Advances in Materials Research, *Vol. 13, No. 4 (2024) 253-267*
https://doi.org/10.12989/amr.2024.13.4.253

Influence of viscosity and locality on a fiber-reinforced thermoelastic solid with two different theories

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(*Received April 2, 2023, Revised January 11, 2024, Accepted January 11, 2024*)

Abstract. The current study attempts to discuss the effects of viscosity and locality on a fiber-reinforced thermoelastic solid. The problem is solved analytically in the context of the three-phase-lag model as well as the Green-Naghdi theory without energy dissipation (G-N II). The method of normal mode analysis is used to obtain analytical expressions for the displacement, stress, and temperature distributions. Compute the physical fields with suitable boundary conditions and perform numerical calculations using MATLAB programming. Comparisons are carried out with the results in the absence and presence of locality as well as viscosity. The locality and viscosity have great effects on all considered physical fields since the amplitudes of these quantities are vary. This procedure remains valid when a nonlocal elastic solid is replaced with an elastic one.

Keywords: nonlocal fiber-reinforced; three-phase lag-model; visco-thermoelastic

1. Introduction

Biot (1965) introduced a generalized thermoelastic theory to overcome the infinite heat propagation rate paradox inherent in the classical thermoelastic coupled dynamical theory. Lord and Shulman (1967), introduced a model called the thermoelastic extension theory, which includes a thermal relaxation time parameter (single-phase hysteresis model). Another model, given by Green and Lindsay (1972), involves two thermal relaxation times, one in the equation of motion and the other in the equation of thermal conductivity, and is called temperature-dependent thermoelasticity (TRDTE). Afterward, Green and Naghdi (1991, 1992, 1993) proposed three more models, hereinafter referred to as G-N I, II, and III models. Tzou (1995) proposed an alternative theory of hyperbolic thermoelasticity, the so-called dual phase-lag model, in which Fourier's law is replaced by a modified approximation of Fourier's law, with two different time transitions of heat flow and temperature gradient. Roy Choudhuri (2007) introduced the heat conduction equation with three-phase lags, replacing Fourier's law of heat conduction with a modified approximation of Fourier's law that introduces three different phase lags of the heat flux vector, the thermal displacement gradient, and temperature gradient. Three-phase-lag model is useful in the problems of heat transfer, heat conduction, nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, and phonon-scattering. The three-phase lags model of thermoelastic material was investigated by Said *et al.* (2020), Othman and Mondal (2020), Othman *et al.* (2019), Marin *et al.* (2017), and Alharbi *et*

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http://www.techno-press.org/?journal=amr&subpage=5 ISSN: 2234-0912 (Print), 2234-179X (Online)

al. (2021) and Othman *et al.* (2023).

The Kelvin-Voigt model is one of the macroscopic models commonly used to describe the viscoelastic behavior of materials. The model represents the delayed elastic response under loading when the deformation varies with time but is recoverable. Viscoelastic materials with complex structural composition and time and temperature-dependent properties, such as elastomers or rubber, are widely utilized in engineering uses such as automobile belts and tires, seals, and biomedical devices. Koltunov (1976) provided important experimental results for determining the mechanical properties of viscoelastic materials. Gupta (2013) studied wave propagation in viscoelastic transversely isotropic media. Othman *et al.* (2009, 2018), Said (2022), and Abouelregal (2023) discussed different types of thermo-viscoelastic problems. Fiber-reinforced polymers are used in many fields due to their excellent properties. Fiber-reinforced materials find applications in a variety of industries where their unique combination of properties, including thermal stability and mechanical strength, is beneficial. Here are some specific applications such as Zhang (2015) and Rajak *et al.* (2019). Stress-deformation analysis of fiber-reinforced materials has been an important topic in solid mechanics for the past few decades. Bayones and Hussien (2017) discussed the propagation of Rayleigh waves under the influence of rotation in viscous, anisotropic, fiber-reinforced thermal media. Sheoran *et al.* (2016) discussed the thermo-viscous problem of the fractal order with temperature-dependent elastic moduli. Bosaeed *et al.* (2019) studied the effects of rotation and initial stress on the propagation of Rayleigh waves in fiber-reinforced magneto-thermo-viscoelastic media. Various vital approaches and real-time applications of problems in thermodynamics and thermoelasticity have been introduced by Craciun *et al.* (2020a, 2020b, 2004). Arefi *et al.* (2019, 2021, 2022), Mohammad-Rezaei Bidgoli *et al.* (2021, 2022a, 2022b, 2023a, 2023b, 2023c) and Ghorbanpour-Arani (2023).

The present problem discussed the influence of viscosity and locality on a fiber-reinforced thermoelastic half-space in the context of the three-phase-lag model as well as the Green-Naghdi theory without energy dissipation (G-N II). The analytical solutions for the field variables of interest are obtained by using the normal mode analysis to obtain the exact expressions for physical variables. That is applicable to a wide range of problems in hydrodynamics. Numerically simulated results are obtained and presented graphically to depict the influence of nonlocal parameter and viscosity on a fiber-reinforced thermoelastic half-space. Comparisons are carried out with the results in the presence and absence of the locality as well as the viscosity. The locality and viscosity have great effects on all considered physical fields since the amplitudes of these quantities are vary (increasing or decreasing) with the increase of the elastic nonlocal parameter. This procedure remains valid when a nonlocal elastic solid is replaced with an elastic one.

2. Formulation of the problem

We consider a nonlocal fiber-reinforced visco-thermoelastic isotropic solid in a half-space $(x \geq 0)$. Plane strain in the xy-plane with the displacement vector $\underline{u} = (u, v, 0)$, $u = u(x, y, t)$, $v = v(x, y, t)$.

The constitutive equations are as in Belfield *et al.* (1983), Said (2020), and Eringen (1974).

$$
\begin{aligned}
(1-\varepsilon^2\nabla^2)\sigma_{ij} &= \lambda^* e_{kk}\delta_{ij} + 2\mu_T^* e_{ij} + \alpha^*\left(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}\right) \\
&\quad + 2(\mu_L^* - \mu_T^*)(a_i a_k e_{kj} + a_j a_k e_{ki}) - \gamma^*\theta\delta_{ij} + \beta^* a_i a_j a_k a_m e_{km}
\end{aligned} \tag{1}
$$

$$\lambda^* = \lambda\left(1+\lambda_0\frac{\partial}{\partial t}\right), \alpha^* = \alpha\left(1+\alpha_0\frac{\partial}{\partial t}\right), \mu_T^* = \mu_T\left(1+\alpha_1\frac{\partial}{\partial t}\right), \mu_L^* = \mu_L\left(1+\alpha_2\frac{\partial}{\partial t}\right),$$
$$\beta^* = \beta\left(1+\beta_0\frac{\partial}{\partial t}\right), \gamma^* = \gamma\left(1+\gamma_0\frac{\partial}{\partial t}\right), \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j}+u_{j,i}\right), e_{kk} = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} \tag{3}$$

where, $\varepsilon = a_0 e_0$ is the elastic nonlocal parameter having a dimension of length, a_0, e_0 respectively, are an internal characteristic length and a material constant, σ_{ij} are the components of stress, e_{ij} are the components of strain, e_{kk} is the dilatation, λ, μ are elastic constants, [see Eringen *et al.* (1972a, 1972b) for details] α, β, $(\mu_L - \mu_T)$ are reinforcement parameters, α_0, α_1, α_2, β_0, γ_0 are the viscoelastic parameters, α_T is the thermal expansion coefficient, $\theta = T - T_0$ where T is the temperature above the reference temperature T_0, δ_{ij} is the Kronecker's delta, and $a \equiv (1,0,0)$ is fiber-direction $a \equiv (a_1, a_2, a_3)$, ${a_1}^2 + {a_2}^2 + {a_3}^2 = 1$.

Eq. (1) then yields

$$(1-\varepsilon^2\nabla^2)\sigma_{xx} = A_1 u_{,x} + A_2 v_{,y} - \gamma(1+\gamma_0\frac{\partial}{\partial t})\theta \tag{4}$$

$$(1-\varepsilon^2\nabla^2)\sigma_{yy} = A_2 u_{,x} + A_3 v_{,y} - \gamma(1+\gamma_0\frac{\partial}{\partial t})\theta \tag{5}$$

$$(1-\varepsilon^2\nabla^2)\sigma_{xy} = A_4(u_{,y} + v_{,x}) \tag{6}$$

where

$$A_1 = \lambda\left(1+\lambda_0\frac{\partial}{\partial t}\right) - 2\mu_T\left(1+\alpha_1\frac{\partial}{\partial t}\right) + 4\mu_L\left(1+\alpha_2\frac{\partial}{\partial t}\right) + 2\alpha\left(1+\alpha_0\frac{\partial}{\partial t}\right) + \beta\left(1+\beta_0\frac{\partial}{\partial t}\right),$$
$$A_2 = \lambda\left(1+\lambda_0\frac{\partial}{\partial t}\right) - \alpha\left(1+\alpha_0\frac{\partial}{\partial t}\right),\ A_3 = \lambda\left(1+\lambda_0\frac{\partial}{\partial t}\right) + 2\mu_T\left(1+\alpha_1\frac{\partial}{\partial t}\right),$$
$$A_4 = \mu_L\left(1+\alpha_2\frac{\partial}{\partial t}\right)$$

The equations of motion in the absence of body force

$$\sigma_{ji,j} = \rho\ddot{u}_i; i,j = 1,2 \tag{7}$$

The heat conduction equation as Roy Choudhuri (2007)

$$k^*\left(1+\tau_v\frac{\partial}{\partial t}\right)\nabla^2\theta + k\left(1+\tau_v\frac{\partial}{\partial t}\right)\nabla^2\dot{\theta} = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)[\rho C_E\left(n_0\ddot{\theta}+n_1\dot{\theta}\right) +$$
$$\gamma^* T_0(1+\gamma_0\frac{\partial}{\partial t})(n_0\ddot{e}+n_1\dot{e})] \tag{8}$$

Where k^* is the additional material constant, k is the coefficient of thermal conductivity, C_E is the specific heat at constant strain, n_0, n_1 are integers, τ_θ is the phase-lag of the temperature gradient, τ_q is the phase-lag of heat flux, τ_v is the phase-lag of the thermal displacement gradient, and ρ is the mass density.

Introducing Eq. (1) in Eq. (7), thus, we have

$$(1-\varepsilon^2\nabla^2)\rho\frac{\partial^2 u}{\partial t^2} = A_1\frac{\partial^2 u}{\partial x^2} + A_5\frac{\partial^2 v}{\partial x\partial y} + A_4\frac{\partial^2 u}{\partial y^2} - \gamma(1+\gamma_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial t} \tag{9}$$

$$(1-\varepsilon^2\nabla^2)\rho\frac{\partial^2 v}{\partial t^2}=A_4\frac{\partial^2 v}{\partial x^2}+A_5\frac{\partial^2 u}{\partial x\partial y}A_3\frac{\partial^2 v}{\partial y^2}-\gamma(1+\gamma_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial y} \tag{10}$$

Where

$$A_5=A_2+A_4$$

Consider the following non-dimensional variables

$$(x',y',\varepsilon',u',v')=\frac{1}{l_0}(x,y,\varepsilon,u,v),\theta'\frac{\gamma\theta}{\lambda+2\mu_T},\sigma'_{ij}=\frac{\sigma_{ij}}{\mu_T},$$
$$\left(t',\tau'_v,\tau'_\theta,\tau'_q,\gamma'_0,\alpha'_0,\alpha'_1,\alpha'_2,\beta'_0,\gamma'_0\right)=\frac{c_0}{l_0}\left(t,\tau_v,\tau_\theta,\tau_q,\gamma_0,\alpha_0,\alpha_1,\alpha_2,\beta_0,\gamma_0\right), \tag{11}$$
$$l_0=\sqrt{\frac{k^*}{\rho C_E T_0}},c_0=\sqrt{\frac{\lambda+2\mu_T}{\rho}}.$$

Using Eq. (11), we get

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}=h_1\frac{\partial^2 u}{\partial x^2}+h_5\frac{\partial^2 v}{\partial x\partial y}+h_4\frac{\partial^2 u}{\partial y^2}-(1+\gamma_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial x} \tag{12}$$

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}=h_4\frac{\partial^2 v}{\partial x^2}+h_5\frac{\partial^2 u}{\partial x\partial y}+h_3\frac{\partial^2 v}{\partial y^2}-(1+\gamma_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial y} \tag{13}$$

$$\left(1+\tau_v\frac{\partial}{\partial t}\right)\nabla^2\theta+d_1\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\nabla^2\dot\theta=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)[\left(d_2\ddot\theta+d_3\dot\theta\right)$$
$$+(1+\gamma_0\frac{\partial}{\partial t})(d_4\ddot e+d_5\dot e)] \tag{14}$$

Where

$$(h_1,h_3,h_4,h_5)=\left(\frac{A_1}{\rho {c_0}^2},\frac{A_3}{\rho {c_0}^2},\frac{A_4}{\rho {c_0}^2},\frac{A_5}{\rho {c_0}^2}\right),d_1=\frac{kc_0}{l_0k^*},d_2=\frac{\rho C_E n_0 {c_0}^2}{k^*},$$
$$d_3=\frac{\rho C_E n_1 c_0 l_0}{k^*},d_4=\frac{T_0 n_0 {c_0}^2\gamma^2}{k^*(\lambda+2\mu_T)},d_5=\frac{T_0 c_0 n_1 l_0\gamma^2}{k^*(\lambda+2\mu_T)}.$$

3. Normal mode analysis method

We solve the problem of a nonlocal fiber-reinforced visco-thermoelastic half-space by using normal mode analysis as Othman *et al.* (2023)

$$\left(u,v,\theta,\sigma_{ij}\right)(x,y,t)=\left(\bar u,\bar v,\bar\theta,\bar\sigma_{ij}\right)(x)e^{iby-mt}. \tag{15}$$

Where $\bar u$, $\bar v$, $\bar\theta$ and $\bar\sigma_{ij}$ are the amplitudes of the fields quantities, $i=\sqrt{-1}$ is the imaginary unit, *m* (complex) is the time constant, and *b* is the wavenumber in the *y*- direction.

Using Eq. (15) in Eqs. (12)-(14), we get

$$(A_7D^2-A_8)\bar u+\left(ib\bar h_5D\right)\bar v-A_6D\bar\theta=0 \tag{16}$$

$$\left(ib\bar h_5D\right)\bar u+(A_9D^2-A_{10})\bar v-A_6ib\bar\theta=0 \tag{17}$$

$$B_1 D\bar{u} + B_1 ib\bar{v} + (B_1 D^2 + B_3)\bar{\theta} = 0 \tag{18}$$

Where

$$(\bar{h}_1, \bar{h}_3, \bar{h}_4, \bar{h}_5) = \left(\frac{\bar{A}_1}{\rho c_0{}^2}, \frac{\bar{A}_3}{\rho c_0{}^2}, \frac{\bar{A}_4}{\rho c_0{}^2}, \frac{\bar{A}_5}{\rho c_0{}^2}\right),$$

$$\bar{A}_1 = \lambda(1-\lambda_0 m) - 2\mu_T(1-\alpha_1 m) + 4\mu_L(1-\alpha_2 m) + 2\alpha(1-\alpha_0 m) + \beta(1-\beta_0 m),$$

$$\bar{A}_2 = \lambda(1-\lambda_0 m) - \alpha(1-\alpha_0 m), \bar{A}_3 = \lambda(1-\lambda_0 m) + 2\mu_T(1-\alpha_1 m),$$

$$\bar{A}_4 = \mu_L(1-\alpha_2 m), \bar{A}_5 = \bar{A}_2 + \bar{A}_4, A_7 = \bar{h}_1 + m^2\varepsilon^2, A_8 = m^2 + b^2\bar{h}_4 + m^2\varepsilon^2 b^2,$$

$$A_9 = \bar{h}_4 + m^2\varepsilon^2, A_{10} = m^2 b^2 \bar{h}_3 + m^2\varepsilon^2 b^2,$$

$$B_1 = \left(1 - \tau_q m + \frac{1}{2}\tau_q^2 m^2\right)(1-\gamma_0 m)(d_4 m^2 - d_5 m), B_2 = m d_1(1 - m\tau_\theta) - (1 - m\tau_\upsilon),$$

$$B_3 = \left(1 - \tau_q m + \frac{1}{2}\tau_q^2 m^2\right)(d_2 m^2 - d_3 m) - m b^2 d_1(1 - m\tau_\theta) + b^2(1 - m\tau_\upsilon)$$

Omitting $\bar{v}(x)$ and $\bar{\theta}(x)$ from Eqs. (16)-(18), we get

$$[D^6 - C_1 D^4 + C_2 D^2 - C_3]\bar{u}(x) = 0,$$

$$C_1 = \frac{-A_6 A_9 B_1 + B_2\left(A_7 A_{10} + A_8 A_9 - \bar{h}_5{}^2 b^2\right) - A_7 A_9 B_3}{A_7 A_9 B_2},$$

$$C_2 = \frac{B_1 A_6\left(2\bar{h}_5{}^2 b^2 - A_7 b^2 - A_{10}\right) + B_2 A_8 A_{10} + B_3(-A_7 A_{10} - A_8 A_9 + \bar{h}_5{}^2 b^2)}{A_7 A_9 B_2}, \tag{19}$$

$$C_3 = \frac{-A_8 A_{10} B_3 - A_6 A_8 b^2 B_1}{A_7 A_9 B_2}.$$

Eq. (19) can be factorized as

$$(D^2 - k_1{}^2)(D^2 - k_2{}^2)(D^2 - k_3{}^2)\bar{u}(x) = 0 \tag{20}$$

where $k_n{}^2$ $(n = 1,2,3)$ are the roots of the characteristic equation

$$k^6 - C_1 k^4 + C_2 k^2 - C_3 = 0 \tag{21}$$

The bounded solution of Eq. (19), is

$$\bar{u}(x) = \sum_{i=1}^{3} N_i e^{-k_i x}. \tag{22}$$

Similarly,

$$\bar{v}(x) = \sum_{i=1}^{3} H_{1i} N_i e^{-k_i x}, \tag{23}$$

$$\bar{\theta}(x) = \sum_{i=1}^{3} H_{2i} N_i e^{-k_i x}, \tag{24}$$

where N_i $(i = 1,2,3)$ are parameters.

$$H_{1i} = \frac{ib(A_8 + \bar{h}_5 k_i^2 - A_7 k_i^2)}{\bar{h}_5 k_i b^2 + A_9 k_i^3 - A_{10} k_i}, H_{2i} = \frac{ib\bar{h}_5 k_i H_{1i} + A_8 - A_7 k_i^2}{A_6 k_i}$$

Using Eqs. (11), and (15) in Eqs. (4)-(6), we obtain

$$\bar{\sigma}_{xx} = \sum_{i=1}^{3} H_{3i} \, N_i e^{-k_i x}, \tag{25}$$

$$\bar{\sigma}_{xy} = \sum_{i=1}^{3} H_{4i} \, N_i e^{-k_i x}, \tag{26}$$

where

$$H_{3i} = \frac{-\bar{A}_1 k_i + \bar{A}_2 ib H_{1i} - (\lambda + 2\mu_T) A_6 H_{2i}}{[1 - \varepsilon^2 (k_i^2 - b^2)]\mu_T}$$

$$H_{4i} = \frac{\bar{A}_4 (ib - k_i H_{1i})}{[1 - \varepsilon^2 (k_i^2 - b^2)]\mu_T}.$$

4. The boundary conditions of the problem

To obtain the parameters N_i (i=1, 2, 3), we consider the following initial and regularity conditions used for solving the current problem at x=0 are

a) A thermal boundary condition for half-space surface exposed to a thermally insulated boundary

$$\frac{\partial \theta}{\partial x} = 0. \tag{27}$$

b) Mechanical boundary condition

$$\sigma_{xx} = -f(y, t) = -f^* e^{iby - mt}. \tag{28}$$

c) Mechanical boundary

$$\sigma_{xy} = 0, \tag{29}$$

where, $f(y, t)$ is the arbitrary function of y, t and f^* is the magnitude of the mechanical force.

By inserting Eqs. (24)- (26) in Eqs. (27)- (29), we have

$$\sum_{i=1}^{3} k_i H_{2i} N_i = 0, \sum_{i=1}^{3} H_{3i} N_i = -f^*, \sum_{i=1}^{3} H_{4i} N_i = 0. \tag{30}$$

Eq. (30) is solved using the inverse of the matrix method as follows

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} k_1 H_{21} & k_2 H_{22} & k_3 H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -f^* \\ 0 \end{pmatrix}. \tag{31}$$

5. Numerical calculations and discussion for the problem

To discuss the theoretical results and compare them in the context of the three-phase-lag model (3PHL) and the Green-Naghdi theory without energy dissipation (G-N II) theory. We now give numerical results for some physical constants

$$\tau_\theta = 7 \times 10^{-3}\ \text{s}, \tau_q = 9 \times 10^{-3}\ \text{s}, \tau_\upsilon = 6 \times 10^{-3}\ \text{s}, k = 15 \times 10^{10}\ N.K^{-1},$$

$$k^* = 386\ N.K^{-1}, n_0 = 1, n_1 = 0, \varepsilon = 0.99, f^* = 0.05, \rho = 7800\ kg.m^{-3},$$

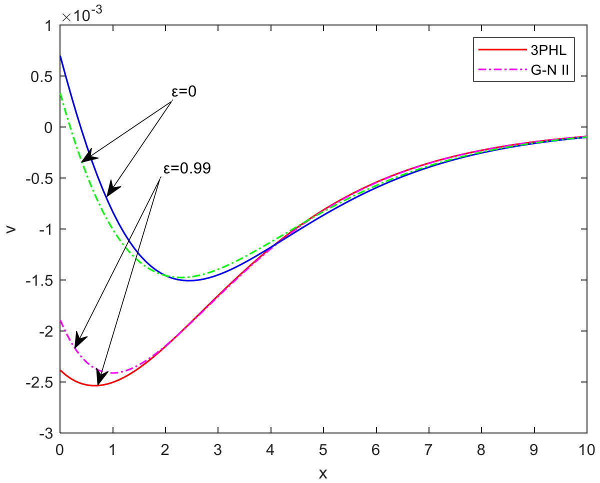

Fig. 1 Vertical displacement distribution *v* for local and nonlocal theories

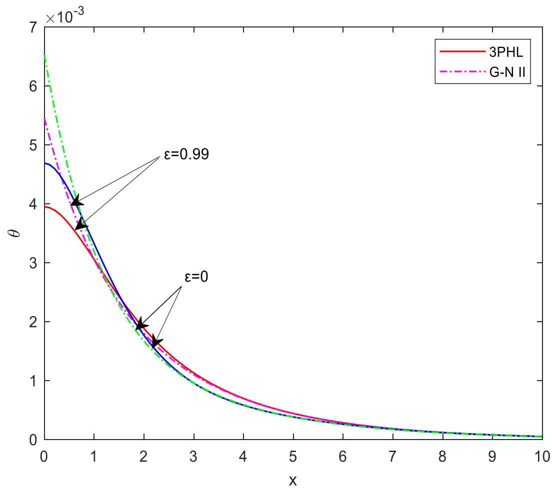

Fig. 2 Thermal temperature distribution θ for local and nonlocal theories

$$
\begin{gathered}
C_E = 383.1\ j.kg^{-1}.K^{-1}, T_0 = 293\ K, \lambda = 5.76 \times 10^{13}\ N.m^{-2}, \beta = 2 \times 10^{13}\ N.m^{-2}, \\
\alpha = 1.78 \times 10^{13}\ K^{-1}, \mu_T = 2.86 \times 10^{13}\ N.m^{-2}, \mu_L = 7.86 \times 10^{13}\ N.m^{-2}, \\
\lambda_0 = 0.96\ s^{-1}, \mu = 3.86 \times 10^{13}\ N.m^{-2}, \alpha_T = 2.78 \times 10^{-4} K^{-1}, \alpha_0 = 0.97\ s^{-1}, \\
\alpha_1 = 0.98\ s^{-1}, \alpha_2 = 0.99\ s^{-1}, \beta_0 = 0.99\ s^{-1}, m_0 = -0.5, m = m_0 + i\zeta, \\
b = 0.5, \zeta = -0.8.
\end{gathered} \tag{1}
$$

At the time value *t*=0.2 s thermal temperature change θ, displacement component *v* as well as stress components σ_{xx} and σ_{xy} in plane *y*=0.5 for the considered problem are calculated based on the three-phase-lag (3PHL) model and Green-Naghdi theory without energy dissipation (G-N II).

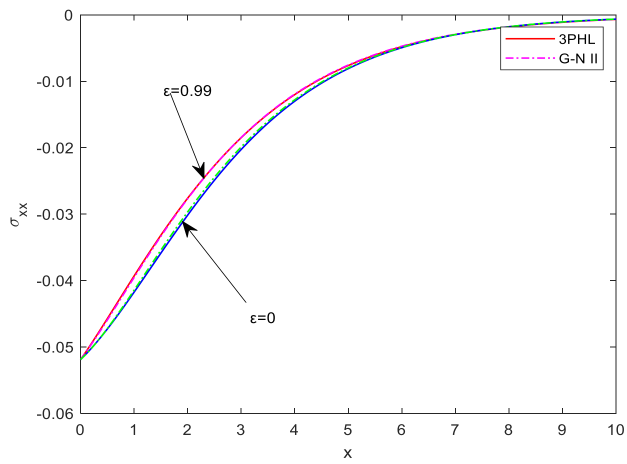

Fig. 3 Distribution of stress component σ_{xx} for local and nonlocal theories

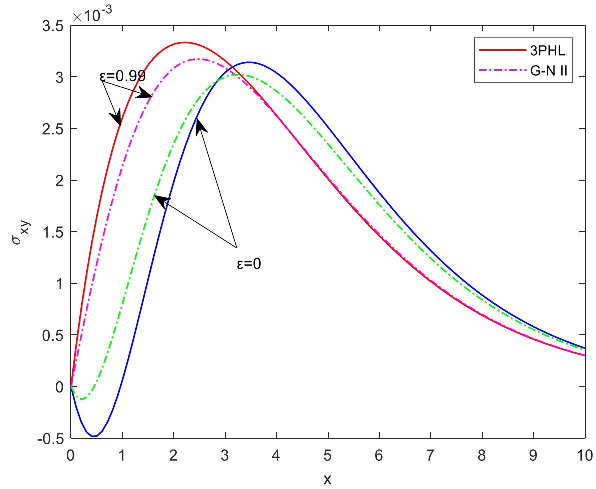

Fig. 4 Distribution of stress component σ_{xy} for local and nonlocal theories

In Figs. 1 to 4, the influence of nonlocal parameters on the thermal temperature θ, variation of the displacement v and stress components σ_{xx}, σ_{xy} are analyzed. Fig. 1 shows that the vertical displacement v changes from positive values for ε=0. It reaches the minimum value and then increases in the range $2.3 \leq x \leq 10$. With nonlocal theory, v starts from a negative value. It reaches its minimum value and then increases in range $2.3 \leq x \leq 10$. Fig. 2 shows that the thermal temperature fluctuation θ starts from positive values for both local and nonlocal theories. The same behavior is assumed in the absence and presence of nonlocal parameters. It is clear that the values of θ associated with the G-N II theory are higher than those under the 3PHL model in the absence and presence of nonlocal parameters. Fig. 3 shows that the fluctuation of the stress component σ_{xx} starts from negative values and obeys the boundary conditions (see Eq. (28)) at x=0. We find that it follows the same path in the presence and absence of non-local parameters and increases. Fig. 4

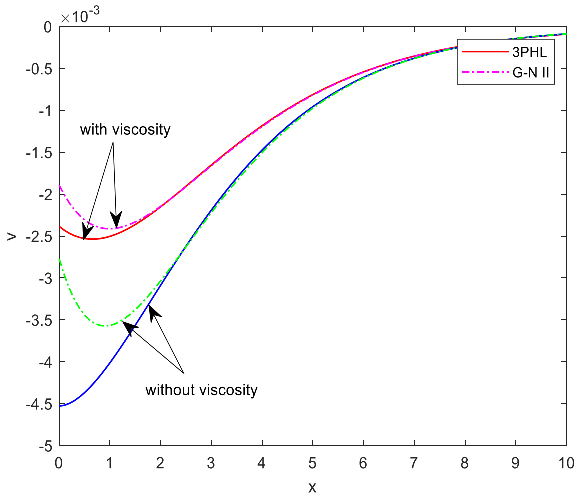

Fig. 5 Vertical displacement distribution *v* with and without viscosity

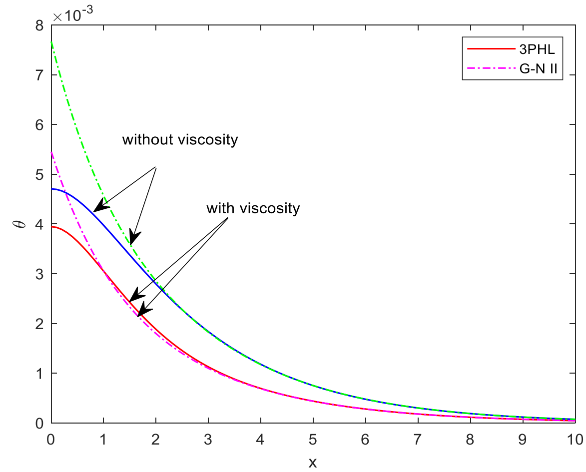

Fig. 6 Thermal temperature distribution θ with and without viscosity

shows that the fluctuations of the stress component σ_{xy} starts from a zero value and obeys the boundary conditions (see Eq. (29)) at *x*=0. In the case of local theory, it decreases to a minimum, then increases and decreases again. In nonlocal theory, it increases until it reaches a maximum and then decreases. Figs. 5-8 shows plots of the vertical displacement component *v* thermal temperature θ, and stress components σ_{xx}, σ_{xy} with and without viscosity. Fig. 5 shows that the vertical displacement distribution *v* starts from negative values. With the viscosity effect, the value starts to drop to the minimum value in the range $0 \le x \le 0.9$ and then increases in the range $0.9 \le x \le 10$. Without the influence of viscosity, the value of *v* drops to the minimum value in the range $0 \le x \le 0.7$ and then increases in the range $0.7 \le x \le 10$. Fig. 6 shows that thermal temperature changes exhibit the same behavior with and without viscosity effects. It starts with a positive value and decreases. Fig. 7 shows that the fluctuations of the stress component σ_{xx} starts from negative

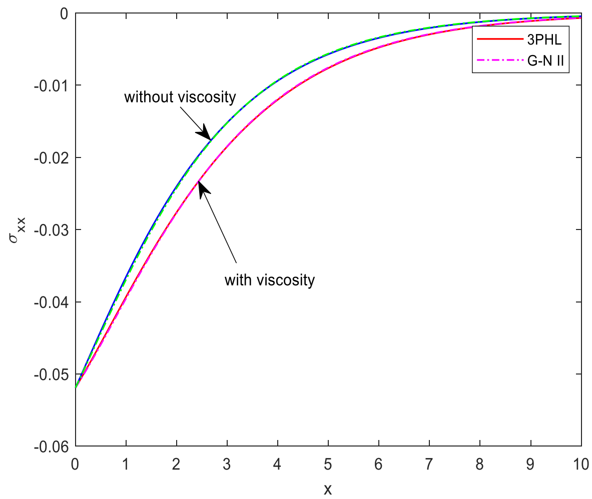

Fig. 7 Distribution of stress component σ_{xx} with and without viscosity

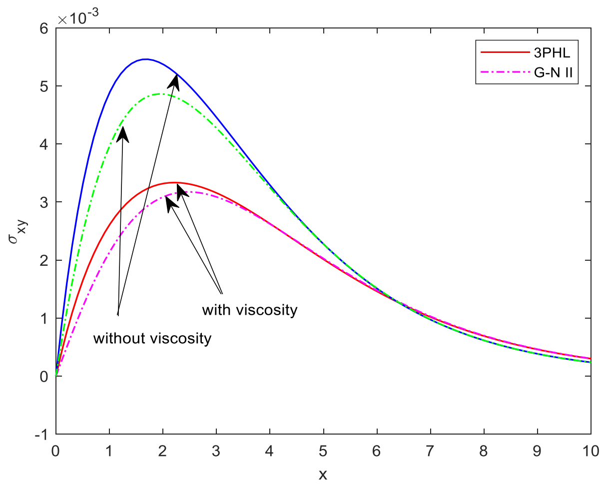

Fig. 8 Distribution of stress component σ_{xy} with and without viscosity

values and obeys the boundary conditions at (see Eq. (28)) at *x*=0. Values of σ_{xx} increase in the range $0.7 \leq x \leq 10$. Fig. 8 shows the fluctuation of the stress component σ_{xy} with and without viscosity starts from a zero value and satisfies the boundary conditions at (see Eq. (29)) at *x*=0. The value of σ_{xy} increases to the maximum value and then decreases. It is also clear that the values of σ_{xy} in the context of the 3PHL model are higher than those under the G-N II model.

Figs. 9-12 show the 3D surface curves of the physical variables, namely the vertical displacement, thermal temperature distribution θ, and stress components σ_{xx}, σ_{xy} in the context of the 3PHL model. These figures are important for studying the dependence of these quantities on the vertical component of displacement. The obtained curves strongly depend on the vertical displacement, and all physical variables are moving in wave propagation.

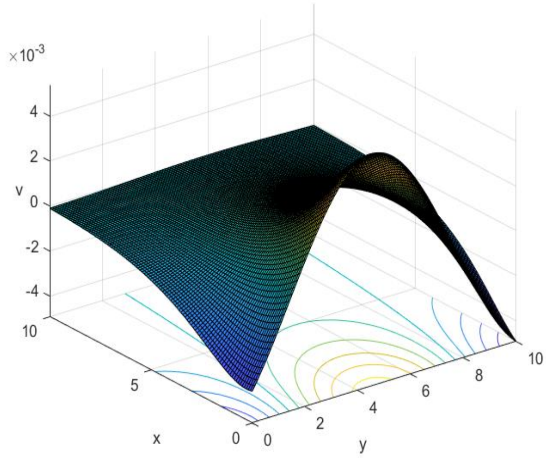

Fig. 9 The vertical displacement distribution *v* in the context of three-phase-lag model

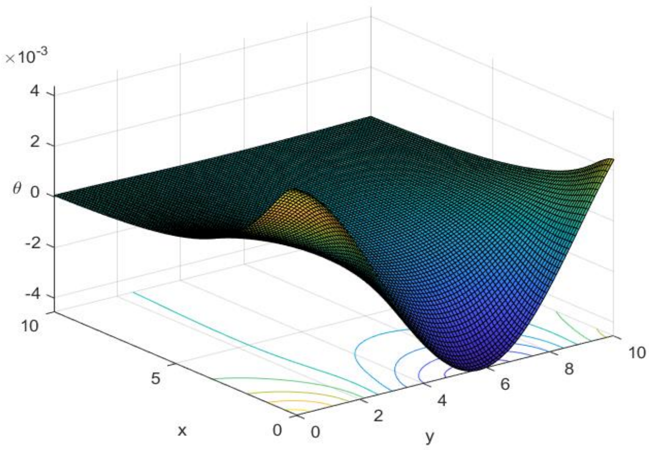

Fig. 10 Distribution of stress component θ in the context of three-phase-lag model

6. Conclusions

The analysis of displacement components, thermal temperature changes, as well as stress components due to viscosity and locality on a fiber-reinforced thermoelastic solid, is an interesting problem of mechanics.

- The value of all physical fields converges to zero with an increase in distance x and all functions are continuous.
- It was noticed that fiber-reinforced thermoelastic materials with viscous have an important role in the distribution of the field variables.
- The locality has a great role in all considered physical fields since the amplitudes of these quantities are varying (increasing or decreasing) with the increase of the elastic nonlocal

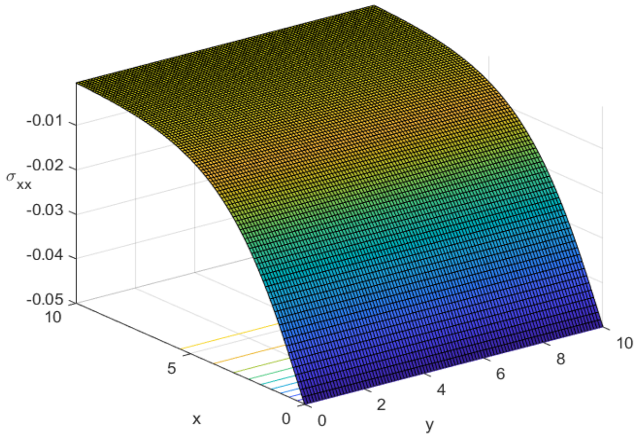

Fig. 11 Distribution of stress component σ_{xx} in the context of three-phase-lag model

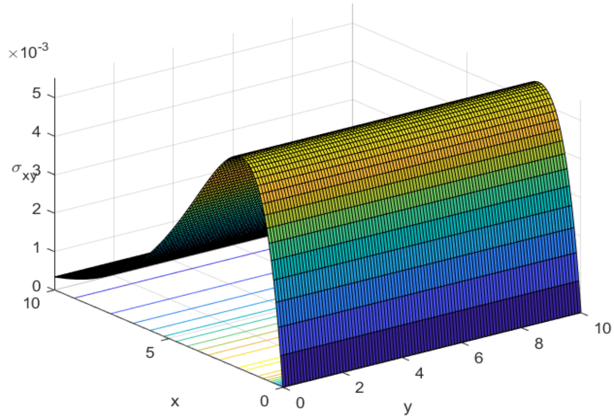

Fig. 12 Distribution of stress component σ_{xy} in the context of three-phase-lag model

parameter.
• The deformation of a body depends on the type of boundary conditions as well as the nature of the applied forces.
• This procedure remains valid when a nonlocal elastic solid is replaced with an elastic one.
• The vertical distance has a great effect on the physical studies fields.
• Normal mode analysis applies to a wide range of problems in thermodynamics and thermoelasticity.

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