Coupled Systems Mechanics, *Vol. 7, No. 5 (2018) 611-626*
DOI: https://doi.org/10.12989/csm.2018.7.5.611

Ocean tide-induced secular variation in the Earth-Moon dynamics

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(Received May 25, 2018, Revised July 19, 2018, Accepted July 20, 2018)

Abstract. We theoretically consider a possible influence of periodic oceanic tides on non-periodic changes in the dynamics of the Earth and Moon over a long time scale. A particular emphasis will be placed on the contribution from rotating tidal waves, which rotate along the inner edge of an oceanic basin surrounded by topographic boundary. We formulate the angular momentum and the mechanical energy of the rotating tidal wave in terms of celestial parameters with regard to the Earth and Moon. The obtained formula are used to discuss how the energy dissipation in the rotating tidal wave should be relevant to the secular variation in the Earth's spin rotation and the Earth-Moon distance. We also discuss the applicability of the formula to general oceanic binary planets subject to tidal coupling.

Keywords: ocean tide; tidal energy dissipation; Kelvin wave; celestial mechanics; lunar orbit

1. Introduction

The ocean tide is the rises and falls of the sea level in a periodic manner. Tide plays an important role in the natural world; for instance, the tidal rhythm of marine organisms (Palmer 1973) and the fortnightly-cycle variation in extra strain on geological faults (Ide *et al.* 2016) are natural phenomena that are strongly governed by tidal cycle in the ocean. Tide can also give a marked influence on our daily life, especially on people enjoying marine sports such as surfing, diving and snorkeling.

From a viewpoint of mechanics, the ocean tide is mainly driven by combination of the two celestial-scale forces that act on sea water (Murray and Dermott 2000): The one is the gravitational force exerted by the Moon and the other is the centrifugal force associated with the Earth's revolution (i.e., revolution around the common center of gravity of the Moon and the Earth). The resultant force, called tidal force, generates oscillatory flow of sea water at the global scale, called tidal current.

It has been broadly accepted that the friction between tidal current and sea floor (Taylor 1919, Jeffreys 1920) as well as tidal wave scatterings in deep ocean (Egbert and Ray 2000) cause dissipation of the mechanical energy of the Earth-Moon system at a rate of several terawatts. Due to the energy dissipation, the Earth's spin has been slowing down gradually and the decline in the

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http://www.techno-press.org/?journal=csm&subpage=8 ISSN: 2234-2184 (Print), 2234-2192 (Online)

Earth's spin angular momentum is transferred to the increment in the orbital angular momentum of the Moon. This angular momentum transfer is believed to elongate the orbital length radius of the Moon at a rate of 3.82 ± 0.07 cm/year, as was confirmed by the Lunar Laser Ranging experiment based on the reflector installed on the Moon (Samain *et al*. 1998, Murphy *et al*. 2012, Murphy 2013). Accumulating this tiny recession over billions of years, which is the time duration that have passed since the birth of the Moon, attains a celestial length scale comparable to the present Earth-Moon distance ($\cong 3.8 \times 10^6$ km). It is interesting to say that the ocean tide, though quite familiar to our every life, gives a dramatic impact on the secular variation in the celestial-body dynamics in a timescale of billions of years (Burns and Matthews 1986).

In the present work, we consider the contribution from "rotating tidal waves" (or so-called "boundary-trapped surface Kelvin waves") to the secular variation in the coupled dynamic of the Earth-Moon system. It is a special class of tidal waves, rotating along the inner edge of topographic boundary such as a coastline or a submarine basin (Pinet 2014). An important feature of the rotating tidal wave is that it is non-dispersive, i.e., the phase velocity of the wave crests is equal to the group velocity of the wave energy for all frequencies. The wave thus retains its shape as it moves in the alongshore direction over time. Furthermore, most rotating tidal waves in the northern (or southern) hemisphere propagate in a counterclockwise (clockwise) direction, wherein the coastline plays a role of a wave guide. The persistency in the rotation direction implies that those tidal waves may be relevant to the energy dissipation or the angular momentum transfer within the Earth-Moon system, while there has been few theoretical attempts to examine the possibility.

To resolve the problems posed above, we have developed a simplified analytic model that describes both the angular momentum and the mechanical energy of rotating tidal waves trapped in oceanic basins on the Earth. In our argument, the velocity of the sea water consisting the tidal waves was evaluated using the shallow-water wave equation (Pinet 2014); the equation is valid under the condition that both the radius of the basin and the wavelength of the tidal wave are sufficiently longer than the mean depth of the sea. We emphasize that the formulation we have developed can apply to not only the Earth-Moon system dynamics but to other coupled celestial dynamics as long as they hold fluid layer on the surface.

2. Tidal force and tidal potential

The tidal force $\mathbf{F}_\mathrm{T}$ associated with the Moon (with mass M_m), exerting on a body of mass m at position $\mathbf{r}$ measured from the center of the Earth, is given by

$$\mathbf{F}_\mathrm{T} = -GmM_\mathrm{m}\left(\frac{\mathbf{d}}{d^2} - \frac{\mathbf{D}}{D^2}\right). \tag{1}$$

Here G is the gravitational constant, $\mathbf{D}$ is the vector from the center of the Moon to the center of Earth, and $\mathbf{d} \equiv \mathbf{D} + \mathbf{r}$ is the vector from the center of the Moon to the body of mass m (See Fig. 1a). We see from Eq. (1) that the tidal force $\mathbf{F}_\mathrm{T}$ is the vector difference of the gravitational attraction to the Moon, $-(GmM_\mathrm{m}/d^2)\mathbf{d}$, and the centripetal force acting on the center of the Earth, $-(GmM_\mathrm{m}/D^2)\mathbf{D}$.

The tidal force $\mathbf{F}_\mathrm{T}$ given by Eq. (1) is the negative gradient of a tidal potential defined by

$$U_\mathrm{T} = -GmM_\mathrm{m}\left(\frac{1}{d} - \frac{r}{D^2}\cos\psi\right) + C\,, \tag{2}$$

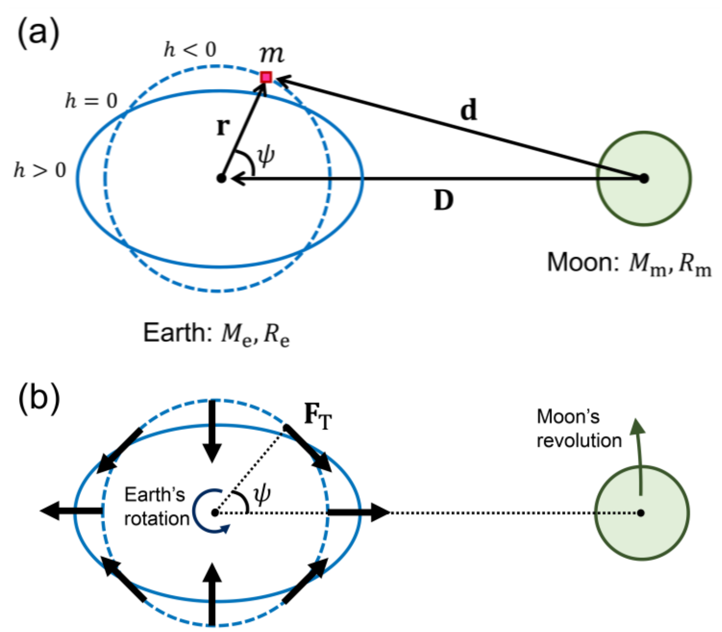

Fig. 1 (a) Tidal bulge formation in the ocean of the Earth. Definition of the vectors used in Eq. (1) is also illustrated. (b) Axially symmetric distribution of the tidal force with respect to the Earth-Moon line.

where C is an arbitrary constant and $\psi = \cos^{-1}[-(\mathbf{r}\cdot\mathbf{D})/(rD)]$. Because of the axial symmetry with respect to the Earth-Moon line, the tidal force depends only on the angle ψ as shown in Fig. 1b. For convenience, we represent d in terms of D, r and ψ as

$$d = \sqrt{D^2 + r^2 + 2Dr\cos\psi}\,, \tag{3}$$

and expand $1/d$ in Eq. (2) in powers of r/D. Since $r/D \ll 1$, it is reasonable to pick up the lowest-order term only, which leads us to

$$U_{\mathrm{T}} \cong -GmM_{\mathrm{m}}\frac{r^2}{D^3}\cdot\frac{1}{2}(3\cos^2\psi - 1) + C\,. \tag{4}$$

The total potential near the surface of the Earth thus reads

$$U = U_T + mgh\,, \tag{5}$$

with $g \equiv GM_{\mathrm{e}}/R_{\mathrm{e}}^2$. Here, M_{e} and R_{e} are the mass and radius of the Earth, respectively, and h is the height above a zero sea level that would be observed in the absence of tides.

The spatial variation in the tidal level (i.e., the ψ-dependence of h) is derived from Eq. (5) by supposing that the surface of the ocean should coincide with the equipotential surface expressed by $U = 0$. We then have

$$h(\psi) = \frac{M_{\mathrm{m}}}{M_{\mathrm{e}}}\cdot\frac{R_{\mathrm{e}}^4}{D^3}\cdot P_2(\cos\psi)\,, \tag{6}$$

where $P_2(x) = (3x^2-1)/2$ is the Legendre polynomial of the second order. Equation (6) indicates that the tidal level is maximized at $\psi = 0$ and $\psi = \pi$, at which the Earth-Moon line intersects the oval-shaped ocean surface; in contrast, it is minimized at $\psi = \pi/2$, i.e., along the circumference perpendicular to the Earth-Moon line (see Fig. 1a).

3. Rotating tidal wave in an oceanic basin

3.1 Formation mechanism

In the preceding section, we have seen that the tidal force induced by the Moon elongates the sea water surface on the Earth, causing the high tide at $\psi = 0,\ \pi$ and low tide at $\psi = \pi/2$. Nevertheless, the argument is quantitatively valid only in an idealized situation that the surface of the Earth is fully covered by a sea-water layer with uniform depth. In reality, the ocean covers only about 70% of the surface of the Earth. Furthermore, the terrain that separates each sea area has a very complicate-shaped coastline and then tidal current is constrained to flow only within topographic boundaries at which tidal waves are scattered. A particular attention should be paid to the presence of oceanic basins. Oceanic basins are large concave portions at the bottom of the ocean, surrounded by continents. In an oceanic basin, water can neither bulge freely nor follow the Moon’s orbital motion. Instead, the basin will forcibly drag the tide bulge because of the relative displacement of the basin to the Moon. As a result, a rotating tidal wave about the node at the center of the basin is formed in the basin (Pinet 2014).

To understand the formation process of the rotating tidal wave, we suppose that there is a circular-shaped oceanic basin in the northern hemisphere of the Earth. Figure 2 illustrates the basin with spatially uniform sea depth, which is surrounded by land on all sides. First, we assume that the Moon is located right above this basin. Then, the sea surface is distorted by the tidal force application and a bulge is formed near the center of the basin. Afterwards, as the Earth rotates, this basin moves from the west to the east, while the bulge remains fixed beneath the Moon. It should be reminded that the period of the Earth’s spin ($\cong$ 24 hours) is much shorter than that of the Moon’s revolution ($\cong$ 28 days). This difference in the rotation period causes relative displacement of the basin to the Moon. Consequently, the bulge is forcibly moved to the western end of the basin (Fig. 2a). As the basin continues to move further east, the bulge is crushed more and more at the western end of the basin. As a result, the sea level in the basin decreases towards the east.

The story is not over yet. Due to the eastward gradient of the sea level, the sea water that had been pressed to the western end will eventually flow east (Fig. 2b). The important point is that the Coriolis force acts on the water flow at this time. If the magnitude of the Coriolis force is strong enough to bias the flow of water to the right, the deflection of the water flow results in the rise of the water at the southern end of the basin (Fig. 2c). At this stage, the sea surface tilts toward the north. Under this inclination, the sea surface starts to flow northward. But the flow is deflected again to the right due to Coriolis force and the eastern end of the basin rises. As these complex water movements continue, the water rise turns around the basin and returns to the initial position, i.e., the western end of the basin. And again, the next lap begins.

When a certain balance is reached, a circling motion of the swell of water will transform to a steady rotating wave in the basin (or called a boundary-trapped surface Kelvin wave), as demonstrated in Fig. 2d. This rotating tidal wave is very similar to the swelling that can be easily observed by stirring the water in a mug cup (Mayer and Krechetnikov 2012). The rotating tidal wave creates a wave peak at one end of the basin and a wave trough on the opposite side and these rotate so as to trace the periphery of the basin. Therefore, if this wave circles the basin in half a

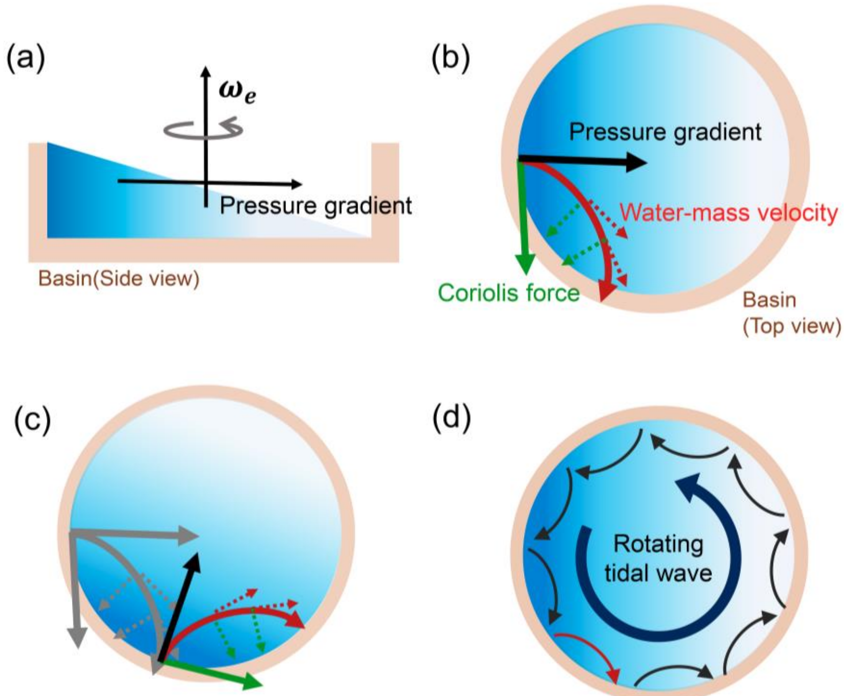

Fig. 2 Diagram of the rotating tidal wave formation in a circular basin locating at the northern hemisphere. (a) High tide at the western edge creates a pressure gradient sloping downward towards the east. (b) Eastward tidal flow is deflected by the Coriolis force and is blocked by continental landmasses. (c) Accumulation of the sea water at the southern edge establish a pressure gradient. (d) This cycle results in a unidirectional rotary flow of tidal waves along the inner edge of the basin.

day, it creates a high tide (wave peak) and low tide (wave trough) of 12 hours cycle at all positions along the edge of the basin. In this way, the rotating tidal wave behaves as a traveling wave when viewed along the circumferential direction. In contrast, when looking at this tidal oscillation in the cross section of the basin, the sea surface moves up and down like a seesaw around the node located in the center of the basin. That is, the rotating tidal wave in the basin behaves as a standing wave when viewed along the radial direction. It is thus concluded that the rotating tidal wave generated in the basin has two properties, a traveling wave and a standing wave.

3.2 Surface Kelvin wave

We have seen that a tide-induced bulge in an oceanic basin can transform into a rotating tidal wave, a sort of the surface Kelvin wave. In oceanography, the surface Kelvin wave is regarded as a special type of gravity wave that is affected by the Earth's spin (Loper 2017). The unique feature of the surface Kelvin wave is its unidirectional propagation. It is known that the surface Kelvin wave moves equatorward along a western boundary, poleward along an eastern boundary and cyclonically around a closed boundary. The rotation direction is counterclockwise if it locates at the northern hemisphere, while clockwise if it does at in the southern hemisphere. The unidirectional property of the surface Kelvin wave originates from that the Coriolis force associated with the Earth's spin is a primary driving force for the surface Kelvin wave to occur.

Fig. 3 demonstrates the spatial distribution of the wave amplitude (top panel) and the periodic time evolution (bottom panel) of the surface Kelvin wave detected by an artificial satellite. We see from the figure that the wave amplitude is concentrated near the coastline and the wave moves along the boundary in a certain period. The maximum of the wave amplitude is roughly estimated as 1.30 m, which will be used in the definition of our simplified model.

The phase velocity of the surface Kelvin wave, c, is determined by

$$c = \sqrt{gH}\,, \tag{7}$$

in the same manner as that of surface gravity waves. The shape of the wave is conserved as the wave travels along the circumferential direction. This implies that the surface Kelvin wave is non-dispersive and that the wave energy is transmitted at the velocity given by Eq. (7). If the sea depth is uniform (typically $H = 4$ km) over the basin considered, the value of c equals to ca. 200 m/s.

Another important feature of the surface Kelvin wave is that the wave amplitude is largest at the boundary and decays exponentially with distance from it. In order words, the amplitude of the surface Kelvin wave is significant only within a distance on the order of the Rossby radius of deformation, μ, from the boundary. Here, the Rossby radius of deformation is one of the key characteristic length scale in the field of geophysical fluid dynamics. It is defined by the ratio of the wave velocity, c, over the absolute-value of the Coriolis parameter, $f_0 \equiv 2\omega_{\mathrm{e}} \sin\alpha$, with ω_{e} being the angular velocity of the Earth's spin and α being the latitude. We thus have

$$\mu = \sqrt{\frac{gH}{f_0^2}}\,. \tag{8}$$

Over the characteristic distance, the tendency of the gravitational force to flatten the sea surface is balanced by the tendency of the Coriolis force to deform the surface. This is why the Kelvin wave trapped within a basin shows exponential decay in the amplitude from the boundary with a decay length of μ.

4. A simplified model of rotating tidal wave

We are now ready to discuss theoretically the mechanics of a rotating tidal wave trapped in a oceanic basin. As a simplified model, we consider a closed basin having a circular boundary with radius R_{b} and uniform sea depth of H. The sea level of the rotating tidal wave in the basin is described by

$$\eta(r,\theta,t) = \frac{\gamma h_0}{2} \cdot \frac{\exp\left(\frac{r}{\mu}\right) - 1}{\exp\left(\frac{R_{\mathrm{b}}}{\mu}\right) - 1} \sin(\theta - \omega t)\,, \tag{9}$$

where r and θ are polar coordinates with respect to the origin at the center of the basin. The constant h_0 is the tidal level difference between the high and low tides, estimated from Eq. (6); it thus reads

$$h_0 = \frac{3}{2} \cdot \frac{M_{\mathrm{m}} R_{\mathrm{e}}^4}{M_{\mathrm{e}} D^3}\,. \tag{10}$$

For the present Earth-Moon system, the value of h_0 is estimated to be 54 cm, which is in fair

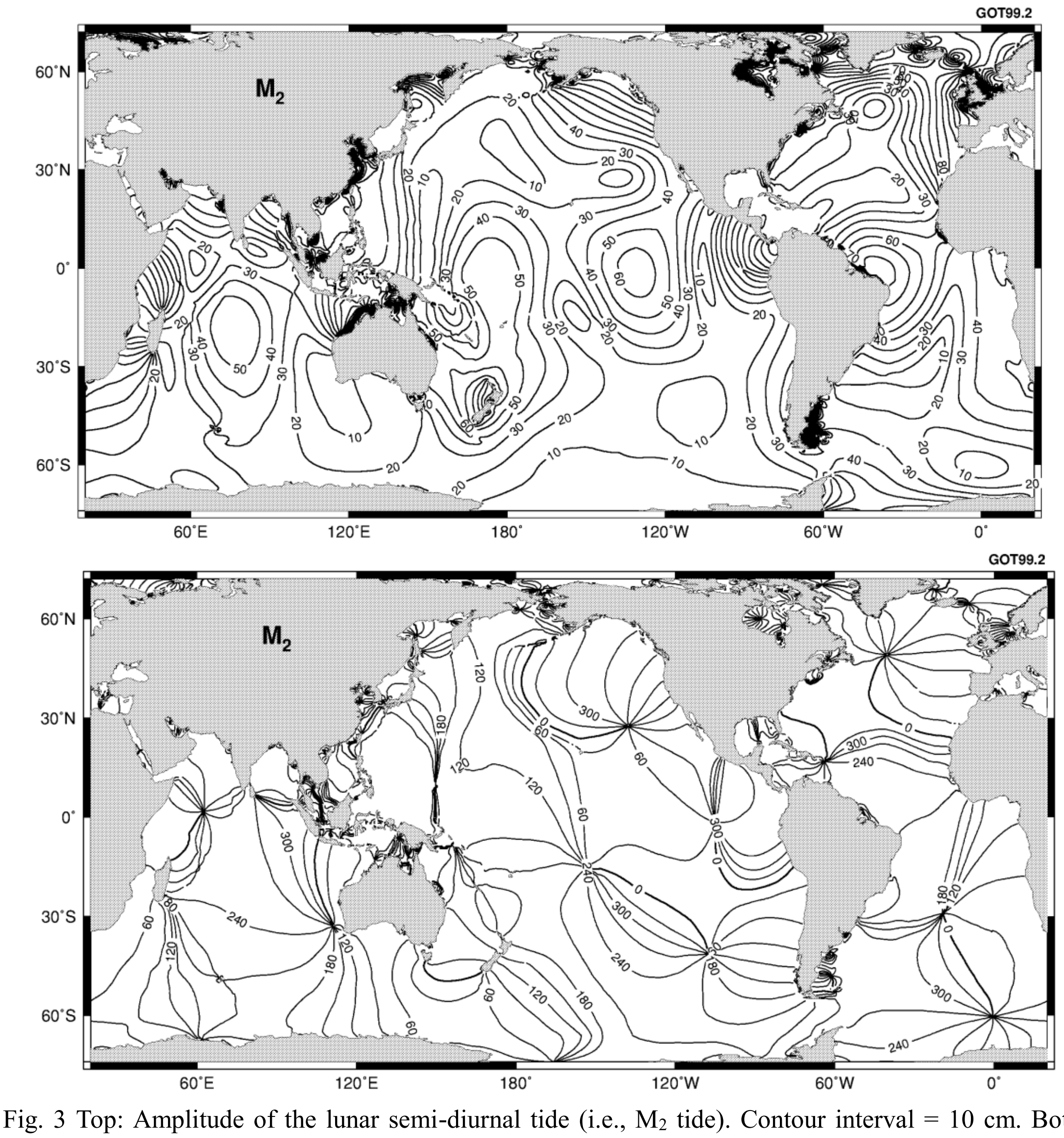

Fig. 3 Top: Amplitude of the lunar semi-diurnal tide (i.e., M_2 tide). Contour interval = 10 cm. Bottom: Greenwich lag of the M_2 tide. The solid lines are cotidal lines spaced at phase interval of 30° (ca. 1 hour). Reprinted from the report written by Ray (1999).

agreement with our daily observation in the sea. On the basis of this value, we have set $\gamma = 2.4$, by which the maximum amplitude of the wave is consistent with real tidal observation demonstrated in Fig. 3. Hereafter the notation of $h \equiv h_0\gamma$ will be used for simplicity.

Note that the expression of $\eta(r, \theta, t)$ was defined by Eq. (9) so that it satisfies the following three conditions: i) The sea level decays exponentially in the radial direction with distance from the circular boundary; ii) The sea level shows a sinusoidal oscillation in the circumferential direction; iii) The sea level at the center of the basin (i.e., $r = 0$) is kept to be zero. It is also possible to prove that when μ/R_b is the order of unity or more, we approximately have

$$\frac{\exp\left(\frac{r}{\mu}\right)-1}{\exp\left(\frac{R_\mathrm{b}}{\mu}\right)-1} \cong \frac{r}{R_\mathrm{b}}, \tag{11}$$

as demonstrated in Fig. 4. Specifically, if we set $H = 3.76 \times 10^3$ m, $R_\mathrm{b} = 1.32 \times 10^6$ m and $f_0 = 10^{-4}$ rad/s, we have $\mu/R_\mathrm{b} \simeq 2.0$; in that case, Eq. (9) can be reduced to

$$\eta(r,\theta,t) = \frac{hr}{2R_\mathrm{b}} \cdot \sin(\theta - \omega t)\,. \tag{12}$$

The value of $H = 3.76 \times 10^3$ m was based on the assumption that the sea water of 1.40×10^{21} kg in mass would occupy the entire ocean with surface area of $3.62 \times 10^{14}\,\mathrm{m}^2$ and uniform depth H. Also, the value of $R_\mathrm{b} = 1.32 \times 10^6$ m was chosen so that the rotation period of the tidal wave tracing the inner edge of the circular basin is in resonance with that of the tidal force: $2\pi R_\mathrm{b} = \sqrt{gH} \cdot T$ with $T = 12$ h. The simplified expression of Eq. (12) will be used in our formulation of the angular momentum and the mechanical energy of rotating tidal waves as described below.

5. Angular momentum of the rotating tidal wave

In this section, we derive the angular momentum of the rotating tidal wave, L_b, in a circular basin with radius R_b and uniform depth H. When the wavelength is sufficiently smaller than the sea depth, we are allowed to use the shallow wave approximation, under which the circumferential component of the fluid velocity, $u_\theta(r,\theta,t)$, is given by

$$u_\theta(r,\theta,t) = \frac{\omega}{kH} \cdot \eta(r,\theta,t)\,, \tag{13}$$

with k being the wavenumber defined by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi R_\mathrm{b}} = \frac{1}{R_\mathrm{b}}. \tag{14}$$

The θ-component of the momentum for an infinitesimal volume element, $dV = r\,dr\,d\theta\,dz$, reads $\rho r u_\theta\,dV$, where ρ is the weight density of the fluid. Summarizing the contributions from all the volume elements, therefore, L_b is represented by

$$L_\mathrm{b} = \iiint \rho r u_\theta\,dV\,, \tag{15}$$

or equivalently

$$L_\mathrm{b} = \frac{\rho\omega}{kH} \int_0^{R_\mathrm{b}} dr \int_0^{2\pi} d\theta \int_0^{\eta(r,\theta,t)} dz \cdot r^2 \eta(r,\theta,t)\,. \tag{16}$$

Substituting the expression of η given by Eq. (12) into Eq. (15) followed by integrations, we have

$$L_\mathrm{b} = \frac{\pi}{20} \cdot \frac{\rho\omega}{H} \cdot h^2 R_\mathrm{b}^4\,. \tag{17}$$

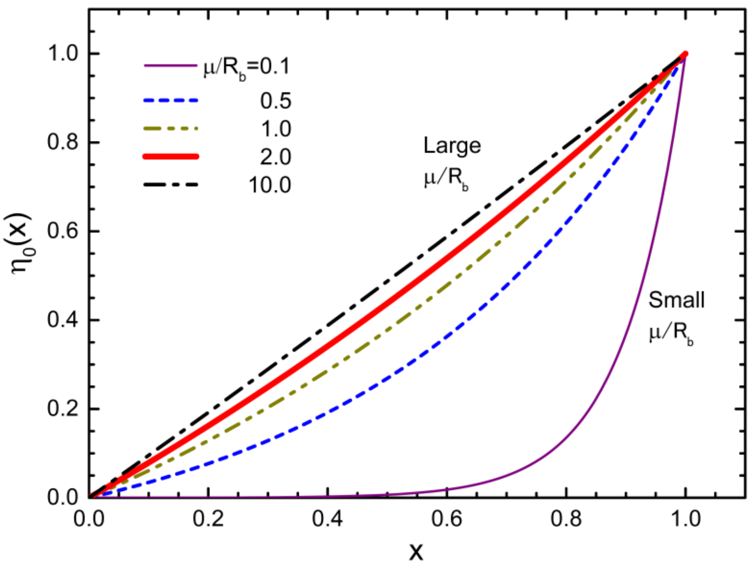

Fig. 4 Radial distribution of the amplitude of the rotating tidal wave modeled by Eq. (11) for various values of the Rossby radius of deformation.

In addition, we recall the dispersion relation of the rotating tidal wave:

$$\omega = ck = \sqrt{gH} \cdot k = \sqrt{\frac{GM_{\rm e}}{R_{\rm e}^2} H} \cdot k \,, \tag{18}$$

We eliminate ω and $h = \gamma h_0$ in Eq. (17) by using the rightmost term in Eq. (18) and Eq. (10) to rewrite $L_{\rm b}$ in terms of celestial parameters as

$$L_{\rm b} = \frac{9\pi\rho\gamma^2}{80} \cdot \frac{M_{\rm m}^2 R_{\rm e}^7 R_{\rm b}^3}{M_{\rm e} D^6} \sqrt{\frac{G}{M_{\rm e} H}} \,. \tag{19}$$

Eq. (19) is the first one of the main results of the present work. This formula describes the angular momentum of a rotating tidal wave trapped in a circular basin as a function of the celestial parameters (i.e., $G, M_{\rm e}, M_{\rm m}, R_{\rm e}, D$) as well as the basin-related parameters (i.e., $R_{\rm b}, H, \gamma$). Given a basin with $H = 3.76 \times 10^3$m and $R_{\rm b} = 1.32 \times 10^6$m, we have $L_{\rm b} = 3.16 \times 10^{19}\,\mathrm{kg \cdot m^2/s}$, for example.

An important finding deduced from Eq. (19) is the negative power-low dependence of $L_{\rm b}$ on D to a six degree. This means that the magnitude of $L_{\rm b}$ can be remarkably enhanced if D is set to be small sufficiently. Such the highly enhanced $L_{\rm b}$ may become responsible to the secular variation in the dynamics of a binary planet, other than the Earth-Moon pair, in which the two planets are very close to each other. In that case, the mechanical friction between the rotating tidal wave and the sea floor will cause to a degree the angular momentum transfer between them and its accumulation over millions or billions of years may result in a feasible change in the coupled dynamics of the binary planet. Equation (19) will give a clue to consideration of the problem, as will be demonstrated in our future work.

6. Mechanical energy of the rotating tidal wave

We next describe the mechanical energy of the rotating tidal wave, designated by E_{b}. It is the sum of the gravitational potential energy, U_{b}, and the kinetic energy, K_{b}, as given by

$$E_{\mathrm{b}} = K_{\mathrm{b}} + U_{\mathrm{b}} \,. \tag{20}$$

We set z to be the axis normal to the spherical sea surface without any tidal deformation. Then, the work done by the tidal force on the sea water, U_{b}, is given by

$$U_{\mathrm{b}} = \frac{\rho G M_{\mathrm{e}}}{R_{\mathrm{e}}^2} \int_0^{R_{\mathrm{b}}} dr \int_0^{2\pi} d\theta \int_0^{\eta(r,\theta,t)} (rz)\, dz \,. \tag{21}$$

A simple calculation yields

$$U_{\mathrm{b}} = \frac{\pi \rho G M_{\mathrm{e}} h^2}{32} \cdot \frac{R_{\mathrm{b}}^2}{R_{\mathrm{e}}^2} \,. \tag{22}$$

Similarly, we have

$$K_{\mathrm{b}} = \frac{\rho}{2} \int_0^{R_{\mathrm{b}}} dr \int_0^{2\pi} d\theta \int_{-H}^{\eta(r,\theta,t)} (r u_\theta^2)\, dz \,. \tag{23}$$

Substituting Eqs. (13) and (18), we obtain

$$K_{\mathrm{b}} = \frac{\rho}{2H} \cdot \frac{G M_{\mathrm{e}}}{R_{\mathrm{e}}^2} \int_0^{R_{\mathrm{b}}} dr \int_0^{2\pi} d\theta \int_{-H}^{\eta(r,\theta,t)} \frac{r^3}{R_{\mathrm{b}}^2} \cdot \left(\frac{h}{2}\right)^2 \sin^2(\theta - \omega t)\; dz \,. \tag{24}$$

Integration with respect to z yields

$$K_{\mathrm{b}} = \frac{\rho}{2} \cdot \frac{G M_{\mathrm{e}}}{R_{\mathrm{e}}^2} \left(\frac{h}{2}\right)^2 \int_0^{R_{\mathrm{b}}} dr \int_0^{2\pi} d\theta \left[\frac{r^3}{R_{\mathrm{b}}^2} \sin^2(\theta - \omega t) + \frac{r^4}{R_{\mathrm{b}}^3} \cdot \frac{h}{2H} \sin^3(\theta - \omega t)\right] . \tag{25}$$

Since $h/H \ll 1$, the second term in the square bracket in Eq. (25) can be neglected and then we have

$$K_{\mathrm{b}} = \frac{\pi \rho G M_{\mathrm{e}} h^2}{32} \cdot \frac{R_{\mathrm{b}}^2}{R_{\mathrm{e}}^2} \,. \tag{26}$$

It is remarked that the result of K_{b} given in Eq. (26) is exactly same as U_{b} given in Eq. (20), as is in agreement with a general feature of linear gravity waves.

In summary, E_{b} is written in terms of celestial parameters as

$$E_{\mathrm{b}} = \frac{\pi \rho G M_{\mathrm{e}} h^2}{16} \cdot \frac{R_{\mathrm{b}}^2}{R_{\mathrm{e}}^2} \,, \tag{27}$$

or equivalently

$$E_{\mathrm{b}} = \frac{9\pi \rho G \gamma^2}{64} \cdot \frac{M_{\mathrm{m}}^2}{M_{\mathrm{e}}} \cdot \frac{R_{\mathrm{e}}^6 R_{\mathrm{b}}^2}{D^6} \,, \tag{28}$$

Equation (28) is the second one of the main results of this article. It gives estimation of the mechanical energy carried by the rotating tidal wave in a basin, as a function of the celestial parameters and the basin-related ones. Again we point out that $E_{\rm b}$ is inversely proportional to the six power of D, thus will increase drastically if we consider the case of a binary planet that approach each other.

7. Angular momentum transfer from the Earth to the Moon

This section describes the mechanism of the angular momentum transfer from the Earth's spin rotation to the Moon's orbital revolution. The transfer is a consequence of the conservation law of the total angular momentum, which requires that the total angular momentum of two coupled celestial bodies must remain constant as far as no external torque is applied to the system. Hence, the angular momentum transfer may occur in general planet-satellite systems, not limited to our Earth-Moon system. In the following, we basically discuss the angular momentum transfer observed in the Earth-Moon system, keeping in mind that the argument holds true for general planet-satellite systems.

From the definition, the total angular momentum of the Earth-Moon system reads

$$L_{\rm total} = M_{\rm e} r_{\rm e}^2 \Omega_{\rm e} + M_{\rm m} r_{\rm m}^2 \Omega_{\rm m} + I_{\rm e}\omega_{\rm e} + I_{\rm m}\omega_{\rm m} + L_{\rm b} \,. \tag{29}$$

Here, the first two terms in the right side represent the orbital angular momenta associated with the revolution of the Earth and Moon, respectively, about their common center of gravity; $r_{\rm e}$ ($r_{\rm m}$) is the distance from the center of gravity to the center of the Earth (Moon) and $\Omega_{\rm e}$ and $\Omega_{\rm m}$ denote the angular velocities during the revolutions. The next two terms are the spin angular momenta originating from the rotations of the two planets; $\omega_{\rm e}$ and $\omega_{\rm m}$ are the associated angular velocities, while $I_{\rm e}$ and $I_{\rm m}$ are their moments of inertia. The last term, $L_{\rm b}$, describes the contribution from the rotating tidal waves trapped in oceanic basins, as derived in Eq. (19); it should be replaced by a component of the angular momentum vector in the direction of Earth's spin axis and thus its magnitude is kept to be constant during the Earth's spin rotation.

In the following discussion, we make an assumption that

$$\Omega_{\rm e} = \Omega_{\rm m} \equiv \Omega \,. \tag{30}$$

That is, the Earth and Moon are assumed to revolve around the center of gravity with the same angular velocity defined by Ω. This implies that

$$G(M_{\rm e} + M_{\rm m}) = D^3 \Omega^2 \,, \tag{31}$$

as followed from Kepler's third law. Using Eqs. (30) and (31), the orbital components given by the first and second terms in the right side of Eq. (29) are simplified as

$$M_{\rm e} r_{\rm e}^2 \Omega_{\rm e} + M_{\rm m} r_{\rm m}^2 \Omega_{\rm m} = M' D^2 \Omega \,, \tag{32}$$

where

$$M' \equiv \frac{M_{\rm e} M_{\rm m}}{M_{\rm e} + M_{\rm m}} \tag{33}$$

is the reduce mass. Besides, we differentiate the both sides of Eq. (31) to obtain

$$3D^2\Omega^2 \frac{dD}{dt} + 2D^3\Omega \frac{d\Omega}{dt} = 0 \,, \tag{34}$$

which yields

$$\frac{d\Omega}{dt} = -\frac{3\Omega}{2D}\cdot\frac{dD}{dt}. \tag{35}$$

The final assumption in our argument is the equality of

$$\omega_{\rm m} = \Omega \tag{36}$$

as a consequence of the tidal locking of the Moon's spin rotation (Souchay 2013).

Summarizing all the results obtained above and considering the conservation of the total angular momentum,

$$\frac{dL_{\rm total}}{dt} = 0, \tag{37}$$

we arrive at the main conclusion:

$$(Z_1 - Z_2)\frac{dD}{dt} = -I_{\rm e}\frac{d\omega_{\rm e}}{dt}, \tag{38}$$

where

$$Z_1 = (M'D^2 - 3I_{\rm m})\frac{\Omega}{2D}, \tag{39}$$

and

$$Z_2 = \frac{27\pi\rho\gamma^2}{40}\cdot\frac{M_{\rm m}^2}{M_{\rm e}^2}\cdot\frac{R_{\rm e}^7 R_{\rm b}^3}{D^7}\cdot\sqrt{\frac{GM_{\rm e}}{H}}. \tag{40}$$

The series of Eqs. (38)-(40) is the third one of the main results of this paper. It should be emphasized that Eq. (38) relates the rate of time evolution in the lunar distance (i.e., dD/dt) with the Earth's spin deceleration (i.e., $d\omega_{\rm e}/dt$). The term Z_2 quantifies the effect of the rotating tidal wave to the secular variation in the coupled dynamics of the Earth and the Moon, while the term Z_1 does the effect of spin and revolution of the two celestial bodies. For the present Earth-Moon system, we have $Z_1 = 3.71\times 10^{25}\ \mathrm{kg\cdot m/s}$ and $Z_2 = 4.93\times 10^{11}\ \mathrm{kg\cdot m/s}$; these values imply that, at least in the present Earth-Moon system, rotating tidal waves plays almost no role in the angular momentum transfer. Nevertheless, since Z_2 is inversely proportional to D^{-7}, the magnitude of Z_2 may increase significantly if we consider a binary planet with small distance.

8. Secular variation in the interplanetary distance

The mechanical energy of the Earth-Moon system, $E_{\rm em}$, is the sum of the spin rotation energy of the Earth and the orbital energy of the system. It is thus given by

$$E_{\rm em} = \frac{I\omega_{\rm e}^2}{2} - G\frac{M_{\rm e}M_{\rm m}}{2D}. \tag{41}$$

Differentiate both sides of Eq. (41) with respect to time to obtain

$$\frac{dE_{\mathrm{em}}}{dt} = I\omega_{\mathrm{e}}\frac{d\omega_{\mathrm{e}}}{dt} + G\frac{M_{\mathrm{e}}M_{\mathrm{m}}}{2D^2}\cdot\frac{dD}{dt}\,. \tag{42}$$

We rewrite the last term in the right side using Kepler's third law of Eq. (32) and then obtain

$$\frac{dE_{\mathrm{em}}}{dt} = I\omega_{\mathrm{e}}\frac{d\omega_{\mathrm{e}}}{dt} + \frac{M'\Omega^2 D}{2}\cdot\frac{dD}{dt}\,. \tag{43}$$

Finally, we substitute Eq. (38) into Eq. (43) to eliminate the term $d\omega_{\mathrm{e}}/dt$ and achieve the key result:

$$\frac{dE_{\mathrm{em}}}{dt} = -\frac{M'\Omega}{2}(\omega_{\mathrm{e}} - \Omega)\cdot D\frac{dD}{dt}\,. \tag{44}$$

We note here that $dE_{\mathrm{em}}/dt < 0$ in the real system, because a portion of the mechanical energy must be dissipated as heat according to the thermodynamic law. In addition, since ω_{e} ($\sim 7.29 \times 10^{-5}$ rad/s) is much faster than Ω ($\sim 2.66 \times 10^{-6}$ rad/s), we have

$$\frac{dD}{dt} > 0\,. \tag{45}$$

This means that the Earth-Moon distance increases with time or equivalently, the Moon is slowly receding from the Earth. Furthermore, we see from Eqs. (38) and (45) that

$$\frac{d\omega_{\mathrm{e}}}{dt} < 0\,. \tag{46}$$

It is thus concluded that in the Earth-Moon system, the rotational period of the Earth is increasing gradually, producing longer days.

The energy dissipation rate, dE_{em}/dt, can be estimated from Eq. (44) by applying the measurement data of $dD/dt \simeq 3.82$ cm/year, which was obtained by the Lunar Laser Ranging experiment (Samain *et al*. 1998, Murphy *et al*. 2012, Murphy 2013). Substituting it as well as other relevant celestial parameters to Eq. (44), it is concluded that dE_{em}/dt takes the value of 3.15 TW. This result will be referred to in the discussion below.

9. Quality factor deduced from the simple model

In the last section, we discuss the consistency of our simplified model for the ocean tide dynamics with those existing, more sophisticated (but complicated) ones by considering the Q-factor (quality factor) of the global ocean. Here, Q-factor is a dimensionless parameter that describes the resonance behavior of an oscillator in general. In typical, Q-factor is defined by the ratio of the energy stored in a damped, forced, harmonic oscillator to the energy dissipated per cycle by damping processes:

$$Q = 2\pi \times \frac{\text{Energy stored in the sysem}}{\text{Energy dissipated per cycle}}\,. \tag{47}$$

Thus, higher Q-factor indicates a lower rate of energy loss relative to the stored energy of the oscillator.

The formula given by Eq. (28) enables us to estimate the Q-factor of the oscillatory tidal motion on the Earth. Now let us estimate the Q-factor of the global ocean. Suppose that there are ten oceanic basins with $R_{\mathrm{b}} = 1.32 \times 10^6$ m, taking into account the real distribution of the rotating tidal waves demonstrated in Fig. 3. Since the energy stored in a single basin accounts for $E_{\mathrm{b}} = 5.74 \times 10^{15}$ J, as followed from Eq. (28), the total energy stored in the ten basins is ten times as much the energy. It is also known that the tidal energy dissipation in the global ocean accounts for ca. 2.0 TW (Egbert and Ray 2000). From these facts, it is inferred that $Q \simeq 7.5$. This result agrees with the existing estimates of Q-factor ($Q = 10 \sim 20$) for ocean tides, indicating the validity of our formula of Eq. (28) for discussing the energetics of ocean tides.

10. Conclusions

We have developed a simplified model that describes the effect of rotating tidal waves to the coupled dynamics of the Earth-Moon system. The model is based on the Coriolis force-driven rotational motion of the sea water trapped in a circular basin, as realized in the real global sea. We have demonstrated that the model allows us to evaluate the quality factor of the tidal oscillation in the sea and to estimate the contribution of the rotating tidal waves to the secular variation in an interplanetary distance on time scale on the order of billions of years.

As a concluding remark, we emphasize that the contribution from rotating tidal waves to the secular variation is likely to become much significant if the two celestial bodies (e.g., a pair of an Earth-like ocean planet and a Moon-like satellite) get much closer as realized in the early Earth-Moon system in billions of years ago (Webb 1982). This is because the short distance between the two celestial bodies will amplify the difference in the tidal level, thus possibly enhance the energy dissipation. The formula we have developed, describing the effect of rotating tidal wave on the secular variation as a function of celestial parameters, therefore, will give a clue to obtain better understanding of tidal effects on the motion of satellites endowed with fluid-layer component (Nimmo and Pappalardo 2016) such as Europa (Carr *et al*. 1998, McCarthy and Cooper 2016), Titan (Folonier and Ferraz-Mello 2017), Enceladus (Roberts and Nimmo 2008, Čadek *et al*. 2016) and various exoplanets that shall be discovered in future.

Acknowledgment

This work was financially supported by JSPS KAKENHI Grant No. 18H03818.

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Nomenclature

The values listed below are cited from the monograph written by Stacey and Davis (2014).

$G = 6.67 \times 10^{-11}$ [$\mathrm{m^3/kg/s^2}$]: Gravitational constant
$R_\mathrm{e} = 6.37 \times 10^{6}$ [m]: Earth's radius
$R_\mathrm{m} = 1.74 \times 10^{6}$ [m]: Moon's radius
$M_\mathrm{e} = 5.97 \times 10^{24}$ [kg]: Earth's mass

$M_{\mathrm{m}} = 7.35 \times 10^{22}$ [kg]: Moon's mass
$I_{\mathrm{e}} = 8.04 \times 10^{37}$ [kg · m^2]: Earth's momentum of inertia
$I_{\mathrm{m}} = 8.68 \times 10^{34}$ [kg · m^2]: Moon's momentum of inertia
$\Omega_{\mathrm{e}} = 2.66 \times 10^{-6}$ [rad/s]: Earth's orbital angular velocity
$\Omega_{\mathrm{m}} = 2.66 \times 10^{-6}$ [rad/s]: Moon's orbital angular velocity
$D = 3.84 \times 10^{8}$ [m]: Earth-Moon distance at present
$\rho = 1.03 \times 10^{3}$ [kg/m^3]: Weight density of the sea water