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Elastic-plastic fracture of functionally graded circular shafts in torsion

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Abstract. Analytical investigations were performed of a longitudinal crack representing a cylindrical surface in circular shafts loaded in torsion with taking into account the non-linear material behavior. Both functionally graded and multilayered shafts were analyzed. It was assumed that the material is functionally graded in radial direction. The mechanical behavior of shafts was modeled by using non-linear constitutive relations between the shear stresses and shear strains. The fracture was studied in terms of the strain energy release rate. Within the framework of small strain approach, the strain energy release rate was derived in a function of the torsion moments in the cross-sections ahead and behind the crack front. The analytical approach developed was applied to study the fracture in a clamped circular shaft. In order to verify the solution derived, the strain energy release rate was determined also by considering the shaft complimentary strain energy. The effects were evaluated of material properties, crack location and material non-linearity on the fracture behavior. The results obtained can be applied for optimization of the shafts structure with respect to the fracture performance. It was shown that the approach developed in the present paper is very useful for studying the longitudinal fracture in circular shafts in torsion with considering the material non-linearity.

Keywords: functionally graded material; torsion; fracture; elastic-plastic behavior; analytical solution

1. Introduction

Recently, the application of functionally graded materials in structures that are subjected to non-uniform service requirements has increased rapidly (Gasik 2010, Nemat-Allal *et al*. 2011, Bohidar *et al*. 2014, Abdelhak *et al*. 2015, Daouadji and Adim 2016, Daouadji *et al*. 2016). This is due mainly to the fact that functionally graded materials allow for spatial optimization of composition and properties in one or more directions during manufacturing. In this way, novel materials that have remarkable advantages over the traditional structural materials can be manufactured. Bending of functionally graded plates and beams has been analyzed recently by Ait Yahia *et al*. 2015, Belabed *et al*. 2014, Bellifa *et al*. 2016, Bennoun *et al*. 2016, Bouderba *et al*. 2013, Bourada *et al*. 2015, Bourada *et al*. 2016, Bousahla *et al*. 2014, Hadji *et al*. 2016, Hamidi *et al*. 2015, Mahi *et al*. 2015). The study of fracture behavior of functionally graded materials plays

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