

Wave behavior at the interface of inviscid fluid and NL bio-thermoelastic diffusive media

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Abstract. This work throws light on the reflection and transmission phenomenon due to incident plane longitudinal wave at a plane interface between inviscid fluid half-space and a nonlocal bio-thermoelastic diffusive half-space. The governing equations are formulated by adopting nonlocal heat conduction and mass diffusion along with dual phase lag (DPL) model. The amplitude ratios are obtained analytically and these amplitude ratios are used to drive energy ratios. The distribution of energy of incident wave among reflected and transmitted waves are obtained. The obtained ratios are impacted by angle of incidence, frequency and different properties of media involved. Numerically examined energy ratios are displayed in the form of graphs to know the effect of nonlocal parameters, lagging times, stiffness and blood perfusion rate.

Keywords: amplitude ratios; bio-thermoelasticity; diffusion; fluid; non local; reflection; transmission

1. Introduction

In clinical treatment, a lot of modern thermo-therapeutics (microwave, laser, focused ultrasound, and radio-frequency) have been widely used. For example, the laser is focused on tumor by an objective lens for thermal therapy. During thermal therapy, delivering the appropriate heat energy to the infected tissue without affecting the healthy tissue is the biggest challenge. Thus an acute need is to understand how the temperature/stress fields affect the kinetics of a thermal treatment.

In living biological tissues, heat transfer analysis is a complicated physiological process due to the inherent characteristics in tissues, e.g. blood circulation, sweating, metabolic heat generation, and heat dissipation via hair or fur. Pennes (1958) established the bioheat transfer model to describe this complex phenomena which is based on Fourier's law of heat conduction. So far lots of research work is done based on Cattaneo (1958), Vernotte (1958) and Tzou (1995) models of heat conduction to understand the thermal behavior of biological tissues.

Later, Roychoudhuri (2007) extended the idea of Tzou on Green Nagdhi III by introducing the third phase-lag τ_v in correspondence to thermal displacement and developed another generalized

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Appendix C

$$\begin{aligned}
d_{11} &= \frac{i\rho^f c_1^2}{\omega}, d_{1j} = \lambda_0 \left(\frac{\xi_R}{\omega}\right)^2 + \rho c_0^2 \left(\frac{dV_j}{\omega}\right)^2 + \rho c_0^2 \frac{n_j}{\omega^2} + b\gamma_2^2 \frac{k_j}{\omega^2}, d_{15} \\
&= (\rho c_0^2 - \lambda_0) \left(\frac{\xi_R}{\omega}\right) \left(\frac{dV_4}{\omega}\right), \\
d_{21} &= 0, d_{2j} = 2\mu \frac{\xi_R}{\omega} \frac{dV_j}{\omega}, d_{25} = \mu \left[\left(\frac{\xi_R}{\omega}\right)^2 - \left(\frac{dV_4}{\omega}\right)^2 \right], \\
d_{31} &= c_1, \frac{dV_\alpha k_n}{i\omega c_0}, d_{3j} = \lambda_0 \left(\frac{\xi_R}{\omega}\right)^2 + \rho c_0^2 \left(\frac{dV_j}{\omega}\right)^2 + \rho c_0^2 \frac{n_j}{\omega^2} + b\gamma_2^2 \frac{k_j}{\omega^2} - ik_n \frac{dV_j}{\omega}, d_{35} \\
&= (\rho c_0^2 - \lambda_0) \frac{\xi_R}{\omega} \frac{dV_4}{\omega} - ik_n \frac{\xi_R}{\omega}, \\
d_{41} &= 0, \quad d_{4j} = in_j \frac{dV_j}{\omega}, d_{45} = 0, \\
d_{51} &= 0, \quad d_{5j} = ik_j \frac{dV_j}{\omega}, \quad d_{55} = 0, \\
g_1 &= d_{11}, \quad g_i = 0, \quad i = 2,3,4,5, \quad j = 2,3,4.
\end{aligned}$$

Appendix D

$$\begin{aligned}
\langle P_{ij}^* \rangle &= -\frac{\omega^4}{2} \text{Re} \left[2\mu \frac{\xi_R}{\omega} \frac{\xi_R}{\omega} \frac{dV_i}{\omega} + \frac{dV_2}{\omega} \left(\lambda_0 \left(\frac{\xi_R}{\omega}\right)^2 + \rho c_0^2 \frac{dV_1}{\omega} + \frac{\rho c_0^2 n_i}{\omega^2} + \frac{\rho c_0^2 k_i}{\omega^2} \right) Z_i \bar{Z}_j \right], \\
\langle P_{i4}^* \rangle &= -\frac{\omega^4}{2} \text{Re} \left[-2\mu \frac{\xi_R}{\omega} \frac{dV_4}{\omega} \frac{dV_i}{\omega} + \frac{\xi_R}{\omega} \left(\lambda_0 \left(\frac{\xi_R}{\omega}\right)^2 + \rho c_0^2 \left(\frac{dV_i}{\omega}\right)^2 + \frac{\rho c_0^2 n_i}{\omega^2} + \frac{\rho c_0^2 k_i}{\omega^2} \right) Z_i \bar{Z}_4 \right], \\
\langle P_{4j}^* \rangle &= \frac{-\omega^4}{2} \text{Re} \left[\left[\mu \left(\left(\frac{\xi_R}{\omega}\right)^2 - \left(\frac{dV_4}{\omega}\right)^2 \right) \frac{\xi_R}{\omega} - \frac{\xi_R}{\omega} \cdot \frac{dV_4}{\omega} + \lambda_0 \frac{dV_j}{\omega} + \frac{\xi_R}{\omega} \frac{dV_4}{\omega} \frac{dV_j}{\omega} \rho c_0^2 \right] Z_4 \bar{Z}_j \right], \\
\langle P_{44}^* \rangle &= \frac{-\omega^4}{2} \text{Re} \left[\left[-\mu \left(\left(\frac{\xi_R}{\omega}\right)^2 - \left(\frac{dV_4}{\omega}\right)^2 \right) \frac{dV_4}{\omega} + 2\mu \frac{\xi_R}{\omega} \frac{\xi_R}{\omega} \frac{dV_4}{\omega} \right] Z_4 \bar{Z}_4 \right], \\
& \quad i, j = 1, 2, 3.
\end{aligned}$$