Interaction and Multiscale Mechanics, Vol. 4, No. 1 (2011) 1-16
DOI: http://dx.doi.org/10.12989/imm.2011.4.1.001

Sensitivity of resistance forces to localized geometrical imperfections in movement of drill strings in inclined bore-holes

V.I. Gulyayev*, S.N. Khudoliy and E.N. Andrusenko

Department of Mathematics, National Transport University, Suvorov str., 1, 01010, Kiev, Ukraine

(Received October 17, 2010, Accepted November 2, 2010)

Abstract. The inverse problem about the theoretical analysis of a drill string bending in a channel of an inclined bore-hole with localized geometrical imperfections is studied. The system of ordinary differential equations is first derived based on the theory of curvilinear flexible elastic rods. One can then use these equations to investigate the quasi-static effects of the drill string bending that may occur in the process of raising, lowering and rotation of the string inside the bore-hole. The method for numerical solution of the constructed equations is described. With the proposed method, the phenomenon of the drill column movement, its contact interaction with the bore-hole surface, and the frictional seizure can be simulated for different combinations of velocities, directions of rotation and axial motion of the string. Geometrical imperfections in the shape of localized smoothed breaks of the bore-hole axis line are considered. Some numerical examples are presented to illustrate the applicability of the method proposed.

Keywords: curvilinear drilling; geometrical imperfections; drill string raising; contact forces; resistance forces.

---

1. Introduction

One important factor that complicates the situation in oil and gas extraction industry is that, as a rule, only 37% of hydrocarbon fuels, which fill pores and cracks of underground deposits, can be extracted with the use of traditional technology, through the drilling of vertical bore-holes. An effective means to enlarge the extraction efficiency is to drill the bore-holes in a curvilinear manner, allowing them to penetrate the oil- and gas-bearing strata along the laminated structure of the underground deposits, and to cover larger zones of fuel output (Choe *et al*. 2005, Chow *et al*. 2003, Kerr 2005, Pourcian 2005).

Another reason for the necessity to develop the curvilinear drill method lies in the fact that the world oil and gas market demands a fast redistribution of supply and demand for the hydrocarbon fuels nowadays. According to the experts' opinions, most of the substantial innovations of the current centenary were concerned with power engineering. Particularly, some are related to the pioneering investigation of industrial extraction of shale gas, whose deposit in the world essentially

---

* Corresponding author, Professor, E-mail: valery@gulyayev.com.ua

exceeds the natural gas reserves. The exploration of shale gas has been made possible with the emergence of new technology for its extraction. It is known that this kind of gas is stored in isolated zones of shale rocks which is characterized by high density and low porosity. The gas-containing zones are separated from each other by partitions. Because of this, the use of traditional vertical bore-holes becomes unacceptable as far as the effectiveness is concerned. Instead, drilling the bore-holes along inclined and horizontal paths turns out to be more beneficial despite the fact that its cost is four to five times larger than that of vertical bore-holes.

But the drilling technology for inclined and horizontal bore-holes is featured by its complexity and is not completely mature. The world statistics indicates that the accident rate is 1 in 3 for driving bore-holes of this type (Mohiuddin *et al.* 2006). It is associated to a large extent with the complexity of mechanical and physical phenomena accompanying the drilling process and the lack of reliable methods of computer simulation that enables us to predict the emergency situations and to prevent them beforehand.

The efforts to simulate the inclined drill problems with the use of computer models present considerable difficulties depending on a number of factors (Iyoho *et al.* 2005). Among them the relatively large length of the drill string is the main concern. Based on the geometrical similarity, the drill columns can be compared to a human hair. The computer simulation of internal and external forces acting on them is usually performed with the use of simplified mathematical models based on the theory of absolutely flexible threads (Aadnoy *et al.* 2003, Aadnoy and Andersen 2001, Sheppard 1987). Analysis of these forces is performed by investigating the geometrical peculiarities of the bore-hole axis line without considering the contribution of elastic forces and contact generated during the raising-lowering operations and the rotation of the drill string. Recently, a number of researches have been presented on the designs of bore-holes with the simplest outlines (Aadnoy and Andersen 2001, Sawaryn and Thorogood 2005, Sawaryn *et al.* 2006, Sheppard 1987).

In the papers by Brett *et al.* (1989) and Stuart *et al.* (2003), a more general approach called the minimum curvature method has been adopted, by considering the well axis outline as a smooth curve made up of segments of straight lines and circular or catenary curves. By this approach, explicit analytical equations are derived to model the drill string (thread) with tension and friction forces for hoisting or lowering operations. In addition, explicit expressions are developed for the drill column to include the effects of drag and torque under the combined axial motion and rotation. Using the equalities, the total drag and torque are derived as the sum of their separate contributions from each section of the hole. Several examples were prepared to demonstrate the use of the analytical models. It was shown that any change in the direction of the well path contributes to increased friction.

The conclusion achieved based on the assumption of well trajectory smoothness and negligible bending stiffness of the drill string tube may underline the weakness of the theory used. In practice, the axial trajectories of the bore-holes cannot be represented as lines with smooth geometry because of the geometrical imperfections involved in drilling. They can be caused by distortions of the bit geometry, dynamic imbalance of the bottom-hole-assembly and physical non-homogeneities of the drilled rock medium. Usually they have the modes of broken lines with smoothed angles and local spirals or cosinusoidal curvings imposed on the designed trajectory of the bore-hole. Depending on their amplitudes and occurrence stages, they can lead to essential local bending of the drill string and emergence of additional contact and friction forces, which can be detected and described only with the use of the elastic curvilinear rod theory.

The second important reason for application of this theory is the possible existence of friction force regulation via the simultaneous axial motion and rotation of the drill string during its lowering and raising (Brett *et al.* 1989, Stuart *et al.* 2003). These procedures can be more precisely simulated only using more accurate elastic rod models, such as the non-linear high order differential equations and, as will be shown below, formulation of the inverse problem for part of the required variables.

Simulation of the resistance forces and quasi-static phenomena associated with the bore-hole drilling allows one to solve the fundamental problems like the provision of the required geometry for the bore-hole axis, decrease of the contact and friction interaction forces between the drill string and bore-hole wall, and avoiding the drill string tube seizure inside the bore-hole (Gulyaev *et al.* 2007a, Gulyaev *et al.* 2007b). Owing to these effects, the drill string tube wearing is reduced, and undesirable curving along the axial line of the bore-hole is excluded. As a consequence, serious emergent events may be prevented during the drilling processes.

This paper considers the problem of quasi-static equilibrium and deformation of elongated (down to 10 km) drill strings in inclined bore-holes with allowance made for the effects of non-uniform gravity and friction forces along with the action of torque. The investigations are based on the statement of direct problems for one part of the variables and inverse problems for the others. Special attention is paid to the question of analysis of friction force influence on the mechanical behavior of drill strings during their raising, lowering and rotating in inclined bore-holes with the geometrical imperfections expressed in the form of broken lines. The key concern herein is the estimation of internal and external forces and moments acting on the drill string.

2. Statement of the problem about elastic drill string dragging in a bore-hole with localized geometrical imperfections

The problem of determination of forces resisting the drill string movement in a bore-hole with geometrical imperfection can arise at the stage of design of a curvilinear bore-hole geometry, when the driving accuracy has to be imposed, as well as during the drill process, when the imperfections are already created and there is a need to analyze the drill string behavior in the bore-hole channel. Let the axial line of the drill string be determined by the equation

$$\boldsymbol{\rho} = \boldsymbol{\rho}(s) \tag{1}$$

in the Cartesian coordinate system *Oxyz*. Here $\boldsymbol{\rho}$ is the radius-vector $\boldsymbol{\rho} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$; $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the appropriate unit vectors; s is the parameter measured by the length of the axial line of the drill string; $x(s)$, $y(s)$, $z(s)$ are the differentiable functions describing the trajectory of the drill string with any geometrical imperfections.

In this paper, the direct problem based on the quasi-static theory of curvilinear flexible rods will be adopted in calculating the internal longitudinal force and torque, while the external forces of contact and friction interaction between the drill string and bore-hole wall will be calculated through the statement of an inverse problem. With this approach, the behavior of the drill string in the curvilinear bore-hole can be described, the zones of possible seizure of the drill string can be detected, and the measures for the jarring and release of the drill string can be designed. Based on the above idea, the theory of curvilinear flexible rods will be adopted to describe the stress-strain state of the drill string, for which the foundations of the theory were presented in Gulyayev *et al.* (1992).

To describe the mechanics of the drill string, it is suitable to use jointly the external and internal geometries, namely, applying the former to individualize the points of the curvilinear tubular rod and the latter to describe its geometry in the deformed state.

The internal geometry of the rod is specified by the coordinate s, measured as the length of the axial line from the initial to the current point, and a moving right-handed coordinate system (u, v, w), whose orientation is rigidly connected with the cross-section of interest at a generic point of the tube axial line. The origin of this system lies at the center of gravity of the cross-sectional area, the u - and v-axes are directed along the principal central axes of inertia of the cross-sectional area, and the w-axis is directed along the tangent to the elastic line. In this case the coordinate s is a concomitant one. The external geometry of the rod determines the location of each point along the elastic line in the fixed inertial coordinate system $Oxyz$.

The Frenet natural trihedron of the elastic line of the rod with unit vectors of the principal normal **n**, binormal **b** and tangent $\boldsymbol{\tau}$ is also introduced, which is determined by the formulas

$$\boldsymbol{\tau} = \frac{d\boldsymbol{\rho}}{ds}, \quad \mathbf{n} = R\frac{d\boldsymbol{\tau}}{ds}, \quad \mathbf{b} = \boldsymbol{\tau} \times \mathbf{n} \tag{2}$$

This system of unit vectors is redundant and for this reason it satisfies six first integrals

$$|\boldsymbol{\tau}| = 1, \quad |\mathbf{n}| = 1, \quad \boldsymbol{\tau} \cdot \mathbf{n} = 0, \quad \boldsymbol{\tau} \times \mathbf{n} = \mathbf{b} \tag{3}$$

which can be used in the calculation for precision verification.

Eq. (1) entirely determines the other geometrical properties of the bore-hole trajectory, including its absolute curvature k_R and k_T torsion (Gulyayev *et al.* 1992)

$$k_R = \sqrt{(x'')^2 + (y'')^2 + (z'')^2}, \quad k_T = k_R^{-2}\begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix} \tag{4}$$

With the above expressions, the curvatures p, q in the (u, v, w) system and elastic torsion r are calculated as

$$p = k_R \sin\chi, \quad q = k_R \cos\chi, \quad r = k_T + \frac{d\chi}{ds} \tag{5}$$

where χ is the angle between the unit vector **n** and axis u.

Consider the quasi-static movement of the drill string inside the bore-hole during the drilling state or the raising-lowering operations. Assuming that the radii of curvature R and torsion of the curve of the drill string, as given in Eq. (1), are so large in comparison with its diameter, the bending of the drill string can be considered to be elastic.

The stress-strain state of an element of the drill string is determined by the principal vectors of internal forces $\mathbf{F}(s)$ and moments $\mathbf{M}(s)$ acting on the cross-section and the distributed vectors of external forces $\mathbf{f}(s)$ and moments $\mathbf{m}(s)$. The external force $\mathbf{f}(s)$ includes the distributed gravity force $\mathbf{f}^{gr}(s)$, the contact interaction $\mathbf{f}^{c}(s)$, and the force $\mathbf{f}^{fr}(s)$ of friction between the surfaces of the drill string and the bore-hole wall. The external distributed moment $\mathbf{m}(s)$ consists only of the moment $\mathbf{m}^{fr}(s)$ due to the friction forces. Thus

$$\mathbf{f} = \mathbf{f}^{gr} + \mathbf{f}^{c} + \mathbf{f}^{fr}, \quad \mathbf{m} = \mathbf{m}^{fr} \tag{6}$$

It is convenient to consider the vectors **F**, **M** in the system of axes (u, v, w). Inasmuch as the drill string is very long in comparison with its diameter and its deformation is caused by bending in the bore-hole channel, its elastic elongation can be neglected. Then the forces F_u, F_v, F_w are purely static factors and can be determined from the equilibrium conditions (Gulyayev *et al*. 1992). The components M_u, M_v, M_w of the principal vector **M** are calculated with the use of Hooke's law. The initial curvature and torsion of the drill string are taken as zero. Then

$$M_u = Ap, \qquad M_v = Aq, \qquad M_w = Cr \tag{7}$$

where $A = EI$, $C = GI_w$ are the bending and torsion stiffness factors; E, G are the elastic and shear moduli of elasticity; I, I_w are the axial and polar moments of inertia of the drill string about the cross-section.

The external and internal forces and moments acting on the element of the drill string obey the following equilibrium equations

$$\frac{d\mathbf{F}}{ds} = -\mathbf{f}, \qquad \frac{d\mathbf{M}}{ds} = -\boldsymbol{\tau} \times \mathbf{F} - \mathbf{m} \tag{8}$$

It is useful to formulate Eq. (8) in the system of axes (u, v, w), because the components of the internal moment, as given in Eq. (7), are present in this system. As the drill string rotates with its movement along the s axis, the total derivatives $d\mathbf{F}/ds$, $d\mathbf{M}/ds$ should be expressed in the following form

$$\frac{d\mathbf{F}}{ds} = \frac{\tilde{d}\mathbf{F}}{ds} + \boldsymbol{\omega}_\chi \times \mathbf{F}, \qquad \frac{d\mathbf{M}}{ds} = \frac{\tilde{d}\mathbf{M}}{ds} + \boldsymbol{\omega}_\chi \times \mathbf{M} \tag{9}$$

where $\tilde{d}\ldots/ds$ is the local derivative; $\boldsymbol{\omega}_\chi$ is the Darboux vector, calculated through the formula

$$\boldsymbol{\omega}_\chi = k_R\mathbf{b} + (k_T + d\chi/ds)\boldsymbol{\tau} \tag{10}$$

Taking into consideration Eqs. (9), (10), one can rewrite Eq. (8) in the following form

$$\frac{\tilde{d}\mathbf{F}}{ds} = -\boldsymbol{\omega}_\chi \times \mathbf{F} - \mathbf{f}, \qquad \frac{\tilde{d}\mathbf{M}}{ds} = -\boldsymbol{\omega}_\chi \times \mathbf{M} - \boldsymbol{\tau} \times \mathbf{F} - \mathbf{m} \tag{11}$$

So the vectors **F**, **M**, $\tilde{d}\mathbf{F}/ds$, $\tilde{d}\mathbf{M}/ds$ and $\boldsymbol{\omega}_\chi$ have the components F_u, F_v, F_w, M_u, M_v, M_w, dF_u/ds, dF_v/ds, dF_w/ds, dM_u/ds, dM_v/ds, dM_w/ds and p, q, r, respectively. Then Eq. (8) can be represented in the scalar form

$$\begin{aligned} dF_u/ds &= -qF_w + rF_v - f_u \\ dF_v/ds &= -rF_u + pF_w - f_v \\ dF_w/ds &= -pF_v + qF_u - f_w \end{aligned} \tag{12}$$

for the forces equilibrium and in the same form

$$\begin{aligned} dp/ds &= (Brq - Cqr + F_v - m_u)/A \\ dq/ds &= (Cpr - Arp - F_u - m_v)/B \\ dr/ds &= (Aqp - Bpq - m_w)/C \end{aligned} \tag{13}$$

for the moments equilibrium.

As a rule, in solving the practical problems for a curvilinear rod with complicated axial line, it is difficult to choose the s variable, which parametrises the geometry of the rod. Thus, one is forced to select, instead, some dimensionless parameter ϑ. In this case, the substitution $ds = Dd\vartheta$ should be performed, where D is the metric multiplier calculated by the following formula

$$D = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

where the dot above a letter denotes differentiation with respect to ϑ.

With this substitution and introduction of the designations $\chi = h_1$, $d\chi/d\vartheta = dh_1/d\vartheta = h_2$, Eqs. (12), (13) are brought to the form

$$\frac{dF_u}{Dd\vartheta} = \left(k_T + \frac{h_2}{D}\right)F_v - k_R \cos h_1 \cdot F_w - f_u^{gr} - f_u^c$$

$$\frac{dF_v}{Dd\vartheta} = k_R \sin h_1 \cdot F_w - \left(k_T + \frac{h_2}{D}\right) \cdot F_u - f_v^{gr} - f_v^c$$

$$\frac{dF_w}{Dd\vartheta} = k_R \cos h_1 \cdot F_u - k_R \sin h_1 \cdot F_v - f_w^{gr} - f_w^{fr} \tag{14}$$

$$\frac{1}{D}\left[\frac{dk_R}{d\vartheta}\sin h_1 + k_R \cos h_1 \cdot h_2\right] = \frac{A-C}{A} \cdot k_R \cos h_1\left(k_T + \frac{h_2}{D}\right) + \frac{F_v}{A}$$

$$\frac{1}{D}\left[\frac{dk_R}{d\vartheta}\cos h_1 - k_R \sin h_1 \cdot h_2\right] = \frac{C-A}{A} \cdot k_R \sin h_1\left(k_T + \frac{h_2}{D}\right) - \frac{F_u}{A}$$

$$\frac{1}{D}\left[\frac{dk_T}{d\vartheta} - \frac{1}{D^2}\frac{dD}{d\vartheta}h_2 + \frac{1}{D}\frac{dh_2}{d\vartheta}\right] = -\frac{m_w^{fr}}{C} \tag{15}$$

In this system, four functions $F_u(\vartheta), F_v(\vartheta), F_w(\vartheta), h_1(\vartheta)$, which determine the stress-strain state of the drill string, are required. The external distributed forces of contact $(f_u^c(\vartheta), f_v^c(\vartheta))$, friction interaction $(f_w^{fr}(\vartheta))$ and friction moment $m_w^{fr}(\vartheta)$ are unknown as well. At the same time, the components $f_u^{gr}, f_v^{gr}, f_w^{gr}$ are considered to be active and given. They can be determined by the equalities

$$f_u^{gr} = -Fg(\gamma_t - \gamma_l)(n_z \cos h_1 + b_z \sin h_1)$$

$$f_v^{gr} = Fg(\gamma_t - \gamma_l)(n_z \sin h_1 - b_z \cos h_1)$$

$$f_w^{gr} = -Fg(\gamma_t - \gamma_l)\,\tau_z \tag{16}$$

where F is the area of the cross-section of the drill string, γ_t is the tube material density, and γ_l is the density of the washing liquid.

To construct the system of constitutive equations, one may rewrite Eq. (15) in the form

$$F_u = (C-A)k_R k_T \sin h_1 + C k_R \frac{h_2}{D} \sin h_1 - \frac{A}{D}\frac{dk_R}{d\vartheta}\cos h_1$$

$$F_v = (C-A)k_R k_T \cos h_1 + C k_R \frac{h_2}{D} \cos h_1 + \frac{A}{D}\frac{dk_R}{d\vartheta}\sin h_1$$

$$\frac{dh_2}{d\vartheta} = \frac{dD}{Dd\vartheta}h_2 - D\frac{dk_T}{d\vartheta} - \frac{D^2}{C}m_w^{fr} \tag{17}$$

Differentiate both parts of the two first equations in Eq. (17) with respect to ϑ and compare the result with the two first equations in Eq. (14). Thereafter, using the last equation of Eq. (17), one can obtain the equations for calculating the distributed contact forces as follows

$$\begin{aligned} f_u^c = {} & m_w^{fr} k_R \sin h_1 + 2Ak_T\frac{dk_R}{Dd\vartheta}\sin h_1 - Ck_T\frac{dk_R}{Dd\vartheta}\sin h_1 - \frac{Ch_2}{D^2}\cdot\frac{dk_R}{d\vartheta}\sin h_1 + \\ & + Ak_R\frac{dk_T}{Dd\vartheta}\sin h_1 - F_w k_R \cos h_1 + Ck_R k_T^2 \cos h_1 - Ak_R k_T^2 \cos h_1 + \\ & + Ck_R k_T \frac{h_2}{D}\cos h_1 + A\frac{d}{Dd\vartheta}\left(\frac{dk_R}{Dd\vartheta}\right)\cos h_1 - f_u^{gr} \\ f_v^c = {} & m_w^{fr} k_R \cos h_1 + 2Ak_T\frac{dk_R}{Dd\vartheta}\cos h_1 - Ck_T\frac{dk_R}{Dd\vartheta}\cos h_1 - \frac{Ch_2}{D^2}\cdot\frac{dk_R}{d\vartheta}\cos h_1 + \\ & + Ak_R\frac{dk_T}{Dd\vartheta}\cos h_1 + F_w k_R \sin h_1 - Ck_R k_T^2 \sin h_1 + Ak_R k_T^2 \sin h_1 - \\ & - Ck_R k_T \frac{h_2}{D}\sin h_1 - A\frac{d}{Dd\vartheta}\left(\frac{dk_R}{Dd\vartheta}\right)\sin h_1 - f_v^{gr} \end{aligned} \tag{18}$$

Generally, at functioning the DS performs axial motion with velocity $\dot{w}$ and rotates with angular velocity ω at a time. Then the total friction force as given below

$$|f^{fr}| = \mu \cdot |f^c| = \mu\sqrt{(f_u^c)^2 + (f_v^c)^2}$$

can be decomposed into the axial and circumferential components

$$f_w^{fr} = \pm\mu \cdot f^c \frac{\dot{w}}{\sqrt{\dot{w}^2 + (\omega d/2)^2}}, \qquad f_\omega^{fr} = \pm\mu \cdot f^c \frac{\omega d}{2\sqrt{\dot{w}^2 + (\omega d/2)^2}} \tag{19}$$

which are proportional to the appropriate components of velocities $\dot{w}$ and $\omega d/2$. Here d is the external diameter of the drill string tube.

The first component in Eq. (19) impedes the axial movement of the drill string, while the second occurs in the circumferential direction, representing the distributed friction moment

$$m_w^{fr} = f_\omega^{fr} \cdot \frac{d}{2} = \pm\mu \cdot f^c \frac{\omega d^2}{4\sqrt{\dot{w}^2 + (\omega d/2)^2}} \tag{20}$$

The signs "±" in Eqs. (19) and (20) should be chosen depending on the directions of movement and rotation of the drill string. The sign "–" in the expression for f_w^{fr} corresponds to the procedure of raising of the drill string, whereas the sign "+" relates to the lowering and process of drilling.

Hence the stress-strain state of the drill string in a curvilinear bore-hole is described by the third order differential equation system

$$\frac{dh_1}{d\vartheta} = h_2$$

$$\frac{dh_2}{d\vartheta} = \frac{dD}{Dd\vartheta}h_2 - D\frac{dk_T}{d\vartheta} - \frac{D^2}{C}m_w^{fr}$$

$$\frac{dF_w}{d\vartheta} = D \cdot k_R \cos h_1 \cdot F_u - D \cdot k_R \sin h_1 \cdot F_v - D \cdot f_w^{gr} - D \cdot f_w^{fr} \tag{21}$$

together with equalities

$$F_u = (C-A)k_R k_T \sin h_1 + Ck_R\frac{h_2}{D}\sin h_1 - \frac{A}{D}\frac{dk_R}{d\vartheta}\cos h_1$$

$$F_v = (C-A)k_R k_T \cos h_1 + Ck_R\frac{h_2}{D}\cos h_1 + \frac{A}{D}\frac{dk_R}{d\vartheta}\sin h_1 \tag{22}$$

The external force factors $f_u^c, f_v^c, f_w^{fr}, f_\omega^{fr}$ and m_w^{fr} are calculated with the aid of Eqs. (18), (19), (20).

Using the equations derived above, one can determine the external and internal force factors initiated in movement of the drill string inside a bore-hole with arbitrary trajectories of axial line. The trajectory can also include any geometrical imperfections, which yet are described by Eq. (1) and are differentiable.

3. Analytical simulation of a bore-hole axial line with localized geometrical breaks

Let us assume that the bore-hole trajectory was originally conceived as a quarter of an ellipse, as shown in Fig. 1

$$x = L\cos\vartheta, \quad y = 0, \quad z = H\sin\vartheta \quad (3\pi/2 \le \vartheta \le 2\pi) \tag{23}$$

where H and L are the depth and the horizontal distance between the top and lower ends of the drill string, respectively.

But in reality it is not possible to ensure exactly the designed outline of the bore-hole axis and some distortions will be introduced into the geometry. The localized breaks with smoothed angles are the most commonly encountered shapes of distortions. The breaks of this sort can be approximated by superimposing apexes of hyperbolas with different sharpnesses (eccentricities) and angles between their asymptotes, as exemplified in Fig. 2. If the asymptotes of the left branches of two similar hyperbolas with different signs are oriented along the negative direction of the Ox axis (Fig. 2(a) and (b)), then their superposition with small shift Δs in plane xOz approximately produces a broken rectilinear line with small shift Δz and curvature radii R_h at the break apexes (Fig. 2(c)). If

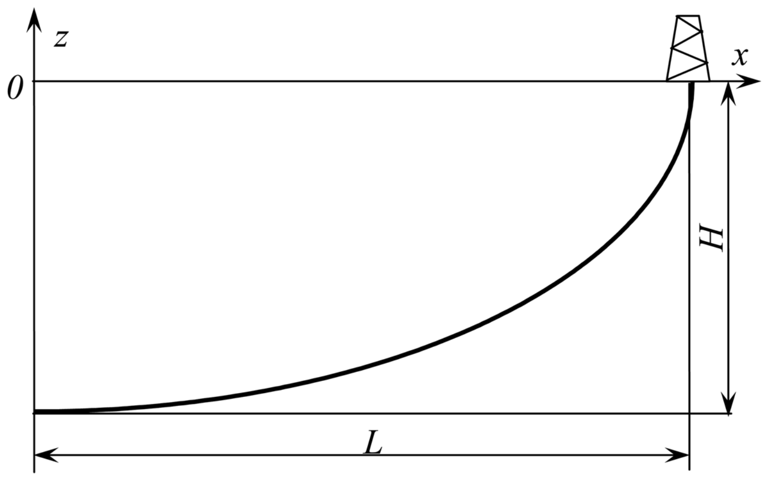

Fig. 1 Curvilinear bore-hole with ideal elliptic trajectory

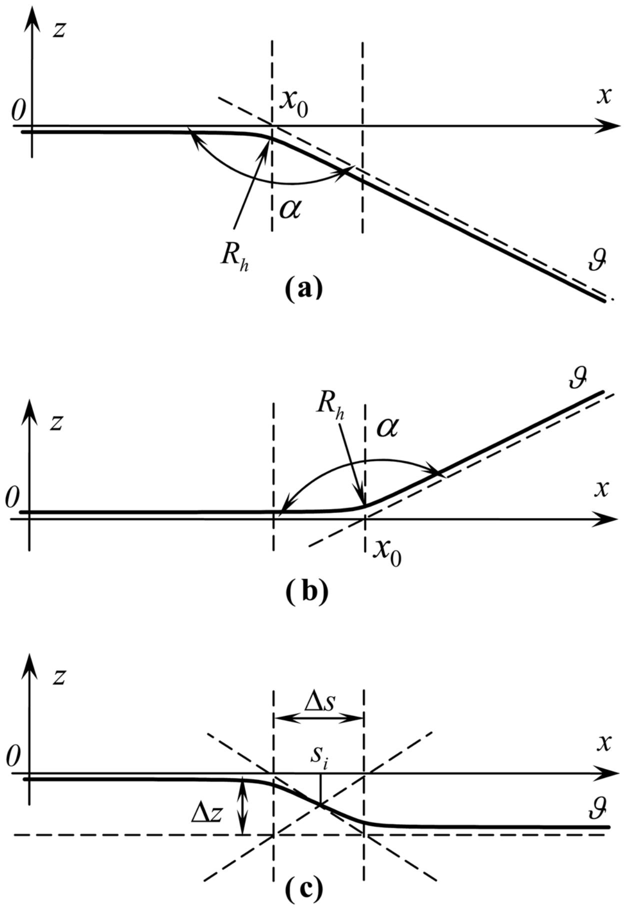

Fig. 2 Modelling a localized break of the bore-hole trajectory

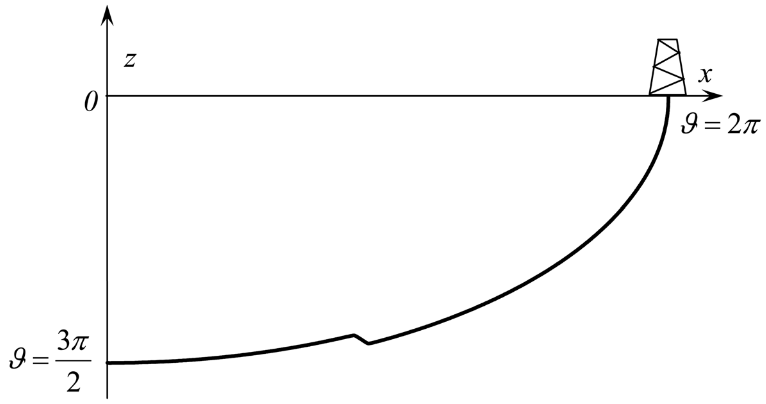

Fig. 3 Trajectory of a curvilinear bore-hole with a localized break

one imposes this line on the initial trajectory as given in Eq. (23), then the ultimate trajectory of the drill string will attain the configuration shown in Fig. 3.

The analytical expression of the hyperbola illustrated in Fig. 2(a) is represented in the form

$$z = -\left\{\frac{\mathrm{tg}\,\alpha(x-x_0)}{2} + \sqrt{\left[\frac{\mathrm{tg}\,\alpha(x-x_0)}{2}\right]^2 - \varepsilon}\right\} \tag{24}$$

where α is the angle between the hyperbola asymptotes, x_0 is the x coordinate of the hyperbola center, and ε is the parameter characterizing the hyperbola eccentricity.

The hyperbola shown in Fig. 2(b) is described by the equation

$$z = \left\{\frac{\mathrm{tg}\,\alpha(x-x_0)}{2} + \sqrt{\left[\frac{\mathrm{tg}\,\alpha(x-x_0)}{2}\right]^2 - \varepsilon}\right\} \tag{25}$$

The breaks can be located also out of the plane, then their expressions are as follows

$$y = \pm\left\{\frac{\mathrm{tg}\,\alpha(x-x_0)}{2} + \sqrt{\left[\frac{\mathrm{tg}\,\alpha(x-x_0)}{2}\right]^2 - \varepsilon}\right\} \tag{26}$$

In the general case, the planar and three-dimensional imperfections can be located at several places and have different radii R_h and angles α. As such, they should be simulated by superposition of different breaks as given in Eqs. (24), (25) or (26) on the design trajectory of Eq. (23).

4. Influence of the bore-hole trajectory breaks on the resistance forces and moments

The elaborated techniques were used for computer simulation of a drill string movement in a bore-hole channel with the assumed geometrical imperfections.

A great variety of determining factors have to be considered in drilling designs. They differ essentially by the bore-hole diameters (up to 40 cm), drill string materials (steel, aluminum,

Table 1 The curvature radii of the bore-hole axis breaks

Problem number	Type of imperfection	Break 1		Break 2		Break 3		Break 4	
		$R_{h,1}$ (m)	$R_{h,2}$ (m)	$R_{h,3}$ (m)	$R_{h,4}$ (m)	$R_{h,5}$ (m)	$R_{h,6}$ (m)	$R_{h,7}$ (m)	$R_{h,8}$ (m)
1	None	–	–	–	–	–	–	–	–
2	Planar	3676	2363	257.5	260.3	63.7	60.5	198.9	233.7
3		107	101	143.8	147.8	38.4	36.2	198.9	233.7
4	3D	2938	2762	223.9	217.4	91.7	90.5	57.9	57.2
5		132	129	91.7	90.0	70.4	69.4	57.9	57.2

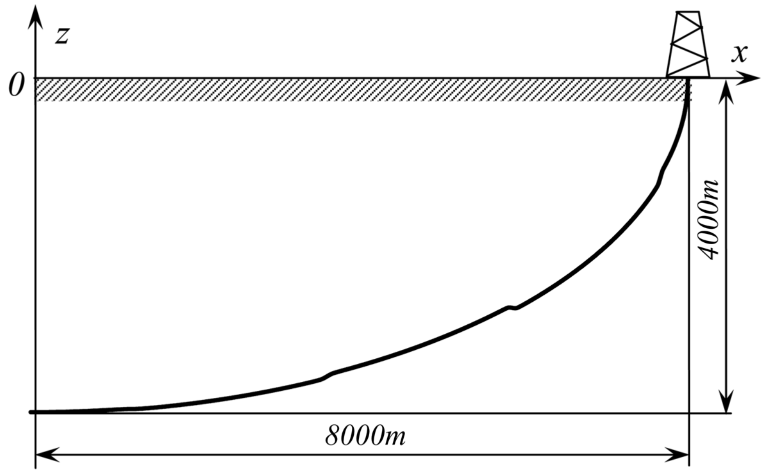

Fig. 4 The bore-hole trajectory with planar imperfections (Problem 2)

titanium, composite), horizontal distances from the rig tower (exceeding 12 km), friction coefficients (μ = $0.2 \div 0.25$), outlines of the bore-hole trajectory, and others. In our example, the following typical determining factors were chosen for the analysis: $L = 8000$ m, $H = 4000$ m, $d = 0.1683$ m, thickness of the drill string tube $\delta = 0.01$ m, $E = 2.1 \cdot 10^{11}$ Pa, $G = 0.8077 \cdot 10^{11}$ Pa, $\gamma_{st} = 7850$ kg /m^3, $\gamma_l = 1500$ kg / m^3, and $\mu = 0.2$, $\nu = 100$.

Altogether five problems were solved, as listed in Table 1. Firstly, the bore-hole without imperfection was considered (Problem 1). Thereupon four breaks were introduced into its trajectory, two problems (2 and 3) were associated with four planar breaks as described by Eqs. (24), (25) and other two problems (4 and 5) were associated with breaks oriented out of the *xOz* plane, as described by Eq. (26).

The break centers s_i were located at points $s_1 = S/8$, $s_2 = 3S/8$, $s_3 = 5S/8$, $s_4 = 7S/8$ for Problems 2 and 4 and $s_1 = 4S/8$, $s_2 = 5S/8$, $s_3 = 6S/8$, $s_4 = 7S/8$ for Problems 3 and 5. The adjacent hyperbola apexes were spaced at points $s_i \pm 50$ m. Here S is the length of the bore-hole with ideal elliptic geometry. It is calculated as follows

$$S = \int_{3\pi/2}^{2\pi} D(\vartheta)d\vartheta = 9688m$$

In all cases, the hyperbola angle is selected as $\alpha = 0.983\,rad$, but the curvature radius $R_{h,j}$ was allowed to be different as listed in Table 1.

As an example, in Fig. 4 the bore-hole trajectory for Problem 2 is shown. The first break is not visually discernible in this sketch, so it is not essential.

Using the initial data presented above, the process of raising was considered for the drill string. The end $\vartheta = 3\pi/2$ was assumed to be free from the axial force F_w and torque M_w, which permitted us to state the Cauchy problem for Eq. (21) with the initial conditions $F_w(3\pi/2) = 0$, $h_1(3\pi/2) = 0$, $h_2(3\pi/2) = -k_T(3\pi/2)$. Integration of Eq. (21), together with Eqs. (22), (18) - (20), was performed by the Runge-Kutta method. The integration step $\Delta\vartheta = (2\pi - 3\pi/2)/8000$ was chosen based on the condition of calculation convergence.

For the purpose of comparison, all the results obtained have been listed in Table 2, including those for the axial force $F_w(S)$; the ratio $F_w(S)/P_t$ of axial force to tension force $P_t = F(\gamma_t - \gamma_l)S = 3001427N$, acting on the drill string tube inserted into a vertical rectilinear bore-hole filled by mud; the total elastic elongation of the drill string tube during its raising in the curvilinear bore-hole

Table 2 The parameters of the stress-strain states of the drill strings

Problem number	Type of imperfection	$F_w(S)\,(N)$	$F_w(S)/P$ –	$\Delta S\,(m)$	$M_w(S)\,(Nm)$	$\phi(S)\,(rad)$
1	None	$1.765 \cdot 10^6$	0.59	5.74	442	0.82
2	Planar	$2.210 \cdot 10^6$	0.74	6.95	803	1.22
3		$2.791 \cdot 10^6$	0.93	8.18	1301	1.66
4	3D	$2.525 \cdot 10^6$	0.84	7.20	1082	1.32
5		$3.191 \cdot 10^6$	1.06	8.59	1642	1.79

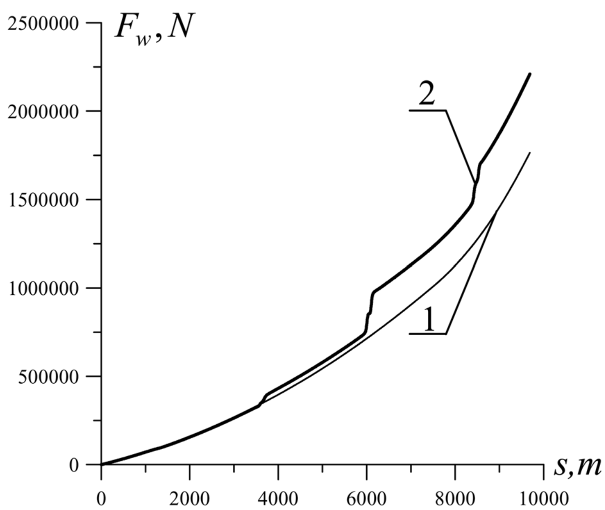

Fig. 5 Internal axial force F_w vs. longitudinal coordinate s for the ideal bore-hole (curve 1) and bore-hole with imperfections (curve 2) (Problem 2)

$$\Delta S = \frac{1}{EF}\int_{3\pi/2}^{2\pi} F_w(\vartheta)Dd\vartheta$$

the torque $M_w(S)$ and the angle $\varphi(S)$ of the elastic twist of the drill string at end $s = S$

$$\varphi(S) = \frac{1}{GI_w}\int_{3\pi/2}^{2\pi} M_w(\vartheta)Dd\vartheta$$

Some of the results obtained for Problem 2 have been plotted schematically in Figs. 5-9. Figs. 5 and 6 show the longitudinal force F_w and torque M_w, respectively, as functions of s. Curves 1 and 2 correspond to Problems 1 and 2, respectively. One can see that if the bore-hole does not have imperfections, functions $F_w(s)$ and $M_w(s)$ are smooth and possess comparatively small values. But the introduction of insignificant imperfections (see Fig. 4) causes discontinuities in the outlines of functions $F_w(s)$, $M_w(s)$ and essential enlargements of their values (curves 2 in Figs. 5 and 6).

As can be seen from Fig. 7, the function of distributed contact force $f^c(s)$ also has discontinuous character; it rises steeply at the imperfection zones.

The functions of distributed friction force $f_w^{fr}(s)$ and moment $m_w^{fr}(s)$ differ from function $f^c(s)$ by coefficients only. So they have the similar characteristics (Figs. 8 and 9).

The resultant bending moment

$$M_R(s) = \sqrt{M_u^2 + M_v^2}$$

depends solely upon the curvature k_R

$$M_R = EIk_R$$

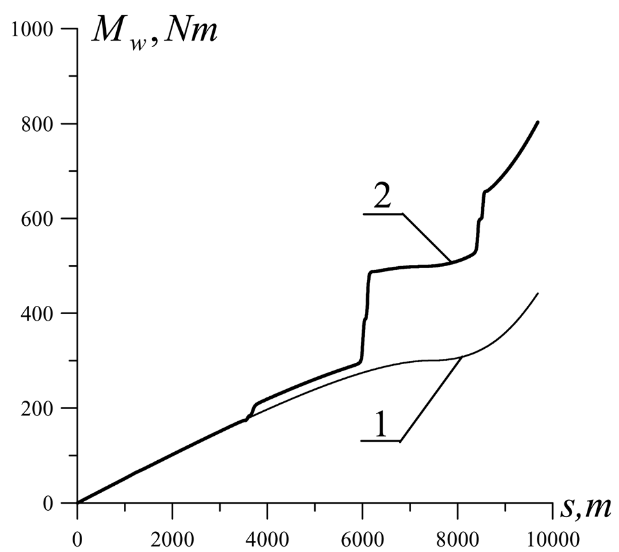

Fig. 6 Internal axial torque M_w vs. longitudinal coordinate s for the ideal bore-hole (curve 1) and bore-hole with imperfections (curve 2) (Problem 2)

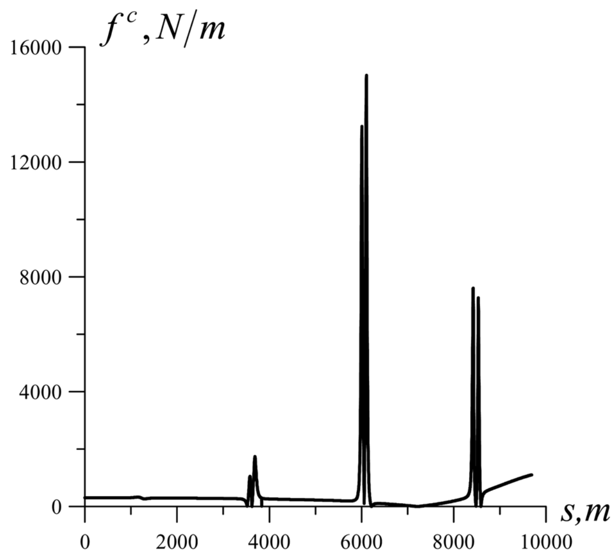

Fig. 7 Distributed contact force f^c vs. longitudinal coordinate s for the bore-hole with imperfections (Problem 2)

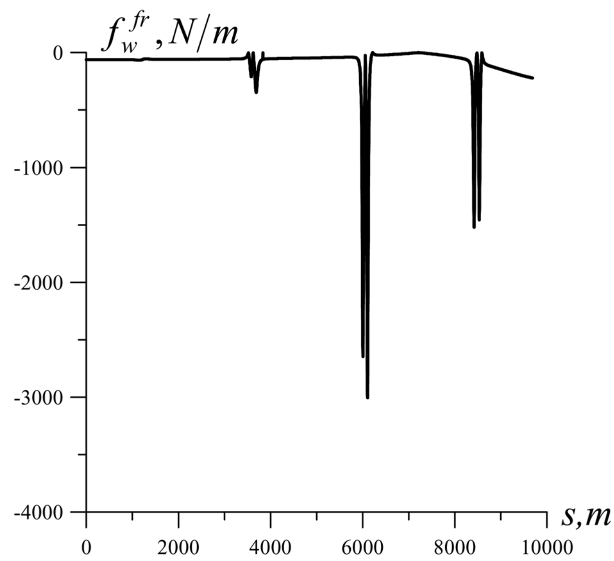

Fig. 8 Distributed friction force f_w^{fr} vs. coordinate s for the bore-hole with imperfections (Problem 2)

As can be seen from Fig. 10, it has nearly discontinuous shape as well.

The calculations above testify that the resistance forces and moments increase somewhat for the three-dimensional imperfections and they also increase when the imperfections are shifted to the upper part of the bore-hole (Problems 3, 5 and 2, 4 in Tables 1 and 2).

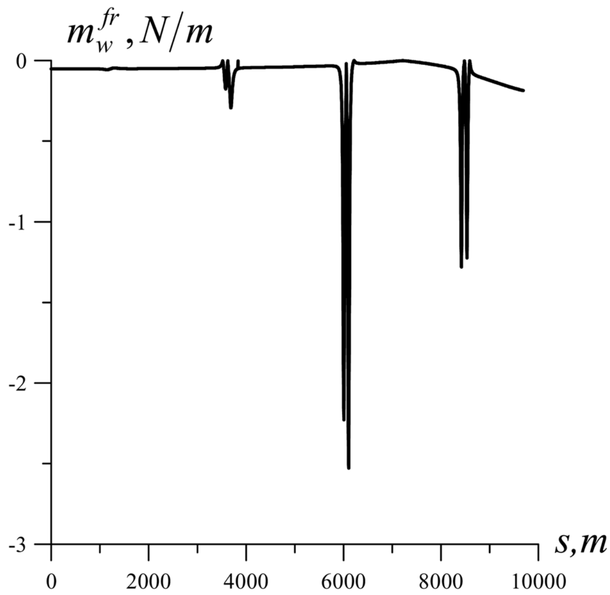

Fig. 9 Distributed friction moment m_w^{fr} vs. coordinate s (Problem 2)

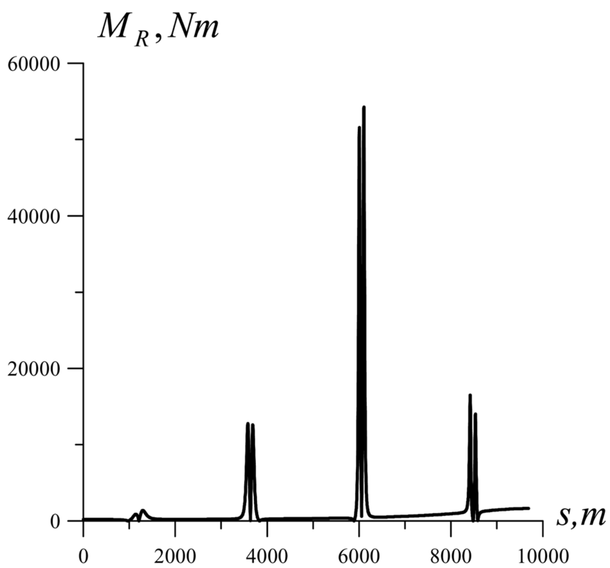

Fig. 10 Internal resultant bending moment M_R vs. coordinate s (Problem 2)

5. Conclusions

1. This paper presents the direct and inverse problems of an elastic rod bending inside a curvilinear channel for evaluating the distortion influence of a bore-hole trajectory on the

contact and friction forces encountered during the movement of a drill string inside the channel.

2. The techniques for geometrical modelling the bore-hole imperfections in the shapes of planar and three-dimensional breaks are presented.
3. Computer simulation of the process of the drill string raising inside a curvilinear bore-hole is performed. The results obtained testify that the functions of contact and friction forces generated by interaction of the drill string with the bore-hole rise steeply in the zones of the imperfection origination. This effect manifests itself more distinctly for imperfections shifted to the upper end of the bore-hole.
4. The present techniques can be used in analysis of the contact and friction force sensitivity to the breaks of bore-hole trajectories at the stages of design and drivage.

References

Aadnoy, B.S., Larsen, K. and Berg, P.C. (2003), "Analysis of stuck pipe in deviated boreholes", *J. Petrol. Sci. Eng.,* **37**, 195-212.

Aadnoy, B.S. and Andersen, K. (2001), "Design of oil wells using analytical friction models", *J. Petrol. Sci. Eng.,* **32**, 53-71.

Brett, J.F., Beckett, A.D., Holt, C.A. and Smith, D.L. (1989), "Uses and limitations of drillsting tension and torgue models for monitoring hole conditions", *SPE Drill. Eng.*, **4**, 223-229.

Choe, J., Schubert, J.J. and Juvkam-Wold, H.C. (2005), "Well-control analyses on extended-reach and multilateral trajectories", *SPE Drill. Completion,* June, 101-108.

Chow, J, Kopp, R.J. and Portney, P.R. (2003), "Energy resources and global development", *Science,* **302**, 1528-1531.

Gulyayev, V.I., Gaidaichuk, V.V. and Koshkin, V.L. (1992), *Elastic deforming, stability and vibrations of flexible curvilinear rods,* Naukova Dumka, Kiev. (in Russian)

Gulyaev, V.I., Gaidaichuk, V.V., Solov'ev, I.L. and Glovach, L.V. (2007a), "Numerical simulation of the resistance forces acting on the curvilinear drill strings", *Probl., Strength*, **5**(389), 55-67.

Gulyaev, V.I., Lugovoi, P.Z., Khudoliy, S.N. and Glovach, L.V. (2007b), "Theoretical identification of resistance forces to the longitudinal displacement of the drill columns in curvilinear wells", *Int. Appl. Mech.*, **43**(11), 80-89.

Iyoho, A.W., Meize, R.A., Millheim, K.K. and Grumrine, M.J. (2005), "Lessons from integrated analysis of GOM drilling performance", *SPE Drill. Completion*, March, 6-16.

Kerr, R.A. (2005), "Bumpy road ahead for world's oil", *Science*, **310**, 1106-1108.

Mohiuddin, M.A., Khan, K., Abdulraheem, A., Al-Majed, A. and Awall, M.R. (2006), "Analysis of wellbore instability in vertical, directional and horizontal wells using field data", *J. Petrol. Sci. Eng.*, **55**, 83-92.

Pourcian, R.D., Fisk, J.H., Descant, F.J. and Waltman, R.B. (2005), "Completion and well-performance results, genesis field, deepwater Gulf of Mexico", *SPE Drill. Completion*, June, 147-155.

Sawaryn, S.J. and Thorogood, J.L. (2005), "A compendium of directional calculations based on the minimum curvature method", *SPE Drill. Completion*, March, 24-36.

Sawaryn, S.J., Sanstrom, B. and McColpin, G. (2006), "The management of drilling-engineering and well-services software as safety-critical systems", *SPE Drill. Completion*, June, 141-147.

Sheppard, M.C. (1987), "Designing well paths to reduce drag and torque", *SPE Drill. Eng.*, December, 344- 350.

Stuart, D., Hamer, C.D., Henderson, C., Gaynor, T. and Chen, D. C-K (2003), "New drilling technology reduces torque and drag by drilling a smooth and wellbore", *SPE IADC Drilling Conference*, Amsterdam, Netherlands, 83-88.