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Decaying temperature and dynamic response of a thermoelastic nanobeam to a moving load

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Abstract. The decaying temperature and dynamic response of a thermoelastic nanobeam subjected to a moving load has been investigated in the context of generalized theory of nonlocal thermoelasticity. The transformed distributions of deflection, temperature, axial displacement and bending moment are obtained by using Laplace transformation. By applying a numerical inversion method, the results of these fields are then inverted and obtained in the physical domain. Also, for a particular two models, numerical results are discussed and presented graphically. Some specific and special results are derived from the current study.

Keywords: thermoelastic nanobeam; moving load; exponentially decaying temperature

1. Introduction

The dynamical reaction of solid and strong materials subjected to moving loads is of strong of engineering and building fields, for example, structural designing, sea industrial, quake designing and tribology. For instance ground movement and stresses are instigated in immersed soils by quick moving vehicular loads or surface impact waves because of explosives. Many researchers have considered dynamic reaction of beams subjected to a moving force.

Tang (1967) presented the steady-state behavior of a plate strip on a foundation of Kelvin material to a moving force. Cheng (1987) discussed the dynamic behavior of thin plates resting on elastic foundations and subjected to moving forces by means of variational calculus. Lin and Trethewey (1990) presented dynamic analysis of beams subjected to dynamic loads induced by the arbitrary movement of a spring-mass-damper system via Bernoulli–Euler theory. Olsson (1991) presented a moving force problem and studied the dynamic response of a simply-supported beam subjected to a uniform moving force. Jaiswal and Iyengar (1993) discussed dynamic response of an infinitely long beam resting on elastic foundation and subjected to a moving load. Lee and Ng (1994) presented dynamic behavior of a cracked beam due to the action of a moving load via Euler beam theory. Chang and Liu (1996) presented the vibration response of a nonlinear beam resting on elastic foundations to a moving force. Lu and Xuejun (1998) dealt with the dynamic response of infinite beam to a moving line force. Hasheminejad and Rafsanjani (2011) studied the

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elasto-dynamic response of a simply-supported beam subjected to a moving transverse load. Rajabi *et al.* (2013) investigated the dynamic response of a functionally graded (FG) simply-supported Euler–Bernoulli beam due to the action of a moving oscillator.

The dynamic response of micro- or nano-structures subjected to moving forces is an available area for many researchers. Şimşek (2010) used the modified couple stress theory to present vibration of an embedded Euler–Bernoulli microbeam due to a moving microparticle. Kiani (2011) presented the vibration response of nanoplates to a moving nanoparticle using Eringen’s nonlocal theory. Yang *et al.* (2013) investigated the dynamic behavior of Timoshenko's beams resting on a six-parameter foundation and subjected to a moving force. Pirmohammadi *et al.* (2014) investigated the vibration response of a single-walled carbon nanotube to a moving harmonic force via the nonlocal elasticity theory. Şimşek *et al.* (2015) dealt with the vibration behavior of a microplate under the action of a moving load. Hosseini and Rahmani (2017) analyzed the dynamic behavior of a FG Euler–Bernoulli nanobeam under a moving constant force.

The point of this article is to develop the governing equations of nonlocal Euler–Bernoulli beams subjected to transverse moving load (Zenkour *et al.* 2014, Abouelregal and Zenkour 2015, 2017, Carrera *et al.* 2015). The thermoelastic modification model of heat conduction is based on the thermal relaxation times is applied. The Laplace transform technique is utilized as a part of the deduction. The effects due to the nonlocal, moving load velocity and the exponential decay parameters will be investigated and represented graphically.

2. Mathematical modeling for nanobeams

The model under consideration is of a simply-supported Euler–Bernoulli nanobeam due to the action of a moving load with velocity υ along the axial direction of the nanobeam (Fig. 1). The initial condition is that the nanobeam is unstrained, unstressed and at initial temperature T_0 over its entirety. The ratio L/h is supposed to be large enough to consider the shear deformation negligible. The displacement field of any point of nanobeam is given by

$$u_1 = u = -z\frac{\partial w}{\partial x}, \quad u_2 = 0, \quad u_3 = w(x,t) \tag{1}$$

Without heat sources the generalized heat conduction equation is given by (Lord and Shulman 1967, Green and Lindsay 1972, Zenkour 2015)

$$K\nabla^2\theta = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\rho C_E\frac{\partial \theta}{\partial t}\right) + \gamma T_0\left(1+\alpha_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial t\partial x} \tag{2}$$

Substituting Eq. (1) into Eq. (2) yields the following equation

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial z^2} = \frac{\rho C_E}{K}\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t} - \frac{\gamma T_0}{K}z\left(1+\alpha_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial^3 w}{\partial t\partial x^2} \tag{3}$$

According to Eringen’s nonlocal elasticity theory (Eringen 1972, 1983), Eringen and Edelen 1972) the stress at a point is a function of strains at all points in the continuum. So, the nonlocal stress-strain equations are expressed as

$$\sigma_x - \xi\frac{\partial^2\sigma_x}{\partial x^2} = -E\left[z\frac{\partial^2 w}{\partial x^2} + \alpha_T\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta\right] \tag{4}$$

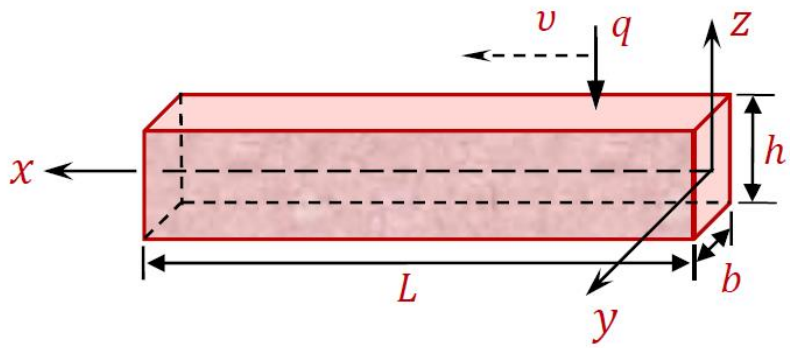


Fig. 1 Schematic diagram for a nanobeam subjected to a moving load

Based on Euler's nanobeam theory, the equation of motion for free vibration of nanobeams subjected to a distributed transverse load including nonlocal elasticity is written as

$$\frac{\partial^2 M}{\partial x^2} = \rho A \frac{\partial^2 w}{\partial x^2} - q(x,t) \tag{5}$$

in which M is given by

$$M(x,t) = \int_{-h/2}^{h/2} z\sigma_x \mathrm{d}z \tag{6}$$

Multiplying Eq. (6) by z and integrating the results over the area A yields

$$M - \xi \frac{\partial^2 M}{\partial x^2} = -EI\left[\frac{\partial^2 w}{\partial x^2} + \alpha_T\left(1 + \tau_1 \frac{\partial}{\partial t}\right) M_T\right] \tag{7}$$

The moment of beam due to the presence of thermal effects is given by

$$M_T(x,t) = \frac{12}{h^3}\int_{-h/2}^{h/2} z\theta(x,z,t)\mathrm{d}z \tag{8}$$

The substitution of Eq. (5) into Eq. (7) gives

$$M = \xi\left(\rho A \frac{\partial^2 w}{\partial x^2} - q\right) - EI\left[\frac{\partial^2 w}{\partial x^2} + \alpha_T\left(1 + \tau_1 \frac{\partial}{\partial t}\right) M_T\right] \tag{9}$$

Now, equation of motion of nonlocal Euler–Bernoulli nanobeam is expressed in terms of transverse displacement w as

$$\left[\frac{\partial^4}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2}{\partial t^2}\left(1 - \xi \frac{\partial^2}{\partial x^2}\right)\right] w - \frac{1}{EI}\left(1 - \xi \frac{\partial^2}{\partial x^2}\right) q + \alpha_T\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial^2 M_T}{\partial x^2} = 0 \tag{10}$$

Eqs. (2) and (11) describe the governing equations of the generalized nonlocal thermoelasticity theories with thermal relaxation times for nanobeams.

It can be seen that some special cases of interest can be obtained and the corresponding nonlocal thermoelastic models can be recovered as:

- The basic equations of the coupled theory of thermoelasticity (CTE theory) for a thermoelastic nanobeams are obtained when $\tau_0 = \tau_1 = 0$.
- For generalized theory of thermoelasticity proposed by Lord and Shulman (1967) (LS theory) with one relaxation time, when $\tau_0 > 0$, $\tau_1 = 0$ and $\alpha_0 = 1$.
- For generalized theory of thermoelasticity Green–Lindsay (1972) (GL theory) with two relaxation times $\tau_1 \geq \tau_0 > 0$ and $\alpha_0 = 0$.

It can be observed that when the parameter ξ is neglected, one can obtain the basic equations of the classical (local) thermoelasticity case.

3. Sinusoidal variation thermal solution

To solve the present problem, we assume that the solution of temperature increment varies in terms of a $\sin\left(\frac{\pi z}{h}\right)$ function. So, the temperature increment $\theta(x, z, t)$ is considered as

$$\theta(x, z, t) = \Theta(x, t) \sin\left(\frac{\pi z}{h}\right) \tag{11}$$

Substituting the above relation into Eqs. (9), (10) and (3) yields the following

$$M = \xi\left(\rho A \frac{\partial^2 w}{\partial t^2} - q\right) - EI\left[\frac{\partial^2 w}{\partial x^2} + \frac{24 T_0 \alpha_T}{\pi^2 h}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\Theta\right] \tag{12}$$

$$\left[\frac{\partial^4}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2}{\partial t^2}\left(1 - \xi \frac{\partial^2}{\partial x^2}\right)\right] w - \frac{1}{EI}\left(1 - \xi \frac{\partial^2}{\partial x^2}\right) q + \frac{24 \alpha_T}{\pi^2 h}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial^2 \Theta}{\partial x^2} = 0 \tag{13}$$

$$\frac{\partial^2 \Theta}{\partial x^2} - \frac{\pi^2}{h^2}\Theta = \frac{\rho C_E}{K}\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial \Theta}{\partial t} - \frac{\gamma T_0 \pi^2 h}{24K}\left(1 + \alpha_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial^3 w}{\partial t \partial x^2} \tag{14}$$

The following non-dimensional parameters are presented

$$\begin{gathered}\{u', w', x', z', L', b', h'\} = \eta c\{u, w, x, z, L, b, h\}, \quad \{t', t_0', \tau_1'\} = \eta c^2\{t, t_0, \tau_1\}, \\ \xi' = \eta^2 c^2 \xi, \quad \Theta' = \frac{\Theta}{T_0}, \quad q' = \frac{Aq}{EI}, \quad M' = \frac{M}{\eta c EI}, \quad c = \sqrt{\frac{E}{\rho}}, \quad \eta = \frac{\rho C_E}{K}\end{gathered} \tag{15}$$

Using the above non-dimensional variables, Eqs. (13)-(15) can be rewritten as (dropping the primes for convenience)

$$M = \xi\left(\frac{12}{h^2}\frac{\partial^2 w}{\partial t^2} - q\right) - \frac{\partial^2 w}{\partial x^2} - \frac{24 T_0 \alpha_T}{\pi^2 h}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\Theta \tag{16}$$

$$\left[\frac{\partial^4}{\partial x^4} + \frac{12}{h^2}\frac{\partial^2}{\partial t^2}\left(1 - \xi \frac{\partial^2}{\partial x^2}\right)\right] w - \left(1 - \xi \frac{\partial^2}{\partial x^2}\right) q + \frac{24 T_0 \alpha_T}{\pi^2 h}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial^2 \Theta}{\partial x^2} = 0 \tag{17}$$

$$\frac{\partial^2 \Theta}{\partial x^2} - \frac{\pi^2}{h^2}\Theta = \left(1 + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial \Theta}{\partial t} - \frac{\gamma \pi^2 h}{24 K \eta}\left(1 + \alpha_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial^3 w}{\partial t \partial x^2} \tag{18}$$

The external concentrated load $q(x, t)$ with constant strength Q_0 is traveling along the nanobeam axis at a constant speed υ. This load may be expressed in the form

$$q(x, t) = Q_0 \delta(x - \upsilon t) \tag{19}$$

4. Solution using Laplace transforms technique

It is difficult to find an exact and analytical solution for the two coupled equations (17) and (18). Therefore, the Laplace transform technique is employed to eliminate time. Applying the Laplace transform technique to Eqs. (16)-(18) and considering zero homogenous initial conditions, the governing equations can be obtained by

$$\bar{M} = -\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - A_3 s^2\right)\bar{w} - A_2\bar{\Theta} - \frac{\xi Q_0}{\upsilon}\mathrm{e}^{-\frac{s}{\upsilon}x} \tag{20}$$

$$\left(\frac{\mathrm{d}^4}{\mathrm{d}x^4} - A_3 s^2\frac{\mathrm{d}^2}{\mathrm{d}x^2} + A_1 s^2\right)\bar{w} = -A_2\frac{\mathrm{d}^2\bar{\Theta}}{\mathrm{d}x^2} + \bar{g}(s)\mathrm{e}^{-\frac{s}{\upsilon}x} \tag{21}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - B_1\right)\bar{\Theta} = -B_2\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2} \tag{22}$$

where

$$\begin{gathered} B_1 = A_4 + s(1+\tau_0 s),\ \ B_2 = s(1+\alpha_0\tau_0 s)A_5,\quad \bar{g}(s) = \frac{Q_0}{\upsilon}\left(1-\xi\frac{s^2}{\upsilon^2}\right) \\ A_1 = \frac{12}{h^2},\quad A_2 = \frac{24T_0\alpha_T}{\pi^2 h}(1+\tau_1 s),\quad A_3 = \frac{12\xi}{h^2},\quad A_4 = \frac{\pi^2}{h^2},\quad A_5 = \frac{\gamma\pi^2 h}{24K\eta} \end{gathered} \tag{23}$$

Eliminating the function $\bar{\Theta}$ from Eqs. (21) and (22), the following differential equation for $\bar{w}$ can be obtained as

$$\left(\frac{\mathrm{d}^6}{\mathrm{d}x^6} - A\frac{\mathrm{d}^4}{\mathrm{d}x^4} + B\frac{\mathrm{d}^2}{\mathrm{d}x^2} - C\right)\bar{w} = \gamma_1\mathrm{e}^{-\frac{s}{\upsilon}x} \tag{24}$$

where

$$A = B_1 + A_2B_2 + A_3s^2,\quad B = s^2(A_1 + A_3B_1),\quad C = A_1B_1s^2,\quad \gamma_1 = \bar{g}(s)\left(\frac{s^2}{\upsilon^2} - B_1\right) \tag{25}$$

Considering the characteristic equation

$$m^6 - Am^4 + Bm^2 - C = 0 \tag{26}$$

with the roots m_1^2, m_2^2 and m_3^2, the analytical solutions for transverse deflection $\bar{w}$ can be expressed as

$$\bar{w} = \sum_{j=1}^{3}\left(C_j\mathrm{e}^{-m_jx} + C_{j+3}\mathrm{e}^{m_jx}\right) + C_7\mathrm{e}^{-\frac{s}{\upsilon}x} \tag{27}$$

where

$$C_7 = \frac{\gamma_1}{\left(\frac{s}{\upsilon}\right)^6 - A\left(\frac{s}{\upsilon}\right)^4 + B\left(\frac{s}{\upsilon}\right)^2 - C} \tag{28}$$

In a similar manner, one can obtain

$$\bar{\Theta} = \sum_{j=1}^{3}\left(F_j\mathrm{e}^{-m_jx} + F_{j+3}\mathrm{e}^{m_jx}\right) + C_8\mathrm{e}^{-\frac{s}{\upsilon}x} \tag{29}$$

From Eq. (22), the relation between the parameters F_j and C_j can be derived as

$$F_j = \beta_j C_j, \quad F_j = \beta_j C_{j+3}, \quad \beta_j = -\frac{B_2 m_j^2}{m_j^2 - B_1}, \quad C_8 = -\frac{B_2 s^2}{s^2 - B_1 v^2} C_7 \tag{30}$$

So

$$\bar{\Theta} = \sum_{j=1}^{3} \beta_j \left(C_j \mathrm{e}^{-m_j x} + C_{j+3} \mathrm{e}^{m_j x} \right) + C_8 \mathrm{e}^{-\frac{s}{v} x} \tag{31}$$

Substituting the expressions of $\bar{w}$ and $\bar{\Theta}$ from Eqs. (27) and (32) into Eq. (23), we get the solution for the bending moment $\bar{M}$ as

$$\bar{M} = -\sum_{j=1}^{3} \left(m_j^2 - A_3 s^2 + A_2 \beta_j \right) \left(C_j \mathrm{e}^{-m_j x} + C_{j+3} \mathrm{e}^{m_j x} \right) - C_9 \mathrm{e}^{-\frac{s}{v} x} \tag{32}$$

where

$$C_9 = \left(\frac{s^2}{v^2} - A_3 s^2 \right) C_7 + A_2 C_8 + \frac{\xi Q_0}{v} \tag{33}$$

Finally, the axial displacement and its normal strain are given, respectively, by

$$\bar{u} = z \sum_{j=1}^{3} m_j \left(C_j \mathrm{e}^{-m_j x} - C_{j+3} \mathrm{e}^{m_j x} \right) + \frac{s}{v} C_7 \mathrm{e}^{-\frac{s}{v} x} \tag{34}$$

$$\bar{e} = -z \sum_{j=1}^{3} m_j^2 \left(C_j \mathrm{e}^{-m_j x} + C_{j+3} \mathrm{e}^{m_j x} \right) - \frac{s^2}{v^2} C_7 \mathrm{e}^{-\frac{s}{v} x} \tag{35}$$

5. Application

In this article, the dimensionless temperature $\Theta(x,t)$ may be expressed as an exponential decay function at the first end of the nanobeam $x = 0$. That is

$$\Theta(x,t) = \Theta_0 \mathrm{e}^{-kt} \quad \text{at} \quad x = 0 \tag{36}$$

The time constant of the decay k must be always positive. The thermal shock problem is available if $k = 0$. The above condition in the Laplace transform domain will be

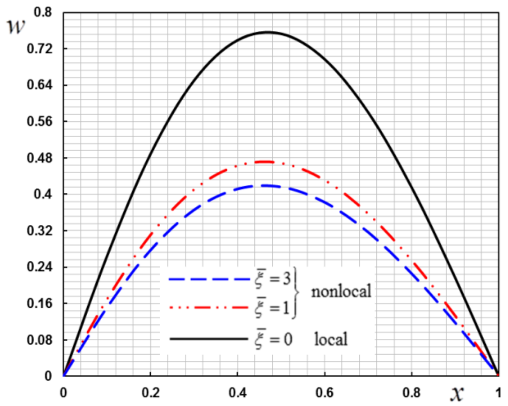
$$\bar{\Theta}(0,s) = \frac{\Theta_0}{k+s} \left(\frac{1 - \mathrm{e}^{-t_0 s}}{t_0 s^2} \right) = \bar{G}(s) \tag{37}$$

The considered nanobeam is assumed to be simply-supported at the axial ends $x = 0, L$. So, one gets in the Laplace transform domain

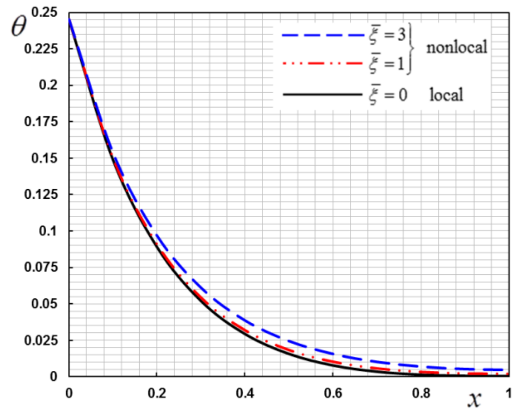
$$\bar{w}(0,s) = \bar{w}(L,s) = 0, \quad \left. \frac{\mathrm{d}^2 \bar{w}(x,s)}{\mathrm{d}x^2} \right|_{x=0,L} = 0 \tag{38}$$

Also, the temperature at the end boundary in the Laplace transform domain satisfies the relation

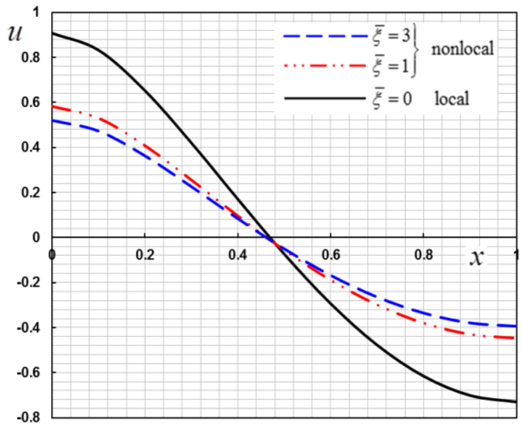
$$\left. \frac{\mathrm{d}\bar{\Theta}}{\mathrm{d}x} \right|_{x=L} = 0 \tag{39}$$



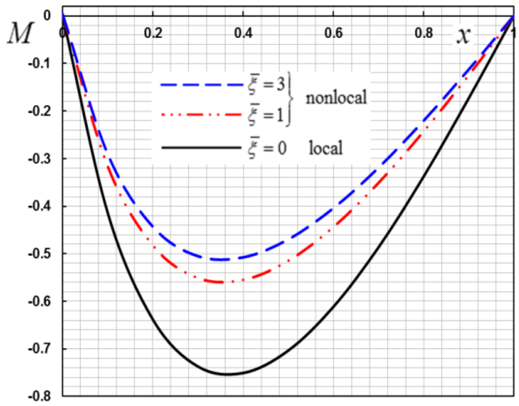
(a) Transverse deflection w versus x



(b) Temperature θ versus x



(c) Axial displacement u versus x



(d) Bending moment M versus x

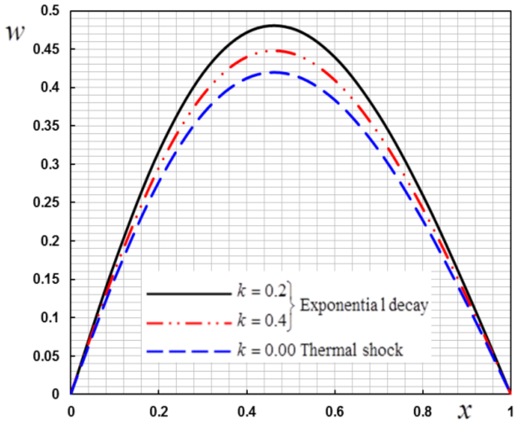
Fig. 2 The transverse deflection, temperature, displacement and moment distributions for different values of the nonlocal thermoelastic parameters $\bar{\xi}$

From Eqs. (27) and (31), we find that the conditions in Eqs. (37)-(39) have been satisfied then we get linear system of six equations in terms of the constants C_i $(i = 1,2,\ldots,6)$. Solving this system gives the final form of the constants C_i. It is difficult to get the inversion of Laplace transform of the complicated solutions for the studied fields in Laplace transform space. Therefore, the results will be analyzed numerically using a method based on Fourier series expansion technique in the following section.

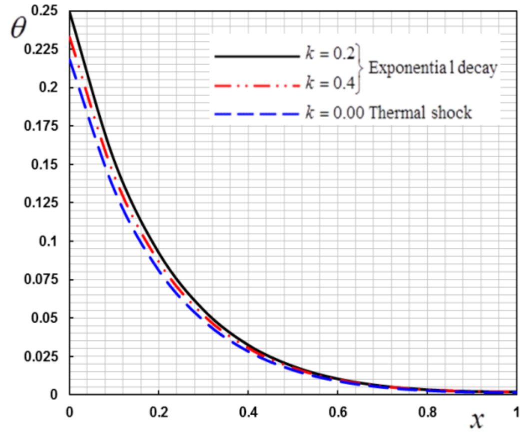
The Riemann-sum approximation method is applied to obtain numerical results for the lateral vibration, thermal temperature, axial displacement, and bending moment in time domain. In this method, any function $\bar{f}(x,s)$ in Laplace domain can be inverted to time domain $f(x,t)$ as

$$f(x,t) = \frac{e^{\zeta t}}{t}\left[\frac{1}{2}\mathrm{Re}\{\bar{f}(x,\zeta)\} + \mathrm{Re}\left\{\sum_{n=0}^{N}(-1)^n \bar{f}\left(x,\zeta+\frac{\mathrm{i}n\pi}{t}\right)\right\}\right], \quad \mathrm{i}=\sqrt{-1} \tag{40}$$

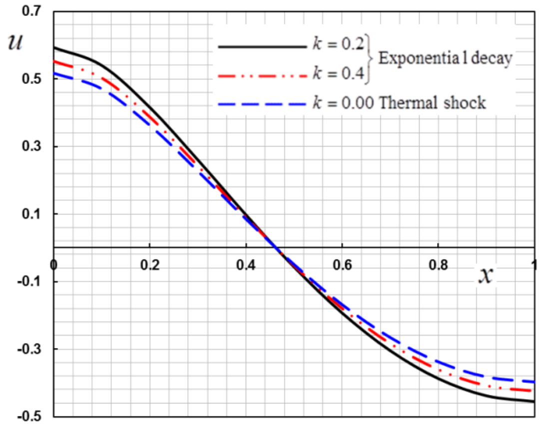
where ζ is an arbitrary real number given experimentally by $\zeta t = 4.7$ (Tzou 1995, 1996).



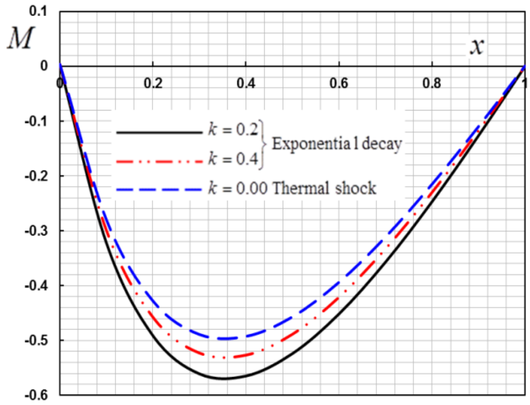
(a) Transverse deflection w versus x



(b) Temperature θ versus x



(c) Axial displacement u versus x



(d) Bending moment M versus x

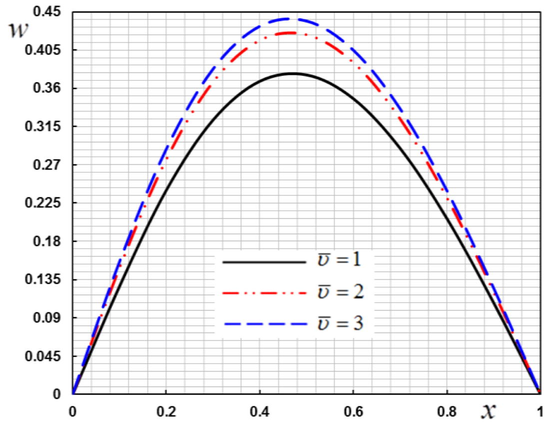
Fig. 3 The transverse deflection, temperature, displacement and moment distributions for different values of the decaying parameter k

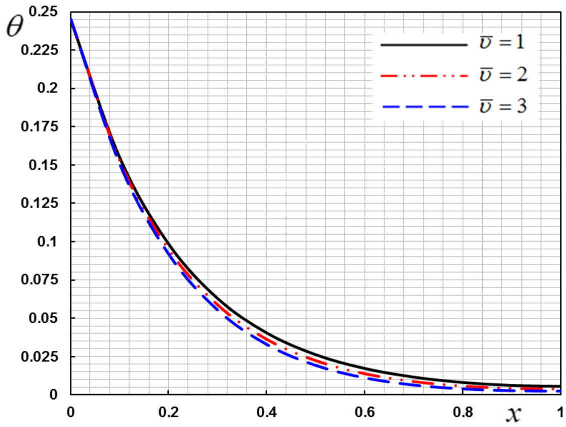
6. Numerical results

In this section, silicon (Si) nanobeam at $T_0 = 293\,K$ is used as an example. The effects of the moving load velocity $\bar{v}$ ($\bar{v} = 10^3\,v$), nonlocal parameter $\bar{\xi}$ ($\bar{\xi} = 10^3\,\xi$), the external moving load strength Q_0 and decaying parameter k on the temperature, displacement, lateral vibration, and moment are analyzed numerically. The basic physical material properties are

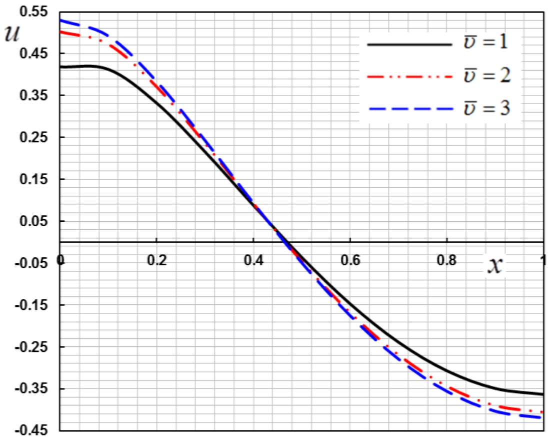
$$E = 169\ \text{GPa}, \quad \nu = 0.22, \quad \rho = 2330\ \text{kg/m}^3, \quad K = 156\ \text{W/(mK)}$$
$$C_E = 713\ \text{J/(kg K)}, \quad \alpha_T = 2.59 \times 10^{-6}\ /\text{(K)} \tag{41}$$

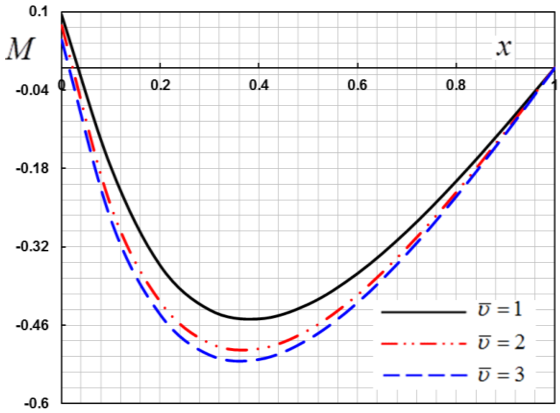
The results are obtained for fixed parameters like $t = 0.1$, $L/h = 10$, $b/h = 0.5$, $L = 1$ and $z = h/3$.





(a) Transverse deflection w versus x (b) Temperature θ versus x





(c) Axial displacement u versus x (d) Bending moment M versus x

Fig. 4 The transverse deflection, temperature, displacement and bending moment distributions for different values of the moving load velocity $\bar{\upsilon}$

6.1 Nonlocal parameter effect

The influence of dimensionless nonlocal parameter $\bar{\xi}$ on dimensionless deflection, temperature, axial displacement, and moment is investigated first in Figs. 2(a)-2(d). In this case, three values of nonlocal parameter, $\bar{\xi}=1$ and $\bar{\xi}=3$ (nonlocal case), and $\bar{\xi}=0$ (local case), are considered. The moving load velocity $\bar{\upsilon}$ and the decaying parameter k remain constants as $\bar{\upsilon}=3$ and $k=0.1$, respectively. The results show that the temperature θ and axial displacement u are decreasing along the axial axis. It is also found that lateral vibration w satisfies the boundary condition at $x=0,L$. From Figs. 2(a), 2(c), 2(d), it is also observed that the amplitude values of deflection w, axial displacement u and bending moment M decrease when nonlocal parameter $\bar{\xi}$ increases. Additionally, Fig. 2(b) shows that the increasing in the value of $\bar{\xi}$ causes

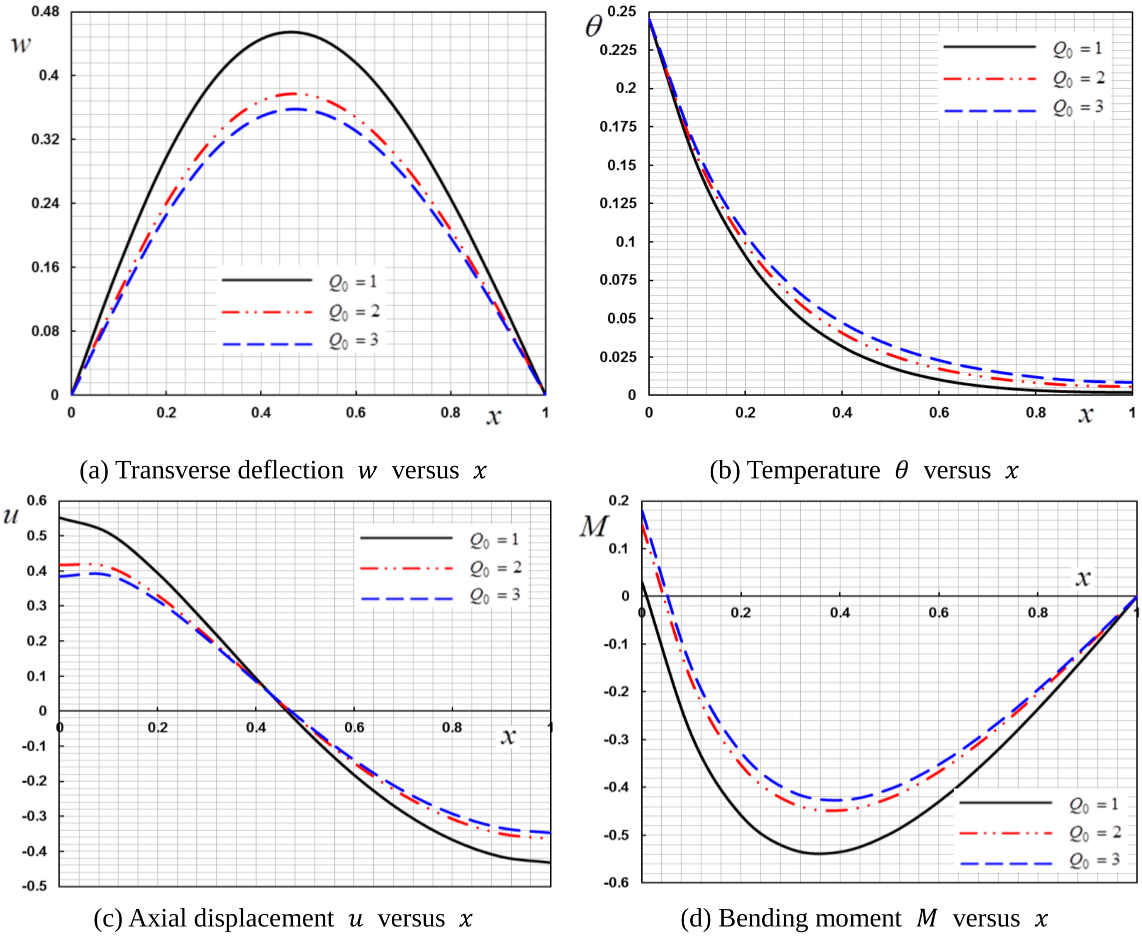


Fig .5 The transverse deflection, temperature, displacement and bending moment distributions for different values of the external moving load strength Q_0

decreasing in the values of temperature θ. It is thus concluded that nonlocal parameter $\bar{\xi}$ has significant effect on all the field quantities. The observations in Figs. 2(a)-2(d) could be explained by the fact that the numerical results in the local generalized thermoelasticity model are different compared to the results in the nonlocal generalized thermoelasticity theory.

6.2 Decaying parameter effect

Figs. 3(a)-3(d) plot the distribution of the lateral vibration, temperature, displacement and bending moment along the axial direction under an applied moving load with different decaying parameter k values when other considered parameter $(\bar{\xi}, \bar{\upsilon}, Q_0)$ remain constants. In the case of $k = 0$, a thermal shock problem is considered. As indicated by Figs. 3(a) and 3(b), the increasing in the value of the decaying parameter causes decreasing in the values of lateral vibration w and temperature θ which is very obvious in the peek points of the curves. Also, the results show that the values of the displacement u start decreasing with the decaying parameter in the range $0 \leq x \leq 0.45$, thereafter increasing to maximum amplitudes in the range $0.45 \leq x \leq 1$ (see Fig. 3(d)). It is also indicated by Fig. 3(d) that the increasing in the value of the parameter k causes increasing in the values of the bending moment M. Comparing the results in Figs. 3(a)-3(d) it can be concluded that the exponential decaying parameter k has a great effects on the distribution of field quantities.

6.3 Moving load velocity effect

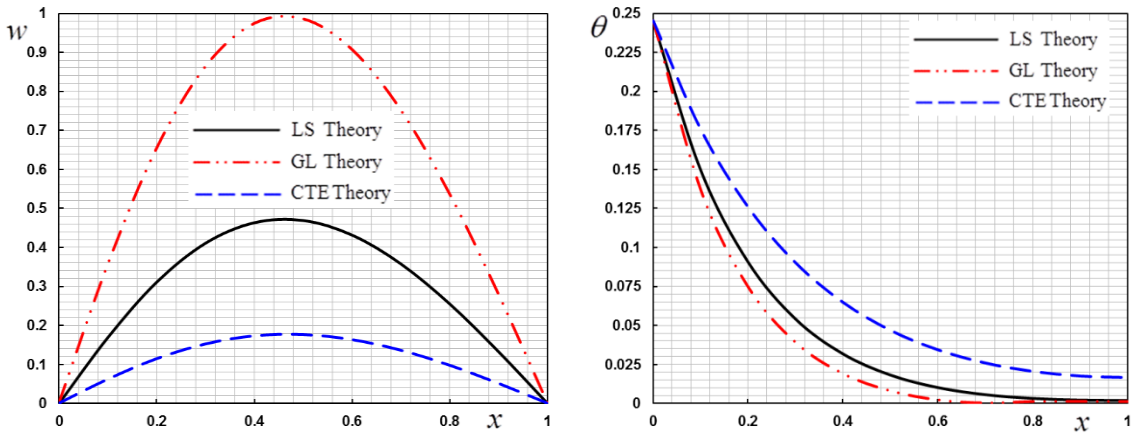
Effects of moving load velocity $\bar{v}$ on the dimensionless field quantities are shown in Figs. 4(a-d), where the parameters $\bar{\xi}$, k and Q_0 are assumed to be constants. It is observed that the magnitudes of the considered fields increase with increasing moving load velocity. On the other hand, it is clear from Fig. 4(b) that the moving load velocity has weak effect on the temperature distributions. It can also be concluded that the values of the lateral vibration, displacement and bending moment fields are sensitive to the values of moving load velocity.

6.4 Moving load strength effect

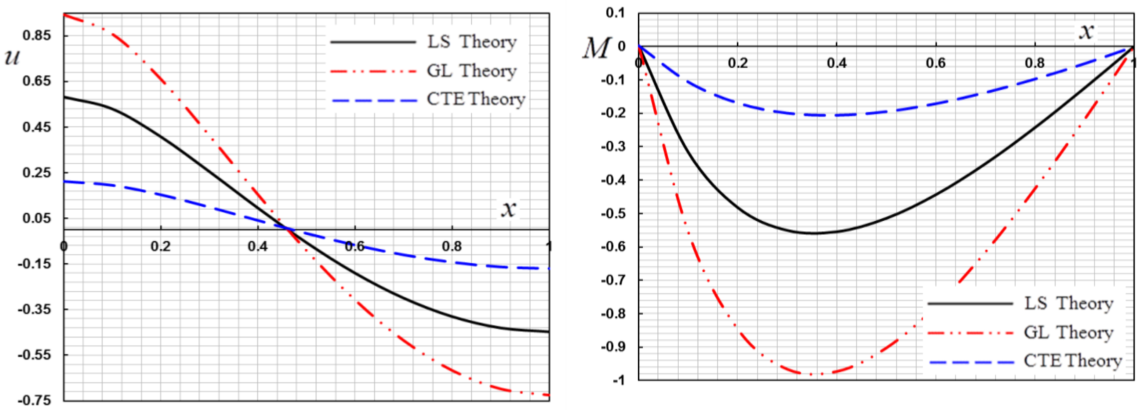
In Figs. 5(a)-5(d) the lateral vibration, the temperature, the displacement, and the bending moment distributions of nanobeams are presented for different values of the magnitude of the external moving load strength Q_0. The effects of the strength of external moving load parameter on transient behaviors of the nanobeam are gotten. We found that, the increasing in the value of the strength Q_0 causes increasing in the values of the lateral vibration, axial displacement and bending moment fields which are very obvious in the peek points of the curves. From Fig. 5(b) we have noticed that, the strength parameter Q_0 has an insignificant effect on the temperature field.

6.5 Different theories of nonlocal thermoelasticity

The graphs in Figs. 6(a)-6(d) represent four curves predicted by three different theories of nonlocal thermoelasticity obtained as a special case of the present work. These computations were carried out in the coupled theory (CTE) by setting ($\tau_1 = \tau_0 = 0$), in Lord-Shulman theory (LS) putting ($\tau_1 = 0,\ \alpha_0 = 1$ and $\tau_0 > 0$) and in the generalized theory of thermoelasticity proposed by Green and Lindsay (GL) when $\tau_1 \geq \tau_0 > 0$ and $\alpha_0 = 0$. The distinction of the reaction in the theories of the coupled thermoelasticity, Green and Lindsay theory and Lord–Shulman theory is dissected in the similar graph. It is noted that the estimations of the studied field in the thermoelastic nanobeam gotten from classical theory were distinctive to that obtained by using LS theory as well as GL theory but in the similar behavior. Despite the fact that the thermal wave proliferates with a limited speed in the coupled theory of thermoelasticity, there are great differences between this theory and the other generalized theories of thermoelasticity. The comparable results of three theories for the nonlocal thermoelasticity show that the thermal relaxation effect of all the studied fields, i.e., τ_0 and τ_1, has a great effect on the propagation of the deflection, the temperature, the displacement, and the bending moment of thermoelastic nanobeam.



(a) Transverse deflection w versus x (b) Temperature θ versus x



(c) Axial displacement u versus x (d) Bending moment M versus x

Fig. 6 The transverse deflection, temperature, displacement and bending moment distributions for different theories of thermoelasticity

7. Conclusions

In this work, the behavior of the deflection, temperature, axial displacement and bending moment of thermoelastic nanobeam due to the action of a moving force are investigated. Numerical techniques based on Laplace transformation has been used. The effects of nonlocal and external load strength parameters on all field quantities have been discussed and presented graphically. According to the results shown in all figures, it is found that the moving load velocity, external load strength and nonlocal parameters have significant effects on all fields. The study of dynamic response of nanobeam structures on moving forces has drawn a lot of attention due to its wide applications in the transportation industry.

References

Abouelregal, A.E. and Zenkour, A.M. (2015), "Thermoelastic problem of an axially moving microbeam subjected to an external transverse excitation", *J. Theor. Appl. Mech.*, **53**(1), 167-178.

Abouelregal, A.E. and Zenkour, A.M. (2017), "Dynamic response of a nanobeam induced by ramp-type heating and subjected to a moving load", *Microsyst. Technol.*, **23**(12), 5911-5920.

Carrera, E., Abouelregal, A.E., Abbas, I.A. and Zenkour, A.M. (2015), "Vibrational analysis for an axially moving microbeam with two temperatures", *J. Therm. Stresses*, **38**(6), 569-590.

Chang, T.P. and Liu, Y.N. (1996), "Dynamic finite element analysis of a nonlinear beam subjected to a moving load", *Int. J. Solids Struct.*, **33**, 1673-1688.

Cheng, X.S. (1987), "Dynamic response of plates on elastic foundations due to the moving loads", *Appl. Math. Mech*., **8**(4), 355-365.

Eringen, A.C. (1972), "Nonlocal polar elastic continua", *Int. J. Eng. Sci.*, **10**, 1-16.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**, 4703-4710.

Eringen, A.C. and Edelen, D.G.B. (1972), "On nonlocal elasticity", *Int. J. Eng. Sci.*, **10**, 233-248.

Green, A.E. and Lindsay, K.A. (1972), "Thermoelasticity", *J. Elast.*, **2**, 1-7.

Hasheminejad, S.M. and Rafsanjani, A. (2011), "Two-dimensional elasticity solution for transient response of simply supported beams under moving loads", *Acta Mech.*, **217**, 205-218.

Hosseini, S.A.H. and Rahmani, O. (2017), "Exact solution for axial and transverse dynamic response of functionally graded nanobeam under moving constant load based on nonlocal elasticity theory", *Mecc.*, **52**, 1441-1457.

Jaiswal, O.R. and Iyengar, R.N. (1993), "Dynamic response of a beam on elastic foundation of finite depth under a moving force", *Acta Mech.*, **96**, 67-83.

Kiani, K. (2011), "Small-scale effect on the vibration of thin nanoplates subjected to a moving nanoparticle via nonlocal continuum theory", *J. Sound Vib.*, **330**(20), 4896-4914.

Lee, H.P. and Ng, T.Y. (1994), "Dynamic response of a cracked beam subject to a moving load", *Acta Mech.*, **106**, 221-230.

Lin, Y.H. and Trethewey, M.W. (1990), "Finite element analysis of elastic beams subjected to moving dynamic loads", *J. Sound Vib.*, **136**(2), 323-342.

Lord, H.W. and Shulman, Y. (1967), "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solid*, **15**, 299-309.

Lu, S. and Xuejun, D. (1998), "Dynamic analysis to infinite beam under a moving line load with uniform velocity", *Appl. Math. Mech*., **19**(4), 367-373.

Olsson, M. (1991), "On the fundamental moving load problem", *J. Sound Vib.*, **145**(2), 299-307.

Pirmohammadi, A.A., Pourseifi, M., Rahmani, O. and Hoseini, S.A.H. (2014), "Modeling and active vibration suppression of a single-walled carbon nanotube subjected to a moving harmonic load based on a nonlocal elasticity theory", *Appl. Phys. A*, **117**(3), 1547-1555.

Rajabi, K., Kargarnovin, M.H. and Gharini, M. (2013), "Dynamic analysis of a functionally graded simply supported Euler–Bernoulli beam subjected to a moving oscillator", *Acta Mech.*, **224**, 425-446.

Şimşek, M. (2010), "Dynamic analysis of an embedded microbeam carrying a moving microparticle based on the modified couple stress theory", *Int. J. Eng. Sci.*, **48**, 1721-1732.

Şimşek, M., Aydin, M., Yurtcu, H.H. and Reddy, J.N. (2015), "Size-dependent vibration of a microplate under the action of a moving load based on the modified couple stress theory", *Acta Mech.*, **226**, 3807-3822.

Tang, S.C. (1967), "Dynamic response of elastic plates on viscoelastic foundations to moving loads", *Ing. Archiv*, **36**(3), 155-163.

Tzou, D.Y. (1995), "Experimental support for the Lagging behavior in heat propagation", *J. Thermophys. Heat Trans*., **9**, 686-693.

Tzou, D.Y. (1996), *Macro- to Microscale Heat Transfer: the Lagging Behavior*, Taylor & Francis, Washington, DC.

Yang, Y., Ding, H. and Chen, L-Q. (2013), “Dynamic response to a moving load of a Timoshenko beam resting on a nonlinear viscoelastic foundation”, *Acta Mech. Sin.*, **29**(5), 718-727.

Zenkour, A.M. (2015), “Three-dimensional thermal shock plate problem within the framework of different thermoelasticity theories”, *Compos. Struct.*, **132**, 1029-1042.

Zenkour, A.M., Abouelregal, A.E. and Abbas, I.A. (2014), “Generalized thermoelastic vibration of an axially moving clamped microbeam subjected to ramp-type thermal loading”, *J. Therm. Stresses*, **37**(11), 1302-1323.

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List of symbols

a	internal characteristic length
$A = bh$	area of nanobeam cross-section
$b\ (-b/2 \leq y \leq b/2)$	width of nanobeam
C_E	specific heat at constant strain
E	Young's modulus
$e = \partial u/\partial x$	normal strain
EI	flexural rigidity
e_0	material constant
$h\ (-h/2 \leq z \leq h/2)$	thickness of nanobeam
$I = bh^3/12$	inertia moment of nanobeam cross-section
K	thermal conductivity
k	time constant of the decay
$L\ (0 \leq x \leq L)$	length of nanobeam
M	flexural moment
M_T	moment of nanobeam due to the presence of thermal effect
$q(x,t)$	distributed transverse load
Q_0	constant strength of external moving load
s	Laplace's variable
T_0	environment temperature
u	axial displacement
w	lateral deflection of nanobeam in the z direction at some position x
$\alpha_T = \alpha_t/(1-2\nu)$	stress-temperature modulus
α_t	thermal expansion coefficient
$\xi = (e_0 a)^2$	nonlocal parameter
$\delta(\cdot)$	Dirac's function
ν	Poisson's ratio
ρ	material density
υ	constant speed of external moving load

$\theta = T - T_0$	excess temperature distribution
Θ_0	thermal constant
σ_x	nonlocal normal stress
τ_0	first thermal relaxation time
τ_1	second thermal relaxation time