Nonlocal dynamic modeling of mass sensors consisting of graphene sheets based on strain gradient theory

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Abstract. The following composition establishes a nonlocal strain gradient plate model that is essentially related to mass sensors laying on Winkler-Pasternak medium for the vibrational analysis from graphene sheets. To achieve a seemingly accurate study of graphene sheets, the posited theorem actually accommodates two parameters of scale in relation to the gradient of the strain as well as non-local results. Model graphene sheets are known to have double variant shear deformation plate theory without factors from shear correction. By using the principle of Hamilton, to acquire the governing equations of a non-local strain gradient graphene layer on an elastic substrate, Galerkin’s method is therefore used to explicate the equations that govern various partition conditions. The influence of diverse factors like the magnetic field as well as the elastic foundation on graphene sheet’s vibration characteristics, the number of nanoparticles, nonlocal parameter, nanoparticle mass as well as the length scale parameter had been evaluated.

Keywords: vibration; mass sensor; refined plate theory; graphene sheets; nonlocal strain gradient

1. Introduction

Graphene happens to be an atomic crystal of extraordinary electronic and mechanical properties, which are typically in two dimensions. A lot of nanostructures that depend on carbon like nanobeams, nanoplates and carbon nanotubes are often understood as deformed graphene sheets (Ebrahimi et al. 2019). Actually, a nanomaterial and nanostructure research show a thorough examination of graphene sheets which happens to be a fundamental matter. Classic theories from literature works have been used for performing analysis of scale-free plates, yet the scale impact on the nanostructures cannot be studied with small sizes by these theories. Hence, non-local elasticity theory is established in light of the small-scale results. Stark contrast to the local theory where the stress condition during stages provided depends exclusively on the stress state of that level. It is important to note that the stress state at that point depends at all stages on the stress state in the non-local theory. The non-local theory of elasticity has been widely employed in investigating nanoscale structures’ mechanical behavior (Ebrahimi and Barati 2016a, b, c, d, e, f). Signs of buckling were observed on single layer graphene sheets exposed to standard in-plane loadings by Pradhan and Murmu (2009). In addition, Pradhan and Kumar (2011) conducted a partially-analytical approach to vibration studies of orthotropic graphene sheets which incorporate non-local effects. The effects of a research conducted by Aksencer and Aydogdu (2011) examine the implementation of the Levy-type method in stability as well as vibrational studies for nanosized plates which include non-local effects. As a result of that, Mohammadi et al. (2014) carried out a review of an orthotropic graphene sheet’s shear buckling, on elastic substrates which include the thermal loading effect. They demonstrated how graphene sheet vibration reaction is greatly affected by the amount of nanoparticle attached to it. Furthermore, Farajpour et al. (2012) investigated the static stability of nonlocal plates which are exposed to standardized edge loads in the plane. On the other hand, Ansari and Sahmani (2013) have implemented simulations of molecular dynamics for investigating the biaxial buckling action for single-layer graphene sheets, dependent on nonlocal elasticity. To derive correct nonlocal parameter values, they compared the findings gathered from molecular dynamics simulations against the nonlocal plate model. According to a higher order shear deformation two-variable theory on the Winkler-Pasternak foundation, Sobhy is studying the vibrational behavior and static bending for single-layered graphene sheets. Furthermore, based on a

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two-variable non-local refined plate hypothesis, Narendar and Gopalakrishnan (2012) performed a stability study of nanoscale orthotropic plaques which was size-dependent. In their research, they mentioned how the two parameter based polished plate model takes into account the transverse shear effects across the plate depth, thus the implementation of shear correction factors is unnecessary. An investigation was conducted by Murmu et al. (2013) which included the effect of unidirectional magnetic fields for single layer nonlocal graphene sheets lying on elastic substrate vibrational behavior. Subsequently, in Bassain et al. (2015) study a nonlocal quasi-3D trigonometric plate model for free micro/nanoscale plate vibration behavior was introduced. Continuously, an examination on free vibrational activity of dual viscoelastic graphene sheets coupled with the visco-pasternak media was completed by Hashemi et al. (2015). According to Reddy’s theory which explains the higher-order deformation of sheets, Ebrahimi and Shafiee (2016) conducted an investigation where they integrated single-layered graphene sheets inside an elastic medium to understand the impacts of initial shear stress on its vibrational behavior. By implementing the Galerkin strip distributed transfer function technique, Jiang et al. (2016) directed an evaluation on vibrational behavior of a mass sensor which was based on a single-layered graphene sheet. Another study worth noting is a study investigated by Arani et al. (2016) where they examined non-local vibration of an axially traveling graphene layer sitting beneath longitudinal magnetic field on the orthotropic visco-Pasternak base. Additionally, Sobhy (2016) used the two-variable plate principle to examine the hygro-thermal vibrational action of coupled graphene sheets through an elastic surface. Zenkour (2016) further conducted transient thermal evaluation of graphene sheets derived from non-local elasticity theory on the viscoelastic foundations. Shi et al. (2019) investigated the kriging proxy model for graphene uncertainty analysis focused on a finite-element system. Georgantzinos (2017) used computer-aided design/computer-aided computational methods to create the right finite element for an effective mechanical analysis of graphene structures. Georgantzinos et al. (2016) used the spring-based finite-element method to evaluate the coupled thermomechanical actions of graphene. Mortazavi et al. (2012) performed simulations of molecular dynamics to examine the thickness and chirality impact on the tensile activity of few-layer graphene.

It ought to be referenced that solid is projected in various sorts and shapes including lightweight, high quality, green, permeable and self-combining concrete. Also, Fresh and solidified properties are two sorts of solid qualities while the new properties are alluded to the significant highlights of cement, for example, droop and usefulness. Then again, solidified properties incorporate flexural quality, compressive quality, shear quality, and consumption obstruction. There are different techniques applied to upgrade these properties, for example, cementitious substitution powders the incorporation of the filaments, and surface insurance (Willam 1975, Shariati et al. 2010, 2011a, b, d, 2014, 2016, 2019c, f, Hamidian et al. 2011, Sinaei et al. 2011, Mohammadhassani et al. 2014a, c, Arabnejad Khanouki et al. 2016, Toghroli et al. 2017, 2018b, Heydari et al. 2018, Nosrati et al. 2018, Shariat et al. 2018, Ziaei-Nia et al. 2018, Li et al. 2019, Luo et al. 2019, Safa et al. 2019, Sajedi and Shariati 2019, Suhairil et al. 2019, Trung et al. 2019b, Xie et al. 2019, Naghipour et al. 2020a, Razavian et al. 2020, Zhao et al. 2020a, b).

There are various kinds of shaped steel utilized in development applications, for example, hot and cold confined segments. Cold encircled steel associations are utilized in mechanical and capacity applications. Consequently, the presentation of rack associations has been examined in various conditions to improve their exhibition (Shah et al. 2015, 2016a, b, c, Chen et al. 2019, Naghipour et al. 2020b).

The shear quality and burden communicating capability of the composite structures can be improved by shear connectors, which are perhaps the most essential pieces of composite frameworks. Since the shear connectors execution can be influenced by high temperatures, a few examinations have been completed to address the quality loss of these components (Shariati et al. 2011c, 2012a, b, e, 2015, Shariati 2013, 2014, Khorrmanian et al. 2016, 2017, Shahabi et al. 2016a, b, Tahmasbi et al. 2016, Hosseinpour et al. 2018, Ismail et al. 2018, Nasrollahi et al. 2018, Paknahad et al. 2018, Wei et al. 2018, Davoodnabi et al. 2019, Razavian et al. 2020, Zhao et al. 2019).

Man-made brainpower strategies and exemplary numerical procedures have been consolidated together which made the new half and half calculations so as to tackle the complex issues. In such manner, trial information can be utilized for streamlining and expectation process. It is beneficial to make reference to few methodologies have been applied in late papers, for example, outrageous learning machine, neural system, and hereditary programming (Shariati et al. 2019a, b, d, e, g, 2020a, c, d, e, f, Arabnejad Khanouki et al. 2011, Daie et al. 2011, Sinaei et al. 2012, Mohammadhassani et al. 2013, 2014b, 2015, Toghroli 2015, Toghroli et al. 2014, 2016, 2018a, Mansouri et al. 2016, 2019, Safa et al. 2016, 2020, Sadeghi Chahnasir et al. 2018, Sari et al. 2018, Sedghi et al. 2018, Katebi et al. 2019, Trung et al. 2019a, Xu et al. 2019, Armaghani et al. 2020, Qi 2020).

Architects ought to think about seismic occasions, which are pulverizing for working in the development procedure. Along these lines, so as to improve the dynamic execution of the structures, various components are planned by analysts. Dampers and diminished segment associations are the two most appropriate components that can be helpful in such manner. What’s more, the auxiliary conduct of shear connectors and rack frameworks has been examined through various investigations, for example, monotonic and full cyclic, half-cyclic, and turned around cyclic tests (Arabnejad Khanouki et al. 2010, Jalali et al. 2012, Shariati 2020, Shariati et al. 2012c, d, 2013, 2017, 2018, 2019a, 2020b, g, Khorami et al. 2017a, b, Zandi et al. 2018, Milovanovic et al. 2019).

It really is obvious that almost all former papers regarding graphene sheets have implemented the non-local elasticity principle for the means of catching limited sized results. However, the principle of non-local elasticity has several drawbacks in effective estimation of nanostructural
mechanical behavior. Since the principle of non-local elasticity cannot investigate the rise in rigidity found in experimental works and the elasticity of the strain gradients. Lim et al. (2015) recently suggested the non-local strain gradient hypothesis to combine all of the longitudinal scales to form one principle. In conjunction with that, the non-local strain gradient hypothesis describes the real effect on the physical as well as mechanical actions of small size systems through the two length scale parameters Li (2016) and Li et al. (2016). Ebrahimí and Barati (2017a, b) decided to apply the non-local strain gradient principle of nanobeam analysis. From the information extracted from their study, it can be note that based on the strain gradient and non-local impacts, mechanical properties of nanostructures are relatively influenced by stiffness-hardening and stiffness-softening mechanisms, respectively. Ebrahimí et al. (2016) recently expanded the non-local strain gradient principle to analyze nanopolyster in order to acquire the wave frequencies over a total of two scale parameters. Thus, it is important that both non-local as well as the strain gradient effects are first incorporated in the graphene sheet’s analysis.

Based on a recently established non-local strain gradient principle, a generalized two-variable plate theory is used to analyze vibration activity for single-layer graphene sheets as mass sensors lying on an elastic surface. It is essential to note that the theory integrates two scale variables which correlate to non-local and strain-gradient effects to catch both stiffness-hardening and stiffness-softening influences. The theory of Hamilton is used to achieve a governing equation of a graphene surface with non-local strain gradients. These equations are obtained by the process Galerkin uses to extract the natural frequencies. The vibration activity of graphene sheets has been shown to be greatly affected by non-local parameter, parameter of the length scale, mass of nanoparticles, elastic base and magnetic field in the plane.

2. Governing equations

The principle of the higher-order refined plate theory consists of the area of displacement shown below as

\[ u_1(x, y, z) = -z \frac{\partial w_b}{\partial x} - f(z) \frac{\partial w_s}{\partial x} \]  
(1)

\[ u_2(x, y, z) = -z \frac{\partial w_b}{\partial y} - f(z) \frac{\partial w_s}{\partial y} \]  
(2)

\[ u_3(x, y, z) = w_b(x, y) + w_s(x, y) \]  
(3)

where the current theory has quite a trigonometric function which can be expressed as

\[ f(z) = z - \frac{h}{\pi} \sin \left( \frac{\pi z}{h} \right) \]  
(4)

Also, \( w_s \) as well as \( w_b \) both represent the shear and bending transverse displacement, respectively. The present plate model’s nonzero strains can be summarized as such

\[ \begin{align*}
\{ \varepsilon_x, \varepsilon_y, \gamma_{xy} \} &= +z \begin{bmatrix} k_x^b & k_x^s & \kappa_x \\ k_y^b & k_y^s & \kappa_y \\ k_{xy} & \kappa_{xy} & \kappa_{xy} \end{bmatrix} + f(z) \begin{bmatrix} k_x^s \\ k_y^s \\ \kappa_{xy} \end{bmatrix} \\
\{ \gamma_{yz}, \gamma_{zx} \} &= g(z) \begin{bmatrix} \gamma_{yz}^s \\ \gamma_{zx}^s \\ \gamma_{zy} \end{bmatrix}
\end{align*} \]  
(5)

whereby \( g(z) = 1 - \int df dz \) and

\[ \begin{align*}
\{ k_x^b, k_y^b, \kappa_{xy} \} &= \begin{bmatrix} \frac{\partial^2 w_b}{\partial x^2} & \frac{\partial^2 w_b}{\partial y^2} & -2\frac{\partial^2 w_b}{\partial x \partial y} \\
\frac{\partial^2 w_s}{\partial x^2} & \frac{\partial^2 w_s}{\partial y^2} & -2\frac{\partial^2 w_s}{\partial x \partial y} \\
-2\frac{\partial^2 w_b}{\partial x \partial y} & -2\frac{\partial^2 w_s}{\partial x \partial y} & 0
\end{bmatrix}, \\
\{ \gamma_{yz}^s, \gamma_{zx}^s, \gamma_{zy} \} &= \begin{bmatrix} \frac{\partial w_s}{\partial y} \\ \frac{\partial w_s}{\partial x} \\ \frac{\partial w_s}{\partial y} \\ \frac{\partial w_s}{\partial x}
\end{bmatrix}
\end{align*} \]  
(6)

Additionally, this is portrayed by Hamilton’s principle

\[ \int_0^t \delta(U - T + V) dt = 0 \]  
(7)

where \( U \) is the strain energy, \( T \) is kinetic energy and \( V \) is work done by external loads. The strain energy variation is quantified by

\[ \delta U = \int \sum_{ij} \sigma_{ij} \delta \epsilon_{ij} dV \]

\[ = \int \left( \sigma_x \delta \epsilon_x + \sigma_y \delta \epsilon_y + \sigma_{xy} \delta \gamma_{xy} + \sigma_{yz} \delta \gamma_{yz} + \sigma_{zx} \delta \gamma_{zx} \right) dV \]  
(8)

Substituting Eqs. (5) and (6) into Eq. (8) forms

\[ \delta U = \int_a^b \int_0^1 \left[ -M_x^b \frac{\partial^2 \delta w_b}{\partial x^2} - M_y^b \frac{\partial^2 \delta w_b}{\partial y^2} - M_x^s \frac{\partial^2 \delta w_s}{\partial x^2} - M_y^s \frac{\partial^2 \delta w_s}{\partial y^2} - 2M_{xy}^s \frac{\partial^2 \delta w_s}{\partial x \partial y} \right] \frac{\partial \delta w_s}{\partial y} + Q_{yz} \frac{\partial \delta w_s}{\partial y} + Q_{zx} \frac{\partial \delta w_s}{\partial x} dxdy \]  
(9)

wherein

\[ (M_x^b, M_y^b) = \int_{-h/2}^{h/2} (z, f) \sigma_1 dz, i = (x, y, xy) \]  
(10)

\[ Q_1 = \int_{-h/2}^{h/2} g \delta_1 dz, i = (xz, yz) \]

The variation of the work done from applied loads can be expressed through

\[ \delta V = \int_a^b \int_0^1 \left( N_x^b \frac{\partial (w_b + w_s)}{\partial x} - \frac{\partial \delta (w_b + w_s)}{\partial x} + N_y^0 \frac{\partial (w_b + w_s)}{\partial y} + 2\delta N_y^0 \right) dxdy \]  
(11a)
where $N^0_0$, $N^0_y$, $N^0_{xy}$ are in-plane applied loads; $k_w$ and $k_p$ are Winkler and Pasternak constants. Additionally, $\eta$ is magnetic parliamentary. Also, $q_{\text{particle}}$ happens to be the transverse force because of attached nanoparticles (more like a bacterium, molecular or buckyball) which has a mass $m$, that happens to be adhered to the location $x = x_0, y = y_0$.

$$ q_{\text{particle}} = - \sum_{j=1}^{N} m_j \delta(x - x_j, y - y_j) \frac{\partial^2 w}{\partial t^2} $$

(11b)

in which $m_j$ is the mass of $j$th attached particle, while $N$ is the amount of concentrated masses. Furthermore, $\delta(x - x_j)$ is Dirac delta function outlined by

$$ \delta(x - x_j) = \begin{cases} \infty & x \neq x_j \\ 0 & x = x_j \end{cases} $$

(11c)

The kinetic energy’s variation is calculated as

$$ \delta K = \int_{0}^{b} \int_{-h/2}^{-h/2} \left[ l_0 \frac{\partial (w_b + w_s)}{\partial t} \frac{\partial \delta(w_b + w_s)}{\partial t} + l_2 \left( \frac{\partial w_b}{\partial x \partial t} \frac{\partial w_b}{\partial y \partial t} + \frac{\partial w_b}{\partial y \partial t} \frac{\partial w_b}{\partial y \partial t} \right) + K_2 \left( \frac{\partial w_b}{\partial x \partial t} \frac{\partial w_b}{\partial x \partial t} + \frac{\partial w_b}{\partial x \partial t} \frac{\partial w_b}{\partial y \partial t} \right) + J_2 \left( \frac{\partial w_b}{\partial x \partial t} \frac{\partial w_b}{\partial x \partial t} + \frac{\partial w_b}{\partial y \partial t} \frac{\partial w_b}{\partial y \partial t} \right) \right] dx dy $$

\[ \begin{array}{l}
(l_0, l_2, J_2, K_2) = \int_{-h/2}^{h/2} (1, z^2, zf, f^2) \rho \, dz \\
(12)
\end{array} \]

Through inputting Eqs. (19)-(22) into Eq. (17) as well as setting the coefficients of $\delta u$, $\delta v$, $\delta w_b$ and $\delta w_s$ to zero, the following Euler-Lagrange expressions should be attained.

$$ \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \left( (N_x^T + N_y^T - \eta \nabla^2) \nabla^2 \right) $$

\[ \begin{array}{l}
(w_b + w_s) + k_p \left[ \frac{\partial^2 (w_b + w_s)}{\partial x^2} + \frac{\partial^2 (w_b + w_s)}{\partial y^2} \right] \\
- k_w (w_b + w_s) - \sum_{j=1}^{N} m_j \delta(x - x_j, y - y_j) \frac{\partial^2 w}{\partial t^2} \\
= l_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} - l_2 \nabla^2 \left( \frac{\partial^2 w_b}{\partial t^2} \right) - J_2 \nabla^2 \left( \frac{\partial^2 w_b}{\partial t^2} \right) \\
\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial Q_{xx}}{\partial x} + \frac{\partial Q_{yy}}{\partial y} - (N_x^T) \end{array} \]

(14)

$$ + N_x^T - \eta \nabla^2 \nabla^2 + k_p \left[ \frac{\partial^2 (w_b + w_s)}{\partial x^2} + \frac{\partial^2 (w_b + w_s)}{\partial y^2} \right] $$

\[ \begin{array}{l}
+ \frac{\partial^2 (w_b + w_s)}{\partial y^2} - k_w (w_b + w_s) - \sum_{j=1}^{N} m_j \delta(x - x_j, y - y_j) \\
y - y_j \frac{\partial^2 w}{\partial t^2} - l_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} - J_2 \nabla^2 \left( \frac{\partial^2 w_b}{\partial t^2} \right) \\
- K_2 \nabla^2 \left( \frac{\partial^2 w_b}{\partial t^2} \right) \end{array} \]

(15)

2.1 Nonlocal strain gradient nanoplate model

The recently created nonlocal strain gradient theory considers the nonlocal stress field as well as the strain gradient effects through presenting two scale parameters. This theory describes the stress field as

$$ \sigma_{ij} = \sigma_{ij}^{(0)} - \frac{\sigma_{ij}^{(1)}}{dx} $$

(18)

in which the stresses $\sigma_{ij}^{(0)}$ and $\sigma_{ij}^{(1)}$ correspond to strain $\varepsilon_{xx}$ along with strain gradient $\varepsilon_{xx,xx}$, respectively through

$$ \sigma_{ij}^{(0)} = \int_{0}^{L} C_{ijkl} a_0(x, x', e_i a) \varepsilon_{kl}(x') dx' $$

(19)

$$ \sigma_{ij}^{(1)} = \int_{0}^{L} C_{ijkl} a_1(x, x', e_i a) \varepsilon_{kl}(x') dx' $$

(20)

whereby $C_{ijkl}$ is the elastic coefficients and $e_i a$ alongside $e_i a$ apprehends the nonlocal effects and $l$ take into consideration the effects of the strain gradient. The constitutive relation of nonlocal strain gradient theory contains the mentioned form below, when the nonlocal functions $a_0(x, x', e_i a)$ and $a_1(x, x', e_i a)$ satisfy the conditions created by Eringen (1983)

$$ [1 - (e_i a)^2 \nabla^2] \left[ 1 - (e_i a)^2 \nabla^2 \right] \sigma_{ij} = C_{ijkl} \left[ 1 - (e_i a)^2 \nabla^2 \right] \varepsilon_{kl} - C_{ijkl} \left[ 1 - (e_i a)^2 \nabla^2 \right] \varepsilon_{kl}^2 $$

(21)

where $\nabla^2$ signifies the Laplacian operator. When taking into account that $e_i = e_0 = e$, the general constitutive relation in Eq. (21) ends up as

$$ [1 - (ea)^2 \nabla^2] \sigma_{ij} = C_{ijkl} [1 - l^2 \nabla^2] \varepsilon_{kl} $$

(22)

Lastly, the constitutive relations of nonlocal strain gradient theory may get portrayed through

$$ \sigma_x \sigma_y \sigma_{xy} \left( 1 - \mu \nabla^2 \right) \sigma_x \sigma_y \sigma_{xy} \left( 1 - \lambda \nabla^2 \right) $$

$$ \begin{pmatrix}
\sigma_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{66} & 0 & 0 \\
0 & 0 & 0 & Q_{44} & 0 \\
0 & 0 & 0 & 0 & Q_{55}
\end{pmatrix} \begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
\gamma_{xx} \\
\gamma_{zz}
\end{pmatrix} $$

(23)
where
\[ Q_{11} = Q_{22} = \frac{E}{1 - \nu^2}, \quad Q_{12} = u Q_{11} \]
\[ Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1 + \nu)} \]

Inputting Eq. (10) in Eq. (23) gives

\[
\begin{pmatrix}
\frac{\partial^2 w_b}{\partial x^2} \\
\frac{\partial^2 w_b}{\partial y^2} \\
-2 \frac{\partial^2 w_b}{\partial x \partial y}
\end{pmatrix}
+ \begin{pmatrix}
D_{11} & D_{12} & 0 \\
D_{12} & D_{22} & 0 \\
0 & 0 & D_{66}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial^2 w_s}{\partial x^2} \\
\frac{\partial^2 w_s}{\partial y^2} \\
-2 \frac{\partial^2 w_s}{\partial x \partial y}
\end{pmatrix}
= \begin{pmatrix}
\frac{\partial^2 w_b}{\partial x \partial y} \\
\frac{\partial^2 w_b}{\partial x \partial y} \\
\frac{\partial^2 w_b}{\partial x \partial y}
\end{pmatrix}
\]

(25)

\[
\begin{pmatrix}
\frac{\partial^2 w_s}{\partial x^2} \\
\frac{\partial^2 w_s}{\partial y^2} \\
-2 \frac{\partial^2 w_s}{\partial x \partial y}
\end{pmatrix}
+ \begin{pmatrix}
H_{11} & H_{12} & 0 \\
H_{12} & H_{22} & 0 \\
0 & 0 & H_{66}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial^2 w_s}{\partial x^2} \\
\frac{\partial^2 w_s}{\partial y^2} \\
-2 \frac{\partial^2 w_s}{\partial x \partial y}
\end{pmatrix}
= \begin{pmatrix}
\frac{\partial^2 w_s}{\partial x \partial y} \\
\frac{\partial^2 w_s}{\partial x \partial y} \\
\frac{\partial^2 w_s}{\partial x \partial y}
\end{pmatrix}
\]

(26)

\[
(1 - \sigma^2 \sigma^2)\begin{pmatrix}
\frac{\partial^2 Q_s}{\partial x^2} & \frac{\partial^2 Q_s}{\partial x \partial y} \\
\frac{\partial^2 Q_s}{\partial y^2} & \frac{\partial^2 Q_s}{\partial y \partial x}
\end{pmatrix}
= (1 - \sigma^2 \sigma^2)\begin{pmatrix}
\frac{\partial^2 Q_s}{\partial x^2} & \frac{\partial^2 Q_s}{\partial x \partial y} \\
\frac{\partial^2 Q_s}{\partial y^2} & \frac{\partial^2 Q_s}{\partial y \partial x}
\end{pmatrix}
\]

(27)

\[
\begin{pmatrix}
D_{11} & D_{12} & 0 \\
D_{12} & D_{22} & 0 \\
0 & 0 & D_{66}
\end{pmatrix}
\int_{-h/2}^{h/2} Q_{11}(\sigma^2 z, \sigma^2 y, \sigma^2) \begin{pmatrix}
1 \\
\nu \\
2
\end{pmatrix}
\frac{dz}{2}
\]

(28)

\[
A_s^{44} = A_s^{55} = \int_{-h/2}^{h/2} g^2 \frac{E}{2(1 + \nu)} \frac{dz}{2}
\]

(29)

The governing equations of nonlocal strain gradient graphene sheet based on the displacement are gathered by placing Eqs. (25)-(27), into Eqs. (14)-(15) as shown below.

\[
-\frac{\partial^4 w_b}{\partial x^4} - \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} = l^2 \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - l^2 \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_b}{\partial x^2 \partial y^2}
\]

(30)

3. Solution by Galerkin's approach

For this portion, Galerkin's approach of solving the governing equations for non-local strain gradient graphene sheets is enforced. Therefore, the field of displacement may get computed as
\[ D_1^4 \frac{\partial^4 w_b}{\partial x^4} + (I(\frac{\partial^6 w_b}{\partial x^6} + \frac{\partial^6 w_b}{\partial x^4 \partial y^2}) + A_5(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2}) + A_{14}(\frac{\partial^4 w_b}{\partial y^4} + \frac{\partial^4 w_b}{\partial y^2 \partial x^2} + \frac{\partial^4 w_b}{\partial x^4 \partial y^2}) - 2(D_1^{12})^2 + 2D_6^{10}(\frac{\partial^2 w_b}{\partial x^2 \partial y^2} - \frac{\partial^2 w_b}{\partial x^4} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2}) - D_2^{12}(\frac{\partial^4 w_b}{\partial x^4 \partial y^2} + \frac{\partial^4 w_b}{\partial x^2 \partial y^2} + \frac{\partial^4 w_b}{\partial x^4} + \frac{\partial^4 w_b}{\partial x^2}) \right) - 2(H_2^{10} + 2H_6^{10})\left(\frac{\partial^4 w_b}{\partial x^4 \partial y^2} - \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - \frac{\partial^4 w_b}{\partial x^2} - \frac{\partial^4 w_b}{\partial x^4} \right) - \left(\frac{\partial^2 w_b}{\partial x^2 \partial y^2} - \frac{\partial^2 w_b}{\partial x^2} \right) = k_2(1 - (\text{eq})^2) \left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} \right) + k_2 \left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} \right) + k_2 \left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} + \frac{\partial^2 w_b}{\partial x^2 \partial y^2} \right)
\]

wherewith \( W_{bnm} \) \( W_{bmn} \) are the unknown coefficients while the functions \( \Phi_{bnm} \) as well as \( \psi_{bn} \) satisfy boundary conditions. According to present plate model, the boundary conditions are

\[ w_b = w_0 = 0 \]
\[ \frac{\partial w_b}{\partial x} = \frac{\partial w_b}{\partial y} = 0 \quad \text{simply-supported edge} \]

Inserting Eqs. (32) and (33) into Eqs. (30)-(31) while ensuring to multiply both sides of the equations through \( \psi_{bnm} \) (i.e., \( i = a, b, s \)) and incorporating over the entire region result in the following simultaneous equations shown below.

\[ \int_0^b \int_0^a \Phi_{bmn} \left[ D_1^{11} \frac{\partial^4 \Phi_{bmn}}{\partial x^4} - \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} - \frac{\partial^4 \Phi_{bmn}}{\partial x^4 \partial y^2} - \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^4 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^4 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi_{bmn}}{\partial x^2 \partial y^2} \right] dx \]
Table 1 Comparison of natural frequency of a graphene sheet for various nonlocal and foundation parameters (\(\alpha h = 10\))

<table>
<thead>
<tr>
<th>(\mu)</th>
<th>(K_w = 0)</th>
<th>(K_w = 100)</th>
<th>(K_w = 0)</th>
<th>(K_w = 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.93861</td>
<td>2.18396</td>
<td>2.78410</td>
<td>2.78410</td>
</tr>
<tr>
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<td>1.54903</td>
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<td>2.31969</td>
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<td>1.36479</td>
<td>2.14629</td>
<td>2.14629</td>
</tr>
<tr>
<td>3</td>
<td>0.78347</td>
<td>1.27485</td>
<td>2.11486</td>
<td>2.11486</td>
</tr>
<tr>
<td>4</td>
<td>0.69279</td>
<td>1.22122</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The function \(\Phi_{mn}\) for simply-supported boundary conditions is outlined by

\[
\text{SS: } \Phi_{mn}(x) = \sin \sin(\lambda_m x) \\
\lambda_m = \frac{m\pi}{a}
\] (37)

The function \(\Psi_n\) may be attained by substituting \(x, m\) and \(a\), respectively by \(y, n\) and \(b\). Setting the coefficient matrix for the abovementioned results in the eigenvalue mentioned below issue

\[
([K] + \omega^2[M]) \begin{bmatrix} W_h \\ W_s \end{bmatrix} = 0
\] (38)

whereby \([M]\) as well as \([K]\) are the mass matrix and stiffness matrix, respectively. Lastly, putting the coefficient matrix to zero provides the natural frequencies. Notably, the calculations presented are reliant on the dimensionless quantities as follows

\[
\tilde{\omega} = \frac{a^2}{h} \sqrt{\frac{\rho}{E}}, \quad K_w = k_w \frac{a^4}{D^*}, \quad K_p = k_p \frac{a^2}{D^*} \quad (39)
\]

4. Numerical results and discussions

This segment has been allocated to analyze the vibration tendencies for the nonlocal strain gradient graphene sheet mass sensors on elastic substrates that according to two-variable shear deformation theory. Consequently, the framework presents two scale coefficients pertaining to strain gradient as well as nonlocal effects in order to obtain an increasingly precise investigation of graphene sheets. Furthermore, the Graphene Sheets’ material properties are: \(E = 1\) TPa, \(\nu = 0.19\) and \(\rho = 2300\) kg/m\(^2\). Additionally, the breadth and length of the graphene sheet are regarded to be \(h = 0.34\) nm, \(a = 10\) nm. Figs. 1 and 2, respectively, provide a visualization as to the distribution of nanoparticles and the configuration of the graphene sheet. The graphene sheet’s natural frequencies get checked to correspond alongside the ones procured by Sobhy (2016) for numerous nonlocal parameters \(\mu = 0, 1, 2, 3, 4\) nm\(^2\) as well as the foundation constants \([K_w, K_p] = \{(0, 0), (100, 0), (0, 20)\}\). As counted in Table 1, the procured frequencies through employing the current Galerkin method are held to be in prime accordance with that of the precise solution introduced by Sobhy (2016). The length scale parameter or strain gradient has been fixed at zero \(\lambda = 0\) for the purposes of the comparative study.

Upon the vibration frequencies of graphene sheet mass parameters \(\mu = 0, 1, 2, 3, 4\) nm\(^2\) as well as the foundation constants \([K_w, K_p] = \{(0, 0), (100, 0), (0, 20)\}\). As counted in Table 1, the procured frequencies through employing the current Galerkin method are held to be in prime accordance with that of the precise solution introduced by Sobhy (2016). The length scale parameter or strain gradient has been fixed at zero \(\lambda = 0\) for the purposes of the comparative study.
sensors, Fig. 3 presents an evaluation of the effects of nonlocal and strain gradient. The previously mentioned figure acknowledges a variety of values of the length scale ($\lambda = 0, 0.1, 0.2$) and nonlocal parameters ($\mu = 0~1$). Upon $\lambda = 0$, it is evident that the natural frequencies of a graphene sheet according to proven nonlocal elasticity theory are capable of being acquired, nonetheless, upon both $\lambda = 0$ and $\mu = 0$, the outcomes founded upon classical continuum mechanics get decreed. It has been noted how an augmentation of nonlocal parameters would result in a reduction in the graphene sheet’s natural frequency and the record posits that the stiffness-softening effect exerted by a nonlocal parameter results in frequencies of a lower vibration. However, the nonlocal parameters impact upon the natural frequencies’ magnitude is dependent on the valuation of the length scale parameter or strain gradient. As a matter of fact, an upsurge of the length scale parameter that accentuates the stiffness-hardening effect because of the strain gradients will cause an upsurge in the graphene sheet’s natural frequency. Additionally, at set length scale and nonlocal parameters, an amelioration in the mass of the...
nanoparticle results in a reduction of the vibration frequencies. Moreover, it is noted that the impact of nonlocal parameters on the vibration frequency of nano-mechanical mass sensors is increasingly significant in regards to smaller nanoparticles.

Fig. 4 presents the variation of dimensionless frequency of graphene sheet against nanoparticle mass for numerous foundation parameters as well as nanoparticle location at \( \mu = 0.2 \) and \( \lambda = 0.1 \). It has been discovered that an increase in mass results in a decrease in the vibration frequencies for every nanoparticle. Furthermore, one can observe the variability of frequency which is evident when the mass that is attached is greater than \( 10^{-21} \) g. Factually, the mass sensitivity of the nano-mechanical mass sensor is capable of recording at least \( 10^{-21} \) g. Also, it is noted how the impact of the location of nanoparticles is prominently enhanced upon an increase of the attached mass. Upon the closing of the nanoparticles proximity to the plate center, there is a reduction in the vibration frequency of the nano-mechanical mass sensor. As a result of that, it can be noticed how the vibration behavior for the nano-mechanical mass sensor relies on values from both the Pasternak and Winkler variables. In actuality, the Pasternak layer contains a lasting relationship with the sheet of graphene, while the Winkler layer contains a discontinued relationship with the graphene sheets. Adding to the parameters for Winkler and Pasternak results in greater frequencies through improving the graphene sheet’s bending rigidity. Although, comparison with the Winkler layer, Pasternak layer displays more growing impact on frequencies.

Fig. 5 demonstrates the variant dimensionless frequencies compared to the attached mass pertaining to various length and width parameters as well as aspect ratios \((a/b)\) at \( K_w = 25, K_p = 5, \mu = 0.2, \lambda = 0.1 \). Notably, graphene sheets with greater lengths contain greater vibration frequencies. This is due to the fact that, graphene sheets...
with reduced sizes happen to be more rigid. Thus, a graphene sheet mass sensor with smaller sizes is much more sensitive with the attached mass. In fact, variation of dimensionless frequency with respects to the mass attached, becomes more significant as the size of graphene sheet reduces.

Magnetic field’s impact on the vibrational frequency of the nano-mechanical mass sensor in related to the mass attached for different scale parameters is presented in Fig. 6 at $K_0 = 25$, $K_p = 5$. It should be remembered how the graphene layer has a stiffness-hardening effect on the in-plane magnetic field. As a matter of fact, increased intensity of the magnetic field results in large frequencies. In conclusion, in-plane magnetic field plays a crucial role on the vibrational behavior of nano-mechanical mass sensor which must be planned to be checked in order to have and sustain their accurate design. While nano-mechanical mass sensor vibration behavior relies on the importance of the scale parameters. Subsequently, if $\mu > \lambda$, the nonlocal influence becomes more predominant, and if $\mu < \lambda$, the impact of the strain gradient is stronger, contributing to vibration frequency enlargement.

Fig. 7 shows the variety of alternative dimensionless frequency of graphene sheet as compared to the length scale parameter for numerous number of nanoparticles at $\mu = 0.2$, $K_0 = 25$, $K_p = 5$, $H_i = 1$. Another important note should be the overall mass of nanoparticles is $10^{-15}$ g and $10^{-20}$ g is considered as an assumption. The graphene sheet shows minimal frequency when one attached mass is positioned at the middle of the layer. Although, when two masses are attached at the location $(x, y) = (0.25, 0.5)$, $(0.75, 0.5)$, they give off larger frequencies as compared to both one attached mass along with four attached masses. Lastly, the location and effective mass have dominant effect on vibration techniques, due to the overall nanoparticle mass which is identical.

5. Conclusions

In order to explore the free vibration tendencies of single-layer graphene sheet mass sensors which rest upon an elastic medium through utilizing a refined two-variable plate theory, this composition has employed the theory of nonlocal strain gradient. Following the introduction of two scale parameters in order to acquire both stiffness-hardening alongside stiffness-softening influences that commensurate with strain gradient and nonlocal effects, Hamilton’s principle is utilized to derive the principal equation of a nonlocal strain gradient graphene sheet. The aforementioned equations are elucidated through making use of Galerkin’s technique in regards to obtaining the natural frequencies. The natural frequency of graphene sheets has accordingly been noted to be reduced following an increase of nonlocal parameters. Contrarily, it has been observed that an upsurge of the length scale parameter which accentuates the stiffness-hardening effect as a result of the strain gradients corresponds with an upsurge of natural frequency. The mass and number of attached nanoparticles are regarded to impact all these observations. Nevertheless, upon proximity of the nanoparticle to the plate center, there is a reduction in the frequency of vibration for the nano-mechanical mass sensor.

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References


Nonlocal dynamic modeling of mass sensors consisting of graphene sheets based on strain gradient theory


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