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Transversely isotropic thick plate with two temperature & GN type-III in frequency domain

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Abstract. This investigation is focused on the variations in transversely isotropic thick circular plate due to time harmonic thermomechanical sources. The homogeneous thick circular plate in presence and absence of energy dissipation and two temperatures has been considered. Hankel transform is used for solving field equations. The analytical expressions of conductive temperature, displacement components, and stress components are computed in the transformed domain. The effects of frequency at different values are represented graphically. Some specific cases are also figured out from the current research.

Keywords: frequency; hankel transformation; thermoelastic; thick circular plate; time harmonic sources; transversely isotropic

1. Introduction

Classical theory (CT) of elasticity is concerned with the systematic study of the stress and strain distribution that develops in an elastic body due to the application of forces or change in temperature. Thermoelasticity in presence of two temperatures represents an overview of both theories i.e., theory of heat conduction and theory of elasticity in solids. Temperature changes cause thermal effects on materials like thermal stress, strain, and deformation. Thermal dependency is the primary contrast of thermoelasticity concerning to the classical theory of thermomechanics. It may also be mentioned that modern laminated media which are being used more and more in engineering and other applications, behave anisotropically locally (thermally and elastically). Thus, there is imperative need to consider the anisotropic media particularly transversely isotropic. However, due to a greater number of elastic and thermal coefficients involved, there are not so many solutions available as there are for isotropic media.

Chen *et al.* (1968a, 1968b, 1969) formulated a two-temperature thermoelasticity of deformable bodies for the conduction of heat depending on two types of temperatures. Green and Naghdi (1991, 1992, 1993) dealt with the linear and the nonlinear theories of thermoelastic body in presence and absence of energy dissipation. Three novel thermoelastic theories were proposed by them, based on entropy equality. Their theories are known as thermoelasticity type I theory, the thermoelasticity type II theory (i.e., thermoelasticity without energy dissipation) and the

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effects of frequency at different values have been represented graphically.

2. Basic equations

The field equations and basic relations for an anisotropic thermoelastic medium in Green-Naghdi type-III theory in absence of heat source and body forces following Chandrasekharaiah (1998), Youssef (2011) and Green and Naghdi (1992) are

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T, \tag{1}$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\,\ddot{u}_i, \tag{2}$$

$$K_{ij}\varphi_{,ij} + K^*_{ij}\dot{\varphi}_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T}, \tag{3}$$

Where

$$T = \varphi - a_{ij}\varphi_{,ij}, \tag{4}$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij}, \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i,j = 1,2,3. \tag{6}$$

$\beta_{ij} = \beta_i\delta_{ij}, \;\; K_{ij} = K_i\delta_{ij}, \;\; K^*_{ij} = K^*_i\delta_{ij}$, i is not summed,

3. Formulation of the problem

We take a transversely isotropic thick circular plate with thickness 2b covering the area D given by $0 \le r \le \infty$,$-b \le z \le b$ and an axisymmetric heat source is used on its axial and radial direction. We take a cylindrical polar co-ordinate system (r,θ,z) with symmetry about Z-axis. The initial temperature in the transversely isotropic thick circular plate is assumed by a constant temperature T_0 and heat flux $g_0F(r,z)$ prescribed on the lower and upper surfaces. For the axisymmetric plane, the field component$(v = 0)$, and $(u, w, and\ \varphi)$ are independent of θ and our research become 2D problem with $\vec{u} = (u, 0, w)$. In addition, the equations for transversely isotropic thermoelastic solid without energy dissipation and with two temperature, using the proper transformation on Eqs. (1)-(3) following Slaughter (2002) are as under

$$C_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r}u\right) + C_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + C_{44}\frac{\partial^2 u}{\partial z^2} + C_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right) - \beta_1\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) - a_3\frac{\partial^2 \varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$(C_{11} + C_{44})\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right) + C_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}\left\{\varphi - a_1\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}\right) - a_3\frac{\partial^2 \varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{8}$$

$$(-\xi^2+\omega^2+\delta_2 D^2)\tilde{u}+(1-\xi)\delta_1 D\widetilde{w}+\left(-(1-\xi)(1-a_3D^2)+a_1\xi^3)\right)\tilde{\varphi}=0 \quad (17)$$

$$(1-\xi)\delta_1 D\tilde{u}+\ (\delta_3 D^2-\xi^2\delta_2+\omega^2)\widetilde{w}-\frac{\beta_3}{\beta_1}D(1+\xi^2 a_1-a_3D^2)\tilde{\varphi}=0, \quad (18)$$

$$\delta_6\omega^2(1-\xi)\tilde{u}+\frac{\beta_3}{\beta_1}\delta_6\omega^2 D\widetilde{w} + \left(\delta_7\omega^2(1+\xi^2 a_1-a_3D^2)+\xi^2(K_1+\delta_4\omega)-D^2(K_3+\delta_5\omega)\right)\tilde{\varphi}=0, \quad (19)$$

where

$$\delta_1=\frac{c_{13}+c_{44}}{c_{11}},\qquad \delta_2=\frac{c_{44}}{c_{11}},\qquad \delta_3=\frac{c_{33}}{c_{11}},\qquad \delta_4=\frac{K_1^*C_1 i}{L},$$

$$\delta_5=\frac{K_3^*C_1 i}{L},\ \ \delta_6=-\ \frac{T_0\beta_1^2}{\rho},$$

$$\delta_7=-\rho C_E C_1^2,\ i=\sqrt{-1}\ .$$

$$\widetilde{t_{zz}}=\ \sum A_i(\xi,\omega)\eta_i cosh(q_i z)+\sum \mu_i\, A_i(\xi,\omega)\sinh(q_i z) \quad (20)$$

$$\widetilde{t_{rz}}=\sum A_i(\xi,\omega)d_i cosh(q_i z)+\xi\sum A_i(\xi,\omega)q_i sinh(q_i z), \quad (21)$$

$$\widetilde{t_{rr}}=\sum A_i(\xi,\omega)R_i cosh(q_i z)+\sum S_i\, A_i(\xi,\omega)\sinh(q_i z), \quad (22)$$

$$\widetilde{t_{\theta\theta}}=\sum A_i(\xi,\omega)M_i cosh(q_i z)+\sum N_i\, A_i(\xi,\omega)\sinh(q_i z), \quad (23)$$

where

$$\eta_i=\ \delta_9\xi-\frac{\beta_3}{\beta_1}(1+a_1\xi^2)\mathrm{l_i}-\frac{\beta_3}{\beta_1}\mathrm{a_3 l_i q_i^2},$$

$$\mu_i=(\delta_9+\delta_3 d_i)q_i,$$

$$M_i=\left(1+\frac{\xi}{2}\right)-\frac{\beta_3}{\beta_1}(1+a_1\xi^2)\mathrm{l_i}+\frac{\beta_3}{\beta_1}\mathrm{a_3 l_i q_i^2},$$

$$N_i=(\delta_8+\delta_9 d_i)q_i,$$

$$R_i=\delta_8\left(1+\frac{\xi}{2}\right)-l_i(1+a_1\xi^2)+\mathrm{a_3}l_i\mathrm{q_i^2},$$

$$S_i=q_i(1+\delta_3 d_i),\qquad i=1,2,3.$$

The non-trivial solution of (17)-(19) yields

$$(AD^6+BD^4+CD^2+E)(\tilde{u},\widetilde{w},\tilde{\varphi})=0. \quad (24)$$

$$d_i = \frac{\delta_2\zeta_8 q_i^4 + (\zeta_8\zeta_1 - \zeta_4\zeta_6 + \delta_2\zeta_9)q_i^2 + \zeta_1\zeta_9 - \zeta_6\zeta_3}{(\delta_3\zeta_8 - \zeta_7\zeta_{11})q_i^4 + (\delta_3\zeta_9 + \zeta_5\zeta_8 - \zeta_7\zeta_{10})q_i^2 + \zeta_5\zeta_9},$$

$$l_i = \frac{\delta_2\delta_3 q_i^4 + (\delta_2\zeta_5 + \zeta_1\delta_3 - \zeta_2^2)q_i^2 + \zeta_1\zeta_5}{(\delta_3\zeta_8 - \zeta_7\zeta_{11})q_i^4 + (\delta_3\zeta_9 + \zeta_5\zeta_8 - \zeta_7\zeta_{10})q_i^2 + \zeta_5\zeta_9}.$$

4. Boundary conditions

We contemplate a cubiform thermal source and normal force of unit magnitude with dispersing of tangential stress at the stress free surface at z = ±b. scientifically, these can be written as

$$\frac{\partial\varphi}{\partial z} = \pm g_o F(r,z), \tag{28}$$

$$t_{zz}(r,z,t) = f(r,t), \tag{29}$$

$$t_{rz}(r,z,t) = 0. \tag{30}$$

By putting the values $\tilde{\varphi}$, $\widetilde{t_{zz}}$, $\widetilde{t_{rz}}$ from (20)-(20) and (27) in boundary conditions (28)-(30) and applying Hankel transform on the resulting equations yields

$$\sum A_i\,(\xi,\omega) l_i q_i \vartheta_i = \pm g_o \tilde{F}(\xi,z), \tag{31}$$

$$\sum A_i(\xi,\omega)\eta_i\theta_i + \sum \mu_i\, A_i(\xi,\omega)\vartheta_i = \tilde{f}(\xi,\omega), \tag{32}$$

$$\sum A_i(\xi,\omega)(\delta_2 q_i\vartheta_i + (1-\xi)l_i\theta_i) = 0. \tag{33}$$

solving (31)-(33) for A_i, and putting in (25)-(27) and (20)-(23) we get the different components of displacement, conductive temperature and stress components as

$$\tilde{u} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1\theta_1 + \chi_2\theta_2 - \chi_3\theta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4\theta_1 - \chi_5\,\theta_2 + \chi_6\theta_3\}, \tag{34}$$

$$\tilde{w} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 d_1\theta_1 + \chi_2 d_2\theta_2 - \chi_3 d_3\theta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4\, d_1\,\theta_1 - \chi_5\, d_2\theta_2 + \chi_6 d_3\theta_3\}, \tag{35}$$

$$\tilde{\varphi} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 l_1\theta_1 + \chi_2 l_2\theta_2 - \chi_3 l_3\theta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 l_1\theta_1 - \chi_5\, l_2\theta_2 + \chi_6 l_3\theta_3\}, \tag{36}$$

$$\begin{aligned}\widetilde{t_{zz}} = &\frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1(\eta_1\theta_1 + \mu_1\vartheta_1) + \chi_2(\eta_2\theta_2 + \mu_2\vartheta_2) - \chi_3(\eta_3\theta_3 + \mu_3\vartheta_3)\}\\ &+ \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4(\eta_1\theta_1 + \mu_1\vartheta_1) - \chi_5\,(\eta_2\theta_2 + \mu_2\vartheta_2) + \chi_6(\eta_3\theta_3 + \mu_3\vartheta_3)\},\end{aligned} \tag{37}$$

As an application of the problem, we take the source function $F(r, z)$ which decays exponentially as moving away from the centre of the thick circular plate in the radial direction and symmetrically increases along the axial directions is specified by

$$F(\mathrm{r,z}) = \mathrm{z}^2\mathrm{e}^{-\delta \mathrm{r}},\ \delta > 0, \tag{40}$$

$$f(r,t) = H(\alpha - r)e^{i\omega t} \tag{41}$$

Where $H(\alpha - r)$ is the Heaviside function.

Applying Hankel Transform, on Eqs. (28) and (29), gives

$$\tilde{F}(\xi, z) = \frac{z^2\delta}{(\xi^2 + \delta^2)^{\frac{3}{2}}} \tag{42}$$

$$\tilde{f}(\xi, \omega) = \frac{\alpha J_1\,(\xi\alpha)}{\xi}e^{i\omega t} \tag{43}$$

The expressions of components of displacement, stress components, can be obtained from Eqs. (34)-(39), by substituting the value of $\tilde{F}$ (ξ,z) and $\tilde{f}$ (ξ,ω) from (42) and (43).

6. Inversion of the transforms

For obtaining the solution in physical domain, invert the Hankel transforms in Eqs. (34)-(39) using

$$f(r,z,\omega) = \int_0^\infty \xi\tilde{f}(\xi,z,\omega)J_n(\xi r)d\xi. \tag{44}$$

and integrate the Eq. (44) as described in Press *et al.* (1986).

7. Numerical results and discussion

To demonstrate our theoretical results and effect of frequency and two temperature, the physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal and Singh (1980) is given as

$$c_{11} = 3.07 \times 10^{11} Nm^{-2},$$

$$c_{12} = 1.650 \times 10^{11} Nm^{-2},$$

$$c_{13} = 1.027 \times 10^{10} Nm^{-2},$$

$$c_{33} = 3.581 \times 10^{11} Nm^{-2}$$

$$c_{44} = 1.510 \times 10^{11} Nm^{-2},$$

although for ω=.25, deviations are very small. In the starting range of distance r, there is a sharp increase in the value of displacement component for the curves when the ω=0.75, whereas there is a sharp decrease in the value of displacement component for ω=0.5. It is clear that two temperature with ω have major effect on the displacement component in all the cases. Behaviour of displacement component u, is oscillatory with variance in the magnitude corresponding to the four different frequencies.

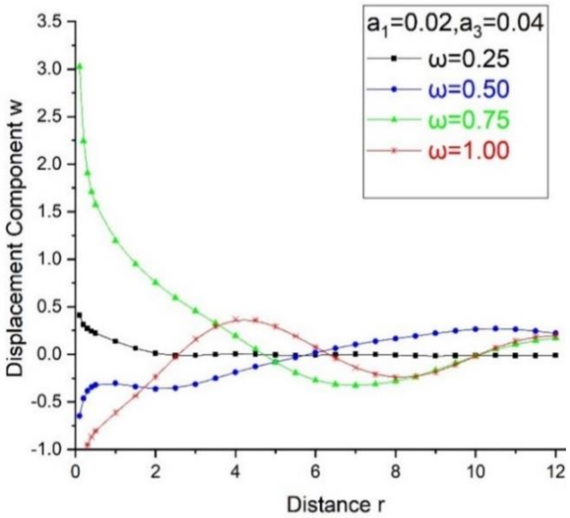

Fig. 2 Variations of displacement component w with distance r

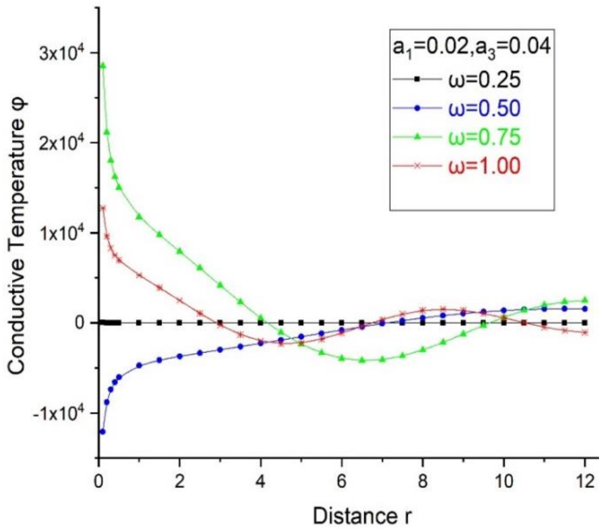

Fig. 3 Depicts the behaviour of conductive temperature φ

Fig. 2 depicts the displacement component w with distance r. In the initial range $0 \leq r \leq 4$of distance r, there is an increase in the value of displacement component when ω=0.50 and ω=1.00

Fig. 4 illustrates the deviations of tangential stress t_{zr} with r. In $0 \leq r \leq 5$ range of r, the value of t_{zr} follow oscillatory pattern for all the curves when the two temperatures are a_1 =0.02, $a_3 =$ 0.04 for *ω*=1.00, *ω*=0.75, *ω*=0.50 and *ω*=0.25.

Fig. 5 shows the deviations of normal stress t_{zz} with r. There is a sharp increase in the value of normal stress t_{zz} for a_1 =0.02, $a_3 =$ 0.04 for *ω*=1.00, *ω*=0.75, *ω*=0.50 and *ω*=0.25.for 0 < r < 2 in the range of r, and then the variations are very small.

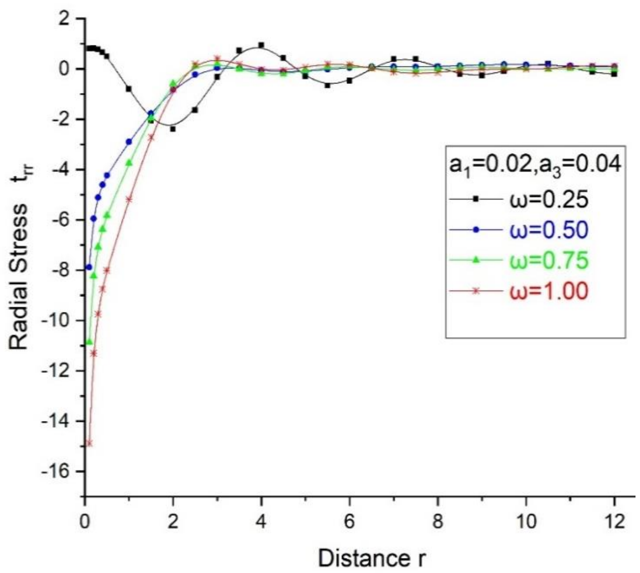

Fig. 6 Variations of radial stress t_{rr} with distance r

Fig. 6 displays the deviations of radial stress t_{rr} with r. There is a sharp increase in the value of radial stress t_{rr} with in the initial range of distance r, when the two temperatures are a_1 =0.02, $a_3 =$ 0.04 for *ω*=1.00, *ω*=0.75, *ω*=0.50 then the variations are very small. For *ω*=0.25 first it shows decrease in the value of radial stress t_{rr} with distance r and then it follows an oscillatory pattern.

8. Conclusions

From the analysis of the graphs, it is clear there is a significant influence of transversely isotropy on the deformation of various displacement components, conductive temperature and various stress components of thick circular plate while relating the influence of frequency *ω* with two temperatures. The effect of time harmonic sources frequency in transversely isotropic thick circular plate with two temperatures plays a significant role in the analysis of the deformed medium. As distance r, varied from the point of use of the time harmonic source, variations of displacement components, conductive temperature and various stress components undergoes sudden changes, causing an inconsistent patterns of curves and shows an oscillatory pattern. The shape of curves shows the impact of frequency *ω* on the body and fulfils the purpose of the study.

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Nomenclature

δ_{ij}	Kronecker delta,
C_{ijkl}	Elastic parameters,
β_{ij}	Thermal elastic coupling tensor,
T	Absolute temperature,

T_0	Reference temperature,
φ	conductive temperature,
t_{ij}	Stress tensors,
e_{ij}	Strain tensors,
u_i	Components of displacement,
ρ	Medium density,
C_E	Specific heat,
a_{ij}	Two temperature parameters,
α_{ij}	Linear thermal expansion coefficient,
K_{ij}	Materialistic constant,
K_{ij}^*	Thermal conductivity,
ω	Frequency