

Analytical crack growth in unidirectional composite flywheel

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Abstract. Scarce research has been published on crack propagation fracture of flywheels manufactured with carbon fiber-reinforced polymers. The present work deals with a calculation method to determine the conditions for which a crack propagates in the axial direction of the flywheel. The assumptions are: flywheels made with just a single thick ply or ply clustering laminates, oriented following the hoop direction; a single crack is analyzed in the plane defined by the hoop and axial directions; the crack starts close to one of the free edges; its axial length is initially large enough so that its tip is far away from that free edge, and the crack expands the entire circumferential perimeter and keeps its concentric position. The developed method provides information for a good design of flywheels. It is concluded that a fracture-based crack propagation criterion generally occurs at a lower speed than a stress-based criterion. Also, that the evolution of failure with thickness using the fracture criterion is exponential, demonstrating that thin flywheels are relatively not sensitive to crack propagation, whereas thick ones are very prone.

Keywords: analytical stress analysis; composite material flywheel; crack growth; failure criteria; finite elements; linear fracture mechanics

1. Introduction

Composite flywheels are one of the few common applications of advanced composites for which the directionality of the fibers is more or less uniform and the thickness very high. Compared with metal parts, they have been increasingly used in many technological sectors due to their low weight and high traction resistance, which allow elevated angular velocities ω and produce a good energy density.

A composite flywheel can be a long cylinder with several layers of different materials. However, in this article, we will study a single and thick layer for simplicity and for being widely used. Cylinders of several layers would behave similarly, but the governing equations are more complex. The rest of the parts, which are generally metallic: hub, center cup, spokes, endcaps, etc., are not included in the present analysis. The flywheel that rotates at ω is under several tensile

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Appendix

The inertial and axial deformations are defined only by two respective coefficients:

$$b = -\frac{\rho\omega^2}{p}; \quad d = (k^2n_H - n_R)\varepsilon_z^u$$

$$U_b = \frac{b}{9 - k^2}; \quad U_d = \frac{d}{1 - k^2}$$

expressions that include the following material coefficients related to transversal isotropy, $\nu_{\theta z} = \nu_{r\theta}$ and $E_r = E_z$:

$$s_{11} = \frac{1 - \nu_{\theta z}^2}{E_\theta}; \quad s_{12} = \nu_{\theta z} \frac{1 + \nu_{rz}}{E_r}$$

$$s_{21} = s_{12}; \quad s_{22} = \frac{1 - \nu_{rz}^2}{E_r}$$

$$\nu_H = \frac{s_{12}}{s_{22}}; \quad \nu_R = \frac{s_{12}}{s_{11}}$$

$$n_H = \nu_{\theta z} + \nu_H \nu_{rz}; \quad n_R = \nu_R \nu_{\theta z} + \nu_{rz}$$

$$k^2 = \frac{s_{22}}{s_{11}}; \quad p = \frac{s_{11}}{s_{11}s_{22} - s_{12}^2}$$

At Eq. (6), the stress coefficients associated only with material properties are:

$$H_A = k^2p(1 - k\nu_H); \quad H_B = k^2p(1 + k\nu_H)$$

$$R_A = p(\nu_R - k); \quad R_B = p(\nu_R + k)$$

From the same equation, the stress coefficients that include material, centrifugal, and axial behavior are:

$$H_b = k^2p(1 + 3\nu_H)U_b; \quad H_d = k^2p(1 + \nu_H)U_d + n_H\varepsilon_z^u$$

$$R_b = p(3 + \nu_R)U_b; \quad R_d = p[(\nu_R + 1)U_d + n_R\varepsilon_z^u]$$