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Transient thermo-mechanical response of a functionally graded beam under the effect of a moving heat source

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Abstract. The transient thermo-mechanical behavior of a simply-supported beam made of a functionally graded material (FGM) under the effect of a moving heat source is investigated. The FGM consists of a ceramic part (on the top), which is the hot side of the beam as the heat source motion takes place along this side, and a metal part (in the bottom), which is considered the cold side. Grading is in the transverse direction, with the properties being temperature-dependent. The main steps of the thermo-elastic modeling included deriving the partial differential equations for the temperatures and deflections in time and space, transforming them into ordinary differential equations using Laplace transformation, and finally using the inverse Laplace transformation to find the solutions. The effects of different parameters on the thermo-mechanical behavior of the beam are investigated, such as the convection coefficient and the heat source intensity and speed. The results show that temperatures, and hence the deflections and stresses increase with less heat convection from the beam surface, higher heat source intensity and low speeds.

Keywords: transient; thermo-mechanical; FGM; beam; moving heat source

1. Introduction

Functionally Graded Materials (FGMs) are a class of composite materials with continuous variation of material properties from one surface to another. The FGM usually consists of a ceramic part and a metal part. The resulting composite material will have the advantages of both materials, where the ceramic part has the thermal resistance ability and the metal part can support the composite with its elasticity. The importance of FGMs comes mainly from their ability to withstand severe temperature gradients. The continuous variation in the microstructure of FGMs distinguishes them from the traditional fiber-reinforced laminated composite materials, where the variation of mechanical properties is not continuous across the interface, which may result in de-bonding of layers at elevated temperatures. The use of FGMs eliminates this problem, which makes them good candidates to replace fiber-reinforced laminated composite materials in applications that involve high temperature gradients.

A heat source moving along a beam is a case that models some practical situations in different engineering applications, such as welding, grinding, metal cutting, firing a bullet in a gun barrel,

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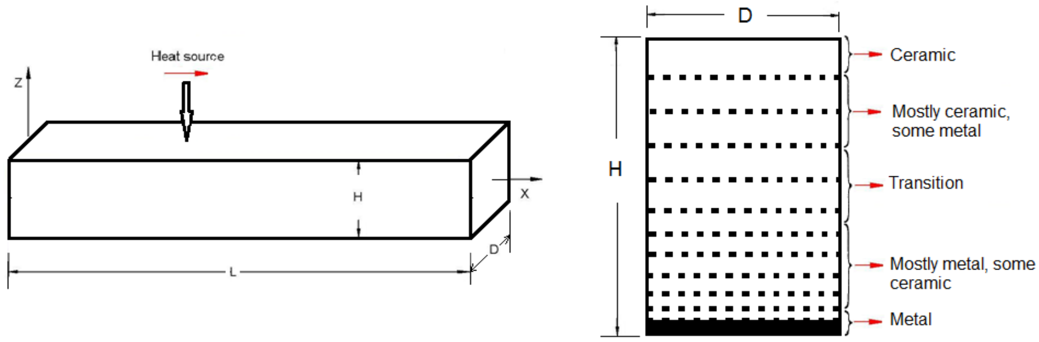

Fig. 1 The simply supported FG beam under the effect of moving heat source

A rectangular simply-supported beam of uniform thickness H and depth D, subjected to a moving heat source with constant intensity and speed is shown in Fig. 1. Initially, the beam is maintained at a uniform temperature, which is equal to the ambient temperature T_∞, then the heat source is applied on the beam starting from one end (x=0) of the beam to the other end (x=L), moving on the top surface.

The beam is made of a functionally graded material (FGM) consisting of a metal part (bottom) and ceramic part (top). Grading is in the transverse direction (z-axis), as shown in Fig. 1. Therefore, the thermal and mechanical properties of the FGM vary in the z-direction of the beam. The smooth transition of the properties in the FGM from metallic to ceramic, as shown in Fig. 1, provides thermal protection as well as structural integrity, reducing the possibilities of failure within the beam. The material properties in the axial direction are the same. In the analysis to follow, the beam is considered to be laminated, i.e., composed of thin layers in the transverse direction, in order to properly model the variation of FGM properties in that direction.

The temperature distribution in the axial direction of the beam depends on different factors such as the heat source intensity and velocity, convection coefficient, beam dimensions and thermal properties. The governing heat equation is given by

$$\rho c\frac{\partial T}{\partial t}=-\frac{\partial q}{\partial x}-\frac{hp}{DH}(T-T_\infty)+g \tag{1}$$

Where the heat flux (q) is given by Fourier's law of heat conduction

$$q=-k\frac{\partial T}{\partial x} \tag{2}$$

Elimination of q between Eqs. (1) and (2) yields

$$\rho c\frac{\partial T}{\partial t}+\frac{hp}{DH}(T-T_\infty)=g+k\frac{\partial^2 T}{\partial x^2} \tag{3}$$

In the above equations, ρ is the mass density, c is the specific heat, h is the convection coefficient, p is the perimeter of the beam, k is the thermal conductivity, $hp/DH(T-T_\infty)$ represents the convection losses from circumferential surface area of the beam, and g represents a moving plane heat source of constant strength releasing its energy continuously while moving along the x-axis with a constant speed υ. Note that Eq. (3) represents the temperature variation along the axial direction of the beam with time, i.e., $T(x,t)$.

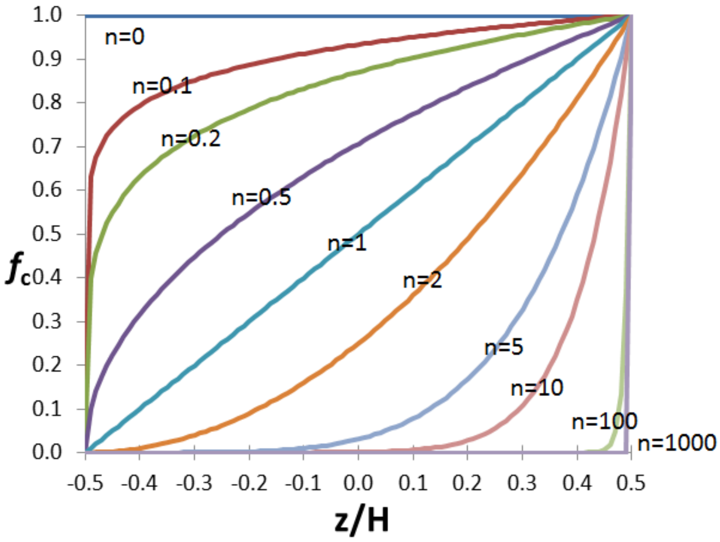

Fig. 2 The effect of the grading parameter *n* on the ceramic volume fraction along the beam

$$C_3 = \frac{\gamma}{\lambda_1^2 - \left(\mathrm{S}/\mathrm{V}\right)^2} \tag{13}$$

$$C_2 = \frac{s}{V\lambda_1}\left(\frac{e^{-\left(\mathrm{S}/\mathrm{V}\right)\zeta_0} - e^{\lambda_1\zeta_0}}{e^{\lambda_1\zeta_0} - e^{-\lambda_1\zeta_0}}\right)C_3 \tag{14}$$

$$C_1 = C_2 + \left(\frac{s}{V\lambda_1}\right)C_3 \tag{15}$$

In order to describe the composition of the beam shown in Fig. 1, the volume fraction of the ceramic, f_c, and that of metal, f_m are needed. The ceramic volume fraction is given by (Praveen and Reddy 1998)

$$f_c = \left(\frac{z}{H} + \frac{1}{2}\right)^n \quad , \quad -\frac{H}{2} \leq z \leq \frac{H}{2}, \qquad 0 \leq n \leq \infty \tag{16}$$

$$f_m = 1 - f_c \tag{17}$$

Where n is the volume fraction exponent, which is a grading parameter that dictates the material variation profile through the thickness of the FGM. Fig. 2 shows the relation between the value of n and the volumetric content of functionally graded material of ceramic and metal, where the lower the value of n means that most of FGM is ceramic and the higher the value indicates the more metal content it has. For n=0, the whole beam is ceramic, while as $n\rightarrow\infty$, the whole beam is metal. At the bottom of the beam where (z/H)=-0.5, the material is metal (f_c=0) for all values of n, while at the top of the beam where (z/H)=0.5, the material is ceramic (f_c=1) for all values of n.

In the previous part, the temperature distribution of top layer of the beam was found. This will be used to find the temperature distribution in the next layers in the transverse direction. The need

The resulting axial stress takes the form

$$\sigma_{xx} = E\varepsilon_x - \mathrm{E}\alpha T(x,z,t) = E\varepsilon_{xo} + zE\varphi - \mathrm{E}\alpha T(x,z,t) \tag{26}$$

The axial force and bending moment resultants, *N* and *M*, are defined by

$$(N, M) = \int_{-H/2}^{H/2} \sigma_{xx}(1, z)dz \tag{27}$$

Substituting and integrating produces the following result

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x0} \\ \varphi \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} \tag{28}$$

Where the beam stiffness coefficients A_{11}, B_{11} and D_{11} are given by the well-known general relation

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-\frac{H}{2}}^{\frac{H}{2}} Q_{ij}(z)(1, z, z^2)dz \qquad (i, j = 1,2,6) \tag{29}$$

In the present analysis, the remaining non-zero stiffness constants are given by

$$Q_{11} = \frac{E(z,T)}{1-\nu^2}, \quad Q_{12} = \frac{\nu E(z,T)}{1-\nu^2} \tag{30}$$

Solving for A_{11}, B_{11} and D_{11}

$$A_{11} = \frac{(E_c - E_m)}{(1-\nu^2)} \tag{31}$$

$$B_{11} = \frac{(HE_c - A_{11})}{(1-\nu^2)} \tag{32}$$

$$D_{11} = \frac{(H^2E_c - B_{11})}{(1-\nu^2)} \tag{33}$$

In the present work, only thermal loads are considered, so *N* and *M* are zero. The thermal force and the thermal moment are defined by

$$(N^T, M^T) = \int_{-H/2}^{H/2} (1, z)\{Q_{11} + Q_{12}\}\alpha T dz \tag{34}$$

And the strains are

$$\begin{Bmatrix} \varepsilon_{x0} \\ \varphi \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix}^{-1} \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} \tag{35}$$

Which can be written in terms of temperature as

$$\begin{Bmatrix} \varepsilon_{xo} \\ \varphi \end{Bmatrix} = \begin{Bmatrix} I_o + I\theta \\ J_o + J\theta \end{Bmatrix} \tag{36}$$

Where I_o, I, J_o, J are used for abbreviation purposes only. For the beam under consideration, the

From Eq. (36)

$$\frac{du_o(\mathrm{x},\mathrm{t})}{dx} = I_o + I\theta(\mathrm{x},\mathrm{t}) \tag{49}$$

The dimensionless form of *u* is

$$\mathrm{U_o}(\zeta,\eta) = \frac{\mathrm{u_o}(\mathrm{x},\mathrm{t})}{\sqrt{\kappa \mathrm{t_o}}} \tag{50}$$

The boundary condition becomes

$$U_o(0,\eta) = 0 \tag{51}$$

Eq. (49) becomes

$$\frac{dU_o(\zeta,\eta)}{d\zeta} = I_o + I\theta(\zeta,\eta) \tag{52}$$

By taking Laplace transform of Eq. (52)

$$\frac{d\bar{U}_o(\zeta,s)}{d\zeta} = \frac{I_o}{s} + I\bar{\theta}(\zeta,s) \tag{53}$$

The solution of Eq. (53) is

$$\bar{U}_o(\zeta,s) = \left(\frac{I_o\zeta}{s}\right) + I\left[\frac{C_1}{\lambda_1}e^{\lambda_1\zeta} - \frac{C_2}{\lambda_1}e^{-\lambda_1\zeta} - \frac{C_3V}{s}e^{-(\mathrm{s}/\mathrm{V})\zeta}\right] + C_6 \tag{54}$$

Where C_6 is

$$C_6 = I\left[\frac{C_1}{\lambda_1} - \frac{C_2}{\lambda_1} - \frac{C_3V}{s}\right] \tag{55}$$

The value of $\bar{W}(\zeta,s)$ was already found in Eq. (44), so the first derivative is equal

$$\frac{\mathrm{d}\bar{W}(\zeta,\mathrm{s})}{\mathrm{d}\zeta} = -\left(\sqrt{\kappa \mathrm{t_o}}\right)\left[\frac{J_o\zeta}{\mathrm{s}} + \mathrm{J}\left(\mathrm{C_1}\frac{\mathrm{e}^{\lambda_1\zeta}}{\lambda_1} - \mathrm{C_2}\frac{\mathrm{e}^{-\lambda_1\zeta}}{\lambda_1} - \mathrm{C_3V}\frac{\mathrm{e}^{-(\mathrm{s}/\mathrm{V})\zeta}}{S}\right) + \mathrm{C_4}\right] \tag{56}$$

3. Solution

In order to determine the temperatures and deflections of the FG beam, Eqs. (12), (44) and (54) are inverted into time domain using the Riemann-sum approximation method. In this method, any function $\bar{f}(\zeta,s)$ is inverted into the time domain as (Tzou 1997)

$$f(\zeta,\eta) = \frac{e^{\varepsilon\eta}}{\eta}\left(\frac{1}{2}\bar{f}(\zeta,\psi) + \mathrm{Re}\sum_{n=1}^{N}\bar{f}(\zeta,\psi + \frac{in\pi}{\eta})(-1)^n\right) \tag{57}$$

Where *Re* is the (real part of) and *i* is the complex number $\sqrt{-1}$. In Eq. (57) $f(\zeta,\eta)$, represents either θ, *W* or *U*. For faster convergence, numerous numerical experiments have shown that the value of Ψ satisfying the relation ($\Psi\eta \approx 4.7$) gives the most satisfactory results.

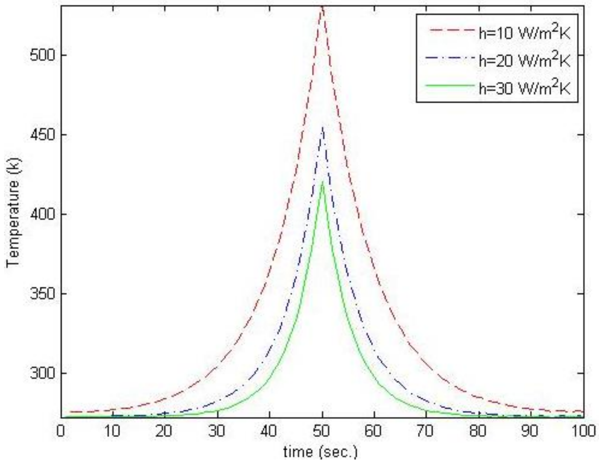

Fig. 4 Variation of beam mid-point temperature with time for different values of the heat convection coefficient

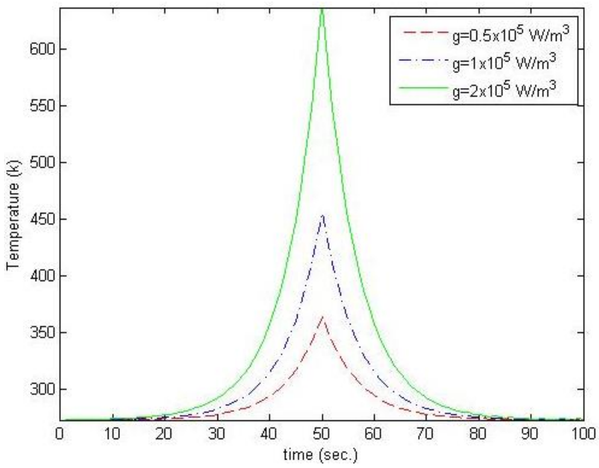

Fig. 5 Variation of beam mid-point temperature with time for different values of the heat source intensity

temperature variations as function of the transverse direction are shown in Fig. 3. As shown in the figure, the temeprature has its maximum value at the top surface of the beam (ceramic), which is the (hot) side where the heat source is acting directly. The heat source is only applied for a short time, so the temperature at the bottom of the beam (metal) is still equal to the ambient temperature (the cold side). By curve fitting of the data in the figure, the values of the constants in Eq. (20) are found and the equation takes the following form

$$T(z) = 300.7e^{19.5z} \tag{58}$$

This equation is considered as a material charactaristic curve where it won't be different if the heat source velocity or intensity are changed.This temperature is added to the temperature variation, as shown by Eq. (21), in order to determine the transient temperature distribution of the beam.

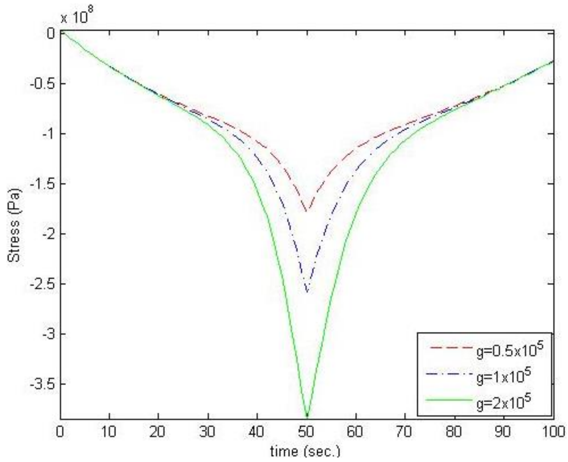

Fig. 8 Variation of stress at the beam mid-point with time for different values of the heat source intensity

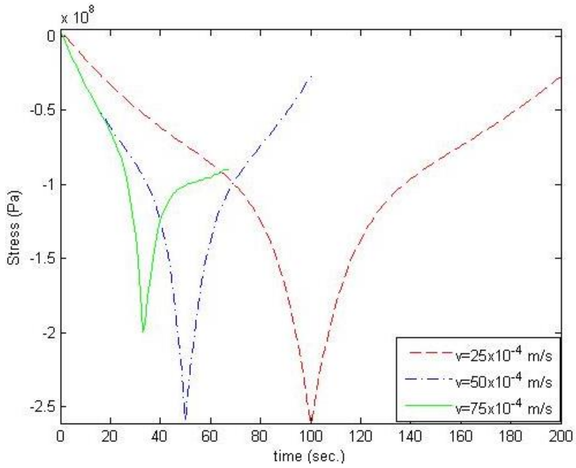

Fig. 9 Variation of stress at the beam mid-point with time for different values of the heat source velocity

location receives more amount of energy as the source speed decreases. This results from the intensity of the released energy that increases as the heat source speed decreases.

4.2 Beam deflections and stresses

Fig. 7 shows the variation of the axial stress at midpoint of the beam with time for different values of heat convection coefficients. Due to the support conditions and the heating process, the stresses are compressive. The behavior in this figure is based on the thermal history which was depicted in Fig. 4. It can be seen that the compressive stress values increase with time and reach the maximum values at t=50 (which is the time at which the heat source passes the midpoint of the beam), and then decrease. It can also be seen that for higher values of the heat convection coefficient, the thermal losses increase and the beam temperature decreases and results in lower values of stress.

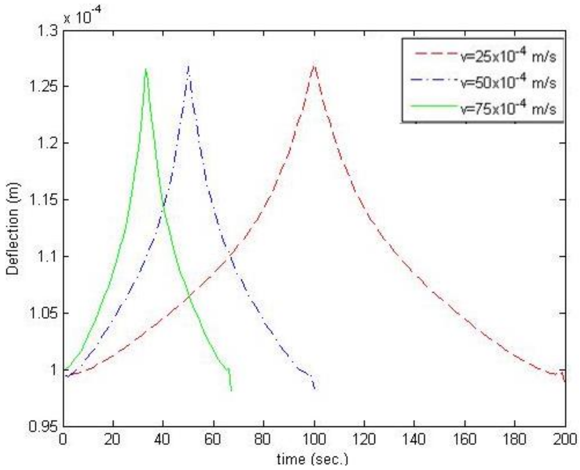

Fig. 12 Variation of the transverse deflection of the beam mid-point with time for different values of the heat source velocity

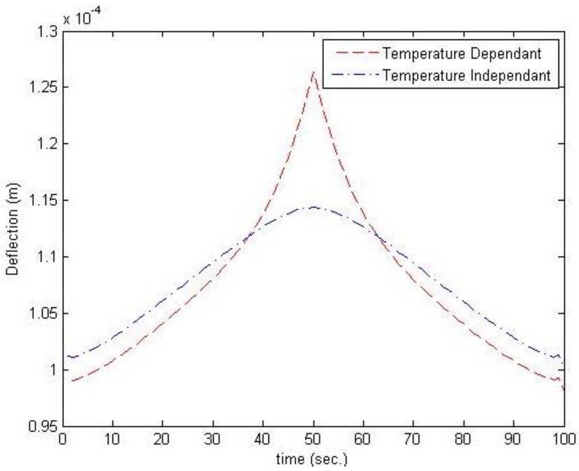

Fig. 13 The transverse deflection beam midpoint for temperature dependent and temperature independent FGM properties

deflection of the beam midpoint. The figure shows an important fact that neglecting the temperature-dependency of the properties results in less accurate results. The more important point here is the underestimation of the value of the maximum deflection.

5. Conclusions

The thermo-mechanical behavior of a simply supported functionally graded material beam under the effect of a moving heat source is investigated. The transient temperatures, deflections and stresses are calculated and presented for some specific cases. The FGM consists of a metal part (bottom) and ceramic part (top) where grading is in the transverse direction, with the properties being temperature-dependent. All material properties are calculated by using the power law. The material properties in the axial direction are the same. The heat source motion takes place along the beam on the top surface where the material is ceramic (the hot side), while the bottom surface where the material is metallic is the cold side.

The temperature distribution is found by superposition of the temperature variation through the transverse direction, within the FGM, and the variation in the axial direction, resulting from the motion of the heat source. Based on the temperature history within the beam, the thermally-induced deflections and stresses are found.

The thermos-elastic modeling approach consisted of formulating the partial differential equations for the temperatures and deflections in time and space, then using Laplace transformation to transform them into ordinary differential equations, and finally using the inverse Laplace transformation to find the solutions.

The temperature, deflection and stress time histories of the beam are presented. The effects of different parameters, such as the convection coefficient and the heat source intensity and speed are investigated. The results show that temperatures, and hence the deflections and stresses increase with less heat convection from the beam surface, higher heat source intensity and low speeds.

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