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Analytical analysis for the forced vibration of CNT surrounding elastic medium including thermal effect using nonlocal Euler-Bernoulli theory

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Abstract. This article studies the free and forced vibrations of the carbon nanotubes CNTs embedded in an elastic medium including thermal and dynamic load effects based on nonlocal Euler-Bernoulli beam. A Winkler type elastic foundation is employed to model the interaction of carbon nanotube and the surrounding elastic medium. Influence of all parameters such as nonlocal small-scale effects, high temperature change, Winkler modulus parameter, vibration mode and aspect ratio of short carbon nanotubes on the vibration frequency are analyzed and discussed. The non-local Euler-Bernoulli beam model predicts lower resonance frequencies. The research work reveals the significance of the small-scale coefficient, the vibrational mode number, the elastic medium and the temperature change on the non-dimensional natural frequency.

Keywords: nanostructures; Winkler's moduls; forced vibration; carbon nanotubes; non-local Euler beam theory

1. Introduction

Carbon nanotube CNTs are cylindrical macromolecules composed of carbon atoms in a periodic hexagonal arrangement discovered by Iijima (1991), which have received tremendous attention from various branches of science. Varieties of experimental, theoretical, and computer simulation approaches indicate that carbon nanotubes (CNTs) possess superior electronic, thermal and mechanical properties (Heireche *et al*. 2009). Other studies have showed that they have good properties so they can be used for nanocomposites. In addition, CNTs are well known for their excellent rigidity, higher than that of steel and any other metal. Zidour (Zidour *et al*. 2014) analysed the Buckling of chiral single-walled carbon nanotubes by using the nonlocal Timoshenko beam theory.

Due to difficulties encountered in experimental methods to predict the responses of nanostructures under different loading conditions, the molecular dynamics (MD) simulations and the continuum mechanics methods are used. But the computational problem when using the (MD) is that the time steps involved in the (MD) simulations are limited by the vibration modes of the

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atoms to be of the order of femto-seconds (10-15 s). The continuum mechanics methods have been effectively used to study mechanical behaviors of not only single-walled carbon nanotubes (SWCNTs) but also MWCNTs (Benferhat *et al*. 2016, Zidour *et al*. 2014, Ben Henni *et al*. 2018, Bouakaz *et al*. 2014, Rabahi *et al*. 2016, Daouadji and Adim 2016, Bensattalah *et al*. 2018a, Aissani *et al*. 2015, Abdelhak *et al*. 2016 and Bensattalah *et al*. 2016). Recently, the continuum mechanics approach has been widely and successfully used to study the responses of nanostructures, such as the static, the buckling, free vibration (Zidour *et al*. 2012, Daouadji *et al*. 2016b, Adim *et al*. 2016a, b, Daouadji 2017, Rabahi *et al*. 2016, Chaht *et al*. 2015, Khelifa *et al*. 2018, Tlidji *et al*. 2014 and Bensatallah *et al*. 2018b), wave propagation (Naceri *et al*. 2011) and thermo-mechanical analysis of (CNTs) (Daouadji and Hadji 2015, Benferhat *et al*. 2016, Hadji *et al*. 2015, Benferhat *et al*. 2016 and Abderezak *et al*. 2018).

At nanoscale, the classical continuum theories are deemed to fail, because the length dimensions at this scale are often sufficiently small such that call the applicability of classical continuum theories into the question. At macroscopic scale, the mechanical characteristics of structures are often significantly different from their behavior at nanoscale. Consequently, many non-local theories that consider the scale effect have been proposed such as deformation gradient theory, micro-polar theory and the nonlocal theory of elasticity (Eringen 1972b), this theories take into account the influence of the screen introducing the intrinsic scale length in the constituent relations. Among the theories mentioned previously, the non-local elasticity theory (Eringen 1983) is considered as a function of strain state of all points in the body.

In the past fifty years, linear and nonlinear problems which appeared in physical, chemistry, mechanics, engineering applications and various of scientific areas are modelled and they are investigated by using so many approximating methods. Some of these numerical methods are Differential Transformation Method (DTM), Homotopy Perturbation Method (HPM), Adomian Decomposition Method (ADM), Variational Iteration Method (VIM) and Homotopy Analysis Method (HAM). Many authors studied linear and nonlinear models to compute approximate solutions and their convergences with Differential Transformation Method (DTM) (Hassan 2002, Sallai *et al*. 2015, Hadji *et al*. 2016a, b, Heireche *et al*. 2008, Ju 2004).

Recently, three-dimensional behaviour of CNT is investigated by some researchers (Yuzhou and Liew 2008, Gupta and Batra 2008). CNT can be used for micro-electro-mechanical and nano-electro-mechanical devices. During these applications, external forces act on CNTs continuously. Because of this fact, understanding their dynamic behavior is a very important task. Forced vibration of CNT-reinforced epoxy is studied by Rajoria (Rajoria and Jalili 2005). According to the authors' best knowledge, forced vibration of CNTs by using continuum beam models was not studied in the previous research. For this reason, we presented in this article studies the free and forced vibrations of the carbon nanotubes (CNTs) embedded in an elastic medium including thermal and dynamic load effects have been extracted via the theory of nonlocal continuum elasticity. The mathematical derivations and numerical investigations are presented and performed while the emphasis is placed on investigating the impact of all different parameters such as nonlocal small-scale effects, high temperature change, Winkler modulus parameter, vibration mode and dynamic load. Comparisons of present approach with the results from the existing literature are provided and the good agreement between the results of the proposed method and those available in literature validated the presented approach.

2. Nonlocal Euler-Bernoulli elastic beam models

The theory of nonlocal continuum elasticity proposed by Eringen (1983) assumed that the stress at a reference point is considered to be a functional of the strain field at every point in the body. In the limit when the effects of strains at points other than x are neglected, one obtains local or classical theory of elasticity. For homogeneous and isotropic elastic solids, the constitutive equation of non-local elasticity can be given by Eringen (1972a). Non-local stress tensor (t) at point (x') is defined by

$$\begin{aligned} &\sigma_{ij,j} = 0 \\ &\sigma_{ij}(x) = \int K(|x - x'|,\ \tau) C_{ijkl}\varepsilon_{kl}(x')dV(x'), \quad \forall x \in V \\ &\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \end{aligned} \tag{1}$$

where C_{ijkl} is the classical, macroscopic stress tensor at point x', σ_{ij} and ε_{ij} are stress and strain tensors respectively. K ($|x - x'|$, τ) is the kernel function and ($\tau = e_0a/l$) is a material constant that depends on internal and external characteristic length (such as the lattice spacing and wavelength), where e_0 is a constant appropriate to each material, a is an internal characteristic length, e.g., length of (C–C) bond, lattice parameter, granular distance, and (l) is an external characteristic length.

Non-local constitutive relations for present nano-beams can be approximated to a one-dimensional form as

$$\left(1 - e_0 a^2 \frac{\partial^2}{\partial x^2}\right)\sigma_x = E\varepsilon_x \tag{2}$$

where E is the Young's modulus and the scale coefficient (e_0a) in the modelling will lead to small-scale effect on the response of structures at nano-size. Assume that the displacement of the beam along the z axis is $w(x, t)$ in terms of spatial coordinate x and time variable t (Fig. 1).

For transverse vibration of nanotube, the equilibrium conditions of the Euler-Bernoulli beam can be written as

$$\frac{\partial V}{\partial x} = \rho A \frac{\partial^2 w}{\partial t^2} - P(x) - F(x,t) \tag{3a}$$

$$V = \frac{\partial M}{\partial x} - N\frac{\partial w}{\partial x} \tag{3b}$$

where V and M are resultant shear force and bending moment of the beam, ρ is the mass density, A is the area of the cross-section of the beam, w is the transverse displacement of the micro-tubules, $f(x, t)$ is the distributed transverse force along the x-axis, $P(x)$ is the inter action pressure per unit axial length between the nanotube and the surrounding elastic medium, and t is the time variable. N is the axial force arising from the thermal effect and it is defined as

$$N = -EA\alpha\theta \tag{4}$$

In addition the pressure per unit axial length, acting on the outermost tube due to the

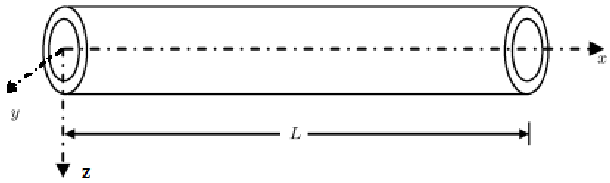

Fig. 1 The illustration of carbon nanotube

surrounding elastic medium, can be described by a Winkler type model.

$$P(x) = -K_{win} w(x) \tag{5}$$

Where the negative sign indicates that the pressure $P(x)$ is opposite to the deflection of the outermost tube and K_{win} is spring constant of the surrounding elastic medium.

The elasto-dynamics differential equation that governs the thermo-mechanical vibration of the nanotube SWCNT based on the nonlocal Euler-Bernoulli beam theory is

$$EI\frac{\partial^4 w}{\partial x^4} + \left(1 - e_0 a^2 \frac{\partial^2}{\partial x^2}\right)\left[\rho A \frac{\partial^2 w}{\partial t^2} - N\frac{\partial^2 w}{\partial x^2} - k_{win} w\right] = \left(f - e_0 a^2 \frac{\partial^2}{\partial x^2} f\right) \tag{6}$$

Since fiending an analytical solution is possible for simply supported boundary conditions for the present problem, the (SWNT) beam is assumed simply supported. For the forced flexural vibrations, the applied dynamic load $f = f(x,t)$ is assumed as a result, the boundary conditions have the following form

$$w(x,t) = \sum_{m=1}^{\infty} W_m . \sin\left(\frac{m\pi x}{L}\right) . \sin(\omega . t) \tag{7a}$$

$$f(x,t) = \sum_{m=1}^{\infty} F_m . \sin\left(\frac{m\pi x}{L}\right) . \sin(\omega . t) \tag{7b}$$

Here ω is the operational frequency of the load. Writing in Eq. (6), setting $e_0 a = 0$, $N = 0$ and $K_{win} = 0$ give equation of motion for forced vibration of CNTs in the local form

$$EI\frac{\partial^4 w}{\partial x^4} + \left[\rho A \frac{\partial^2 w}{\partial t^2}\right] = f \tag{8a}$$

The simply supported boundary conditions can be written as

$$w = M = 0 \qquad \text{at} \qquad x = 0, L \tag{8b}$$

Following displacement field satisfies equation of motion (equation 6) and boundary conditions (Eq. (8)). Inserting Eq. (7) into Eq. (6) and solving Wm gives the amplitude of CNT in the following form

$$\lambda_{LE} = \left(\frac{F_O L^4}{EI} + \beta^4 \right)^{0.25} \tag{9}$$

where $\beta = m\pi$, without omitting all parameters e_0a, N and k in Eq. (6) and doing similar operations mentioned above, the following amplitude can be found for the nonlocal case

$$\lambda_{NE} = \left[\frac{F_O L^4 (1-\xi)}{EI(1+\xi)} + \frac{\left(\beta^4 - \beta^2 \overline{N}(1+\xi) + \mu(1-\xi)\right)}{(1+\xi)} \right]^{0.25} \tag{10}$$

The non-dimensional parameters for the Euler-beam on the Winkler foundation are defined as

$$\xi = \frac{\beta^2 e_0 a^2}{L^2}, \quad \lambda^4 = \frac{\rho A \omega^2 L^4}{EI}, \quad \mu = \frac{k L^4}{EI}, \quad \overline{N} = \frac{N.L^2}{EI}, \quad \varepsilon = \frac{e0a}{L} \tag{11}$$

The Young's modulus, thermal expansion coefficients of CNTs and the spring constant of polymer matrix, under temperature changes environments, which may be a function of temperature change as follows

$$\begin{aligned} &E = E^0(1 - 0.0005\theta) \\ &N = -EA\alpha\theta\alpha = \alpha^0(1 + 0.002\theta) \\ &k = k^0(1 - 0.0003\theta) \end{aligned} \tag{12}$$

where E^0 and α^0 express elastic modulus and thermal expansion coefficients of CNTs under a room temperature environment, respectively. k^0 is spring constant of polymer matrix under a room temperature environment.

3. Results and discussion

In the present study the impact of all parameters such as nonlocal small-scale effects, high temperature change, Winkler modulus parameter, vibration mode for four types of boundary conditions e.g. simply supported, clamped- simply, clamped ends and Cantilever beam on first,

Table 1 First three non-dimensional frequency λ of nonlocal Euler-Bernoulli for simply supported beam, with ($k = 0$, $\theta = 0$, $F_0 = 0$, $L/d = 10$)

r	Mode 1		Mode 2		Mode 3	
	Wang 2007	Present	Wang 2007	Present	Wang 2007	Present
0	3.1416	3.141593	6.2832	6.283185	9.4248	9.424777
0,1	3.0685	3.068531	5.7817	5.781668	8.0400	8.039987
0,3	2.6800	2.679996	4.3013	4.301343	5.4422	5.442246
0,5	2.3022	2.302231	3.4604	3.460401	4.2941	4.294061
0,7	2.0212	2.021245	2.9585	2.958479	3.6485	3.648549

second and third frequencies of the SWCNTs have been studied.

The material properties used in the present study are the mass density ρ = 2300 kg/m^3, the Poisson ratio υ = 0.19, the Young's modulus E = 5.5 MPa and the thermal expansion coefficients at high temperature α_0 = 1.1×10^{-6} K^{-1} (Yao and Han 2006). Another, Young's modulus, thermal expansion coefficients of CNTs and the spring constant of polymer matrix, under temperature changes environments, which may be a function of temperature change as present in Eq. (12). In the beginning of this study, the validation our model local Euler-Bernoulli for simply supported beam boundary condition is presented in Table 5, with case α = 0, μ = 0, θ = 0, k = 0, F_0 = 0 and L/d = 10. The temperature effect and Winkler modulus parameter have been considered here. The details of first three non-dimensional frequency λ for four kinds of boundary conditions with and without elastic medium using nonlocal Euler-Bernoulli beam model are listed in table 2. The ratio of the length to the diameter, (L/d), is 10 and the scale coefficients (ε = 0, 0.5, 1).

The results show the dependence of the frequency on the mode number and the elastic medium. It is noted that the frequency increases when elastic medium is neglected. This increasing is attributed to the stiffness of the elastic medium. With higher values of mode number the rate of increase of frequency reduces, and becomes more significant with the higher of small-scale parameter. This is interpreted as the small-scale effect makes the CNTs more flexible as CNT being assumed as atoms linked by springs, the external elastic medium "grips" the SWCNTs and forces it to be stiffer. In addition, it is clearly that the frequency increases when the vibrational mode number increases.

The results in Table 3 show the dependence of the frequency of first three non-dimensional frequency λ with temperature change using nonlocal Euler-Bernoulli beam model. The ratio of the length to the diameter, (L/d), is 10 and the scale coefficients, (ε = 0, 0.5, 1). We consider the temperature change values as θ = 20, 40 and 60.

It is noted that the first three non-dimensional frequency λ decreases as the temperature change θ increases, and becomes more significant with the small-scale parameter. This dependence of the frequency on the temperature change appraises in simply supported beam.

Table 2 The effect of Winkler modulus parameter on first three non-dimensional frequency λ of nonlocal Euler-Bernoulli for simply supported beam ~~and~~ (L/d = 10, θ = 0, F_0 = 0)

ε	Without elastic medium			With elastic medium		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
0	3.141593	6.283185	9.424778	3.102895	6.278431	9.423370
0.5	2.302231	3.460401	4.294057	2.198958	3.431617	4.279104
1	1.730201	2.491001	3.061400	1.435812	2.411026	3.019494

Table 3 The effect of temperature change on First three non-dimensional frequency λ of nonlocal Euler-Bernoulli for simply supported beam with (k = 0.1, F_0 = 0 and L/d = 10)

ε	θ = 20			θ = 40			θ = 60		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
0	3.140149	6.282464	9.424297	3.138592	6.281686	9.423779	3.136921	6.280853	9.423224
0.5	2.298556	3.456074	4.288967	2.294578	3.451396	4.283459	2.290293	3.446363	4.277537
1	1.721497	2.479342	3.047268	1.711975	2.466599	3.031828	1.701592	2.452723	3.015015

Table 4 The effect of dynamic load on First three non-dimensional frequency λ of nonlocal Euler-Bernoulli for simply supported beam With (k = 0.1 end L/d = 10)

ε	$F_0 = 10^{-3}$			$F_0 = 10^{-2}$			$F_0 = 10^{-1}$		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
0	3.181725	6.289413	9.427180	3.178361	6.288978	9.427050	3.144121	6.284625	9.425759
0.5	2.268336	3.446134	4.2910530	2.272248	3.448288	4.292303	2.310303	3.469618	4.304749
1	1.5311616	2.4412869	3.0504308	1.555180	2.448323	3.054150	1.75006	2.515568	3.090617

In Table 4 show the dependence of the frequency of first three non-dimensional frequency λ with dynamic load change using nonlocal Euler-Bernoulli beam model. The ratio of the length to the diameter, L/d = 10, and the scale coefficients, (ε = 0, 0.5, 1). We consider the temperature change values as $F_0 = 10^{-3}$ nN, $F_0 = 10^{-2}$ nN, $F_0 = 10^{-1}$ nN. It is noted that the first three non-dimensional frequency λ decreases as the dynamic load change F_0 increases, and becomes more significant with the small-scale parameter. This dependence of the frequency on the temperature change appraises in simply supported beam.

The results are including and excluding, in order to study the effect of the small-scale parameter, the elastic medium and the change of temperature on the vibrations of single-walled nanotubes.

The Fig. 2 shows the dependence of the frequency ratios on the small scale and vibrational mode number of (SWCNTs) embedded in an elastic medium using thermal effects. The frequency ratio serves as an index to assess quantitatively the scale effect on CNT vibration solution. It is observed from Fig. 2 that the frequency ratios are less than unity. This means that the application of the local Euler-Bernoulli beam model for CNT analysis would lead to an over prediction of the frequency if the scale effect between the individual carbon atoms in CNTs is neglected. However for larger values of α, this dependence becomes very largest. However the small scale effect makes the beam more flexible. In addition, it is clearly that as the vibrational mode number increases, this dependence becomes very largest. This significance in higher modes is attributed to the influence of small wavelength for higher modes. For smaller wavelengths, interactions between atoms are

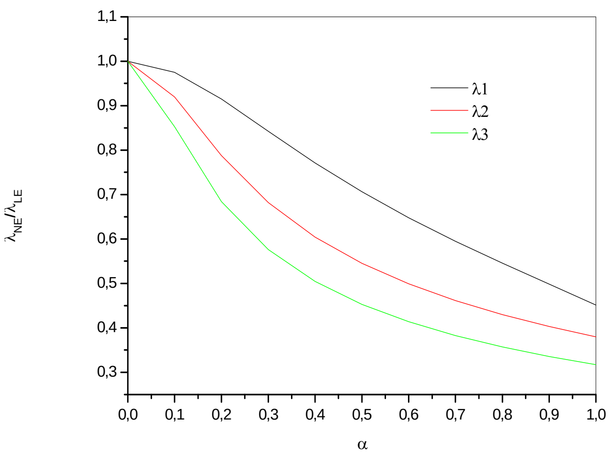

Fig. 2 Small scale effect on different frequency modes for simply supported beam and (k = 0.1, θ = 40, L/d = 10)

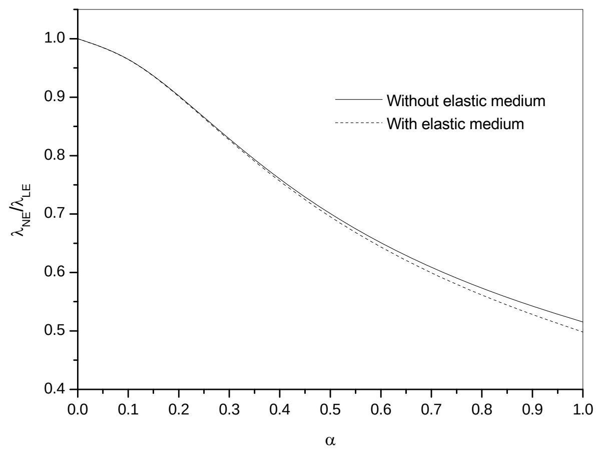

Fig. 3 The effect of elastic medium on first non-dimensional frequency of a short-SWCNT for simply supported beam with different parameter α and (θ = 40, L/d = 10)

increasing and this leads to an increasing in the small scale effect. The effect of elastic medium on first and third non-dimensional frequency of a short-SWCNT for simply supported beam with different parameter α is shown in Fig. 3 with the aspect ratio is L/d = 10 and θ = 40. It can be seen that the difference between the frequency ratios with and without elastic medium is very weak forsmall values of α and for the higher values this difference become clear. In addition, the range of the frequency ratios without elastic medium is the smallest for frequency ratios with elastic medium because the elastic medium grips the CNT and forces it to be stiffer.

The Fig. 4 illustrates dependence of the first and third non-dimensional frequency ratios on the temperature change with the aspect ratio L/d = 10 and with elastic medium. It is clearly seen from Fig. 4 that the frequency ratios are less than unity and the difference between the frequency ratios

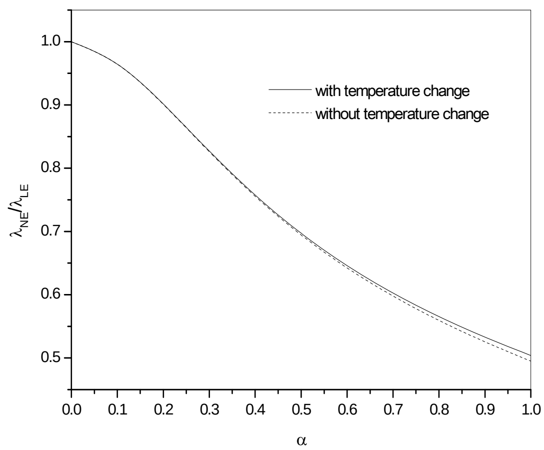

Fig. 4 The effect of temperature on first non-dimensional frequency of a short-SWCNT for simply supported beam with different parameter α *a*nd (k_{win} = 0.1, L/d = 10)

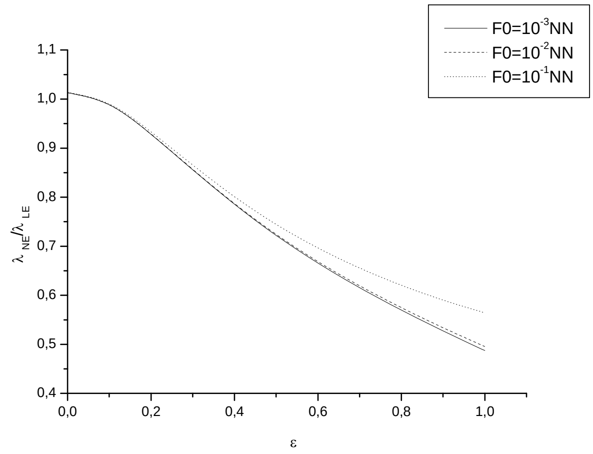

Fig. 5 The effect of dynamic load on non-dimensional frequency of a short-SWCNT for clamped ends with different parameter α *a*nd (m = 1, k = 0.1, L/d = 10)

with and without temperature change is clear for higher values of α and in first mode.

The Fig. 5 illustrates influence of dynamic load of the first and third non-dimensional frequency ratios with the aspect ratio is L/d = 10 and with elastic medium. It is clearly seen from Fig. 5 that the frequency ratios are less than unity and the difference between the frequency ratios at temperature 40 is clear augment if dynamic load augment for higher values of ε and in first mode.

4. Conclusions

Based on local and non-local Euler–Bernoulli beam theory, free and forced vibration of CNT embedded in an elastic medium including thermal and dynamic load effects is studied. Non-dimensional frequency ratios for the local and non-local Euler beam models are given for SWCNTs and DWCNTs. It is found that the non-local models give higher amplitudes when compared with the local Euler–Bernoulli beam models. This study can be extended to other boundary conditions and Timoshenko beam theory. The presented investigation may be helpful in the application of (CNTs), such as nano-electronics, nano-devices, mechanical sensors and nano-composites.

References

Abdelhak, Z., Hadji, L., Daouadji, T.H. and Adda, B. (2016), "Thermal buckling response of functionally graded sandwich plates with clamped boundary conditions", *Smart Struct. Syst., Int. J.*, **18**(2), 267-291.

Abderezak, R., Daouadji, T.H., Abbes, B., Rabia, B., Belkacem, A. and Abbes, F. (2017), "Elastic analysis of interfacial stress concentrations in CFRP-RC hybrid beams: Effect of creep and shrinkage", *Adv. Mater. Res., Int. J.*, 6(3), 257-278.

Abderezak, R., Daouadji, T.H., Rabia, B. and Belkacem, A. (2018), "Nonlinear analysis of damaged RC beams strengthened with glass fiber reinforced polymer plate under symmetric loads", *Earthq. Struct., Int. J.*, **15**(2), 113-122.

Adim, B., Daouadji, T.H. and Rabahi, A. (2016a), "A simple higher order shear deformation theory for mechanical behavior of laminated composite plates", *Int. J. Adv. Struct. Eng.* (*IJASE*), **8**, 103-117.

Adim, B., Daouadji, T.H., Abbes, B. and Rabahi, A. (2016b), "Buckling and free vibration analysis of laminated composite plates using an efficient and simple higher order shear deformation theory", *J. Mech. Industry*, **17**(5), 512.

Aifantis, E.C. (1984), "On the micro structural origin of certain inelastic models", *J. Eng. Mater. Technol.*, **106**(4), 326-330.

Aissani, K., Bachir Bouiadjra, M., Ahouel, M. and Tounsi, A. (2015), "A new nonlocal hyperbolic shear deformation theory for nanobeams embedded in an elastic medium", *Struct. Eng. Mech.*, *Int. J.*, **55**(4), 743-762.

Amara, K., Tounsi, A. and Mechab, I. (2010), "Nonlocal elasticity effect on column buckling of multi walled carbon nanotubes under temperature field", *Appl. Math. Model.*, **34**(12), 3933-3942.

Ansari, R. and Ramezannezhad, H. (2011), "Nonlocal Timoshenko beam model for the large-amplitude vibrations of embedded multi walled carbon nanotubes including thermal effects", *Physica E: Low dimens. Syst. Nanostruct.*, **43**(6), 1171-1178.

Ansari, R. and Sahmani, S. (2012), "Small scale effect on vibrational response of single-walled carbon nanotubes with different boundary conditions based on nonlocal beam models", *Commun. Nonlinear Sci. Numer. Simul.*, **17**(4), 1965-1979.

Ansari, R., Gholami, R., Hosseini, K. and Sahmani, S. (2011), "A sixth-order compact finite difference method for vibration analysis of nano-beams embedded in an elastic medium based on nonlocal beam theory", *Math. Comput. Model.*, **54**(11), 2577-2586.

Baughman, R.H., Zakhidov, A.A. and De Heer, W.A. (2002), "Carbon nanotubes--the route toward applications", *Science*, **297**(5582), 787-792.

Belkacem, A., Tahar, H.D., Abderrezak, R., Amine, B.M., Mohamed, Z. and Boussad, A. (2018), "Mechanical buckling analysis of hybrid laminated composite plates under different boundary conditions", *Struct. Eng. Mech.*, *Int. J.*, **66**(6), 761-769.

Ben Henni, M., Daouadji, T.H., Abbès, B., Li, Y.M. and Abbes, F. (2018), "Analytical and Numerical Results for Free Vibration of Laminated Composites Plates", *Int. J. Chem. Molecul. Eng.*, **12**(6), 300-304.

Benferhat, R., Daouadji, T.H. and Said-Mansour, M. (2014), "A Higher Order Shear Deformation Model for Bending Analysis R. of Functionally Graded Plates", *Trans Indian Inst. Metals*, **68**(1), 7-16.

Benferhat, R., Daouadji, T.H. and Mansour, M.S. (2016), "Free vibration analysis of FG plates resting on the elastic foundation and based on the neutral surface concept using higher order shear deformation theory", *Comptes Rendus Mécanique*, **344**(9), 631-641 .

Bensattalah, T., Daouadji, T.H., Zidour, M., Tounsi, A. and Bedia, E.A. (2016), "Investigation of thermal and chirality effects on vibration of single-walled carbon nanotubes embedded in a polymeric matrix using nonlocal elasticity theories", *Mech. Compos. Mater.*, **52**(4), 555-568.

Bensattalah, T., Bouakkaz, K., Zidour, M. and Daouadji, T.H. (2018a), "Critical buckling loads of carbon nanotube embedded in Kerr's medium", *Adv. Nano Res.*, *Int. J.*, **6**(4), 339-356.

Bensattalah, T., Zidour, M., Meziane, M.E., Daouadji, T.H. and Tounsi, A. (2018b), "Critical buckling load of carbon nanotube with non-local Timoshenko beam using the differential transform method", *Int. J. Civil Environ. Eng.*, **12**(6), 637-644.

Bouakaz, K., Daouadji, T.H., Meftah, S.A., Ameur, M., Tounsi, A. and Bedia, E.A. (2014), "A numerical analysis of steel beams strengthened with composite materials", *Mech. Compos. Mater.*, **50**(4), 685-696.

Cao, G. and Chen, X.I. (2007), "The effects of chirality and boundary conditions on the mechanical properties ofsingle-walled carbon nanotubes", *Int. J. Solids Struct.*, **44**, 5447-5465.

Chaht, F.L., Kaci, A., Houari, M.S.A., Tounsi, A., Bég, O.A. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, *Int. J.*, **18**(2), 425-442.

Chedad, A., Daouadji, T.H., Abderezak, R., Belkacem, A., Abbes, B., Rabia, B. and Abbes, F. (2017), "A high-order closed-form solution for interfacial stresses in externally sandwich FGM plated RC beams", *Adv. Mater. Res.*, *Int. J.*, **6**(4), 317-328.

Chow, T.L. (2013), *Classical Mechanics*, CRC Press, Boca Raton, FL, USA.
Daouadj, T.H. and Adim, B. (2017), "Mechanical behaviour of FGM sandwich plates using a quasi-3Dhigher order shear and normal deformation theory", *Struct. Eng. Mech.*, *Int. J.*, **61**(1), 49-63.
Daouadji, T.H. (2017), "Analytical and numerical modeling of interfacial stresses in beams bonded with a thin plate", *Adv. Computat. Des.*, 2(1), 57-69.
Daouadji, T.H. and Adim, B. (2016), "Theoretical analysis of composite beams under uniformly distributed load", *Adv. Mater. Res.*, *Int. J.*, **5**(1), 1-9.
Daouadji, T.H. and Hadji, L. (2015), "Analytical solution of nonlinear cylindrical bending for functionally graded plates", *Geomech. Eng.*, *Int. J.*, **9**(5), 631-644.
Daouadji, T.H., Rabahi, A., Abbes, B. and Adim, B. (2016a), "Theoretical and finite element studies of interfacial stresses in reinforced concrete beams strengthened by externally FRP laminates plate", *J. Adhes. Sci. Technol.*, **30**(12), 1253-1280.
Daouadji, T.H., Benferhat, R. and Adim, B. (2016b), "Bending analysis of an imperfect advanced composite plates resting on the elastic foundations", *Coupl. Syst. Mech.*, *Int. J.*, **5**(3), 269-285.
Ebrahimi, F. and Nasirzadeh, P. (2015), "Small-scale effects on transverse vibrational behavior of single-walled carbon nanotubes with arbitrary boundary conditions", *Journal of Engineering Solid Mechanics*, **3**(2), 131-141.
Eringen, A.C. (1972a), "Linear theory of nonlocal elasticity and dispersion of plane waves", *Int. J. Eng. Sci.*, **10**(5), 425-435.
Eringen, A.C. (1972b), "Nonlocal polar elastic continua", *Int. J. Eng. Sci.*, **10**(1), 1-16.
Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710.
Ghorbanpourarani, A., Mohammadimehr, M., Arefmanesh, A. and Ghasemi, A. (2010), "Transverse vibration of short carbon nanotubes using cylindrical shell and beam models", *Proceedings of the Institution of Mechanical Engineers*, *Part C*: *Journal of Mechanical Engineering Science*, **224**(3), 745-756.
Gupta, S.S. and Batra, R.C. (2008), "Continuum structures equivalent in normal mode vibrations to single-walled carbon nanotubes", *Comput. Mater. Sci.*, **43**(4), 715-723.
Hadji, L., Khelifa, Z., Daouadji, T.H. and Bedia, E.A. (2015), "Static bending and free vibration of FGM beam using an exponential shear deformation theory", *Coupl. Syst. Mech.*, *Int. J.*, **4**(1), 99-114.
Hadji, L., Daouadji, T.H., Meziane, M.A.A., Tlidji, Y. and Bedia, E.A. (2016a), "Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech.*, *Int. J.*, **57**(2), 315-325.
Hadji, L., Daouadji, T.H. and Bedia, E.A. (2016b), "Dynamic behavior of FGM beam using a new first shear deformation theory", *Eartq. Struct.*, *Int. J.*, **10**(2), 451-461.
Hassan, I.A.H. (2002), "On solving some eigen value problems by using a differential", *Appl. Math. Computat.*, **127**(1), 1-22.
Heireche, H., Tounsi, A., Benzair, A. and Mechab, I. (2008), "Sound wave propagation in single-walled carbon nanotubes with initial axial stress", *J. Appl. Phys.*, **104**, 014301.
Heireche, H., Tounsi, A., Benzair, A., Maachou, M. and Bedia, E.A. (2009), "Sound wave propagation in single-walled carbon nanotubes using nonlocal elasticity", *Physica E*, **40**, 2791-2799.
Iijima, S. (1991), "Helical microtubules of graphitic carbon", *Nature*, **354**(6348), 56-58.
Jiang, H., Liu, B., Huang, Y. and Hwang, K.C. (2004), "Thermal expansion of single wall carbon nanotube", *Journal of Engineering of Material and Technology*, **26**, 265-270.
Ju, S.P. (2004), "Application of differential transformation to transient advective–dispersive transport equation", *Appl. Math. Computat.*, **155**(1), 25-38.
Khelifa, Z., Hadji, L., Daouadji, T.H. and Bourada, M. (2018), "Buckling response with stretching effect of carbon nanotube-reinforced composite beams resting on elastic foundation", *Struct. Eng. Mech.*, *Int. J.*, **67**(2), 125-130.
Kiani, K. (2010), "A meshless approach for free transverse vibration of embedded single-walled nanotubes with arbitrary boundary conditions accounting for nonlocal effect", *Int. J. Mech. Sci.*, **52**(10), 1343-1356.
Kiani, K. and Mehri, B. (2010), "Assessment of nanotube structures under a moving nano-particle using

nonlocal beam theories", *J. Sound Vib*., **329**(11), 2241-2264.

Lee, H.L. and Chang, W.J. (2009a), "Vibration analysis of fluid-conveying double-walled carbon nanotubes based on nonlocal elastic theory", *J. Phys*.: *Condens*. *Matter*., **21**, 448001.

Lee, H.L. and Chang, W.J. (2009b), "Free transverse vibration of the fluid-conveying single-walled carbon nanotube using nonlocal elastic theory", *J. Appl*. *Phys*., **103**, 024302.

Maachou, M., Zidour, M., Baghdadi, H., Ziane, N. and Tounsi, A. (2011), "A nonlocal Levinson beam model for free vibration analysis of zigzag single-walled carbon nanotubes including thermal effects", *Solid State Commun*., **151**, 1467-1471

Naceri, M., Zidour, M., Semmah, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2011), "Sound wave propagation in armchair single walled carbon nanotubes under thermal environment", *J. Appl*. *Phys*., **110**, 124322. DOI: 10.1063/1.3671636

Rabahi, A., Daouadji, T.H., Abbes, B. and Adim, B. (2016), "Analytical and numerical solution of the interfacial stress in reinforced-concrete beams reinforced with bonded prestressed composite plate", *Journal of Reinforced Plastics and Composites*, **35**(3), 258-272.

Rabia, B., Abderezak, R., Daouadji, T.H., Abbes, B., Belkacem, A. and Abbes, F. (2018), "Analytical analysis of the interfacial shear stress in RC beams strengthened with prestressed exponentially-varying properties plate", *Adv. Mater. Res*., *Int. J*., **7**(1), 29-44.

Rajoria, H. and Jalili, N. (2005), "Passive vibration damping enhacement using carbon nanotube-epoxy reinforced composites", *Compos*. *Sci*. *Technol*., **65**, 2079-2093.

Sallai, B., Hadji, L., Daouadji, T.H. and Adda, B. (2015), "Analytical solution for bending analysis of functionally graded beam", *Steel Compos*. *Struct*., *Int*. *J*., **19**(4), 829-841.

Silvestre, N. (2008), "Length dependence of critical measures in single-walled carbon nanotubes", *Int*. *J*. *Solids Struct*., **45**, 4902-4920.

Sudak, L.J. (2003), "Column buckling of multi walled carbon nanotubes using nonlocal continuum mechanics", *J. Appl*. *Phys*., 94(11), 7281-7287.

Tlidji, Y., Daouadji, T.H., Hadji, L., Tounsi, A. and Bedia, E.A.A. (2014), "Elasticity solution for bending response of functionally graded sandwich plates under thermo mechanical loading", *J. Thermal Stress*., **37**, 852-869.

Tounsi, A., Daouadji, T.H. and Benyoucef, S. (2008), "Interfacial stresses in FRP-plated RC beams: Effect of adherend shear deformations", *Int*. *J*. *Adhes*. *Adhes*., **29**(4), 343-351.

Yao, X.H. and Han, Q. (2006), "Buckling analysis of multi-walled carbon nanotubes under tensional load coupling with temperature change", *Journal of Engineering of Material and Technology*, **128**, 419-428.

Yuzhou, S. and Liew, K.M. (2008), "The buckling of single-walled carbon nanotubes upon bending: the higher order gradient continuum and mesh-free method", *Comput*. *Methods Appl*. *Mech*. *Eng*., **197**, 3001-3013.

Zidour, M., Benrahou, K.H., Semmah, A., Naceri, M., Belhadj, H.A., Bakhti, K. and Tounsi, A. (2012), "The thermal effect on vibration of zigzag single walled carbon nanotubes using nonlocal Timoshenko beam theory", *Computat*. *Mater*. *Sci*., **51**(1), 252-260.

Zidour, M., Daouadji, T.H., Benrahou, K.H., Tounsi, A., Bedia, E.A.A. and Hadji, L. (2014), "Buckling analysis of chiral single walled carbon nanotubes by the nonlocal Timoshenko beam theory", *Mech*. *Compos*. *Mater*., **50**(1), 95-104.

Zoubida, K., Daouadji, T.H., Hadji, L., Tounsi, A. and El Abbes, A.B. (2016), "A new higher order shear deformation model of functionally graded beams based on neutral surface position", *Transact*. *Indian Inst*. *Metals*, **69**(3), 683-691.

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