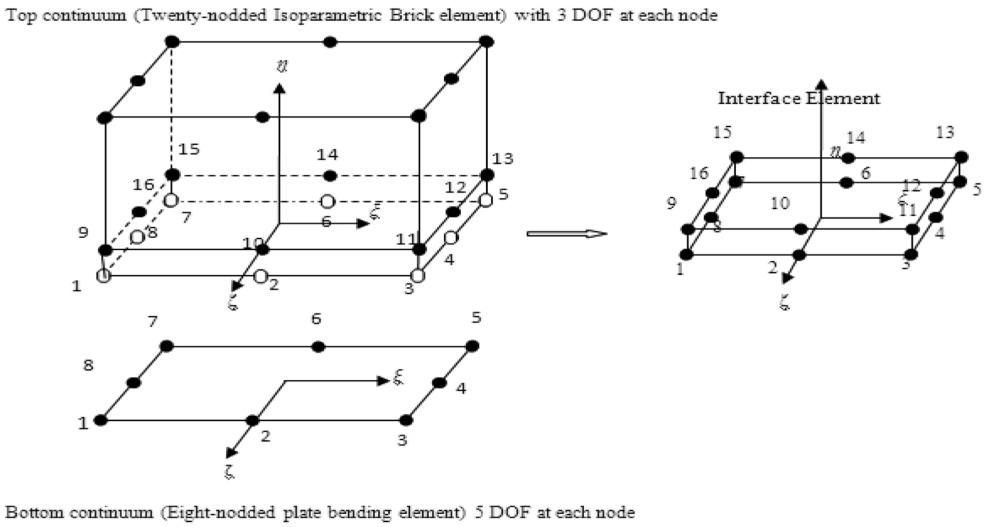
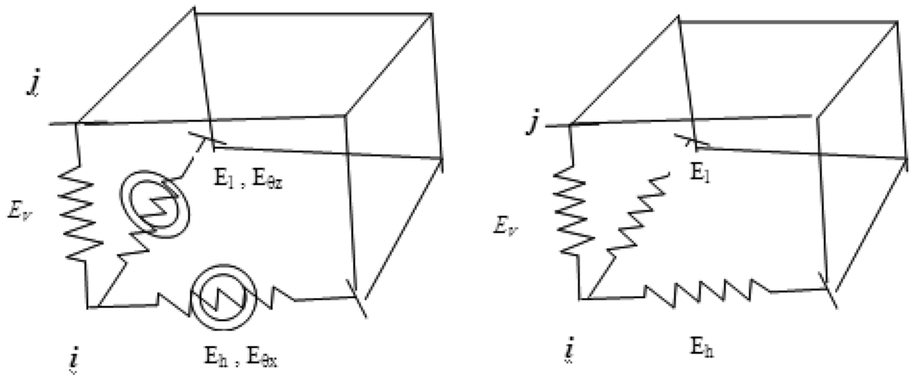


Fig. 1 Interface element between brick and plate bending elements

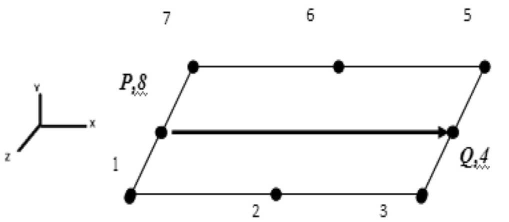


(a) Nodes adjacent to plate (b) Nodes adjacent to brick

Fig. 2 Mechanical representation of the interface element in each node

and 2 rotational springs to plate's elements as shown in Fig. 2(a). Mechanical action for nodes adjacent to the brick is represented by 3 orthogonal springs connected in the horizontal, vertical and lateral directions as shown in Fig. 2(b).The horizontal and lateral spring represent the bond stiffness in the plane parallel to FRP plate and act as bond between two bonded materials, vertical and two rotational bonds also can be applied if bond-slip relation of vertical deformation and two rotations can be found for adhesive material. The procedures for the derivation of stiffness matrix and forces are given below.

Let *X*, *Y*, *Z*, and *X'*, *Y'*, *Z'*, be the global and the local co-ordinate systems respectively. The direction cosines of the local axis (*X'*, *Y'*, *Z'*) with respect to global axes (*X*, *Y*, *Z*) are (*l*, *m*, *n*), (*p*,



$$l = \frac{X_Q - X_P}{L} \qquad m = \frac{Y_Q - Y_P}{L} \qquad n = \frac{Z_Q - Z_P}{L} \qquad p = \frac{-m}{\sqrt{1-n^2}} \qquad q = \frac{l}{\sqrt{1-n^2}}$$

$$r = \frac{-\ln}{\sqrt{1-n^2}} \qquad s = \frac{-mn}{\sqrt{1-n^2}} \qquad t = \sqrt{1-n^2} \qquad L = \sqrt{(X_Q - X_p)^2 + (Y_Q - Y_P)^2 + (Z_Q - Z_P)^2}$$

If l = m = 0 and n = 1, the direction cosines take the following values: *p=0; q=l; t=0; r=-1; s=0.*

Fig. 3 Direction cosines of the line *PQ*

q, θ) and (r, s, t). Let line *PQ* which connects the nodes 8 to 4 be the element line to define the cosine direction as shown in Fig. 3. Derivation for one link node translation is similar to derivation for link element developed by Ahmad (1987). Ahmad element, linked bar element to the adjacent continuum only in one node. In this study element extended for interface element between brick and plate bending elements adding two rotational springs for each link in adjacent to plate element and assembly of eight links to one stiffness matrix as interface stiffness matrix.

With this definition for the direction cosines, the local *X'* is always tangential to line element *PQ*, with the other two transitional directions being orthogonal to it. Let ΔS_h, ΔS_v, ΔS_l, $\Delta\theta_x$ and $\Delta\theta_z$ be the incremental slips in the horizontal, vertical, lateral, rotational around x and rotational around y directions of the plate bending element.

It is assumed that top layer of the interface element has 3 DOF and bottom layer has 5 DOF that can be used as an interface element between two plate and brick elements. The incremental relationship between the slip and the nodal displacements can then be written as

$$\begin{Bmatrix} \Delta S_h \\ \Delta S_v \\ \Delta S_l \\ \Delta\theta_x \\ \Delta\theta_z \end{Bmatrix}_{5\times 1} = \begin{bmatrix} -l & -m & -n & 0 & 0 & l & m & n \\ -p & -q & -\theta & 0 & 0 & p & q & \theta \\ -r & -s & -t & 0 & 0 & r & s & t \\ 0 & 0 & 0 & l & m & 0 & 0 & 0 \\ 0 & 0 & 0 & r & s & 0 & 0 & 0 \end{bmatrix}_{5\times 8} \begin{Bmatrix} \Delta Ui \\ \Delta Vi \\ \Delta Wi \\ \theta x_i \\ \theta z_i \\ \Delta Uj \\ \Delta Vj \\ \Delta Wj \end{Bmatrix}_{8\times 1} \tag{1}$$

Or

$$\{\Delta S^e\} = [T]\{\Delta U^e\} \tag{2}$$

Where T is the transformation matrix, i and j are the element nodes.

The local incremental bond stress and bond-slip may be written as

$$\{\Delta \sigma_b^e\} = [E]\{\Delta S^e\} \tag{3}$$

Or in the matrix form

$$\{\Delta \sigma_b^e\} = \begin{bmatrix} E_h & 0 & 0 & 0 & 0 \\ 0 & E_v & 0 & 0 & 0 \\ 0 & 0 & E_l & 0 & 0 \\ 0 & 0 & 0 & E_{\theta x} & 0 \\ 0 & 0 & 0 & 0 & E_{\theta z} \end{bmatrix} \begin{Bmatrix} \Delta S_h \\ \Delta S_v \\ \Delta S_l \\ \Delta \theta_x \\ \Delta \theta_z \end{Bmatrix}$$

Where E_h, E_v, E_l, $E_{\theta x}$, $E_{\theta z}$ are the bond-slip module in the five directions. These are obtained by using idealized form of bond-slip curves of slip between plates and concrete

$$E_h = \frac{\Delta \sigma_b}{\Delta S} \tag{3a}$$

Where $\Delta \sigma_b$ and ΔS are the incremental bond stress and slip from a specified bond-slip curve.

E_v, E_l, $E_{\theta x}$ and $E_{\theta z}$ can also be obtained using idealized form of bond-slip curves of slip between plates and concrete in these directions.

Assuming bond stresses as average stresses along the area of the each node with area of A_i, the incremental nodal force and the stress relation can be written in the following form

$$\Delta P_{ei} = A_i T^T \Delta \sigma_b^e \tag{4}$$

Identical loading surface at each node for 8 nodes of isoparametric plate bending element are assumed. Then

$$A_i = (A/8)$$

$$\Delta P_{ei} = (A/8)T^T \Delta \sigma_b^e \qquad \text{For } i = 1, 2, 8$$

$$\Delta P^e = [\Delta P_u^i \; \Delta P_v^i \; \Delta P_w^i \; \Delta M\theta_x^i \; \Delta M\theta_z^i \; \Delta P_u^j \; \Delta P_v^j \; \Delta P_w^j]^T$$

A = Surface area of the plate

Now the relationship between the incremental nodal forces and the incremental displacements can be found by substituting Eqs. (2) and (3) in Eq. (4)

$$\Delta P_e = (A/8)T^T E_b T \Delta U^e$$

$$\Delta P_e = K_b^e \Delta U^e \tag{5}$$

Where $$[K]_{8\times 8} = (A/8)[T]^T[E_b][T]$$

Or in the matrix form

$$[K]_{8\times 8} = A_i \begin{bmatrix} -l & -p & -r & 0 & 0 \\ -m & -q & -s & 0 & 0 \\ -m & -\theta & -t & 0 & 0 \\ 0 & 0 & 0 & l & r \\ 0 & 0 & 0 & m & s \\ l & p & r & 0 & 0 \\ m & q & s & 0 & 0 \\ n & \theta & t & 0 & 0 \end{bmatrix}_{8\times 5} \begin{bmatrix} E_h & 0 & 0 & 0 & 0 \\ 0 & E_v & 0 & 0 & 0 \\ 0 & 0 & E_l & 0 & 0 \\ 0 & 0 & 0 & E_{\theta x} & 0 \\ 0 & 0 & 0 & 0 & E_{\theta z} \end{bmatrix}_{5\times 5} \begin{bmatrix} -l & -m & -n & 0 & 0 & l & m & n \\ -p & -q & -\theta & 0 & 0 & p & q & \theta \\ -r & -s & -t & 0 & 0 & r & s & t \\ 0 & 0 & 0 & l & m & 0 & 0 & 0 \\ 0 & 0 & 0 & r & s & 0 & 0 & 0 \end{bmatrix}_{5\times 8}$$

$[K]_{8\times 8}$ = plate-link stiffness matrix for each link node

The explicit form of K, for each link node is given as follow.

$$[K]_{8\times 8} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & 0 & 0 & -k_{11} & -k_{12} & -k_{13} \\ & k_{22} & k_{23} & 0 & 0 & -k_{12} & -k_{22} & -k_{23} \\ & & k_{33} & 0 & 0 & -k_{13} & -k_{23} & -k_{33} \\ & & & k_{44} & k_{45} & 0 & 0 & 0 \\ & & & & k_{55} & 0 & 0 & 0 \\ & SYM & & & & k_{11} & k_{12} & k_{13} \\ & & & & & & k_{22} & k_{23} \\ & & & & & & & k_{33} \end{bmatrix}_{8\times 8} \quad (6)$$

$l, m, n, p, q, o, r, s, t$ = direction cosines

$k_{11} = (A/8)(l^2E_h + p^2E_v + r^2E_l)$ $\quad$ $k_{12} = (A/8)(lmE_h + pqE_v + rsE_l)$

$k_{22} = (A/8)(m^2E_h + q^2E_v + s^2E_l)$ $\quad$ $k_{13} = (A/8)(\ln E_h + rtE_v)$

$k_{33} = (A/8)(n^2E_h + t^2E_v)$ $\quad$ $k_{23} = (A/8)(mnE_h + stE_l)$

$k_{44} = (A/8)(l^2E_{\theta x} + r^2E_{\theta z})$ $\quad$ $k_{45} = (A/8)(lmE_{\theta x} + rsE_{\theta z})$

$k_{55} = (A/8)(m^2E_{\theta x} + s^2E_{\theta z})$

Eq. (6) is a stiffness matrix of link element for connecting the top and bottom continue just in one node. The finial stiffness matrix of interface element includes the 8 link elements stiffness matrices that will be 64 × 64 stiffness matrix as follow

$$[K]_{64\times 64} = \begin{bmatrix} [K1]_{8\times 8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & [K1]_{8\times 8} & 0 & 0 & 0 & 0 & 0 & 0 \\ & & [K1]_{8\times 8} & 0 & 0 & 0 & 0 & 0 \\ & & & [K1]_{8\times 8} & 0 & 0 & 0 & 0 \\ & & & & [K1]_{8\times 8} & 0 & 0 & 0 \\ & SYM & & & & [K1]_{8\times 8} & 0 & 0 \\ & & & & & & [K1]_{8\times 8} & 0 \\ & & & & & & & [K1]_{8\times 8} \end{bmatrix}_{64\times 64} \tag{7}$$

2.2 Constitutive modeling for bond-slip between concrete and FRP plates

An accurate local bond–slip model is important in the modeling of FRP-strengthened RC structures. A few models can be found for bond-slip model between FRP plates and concrete. A review of existing bond strength models and bond–slip models can be found in references Lu *et al.* (2005) and Sayed-Ahmed *et al.* (2009).

One of the most accurate bond stress–slip is that proposed by Lu *et al.* (2005). The behavior of the FRP-Concrete interface is simulated as a relationship between the local shear stress, τ, and the relative displacement, s. Three different bond–slip relations have been suggested by these writers; they are referred as precise, simplified and the bilinear models. In this study, the bilinear model, as shown in Fig. 4, is adopted for its simplicity and difference in the results compare to other two models is not considerable, as has been shown by Lu *et al.* (2005). If τ_{max} be the maximum bond stress and s_0 the corresponding slip, then

$$\tau = \tau_{max}\left(\frac{s}{s_0}\right) \qquad \text{if} \qquad s \le s_0 \tag{8}$$

$$\tau = \tau_{max}\frac{(s_{max}-s)}{(s_{max}-s_0)} \qquad \text{if} \qquad s_0 < s \le s_{max} \tag{9}$$

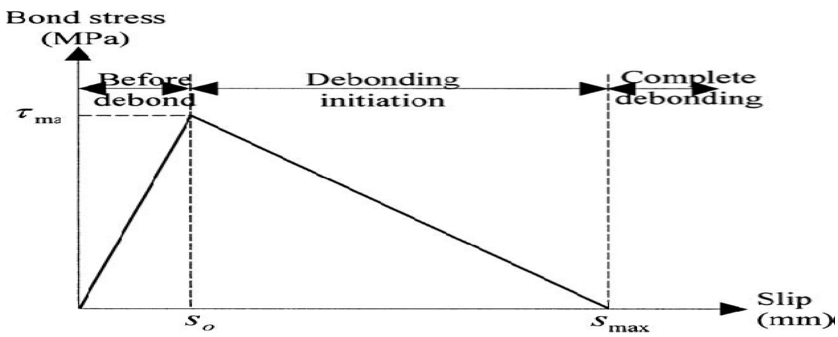


Fig. 4 Bilinear bond–slip model by Lu *et al.* (2005)

Where

$$\tau_{\max} = 1.5B_w f_t$$

$$s_0 = 0.0195B_w f_t$$

$$B_w = \sqrt{\left(2.25 - \frac{b_f}{b_c}\right) / \left(1.25 + \frac{b_f}{b_c}\right)}$$

$$s_{\max} = \left(\frac{2G_f}{\tau_{\max}}\right)$$

$$G_f = 0.308B_w^2\sqrt{f_t}$$

Where b_c and b_f being the widths of the concrete prism and the FRP plate respectively, G_f the interfacial fracture energy and f_t concrete tensile strength. It should be mentioned here range of $s \le s_0$ has been used in this analysis because of the linearity. More detail of bond slip can be found Lu *et al*. (2005).

A finite element computer code has been written in FORTRAN language by Noorzaei (1991) and developed by Kohnehpooshi (2011), compatible with FORTRAN power station environment. The following 3-D elements have been further added to the element library of the element code. Some of the added elements include are:

a: 27 node lagrangian brick element
b: Three dimensional 2 and 3 node bar elements
c:Three dimensional truss-linkage element
d: Three dimensional interface element for two plate bending elements
e: Three dimensional interface element between bricks or membrane shell elements
f: Three dimensional interface element between brick and plate bending elements

3. Analysis of FRP strengthened reinforced concrete beams

3.1 Finite element model

To show the application of the proposed element a numerical example based on the experimental work of Rahimi and Hutchinson (2001) and later numerical studies of Coronado and Lopez (2006) has been selected for numerical analysis.

Only series B correspond to RC beams tested by Rahimi and Hutchinson (2001) under four point bending selected for this analysis. Beams B1, B2 were used as control specimens. Material properties are described in Table 1. Geometric properties for these beams are presented in Fig. 5 and Table 2. Additional strengthening details can be obtained from Rahimi and Hutchinson (2001).

A beam specimen as shown in Fig. 5 is modeled with the three-dimensional finite element.

3.2 Verification of proposed model

Comparison of normal deflection in the linear region between the experimental works of Rahimi

Table 1 Properties of materials used

Property	Concrete	GFRP	CFRP	Epoxy	Steel
Modulus of elasticity, E (GPa)	25	36	127	7	210
Yield strength, f_y (MPa)	--	--	--	--	575
Tensile strength, f_t (MPa)	3	1074	10532	25	--
Compressive strength, $f'c$ (MPa)	61	--	--	70	--
Poisson's ratio	0.2	0.3	0.3	0.3	0.3

Table 2 Geometric properties of beams

Beam No	Type of external reinforcement and plate thickness	Cross-sectional area of plate (mm^2)
B1-B2	None	-----
B3-B4	CFRP laminate, 2-ply (0.4 mm)	60
B5-B6	CFRP laminate, 6-ply (1.2 mm)	180
B7-B8	GFRP laminate, 12-ply (1.8 mm)	270

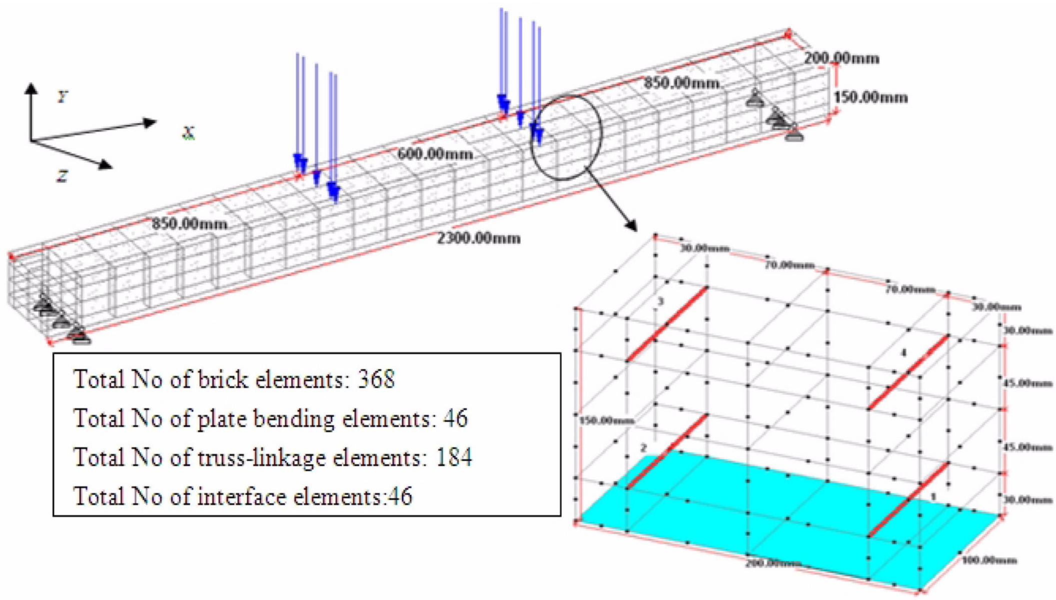


Fig. 5 Three-dimensional beam geometry, finite element mesh and boundary conditions

(2001), numerical study of Coronado and Lopez (2006) and present study are shown in Fig. 6. As it is depicted in Fig. 6 almost similar results for deflection in the linear region can be found. It should be mentioned, present analysis is linear. This is because the main aim of study is to validate the proposed element, so linear analysis seems to be enough.

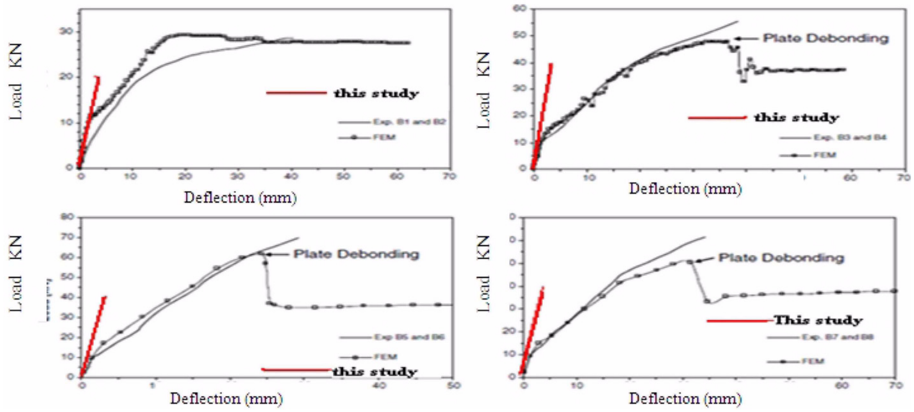
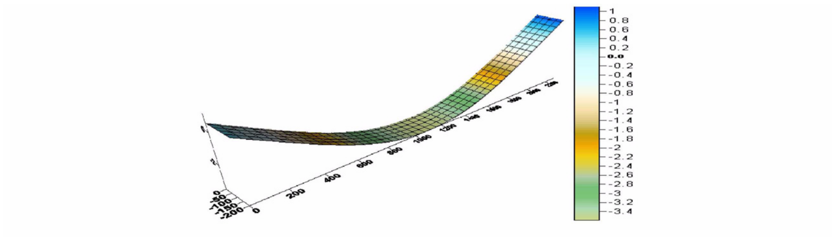


Fig. 6 Load mid-span deflection for, experimental work of Rahimi (2001), numerical study of Coronado and Lopez (2006) and present study



(a) B1-2

(b) B3-4 Full bond analysis (c) B3-4 Interface analysis

Fig. 7 Deflections of Beams (mm) along the length: (a) B1-2, (b) B3-4 with full bond between FRP and concrete and (c) B3-4 with interface element

4. Results

4.1 Y deflection

Deflection of beams B1-2, B3-4 with full bond modeling and B3-4 with bond model (Interface element) are plotted in Figs. 7(a) to 7(c). It is seen from that figures that maximum deflection of control beams B1-2 under 40 KN is 3.4 mm. This amount decreased to 2.6 mm for beams B3-4,

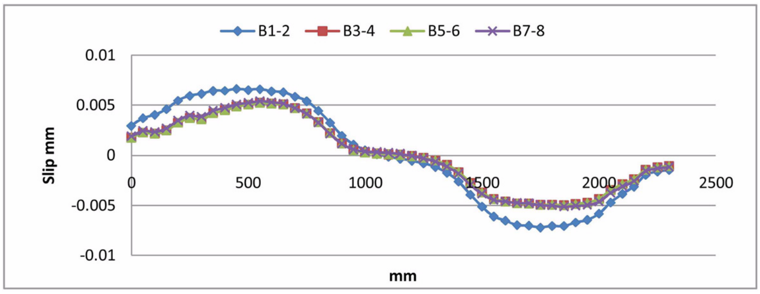
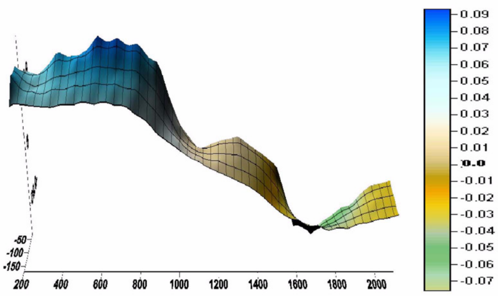
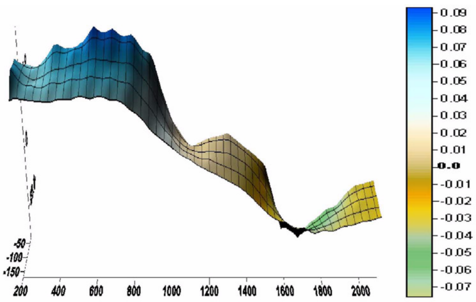


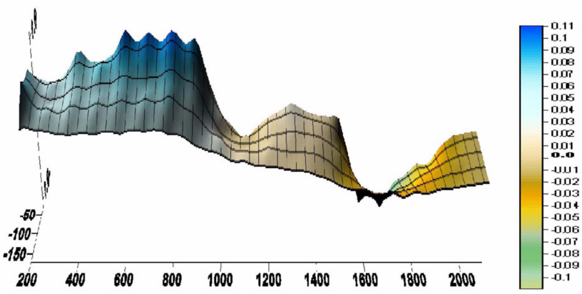
Fig. 8 Slip of bar No 1 for all types of beams along the bar's length



(a) B3-4



(b) B5-6



(c) B7-8 Slip

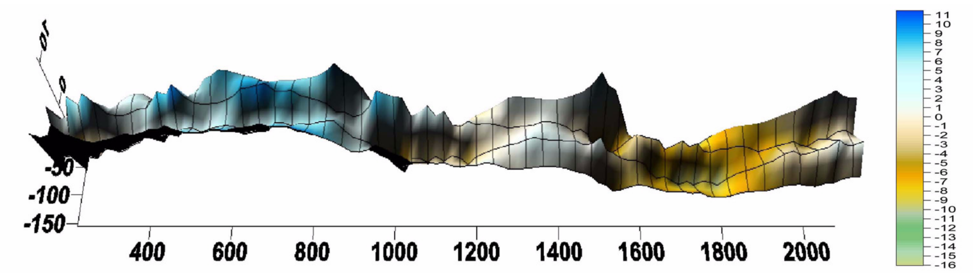
Fig. 9 Slip (mm) between FRP plate and concrete in adhesive region for all type of strengthened beams

equal to 27 % decreasing in deflection. Both type of beams B3-4 with and without bond assumption almost show same deflection.

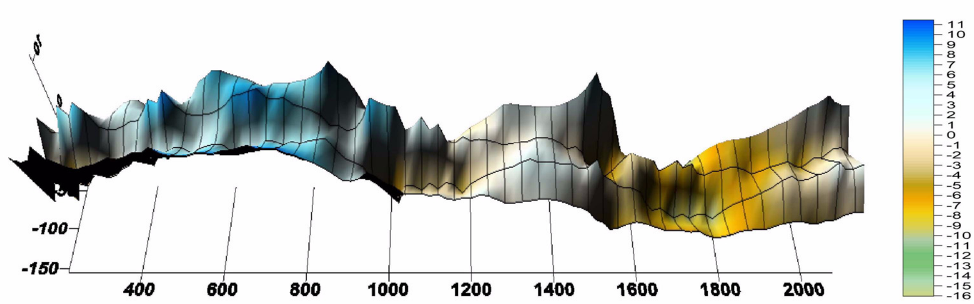
4.2 Slip analysis between bars and concrete

Bar 1 in the tension side of the all mentioned beams was selected to study the effect of strengthening on slip between bars and concrete. The reinforced bars 1 and 2 are in tension in the bottom side of beam, and bars 3 and 4 are in compression in the top of the beam as shown in Fig. 5. Slip of the bar 1 for all analyzed beams along the bar length is shown in Fig. 8.

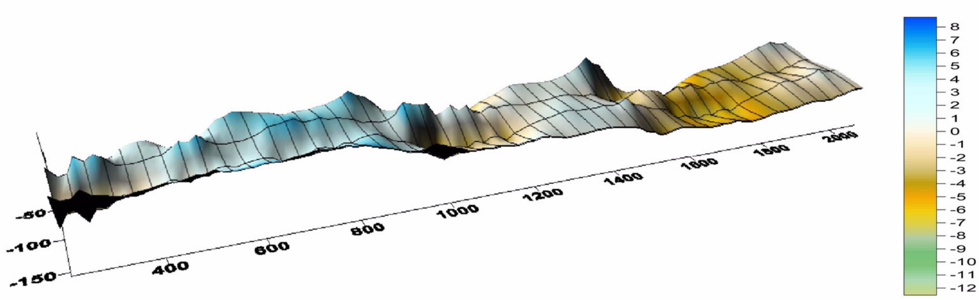
The results show, that there is clear slip between the concrete and steel i.e. the slip is directly related to shear stresses in concrete. As can be seen in the Fig. 8, slips of the B3 to B8 types have been significantly reduced. It also shows that strengthening in flexural region has major effect to reduce the slip in the linear regions. It can be seen from Fig. 8, that B3 to B8 types of the beams almost have the same amount of slip. It shows the increase or decrease the thickness of FRP plates has no obvious effect in the slip reduction between bar and concrete.



(a) B3-4



(b) B5-6



(c) B7-8

Fig. 10 Shear stresses τ_{xz} (N/mm^2) in the adhesive region

4.3 Slip analysis between plate layers and concrete

Slip between plates and concrete beam in the adhesive area are plotted for all type of strengthened beams in Figs. 9(a) to 9(c). As they are shown in these Figures there are clear distributions of slip between concrete and FRP plates i.e. slip is related to the shear stress. In the region of maximum shear the beams have maximum slip and in the region of zero shear stress, zero slip can be considered. Almost same distribution of slip for all types of strengthening can be seen. It shows that thickness of FRP material or changing between CFRP and GFRP have no significant effect to reduce or increase the slip between concrete and plates. Figs. 9(a) to 9(c) show the ability of the proposed element to model interfacial behavior between FRP plate and concrete.

4.4 Stresses

4.4.1 Shears stress τ_{xz}

Figs. 10(a) to 10(c) show the three dimensional variation of the shear stress τ_{xz}. As it is known the shear stresses τ_{xz} and in plane slip are dependent together. The shape of slip and these in-plane shear stresses are almost similar in shape and they are related with some coefficient in the linear region. In the mid span between two concentrated load where beams have pure bending, it is clear that value of shear stresses should be zero, as depicted in Figs. 10(a) to 10(c). Shear stresses are maximum and symmetric with different sign between loading points and supports. Almost same value of shear distribution can be seen in the figures for all types of strengthened beams. The results show although the strengthening reduce considerable amount of deflection so reduce the stresses, compare to the beams without strengthening, but the type of strengthening have no significant effect on the values of reductions.

5. Conclusions

In this study a special proposed three-dimensional interface element has been formulated for stress and displacement analyses in bonded joint between two adjacent plate bending element and brick element. Interface element has 16 nodes with 5 DOF in each node adjacent to plate bending elements and 3 DOF in each node adjacent to brick elements. The interface element has ability to transfer three translations from each side of interface element and two rotations in the side adjacent to the plate element. Stiffness matrix of this element was formulated and implemented in three-dimensional finite element code.

Distributions of the deflection, slip, normal and shear stresses have been plotted. Verification of the developed interface element has been achieved. Based on the experimental and some numerical results have been found there are reasonable distribution of slip, deflection and shear stress in the FRP plate. These results show that this interface element is effective and can be used for structural component with this type of interface elements.

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