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Flexoelectric effects on dynamic response characteristics of nonlocal piezoelectric material beam

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Abstract. Flexoelectric effect has a major role on mechanical responses of piezoelectric materials when their dimensions become submicron. Applying differential quadrature (DQ) method, the present article studies dynamic characteristics of a small scale beam made of piezoelectric material considering flexoelectric effect. In order to capture scale-dependency of such piezoelectric beams, nonlocal elasticity theory is utilized and also surface effects are included for better structural modeling. Governing equations have been derived by utilizing Hamilton's rule with the assumption that the scale-dependent beam is subjected to thermal environment leading to uniform temperature variation across the thickness. Obtained results based on DQ method are in good agreement with previous data on pizo-flexoelectric beams. Finally, it would be indicated that dynamic response characteristics and vibration frequencies of the nano-size beam depends on the existence of flexoelectric influence and the magnitude of scale factors.

Keywords: flexoelectricity; flexoelectric effect; response; piezoelectric; wave; beam

1. Introduction

Flexoelectric effect has a major role on responses of piezoelectric materials when their dimensions become submicron. The flexo-electricity is associated with a specific electrical-mechanical coupling phenomena among polarizations and strains gradients (Jiang *et al*. 2013, Barati 2017, Besseghier *et al*. 2017, Mouffoki *et al*. 2017). Actually, inflicting the strain gradients to a dielectric may exert special electric polarizations via changing the inversion symmetries. In many studies, it has been shown that the flexo-electricity leads to inherent scale influences as the dimension of smart material reduces (Zhang *et al*. 2014a, b, Liang *et al*. 2014, 2015, Yang *et al*. 2015, Shafiei *et al*. 2017, Mirjavadi *et al*. 2017a, b, 2018a, b, 2019a, b).

Recent studies focus on engineering structures at nano-scales due to their involvement in nano-mechanical systems or devices (Azimi *et al*. 2017, 2018) However, the main issue in these studies is to select an appropriate elasticity theory accounting for small scale impacts. The impact of size-dependency might be considered with the help of a scale parameter involved in non-local theory of

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elasticity Eringen (1983). The word “non-local” means that the stresses are not local anymore (Ahmed *et al*. 2019, Alasadi *et al*. 2019, Alimirzaei *et al*. 2019, Adda Bedia *et al*. 2019). This is because we are talking about a stress field of nano-scale structure. Many authors are aware of these facts and they are using this theory to analysis mechanical characteristics of small size engineering structures (Ke and Wang 2012, Liu *et al*. 2013, 2014, Ke *et al*. 2015, Akgoz and Civalek 2013, Akbas 2016, Semmah *et al*. 2014, 2019, Fernández-Sáez *et al*. 2016, Civalek and Demir 2016, Bensaid and Guenanou 2017).

Among different types of smart materials, the piezo-electric material represents superb possible application in smart structures/systems and also nano-sized devices owning to giving wonderful mechanical and electrical coupling performances. Applying electrical fields to nano-dimension beams yields elastic deformations and changed vibrational properties. Due to the reason that performing experiment on piezoelectric nano-dimension beams are effortful yet, many scholars have represented their theoretical models taking into account small scales influences. Employing nonlocal theory of elasticity, one may be able to incorporate the small scales influences in theoretical model of nano-dimension beams. The theory recommends a scale factor called nonlocal parameter for describing that the stress fields at nano scales have a nonlocal character. However, at nano-scale surface effects (Gurtin and Murdoch 1975) become prominent as proved by many authors. The surface layers can accurately describe the nano-dimension character of the beams (Wang and Wang 2011, 2012, Yan and Jiang 2011, Zhang *et al*. 2013, Li and Pan 2016).

Flexoelectric effect has a major role on mechanical responses of piezoelectric materials when their dimensions become submicron. The flexo-electricity is associated with a specific electrical-mechanical coupling phenomena among polarizations and strains gradients (Jiang *et al*. 2013, Barati 2017, Besseghier *et al*. 2017, Mouffoki *et al*. 2017). Actually, inflicting the strain gradients to a dielectric may exert special electric polarizations via changing the inversion symmetries. In many studies, it has been shown that the flexo-electricity leads to inherent scale influences as the dimension of smart material reduces (Zhang *et al*. 2014a, b, Liang *et al*. 2014, 2015, Yang *et al*. 2015).

As mentioned, flexoelectric effect has a major role on mechanical responses of piezoelectric materials when their dimensions become submicron. Applying differential quadrature (DQ) method, the present article studies dynamic characteristics of a small scale beam made of piezoelectric material considering flexoelectric effect. In order to capture scale-dependency of such piezoelectric beams, nonlocal elasticity theory is utilized and also surface effects are included for better structural modeling. Governing equations have been derived by utilizing Hamilton’s rule with the assumption that the scale-dependent beam is subjected to thermal environment leading to uniform temperature variation across the thickness. Obtained results based on DQ method are in good agreement with previous data on pizo-flexoelectric beams. Finally, it would be indicated that dynamic response characteristics and vibration frequencies of the nano-size beam depends on the existence of flexoelectric influence and the magnitude of scale factors.

2. Flexoelectric effects on nonlocal constitutive relations

Here, the nano-size beam has been assumed to be fabricated from PZT-5H piezoelectric material. The geometry of the considered nano-size beam has been illustrated in Fig. 1. To describe nonlocal relations for a flexoelectric nano-size beam, one must define stress components σ_{ij} and polarization components P_i as functions of strains ε_{ij} and electrical field E_k as

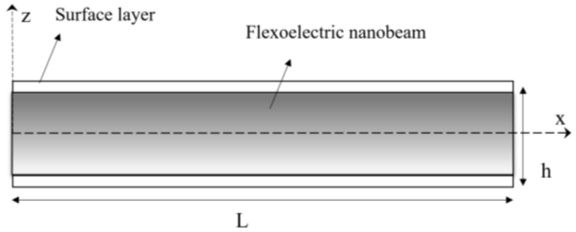

Fig. 1 Geometry of flexoelectric nano-size beam with surface layers

$$\sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k + f_{klij}\frac{\partial E_k}{\partial x_l} - C_{ijkl}\alpha_{kl}\Delta T \tag{1a}$$

$$P_i - (e_0 a)^2 \nabla^2 P_i = \varepsilon_0 \chi_{ij} E_j + e_{ikl}\varepsilon_{kl} + f_{ijkl}\frac{\partial \varepsilon_{kl}}{\partial x_j} - p_i \Delta T \tag{1b}$$

Here, C_{ijkl}, e_{kij} and k_{ik} respectively define elastic, piezo-electrical and dielectrics constants (Aboudi 2001). In the present study, flexoelectirc factor has been denoted by f_{ijkl}, while temperature variation has been denoted by ΔT (Abualnour *et al*. 2019). The below equation is written for defining the energy densities of electrical enthalpy (Zhang *et al*. 2014a, b)

$$H = -\frac{1}{2}a_{kl}E_k E_l + \frac{1}{2}c_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - e_{kij}E_k\varepsilon_{ij} - \frac{1}{2}f_{klij}(E_k\frac{\partial \varepsilon_{ij}}{\partial x_l} - \varepsilon_{ij}\frac{\partial E_k}{\partial x_l}) \tag{2}$$

Next, the general nonlocal formulations for stresses and electrical displacements (D_i) and densities (Q_{ij}) might be stated as

$$(1 - (e_0 a)^2 \nabla^2)\sigma_{ij} = \frac{\partial H}{\partial \varepsilon_{ij}} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k + \frac{f_{klij}}{2}\frac{\partial E_k}{\partial x_l} - C_{ijkl}\alpha_{kl}\Delta T \tag{3a}$$

$$(1 - (e_0 a)^2 \nabla^2)\tau_{ijl} = \frac{\partial H}{\partial(\partial \varepsilon_{ij}/\partial x_l)} = -f_{ijkl}E_k \tag{3b}$$

$$(1 - (e_0 a)^2 \nabla^2)D_i = -\frac{\partial H}{\partial E_i} = a_{ij}E_j + e_{ikl}\varepsilon_{kl} + \frac{f_{ijkl}}{2}\frac{\partial \varepsilon_{kl}}{\partial x_j} - p_i \Delta T \tag{3c}$$

$$(1 - (e_0 a)^2 \nabla^2)Q_{ij} = \frac{\partial H}{\partial(\partial E_i/\partial x_j)} = -\frac{f_{ijkl}}{2}\varepsilon_{kl} \tag{3d}$$

In order to incorporate surface layers influences, one must define the surface energy (U_s) based on below relation (Zhang *et al*. 2014a, b)

$$U_s = \Gamma_{\alpha\beta}\varepsilon^s_{\alpha\beta} - \frac{1}{2}a^s_{\gamma\kappa}E^s_\gamma E^s_\kappa + \frac{1}{2}c^s_{\alpha\beta\gamma\kappa}\varepsilon^s_{\alpha\beta}\varepsilon^s_{\gamma\kappa} - e^s_{\kappa\alpha\beta}E^s_\kappa\varepsilon^s_{\alpha\beta} \tag{4}$$

So that $\Gamma_{\alpha\beta}$ defines the surfaces residual stresses, $a^s_{\gamma\kappa}$ and $c^s_{\alpha\beta\gamma\kappa}$ define the surfaces permittivity and surfaces elastic factors. Then, $e^s_{\kappa\alpha\beta}$ and E^s_κ define the surfaces piezo-electrical tensors and surfaces electrical fields. Next, the general nonlocal formulations for stresses ($\sigma^s_{\alpha\beta}$) and electrical displacements (D^s_γ) of surfaces might be stated as

$$(1-(e_0a)^2\nabla^2)\sigma^s_{\alpha\beta} = \frac{\partial U_s}{\partial \varepsilon_{\alpha\beta}} = \Gamma_{\alpha\beta} + c^s_{\alpha\beta\gamma\kappa}\varepsilon^s_{\gamma\kappa} - e^s_{\kappa\alpha\beta}E^s_\kappa \tag{5a}$$

$$(1-(e_0a)^2\nabla^2)D^s_\gamma = -\frac{\partial U_s}{\partial E^s_\gamma} = a^s_{\gamma\kappa}E^s_\kappa + e^s_{\gamma\alpha\beta}\varepsilon^s_{\alpha\beta} \tag{5b}$$

3. Beam formulation

In order to develop the formulation for linear vibrations of nonlocal beam, well-known classical beam theory among different beam models has been used in the present paper (Bourada *et al*. 2019, Boukhlif *et al*. 2019, Boulefrakh *et al*. 2019, Boutaleb *et al*. 2019, Berghouti *et al*. 2019, Draoui *et al*. 2019, Chaabane *et al*. 2019, Zarga *et al*. 2019, Medani *et al*. 2019, Mahmoudi *et al*. 2019, Draiche *et al*. 2019, Khiloun *et al*. 2019, Tlidji *et al*. 2019, Karami *et al*. 2019a, b, Addou *et al*. 2019). Thus, the displacements of beam (u_1, u_2 = 0, u_3) may be written based on axial (u) and transverse (w) field variables as

$$u_1(x,y,z) = u - z\frac{\partial w}{\partial x} \tag{6a}$$

$$u_3(x,y,z) = w \tag{6b}$$

For the classic beam model, the strain field including its gradient might be expressed by

$$\varepsilon_{xx} = \frac{\partial u_1}{\partial x} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}, \quad \eta_{xxz} = \frac{\partial \varepsilon_{xx}}{\partial z} = -\frac{\partial^2 w}{\partial x^2}. \tag{7}$$

For a piezo-flexoelectric nano-size beam, the governing equations based on classic beam theory and nonlocal stress effects may be expressed by

$$\frac{\partial (N_{xx}+N^s_{xx})}{\partial x} = I_0\frac{\partial^2 u}{\partial t^2} \tag{8}$$

$$\frac{\partial^2 (M_{xx}+M^s_{xx})}{\partial x^2} + \frac{\partial^2 P_{xxz}}{\partial x^2} + (2b\sigma_0 - bN^T)\nabla^2 w = I_0\frac{\partial^2 w}{\partial t^2} - I_2\nabla^2(\frac{\partial^2 w}{\partial t^2}) \tag{9}$$

where $N^T = c_{11}\alpha_1 h\Delta T$ is thermal load and

$$(I_0, I_1, I_2) = b\int_{-h/2}^{h/2}(1,z,z^2)\rho\, dz = b\{\rho h, 0, \frac{\rho h^3}{12}\} \tag{10}$$

Also, N_{xx} and M_{xx} are in-plane force and bending moment defined as

$$(N_{xx}, M_{xx}) = \int_A (1, z)\sigma_{xx}\, dA, \tag{11}$$

$$P_{xxz} = \int_A \tau_{xxz}\, dA. \tag{12}$$

Also, the boundary conditions are

$$u = 0, or (N_{xx} + N_{xx}^s)n_x = 0 \tag{13a}$$

$$w = 0, or n_x\left(\frac{\partial(M_{xx} + M_{xx}^s)}{\partial x} + \frac{\partial P_{xxz}}{\partial x} - N^T \frac{\partial w}{\partial x}\right) = 0 \tag{13b}$$

$$\frac{\partial w}{\partial x} = 0, or (M_{xx} + M_{xx}^s)n_x = 0 \tag{13c}$$

All ingredients of stress field, electrical field displacement (D_z) and electric density (Q_{zz}) for a size-dependent beam relevant to nonlocal theory may be written as

$$\sigma_{xx} - (e_0 a)^2 \nabla^2 \sigma_{xx} = c_{11}\varepsilon_{xx} + e_{31}\frac{\partial \varphi}{\partial z} - \frac{f_{31}}{2}\frac{\partial^2 \varphi}{\partial z^2} - c_{11}\alpha_1 \Delta T \tag{14}$$

$$\tau_{xxz} - (e_0 a)^2 \nabla^2 \tau_{xxz} = +\frac{f_{31}}{2}\frac{\partial \varphi}{\partial z} \tag{15}$$

$$D_z - (e_0 a)^2 \nabla^2 D_z = e_{31}\varepsilon_{xx} - k_{33}\frac{\partial \varphi}{\partial z} + \frac{f_{31}}{2}\eta_{xxz} + p_3 \Delta T \tag{16}$$

$$Q_{zz} - (e_0 a)^2 \nabla^2 Q_{zz} = -\frac{f_{31}}{2}\varepsilon_{xx} \tag{17}$$

So that φ denotes the electro-static potential and $E_z = -\partial\varphi/\partial z$. Next, the nonlocal formulations accounting for surface layers might be defined as

$$\sigma_{xx}^s - (e_0 a)^2 \nabla^2 \sigma_{xx}^s = \sigma_{xx}^0 + c_{11}^s \varepsilon_{xx} + e_{31}^s \frac{\partial \varphi}{\partial z} \tag{18}$$

Based on the assumption of open circuit conditions, the electrical displacements on surfaces become zero. Thus, it is possible to derive the electrical field component by

$$E_z = -\left(\frac{e_{31}}{k_{33}}\frac{\partial u}{\partial x}\right) + \left(z\frac{e_{31}}{k_{33}} + \frac{f_{31}}{k_{33}}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) \tag{19}$$

Next, the gradient of electrical field might be expressed by

$$E_{z,z} = \frac{e_{31}}{k_{33}} \frac{\partial^2 w}{\partial x^2} \tag{20}$$

Utilizing Eqs. (14) and (15), the nonlocal stresses of the bulk and surfaces may be described based on below relation

$$\sigma_{xx} - (e_0 a)^2 \nabla^2 \sigma_{xx} = (c_{11} + \frac{e_{31}^2}{k_{33}}) \frac{\partial u}{\partial x} - (c_{11} + \frac{e_{31}^2}{k_{33}}) z \frac{\partial^2 w}{\partial x^2} - (\frac{e_{31} f_{31}}{2k_{33}}) \frac{\partial^2 w}{\partial x^2} - c_{11} \alpha_1 \Delta T \tag{21}$$

$$\tau_{xxz} - (e_0 a)^2 \nabla^2 \tau_{xxz} = (\frac{e_{31} f_{31}}{2k_{33}}) \frac{\partial u}{\partial x} - (\frac{e_{31} f_{31}}{2k_{33}}) z \frac{\partial^2 w}{\partial x^2} - (\frac{f_{31}^2}{2k_{33}}) \frac{\partial^2 w}{\partial x^2} \tag{22}$$

$$\sigma_{xx}^s - (e_0 a)^2 \nabla^2 \sigma_{xx}^s = \sigma_{xx}^0 + (c_{11}^s + \frac{e_{31}^s e_{31}}{k_{33}}) \frac{\partial u}{\partial x} - (c_{11}^s + \frac{e_{31}^s e_{31}}{k_{33}}) z \frac{\partial^2 w}{\partial x^2} - (\frac{e_{31}^s f_{31}}{k_{33}} \frac{\partial^2 w}{\partial x^2}) \tag{23}$$

By integration from Eqs. (21)-(23) over the thickness of nano-size beam, the below resultants for the nano-size beam would be derived

$$N_{xx} - (e_0 a)^2 \nabla^2 N_{xx} = A_{11} \frac{\partial u}{\partial x} - B_{11} \frac{\partial^2 w}{\partial x^2} - N_{xx}^T \tag{24}$$

$$M_{xx} - (e_0 a)^2 \nabla^2 M_{xx} = -C_{11} \frac{\partial^2 w}{\partial x^2} \tag{25}$$

$$P_{xxz} - (e_0 a)^2 \nabla^2 P_{xxz} = B_{11} \frac{\partial u}{\partial x} - D_{11} \frac{\partial^2 w}{\partial x^2} \tag{26}$$

So that

$$\begin{gathered} A_{11} = (c_{11} + \frac{e_{31}^2}{k_{33}}) bh, B_{11} = (\frac{e_{31} f_{31}}{2k_{33}}) bh, \\ C_{11} = (c_{11} + \frac{e_{31}^2}{k_{33}}) b \frac{h^3}{12}, D_{11} = (\frac{f_{31}^2}{2k_{33}}) bh, \end{gathered} \tag{27}$$

Next, it is possible to express the force/moment stress resultants of surfaces based on below relations

$$N_{xx}^s - (e_0 a)^2 \nabla^2 N_{xx}^s = A_{11}^s \frac{\partial u}{\partial x} - B_{11}^s \frac{\partial^2 w}{\partial x^2} \tag{28}$$

$$M_{xx}^s - (e_0 a)^2 \nabla^2 M_{xx}^s = F_{11}^s \frac{\partial u}{\partial x} - C_{11}^s \frac{\partial^2 w}{\partial x^2} \tag{29}$$

in which

$$A_{11}^s = 2(c_{11}^s + \frac{e_{31}e_{31}^s}{k_{33}})h, B_{11}^s = (c_{11}^s + \frac{e_{31}e_{31}^s}{k_{33}})\frac{bh^2}{2} + 2(\frac{e_{31}^s f_{31}}{k_{33}})h,$$
$$F_{11}^s = (c_{11}^s + \frac{e_{31}e_{31}^s}{k_{33}})\frac{bh^2}{2}, C_{11}^s = (c_{11}^s + \frac{e_{31}e_{31}^s}{k_{33}})\frac{h^3}{6} + (\frac{e_{31}^s f_{31}}{k_{33}})\frac{bh^2}{2}, \tag{30}$$

The governing equations for a flexoelectric nano-scale beam based upon displacement components would be obtained by inserting Eqs. (24)-(29), into Eqs. (12) as

$$(A_{11} + A_{11}^s)\frac{\partial^2 u}{\partial x^2} - (B_{11} + B_{11}^s)\frac{\partial^3 w}{\partial x^3} - I_0\frac{\partial^2 u}{\partial t^2} + (e_0 a)^2\nabla^2(+I_0\frac{\partial^2 u}{\partial t^2}) = 0 \tag{31}$$

$$(B_{11} + F_{11}^s)\frac{\partial^3 u}{\partial x^3} - (C_{11} + C_{11}^s + D_{11})\frac{\partial^4 w}{\partial x^4} + 2b\sigma_0(\frac{\partial^2 w}{\partial x^2})$$
$$-(e_0 a)^2 2b\sigma_0(\frac{\partial^2}{\partial x^2})(\frac{\partial^2 w}{\partial x^2}) - bN^T(\frac{\partial^2 w}{\partial x^2}) + (e_0 a)^2 bN^T(\frac{\partial^2}{\partial x^2})(\frac{\partial^2 w}{\partial x^2})$$
$$-I_0\frac{\partial^2 w}{\partial t^2} + I_2\nabla^2(\frac{\partial^2 w}{\partial t^2}) + (e_0 a)^2\nabla^2(+I_0\frac{\partial^2 w}{\partial t^2} - I_2\nabla^2(\frac{\partial^2 w}{\partial t^2})) = 0 \tag{32}$$

4. Solution by differential quadrature method (DQM)

In the present chapter, differential quadrature method (DQM) has been utilized for solving the governing equations for nanobeam. According to DQM, at an assumed grid point (x_i, y_j) the derivatives for function F are supposed as weighted linear summation of all functional values within the computation domains as

$$\frac{d^n F}{dx^n}\Big|_{x=x_i} = \sum_{j=1}^{N} c_{ij}^{(n)} F(x_j) \tag{33}$$

where

$$C_{ij}^{(1)} = \frac{\pi(x_i)}{(x_i - x_j)\,\pi(x_j)} \qquad i,j = 1,2,\dots,N, \qquad i \neq j \tag{34}$$

in which $\pi(x_i)$ is defined by

$$\pi(x_i) = \prod_{j=1}^{N}(x_i - x_j), \quad i \neq j \tag{35}$$

And when $i = j$

$$C_{ij}^{(1)} = c_{ii}^{(1)} = -\sum_{k=1}^{N} C_{ik}^{(1)}, \quad i = 1,2,\dots,N, \quad i \neq k, \qquad i = j \tag{36}$$

Then, weighting coefficients for high orders derivatives may be expressed by

$$\begin{aligned} C_{ij}^{(2)} &= \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(1)} \\ C_{ij}^{(3)} &= \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(2)} = \sum_{k=1}^{N} C_{ik}^{(2)} C_{kj}^{(1)} \\ C_{ij}^{(4)} &= \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(3)} C_{kj}^{(1)} \qquad i,j = 1,2,\ldots,N. \end{aligned} \tag{37}$$

According to presented approach, the dispersions of grid points based upon Gauss-Chebyshev-Lobatto assumption are expressed as

$$x_i = \frac{a}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right] \qquad i = 1,2,\ldots,N, \tag{38}$$

Next, the time derivative for displacement components may be determined by

$$u(x,t) = U(x)e^{i\omega t} \tag{39}$$

$$w(x,t) = W(x)e^{i\omega t} \tag{40}$$

where U and W denote vibration amplitudes and ω defines the vibrational frequency. Then, it is possible to express obtained boundary conditions as

- Simply-supported (S):

$$w = N_{xx} = M_{xx} = 0 \qquad \text{at} \qquad x = 0, L \tag{41}$$

- Clamped (C):

$$w = \frac{\partial w}{\partial x} = 0 \qquad \text{at} \qquad x = 0, L \tag{42}$$

Now, one can express the modified weighting coefficients for all edges simply-supported as

$$\begin{aligned} \bar{C}_{1,j}^{(2)} &= \bar{C}_{N,j}^{(2)} = 0, \qquad i = 1,2,\ldots,M, \\ \bar{C}_{i,1}^{(2)} &= \bar{C}_{1,M}^{(2)} = 0, \qquad i = 1,2,\ldots,N. \end{aligned} \tag{43}$$

and

$$\bar{C}_{ij}^{(3)} = \sum_{k=1}^{N} C_{ik}^{(1)} \bar{C}_{kj}^{(2)} \qquad \bar{C}_{ij}^{(4)} = \sum_{k=1}^{N} C_{ik}^{(1)} \bar{C}_{kj}^{(3)} \tag{44}$$

By placing Eqs. (39)-(40) into Eqs. (31)-(32) and performing some simplifications leads to the following system based on mass matrix[M] and stiffness matrix [K] as

$$\{[K] + \omega^2[M]\}\begin{Bmatrix} U \\ W \end{Bmatrix} = 0 \tag{45}$$

Also, dimensionless quantities are selected as

$$\bar{\omega} = \omega\frac{L^2}{h}\sqrt{\frac{\rho}{c_{11}}}, \qquad \mu = \frac{(e_0 a)}{L} \tag{46}$$

5. Discussions on results

The present section studies dynamic characteristics of a small scale beam made of piezoelectric material considering flexoelectric effect. In order to capture scale-dependency of such piezoelectric beams, nonlocal elasticity theory is utilized and also surface effects are included for better structural modeling. In this study, we used the data for all material properties based on the work of Ebrahimi and Barati (2017). Verification of obtained frequencies for flexo-electric nanobeam has been done in Fig. 2 by comparing obtained vibration frequencies with those of Galerkin’s method reported by Ebrahimi and Barati (2017). This figure shows the excellent accuracy of presented DQ solution with previous data.

Nonlocal and temperature variation impacts on vibrational frequency of flexoelectric nano-size beam with simply-supported edge conditions have been shown in Fig. 3 when L/h = 20. It can be observed that the vibrational frequency of flexoelectric nano-size beam has a reducing trend with respect to nonlocal factor which means that structural stiffness has been reduced due to nonlocal influences. Another observation is that temperature rise results in smaller vibration frequencies at a prescribed nonlocal factor. So, thermal environment influences are prominent on dynamic characteristics of nonlocal flexoelectric nano-scale beams.

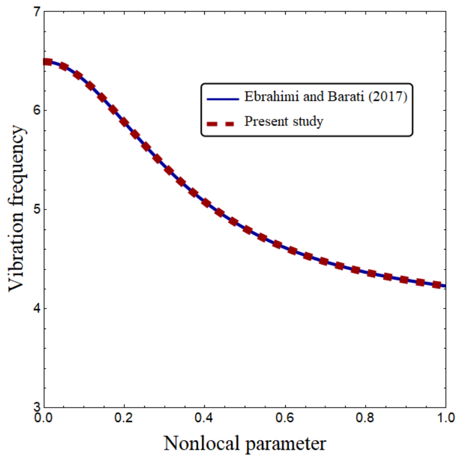

Fig. 2 Verification of obtained frequencies for flexo-electric nanobeam ($\Delta T = 0$, $L/h = 20$)

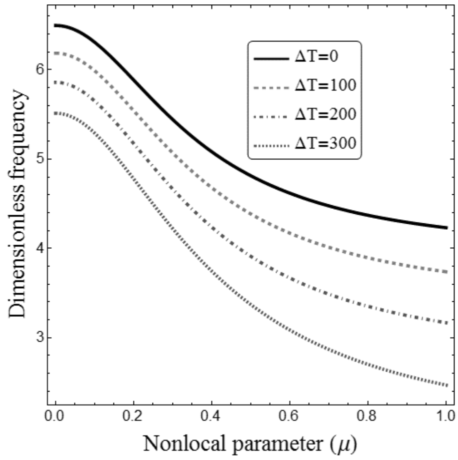

Fig. 3 Nonlocal and temperature variation impacts on vibrational frequency of flexoelectric nano-size beam (L/h = 20)

Fig. 4 illustrates the surfaces and flexo-electric impacts on vibrational frequency of a piezo-electrical nano-size beam when nonlocal factor is set as μ = 0.1. To this end, vibration frequency has been plotted versus thickness-to-length ratio (h/L) of the beam. The results based on nonlocal theory have been denoted by NL. When the surface impacts ae included, the results have been denoted by SE. One can observe from the figure that discarding the surface effects yields smaller vibrational frequency. Actually, incorporation of surfaces impacts improves the structural stiffness of the nano-size beam and vibrational frequencies enhance. Another finding is that flexoelectric effects lead to greater vibrational frequency, particularly at lower beam thicknesses. A conclusion from this figure that for NL modeling of flexoelectric beams, vibrational frequency is independent of thickness value.

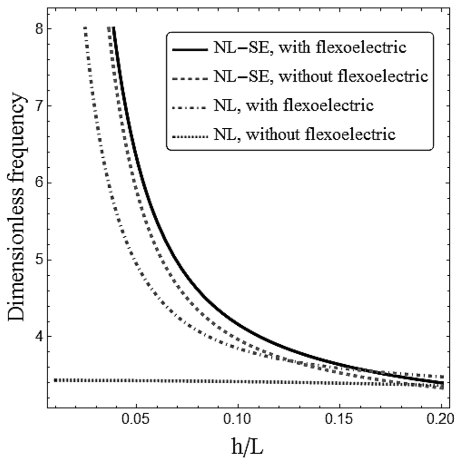

Fig. 4 Surface and flexoelectric impacts on vibrational frequency variation with respect to thickness (μ = 0.1)

In Fig. 5, flexoelectric and nonlocal impacts on vibrational frequency of nonlocal piezoelectric beams for different boundary conditions have been illustrated when L/h = 20. It is assumed that the nanobeams are exposed to uniform temperature variation of ΔT = 200K. For all types of boundary conditions, increase of nonlocal parameter yields smaller vibrational frequencies since the total stiffness of the nanobeam is reduced. So, nonlocal stress field which captures long range atomic interaction has a great influence on vibration characteristics of piezo-flexoelectric nanobeams. However, C-C type of boundary conditions has greater vibrational frequency than S-S and C-S. Thus, boundary condition has a main influence on vibrations of flexoelectric nano-size beams.

Fig. 6 indicates vibrational frequency variation of nonlocal flexoelectric beams with respect to temperature rise across the thickness. In this figure different values for nonlocal factor have been selected. It is obvious that temperature rise results in smaller vibrational frequency. At a particular

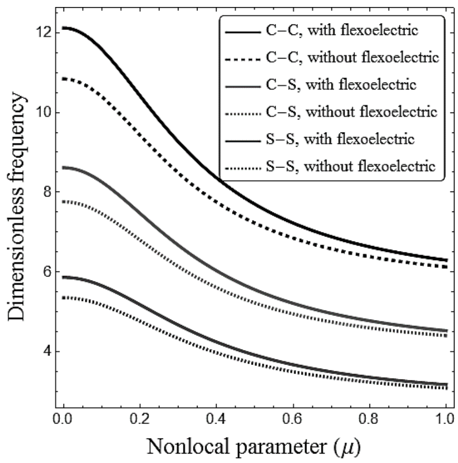

Fig. 5 Flexoelectric and nonlocal impacts on vibrational frequency of nonlocal piezoelectric beams for different boundary conditions (L/h = 20, ΔT = 200K)

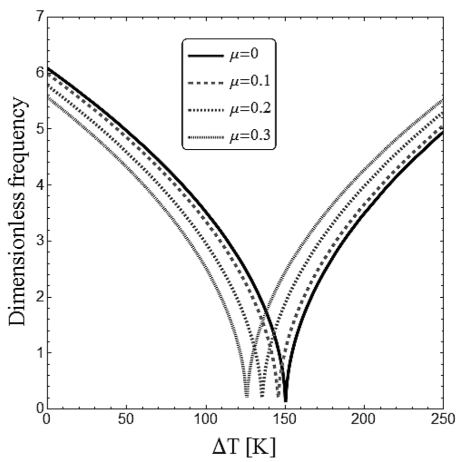

Fig. 6 Variation of vibrational frequency of nonlocal piezoelectric beams according to temperature rise for various nonlocal factors (L/h = 20)

value for temperature rise, the frequency may be zero showing that the buckling occurred. After buckling, temperature rise results in greater vibrational frequency. In the region before buckling, as the nonlocal factor increases the value of vibrational frequency decreases. It means that the buckling point at larger values for nonlocal factor shifts to the left.

6. Conclusions

Applying differential quadrature (DQ) method, the present article studied dynamic characteristics of a small scale beam made of piezoelectric material considering flexoelectric effect. In order to capture scale-dependency of such piezoelectric beams, nonlocal elasticity theory was utilized and also surface effects were included for better structural modeling. Governing equations were derived by utilizing Hamilton’s rule with the assumption that the scale-dependent beam is subjected to thermal environment leading to uniform temperature variation across the thickness. It was observed that the vibrational frequency of flexoelectric nano-size beam has a reducing trend with respect to nonlocal factor which means that structural stiffness has been reduced due to nonlocal influences. Another observation was that temperature rise results in smaller vibration frequencies at a prescribed nonlocal factor. Also, discarding the surface effects led to smaller vibrational frequency. Another finding was that flexoelectric effects led to greater vibrational frequency, particularly at lower beam thicknesses.

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