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# Triangular units based method for simultaneous optimizations of planar trusses

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**Abstract.** Simultaneous optimization of trusses which concurrently takes into account design variables related to the size, shape and topology of the structure is recognized as highly complex optimization problems. In this class of optimization problems, it is possible to encounter several unstable mechanisms throughout the solution process. However, to obtain a feasible solution, these unstable mechanisms somehow should be rejected from the set of candidate solutions. This study proposes triangular unit based method (TUBM) instead of ground structure method, which is conventionally used in the topology optimization, to decrease the complexity of search space of simultaneous optimization of the planar truss structures. TUBM considers stability of the triangular units for 2 dimensional truss systems. In addition, integrated particle swarm optimizer (iPSO) strengthened with robust technique so called improved fly-back mechanism is employed as the optimizer tool to obtain the solution for these class of problems. The results obtained in this study show the applicability and efficiency of the TUBM combined with iPSO for the simultaneous optimization of planar truss structures.

**Keywords:** triangular unit based method; particle swarm optimization; simultaneous optimization; truss structures

## 1. Introduction

Simultaneous optimization requires handling all design variables corresponding to the cross-sectional areas of members, the nodal coordinates, and the members' connectivity all together. Such an optimization problem tends to become very complex since corresponding design variables can either be discrete or continuous, or even combination of them (Tang *et al.* 2005, Achtziger 2007, Miguel *et al.* 2013, Silih *et al.* 2010, Deb and Gulati 2001, Torii *et al.* 2011, Mortazavi and Toğan 2016). To perform the topology optimization, generally a conventional ground structure method (GSM) is used. GSM starts with a structure including high number of members (ground structure), and during the optimization process successively number of members is reduced till an optimum state is reached. Generally, this reduction is performed considering the problem constraints (Deb and Gulati 2001). On the other hand, in the topology optimization, members deleting may produce several unstable mechanisms. Therefore, to obtain feasible results extra

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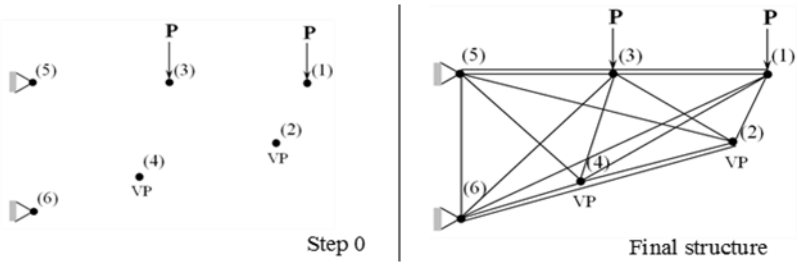



Fig. 1 Creating ground structure using GSM

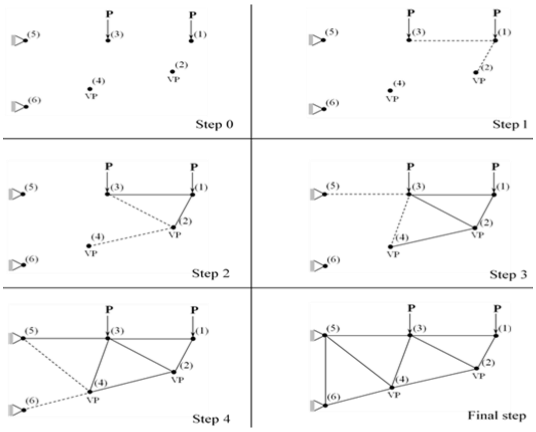



Fig. 2 Creating ground structure using TUBM

geometric constraints should be satisfied.

It seems that it might be unavoidable to use the mixed design variables (e.g. continuous and discrete) in the simultaneous optimization problem. Therefore, an optimization algorithm to be selected to carry out such an optimization problem should be capable of working with both discrete and continuous sets of variables. Due to this, employing a metaheuristic algorithm as an optimization engine rather than the mathematical ones, which are suitable for structural optimization problem with continuous design variables, will be meaningful. In last decades, numerous metaheuristic algorithms simulating the natural events have been consecutively proposed and widely used in engineering problems (Camp and Bichon 2004, Lee *et al.* 2005, Sadollah *et al.* 2012, Sönmez 2011, Toğan 2012, 2013, Hasançebi *et al.* 2013, Fan and Yan 2014, Zheng 2015, Abedinpourshotorban *et al.* 2016, Nabil 2016).

In this study, to reduce the complexity of the search space of the simultaneous optimization of planar trusses, a new method so called triangular units based method (TUBM) is presented. Since the optimization problems to be investigated in the current study include the both discrete and continuous design variables, a metaheuristic algorithm known as integrated Particle Swarm Optimization (iPSO) is applied. iPSO uses the improved fly-back mechanism to handle the problem constraints and applies concept of weighted particle to improve the guidance mechanism of standard PSO (Mortazavi and Toğan 2016). To exhibit the advantages of TUBM and iPSO, some examples associated with the simultaneous optimization of planar truss structures solved previously with distinct metaheuristic algorithms are examined. As a result, TUBM and iPSO produce better solutions for the optimization cases.

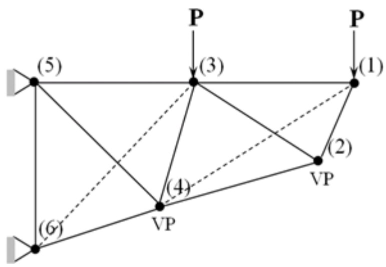



Fig. 3 A sample structure generated by TUBM with two variable points (VPs) and two free elements

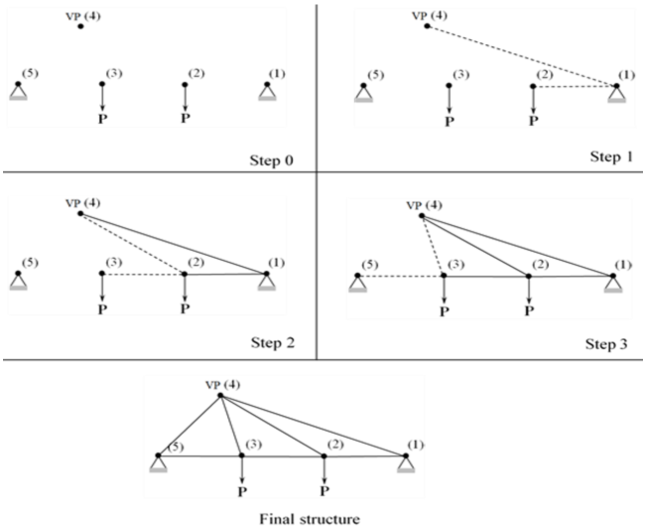



Fig. 4 Prevention of overlapping in TUBM

## 2. Triangular units based method (TUBM) for topology optimization of the planar trusses

Ground structure method (GSM) is mostly used for the topological optimization of truss structures (Deb and Gulati 2001). In this method, an initial structure known as ground structure is generated by large number of elements. Then, during optimization process, by eliminating unnecessary elements, the optimum topology is obtained. Fig. 1 demonstrates the way constructing the ground structure in the GSM. Overlapped members are shown with a little gap in this figure (e.g., element from node 1 to node 5) and VP indicates the variable point. The major shortcoming of this method is to generate numerous elements between available nodes, and this highly increases the number of problem variables. On the other hand, many overlaps which are practically meaningless might occur.

In this study, to decrease the complexity of problem and to prevent the overlaps, triangular unit based method (TUBM) is introduced. This method, which is applicable for planar trusses, uses the stability specification of triangular units to crate the initial planar truss structure. TUBM mainly has two basic phases to establish its own ground structure: Firstly, it generates an initial layout by making triangular units, and finally, it adds a number of extra elements to the initial structure.

Such that, for a planar system TUBM initially sorts the nodes based on their x coordinates in ascending or descending order. In this level, if some points have same x coordinates, their y component takes as the criterion of sorting. TUBM, then establish the basic structure by starting from first point (the point with the highest x coordinates) and connecting each point to its two prior adjacent points, till reaching to the last point (the point with lowest x coordinate). This automatic numbering and construction process are shown in Fig. 2. In this figure the step 0 indicates the initial state of restraint and loading points, while two variable points (VP) are also added randomly to the search space.

Subsequently, TUBM adds the additional elements which randomly placed in initial configuration. The number of this element is determined by user at the start of procedure. For example, if two variable elements are added to the initial structure (final step in Fig. 2) the final topology can be as the configuration shown in Fig. 3. In this figure additional elements are shown with dash line.

By comparing the Figs. 1 and 3, it can be seen that the number of elements in TUBM is much less than GSM. Eventually, to prevent any overlap in initial structure, TUBM checks whether three or more nodes lie on the same direction, if so, it searches another appropriate node(s) to create triangle panels (see Fig. 4). For instance, in Fig. 4, the point number 1 should be firstly connected to node 2, and then to node 3. But, because of this configuration causes to overlap in the structure, the TUBM tries to find another appropriate point (e.g., point number 4). If TUBM does not find such a point it is not avoidable from the overlapping.

## 3. Formulation of the simultaneous optimization problem

In the simultaneous optimization of truss structures, generally, size, shape and topology parameters of the trusses are taken as decision variables in the optimization process. For this class of optimization problems corresponding constraints and objective function are mathematically formulated as following

$$
\begin{aligned}
&\text{find} \quad \mathbf{X} = \{x_1, \ldots, x_{ndv}\} \\
&\text{min.} \quad W(\mathbf{X}) = \sum_{e=1}^{ne} L_e \rho_e A_e \\
\text{subject to} \quad & g_1(\mathbf{X}) = d_k(\mathbf{X}) \le d_{\max,k} \\
& g_2(\mathbf{X}) = \sigma_e(\mathbf{X}) \le \sigma_{a,e} \quad \text{(for tension and compression)} \\
& g_3(\mathbf{X}) = x_{\min,e} \le x_e \le x_{\max,e} \\
& g_4(\mathbf{X}) = \text{kinematic stability} \\
& g_5(\mathbf{X}) = \mu_{\min,p} \le \mu_p \le \mu_{\max,p}
\end{aligned}
\tag{1}
$$

in which, $\mathbf{X}$ is a vector of the design variables; *ndv* is total number of independent design variables; $W(\mathbf{X})$ is the objective function, which gives weight of the structure; $L_e$, $\rho_e$, and $A_e$ are the length, material density and cross-sectional area of the $e^{\text{th}}$ element; *ne* is the total number of elements in the structure; $d_k$ and $d_{max,k}$ are the available and allowable displacements for node *k*, respectively; $\sigma_e$ and $\sigma_{a,e}$ are the available and allowable tension or compression stresses for the $e^{\text{th}}$ member. Also, $x_{min,e}$ and $x_{max,e}$ are the lower and upper bounds for the cross sectional area of $e^{\text{th}}$ member. Eventually, $\mu_{min,p}$ and $\mu_{max,p}$ demonstrate the maximum and minimum limits for $p^{\text{th}}$ nodal coordinate.

In this investigation, in order to perform topology optimization, cross-sectional areas are considered in both negative and positive. Such that, the member with the negative cross section should be deleted while positive values are applied for valid elements. Moreover, the concept of critical area ($\varepsilon$) is applied. If cross section of any member gets less that ε area, it will be removed from the structure.

In this paper a metaheuristic algorithm based on the swarm intelligence is employed as an optimization method. The main concepts of the algorithm are briefly explained in next section.

## 4. Integrated particle swarm optimizer (iPSO)

The standard particle swarm optimizer (PSO) is the metaheuristic algorithm which mimics the behavior of animals (e.g., the colony of fish and birds) to find food sources or to avoid from enemies in the nature (Kennedy and Eberhart, 2001). It is claimed in various applications of PSO that it has some drawbacks e.g., staggering of the convergence in later stages of the process. So, the different forms of PSO are developed to improve its performance (Van den Bergh and Engelbrecht 2003, He *et al.* 2004, Li *et al.* 2009, Li *et al*. 2014). The iPSO used in the current study is different form of the conventional PSO, which is strengthened with improved fly-back method and weighted particle concept. Both of them are briefly explained in following (see more Mortazavi and Toğan 2016).

### *4.1 Improved fly-back mechanism*

To keep all particles in the feasible region during the whole optimization progress a method called fly-back method is introduced by He *et al.* (2004). The particle violating any type constraints returns to its prior best position in accordance with this method. To improve its performance, the fly-back method is modified in order to benefit from the information stored in the weighted particle and to consider type of the violated constraints.

The improved fly-back mechanism proceeds in three main steps:

Step 1 Check the updated particle for the variables-constraints

Step 2 For violation, find the components of the particle causing the violation. Then replace them with those that are available in the weighted particle.

Step 3 Determine whether the new particle is feasible (does not violate any of the problem constraints) and produces a better objective function value than old particle or not. If so, change it with the old one.

### *4.2 Weighted particle*

In the standard PSO process, when current particle lies so close to its own prior best position stored in $P_{best}$ matrix or global best particle ($G_{best}$) or even both of them, the guidance effect of these two particles is highly reduced or even vanished due to this proximity. To escape from this condition a new particle called weighted particle as proposed by Li *et al.* (2014) is incorporated into velocity updating process of the PSO. The weighted particle is formulated as follow

$$x^W = \sum_{i=1}^{M} \bar{c}_i^W x_i^P \tag{2}$$

where,

$$\bar{c}_i^W = \left( \hat{c}_i^W \Big/ \sum_{i=1}^{M} \hat{c}_i^W \right) \tag{3}$$

$$\hat{c}_i^W = \frac{\max\limits_{1\le k\le M}\left(f(x_k^P)\right) - f(x_i^P) + \varepsilon}{\max\limits_{1\le k\le M}\left(f(x_k^P)\right) - \min\limits_{1\le k\le M}\left(f(x_k^P)\right) + \varepsilon} \qquad i = 1, 2, \ldots, M \tag{4}$$

In Eqs. (2) to (4), $M$ is the number of particles; $x^W$ is the position of weighted particle; $\hat{c}_i^W$ is the weighted constant of each particle; $f$ indicates the objective function of the problem. $\max\limits_{1\le k\le M}\left(f(x_k^P)\right)$ and $\min\limits_{1\le k\le M}\left(f(x_k^P)\right)$, respectively, are the worst and the best fitness values in the $P_{best}$; $\varepsilon$ is the small positive number to prevent division by zero condition. As can be seen from Eq. (3), the weighted particle, indeed, is the weighted average of all particles in the swarm.

To escape from trapping into local minima due to cited proximity, the weighted particle is an appropriate point which can conduct the particles toward it. Thus, it should somehow be participated in velocity updating process of the particles. Considering the weighted particle, the velocity updating process for the iPSO is

$$\left.\begin{array}{ll} {}^{t+1}v_i = 0 & \\ {}^{t+1}x_i = {}^{t}x_i + \varphi_{4i}\left({}^{t}x^{\mathrm{W}} - {}^{t}x_i\right) & where,\ i \ge 1, \\ & \varphi_{4i} = C_4 \times \mathrm{rand}_{4i} \end{array}\right\} \text{ if } \mathrm{rand}_{0i} \le \alpha \tag{5}$$

$$\left.\begin{array}{ll} {}^{t+1}v_i = w_i \times {}^{t}v_i + \left(\varphi_{1i} + \varphi_{2i} + \varphi_{3i}\right)\left({}^{t}x_j^P - {}^{t}x_i\right) + & \\ \varphi_{2i}\left({}^{t}x^G - {}^{t}x_j^P\right) + \varphi_{3i}\left({}^{t}x^W - {}^{t}x_j^P\right) & \\ {}^{t+1}x_i = {}^{t}x_i + {}^{t+1}v_i & where,\ i \ge 1,\ j \le M, \\ & \varphi_{1i} = C_1 \times \mathrm{rand}_{1i}, \\ & \varphi_{2i} = C_2 \times \mathrm{rand}_{2i}, \\ & \varphi_{3i} = C_3 \times \mathrm{rand}_{3i} \end{array}\right\} \text{ if } \mathrm{rand}_{0i} > \alpha \tag{6}$$

in which, superscripts "$t$" and "$t$+1" , respectively, are the current and the next iteration; ${}^{t+1}v_i$ is the updated velocity; $w_i$ is the inertia factor of current velocity, and ${}^{t}v_i$ is the present velocity of $i$th particle. $C_1$=-( $\varphi_{2i}$ + $\varphi_{3i}$ ), $C_2$=1, $C_3$=2, and $C_4$=2 are the acceleration factors; $\mathrm{rand}_{ki}$, where $k \in \{0,1,2,3,4\}$, is the random number selected from interval of (0,1). ${}^{t}x_j^P$ is the randomly selected particle from current $P_{best}$. ${}^{t}x^G$ is the global best particle up to recent iteration. ${}^{t+1}x_i$ and ${}^{t}x_i$ , respectively, are the updated position and current position of the $i$th particle. ${}^{t}x^W$ is the weighted particle calculated for current iteration. Based on the studies carried out by Li *et al.* (2014) and Mortazavi and Toğan (2016), $w$ is randomly selected from (0.5, 0.55) and α=0.4.

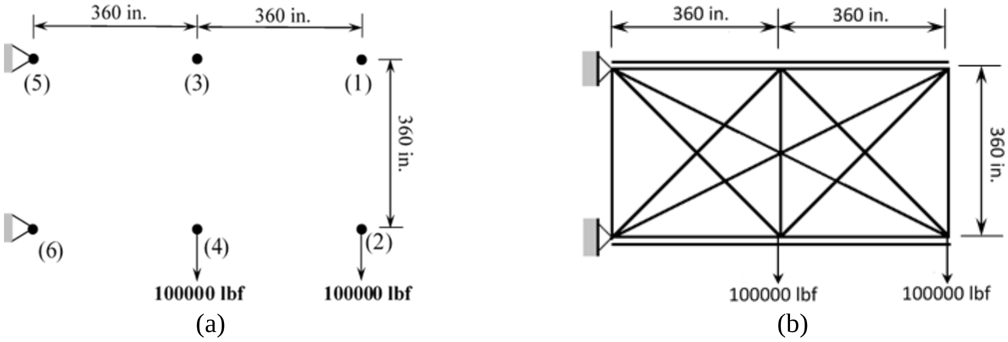



Fig. 5(a) Initial system, (b) Ground structure generated using GSM

The updated position (${}^{t+1}x_i$) and weighted particle ($x^w$) in Eqs. (2), (5) and (6) are modified for the problems with discrete search space as below

$$x^W = INT\left[\sum_{i=1}^{M} \bar{c}_i x_i^P\right] \tag{7}$$

$${}^{t+1}x_i = INT\left[{}^{t}x_i + \varphi_{4i}\left({}^{t}x^{\mathrm{W}} - {}^{t}x_i\right)\right] \qquad \text{if } \operatorname{rand}_{0i} \le \alpha \tag{8}$$

$${}^{t+1}x_i = INT\left[{}^{t}x_i + {}^{t+1}v_i\right] \qquad \text{if } \operatorname{rand}_{0i} > \alpha \tag{9}$$

where the operator *INT* [.] takes the integer part of any scalar variable. The rest of the calculation is same with Eqs. (5) and (6).

## 5. Numerical examples

In this section some numerical examples addressed as benchmark problems available in the literatures are examined. All examples are resolved using TUBM combine with iPSO to test their performance on simultaneous optimization problem of trusses in comparison with conventional GSM and the other techniques available in the literatures.

### *5.1 Fifteen member, six nodes planar truss*

In this example, the 15-member truss structure generated using GSM, is considered for simultaneous optimization. This example is already solved by Deb and Gulati (2001) using GA method, by Luh and Lin (2008) using ant system (AS) and Li (2015) using species-conserving genetic algorithm (SCGA). The basic nodes are shown in Fig. 5(a), which there are two restraint nodes, two load points and two additional points. During the optimization process, the load and restraint points should not be eliminated, but extra points can be removed. It is remarkable that the maximum number of producible elements with available nodes is equal to $(\frac{6}{2})$=15. Ground

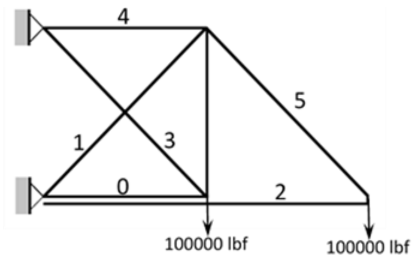



Fig. 6 Optimized structural configuration of the 15-members, 6-nodes truss system found with GSM and iPSO

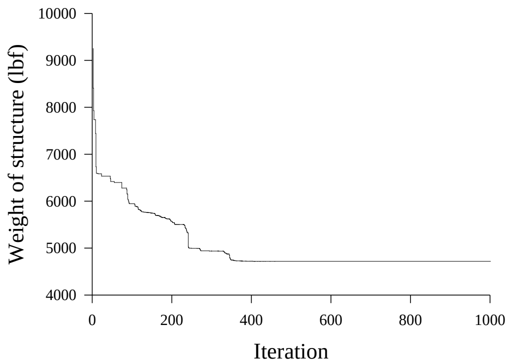


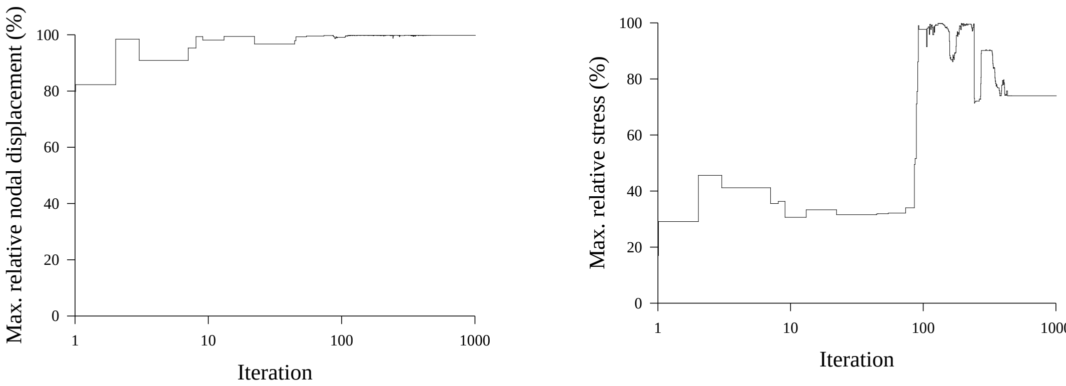

Fig. 7 Weight convergence history for size, shape and topology optimization of the 15-members, 6-nodes truss structure with GSM and iPSO

Table 1 Comparison of optimization results with literature for the 15-members, 6-nodes truss problem with overlaps

| Member number | Area of members ($in^2$) | | | | |
|---|---|---|---|---|---|
| | Deb and Gulati (2001) GSM and GA* | Deb and Gulati (2001) GSM and GA+ | Luh and Lin (2008) GSM and AS | Li (2015) GSM and SCGA | This study GSM and iPSO |
| 0 | 5.172 | 5.219 | 5.428 | 5.294 | 5.394 |
| 1 | 20.054 | 20.310 | 20.549 | 20.277 | 20.305 |
| 2 | 14.845 | 14.593 | 14.308 | 14.339 | 14.446 |
| 3 | 7.821 | 7.772 | 7.617 | 7.714 | 7.6415 |
| 4 | 28.286 | 28.187 | 28.876 | 28.158 | 28.7681 |
| 5 | 20.446 | 20.650 | 20.265 | 21.025 | 20.3825 |
| Weight (lb) | 4733.443 | 4731.650 | 4730.824 | 4732.121 | 4728.772 |
| Total number of function evaluation | - | 85050 | 41000 | - | 50000 |

*population size 300; + population size 450

structure for GSM approach is shown in Fig. 5(b) with giving load and restraint conditions. All these 15 cross-sectional areas are taken as the problem design variables.




(a) Maximum relative nodal displacement (b) Maximum relative axial stress

Fig. 8 Relative deflection and stress histories for size, shape and topology optimization of the 15-members, 6-nodes truss structure with GSM and iPSO

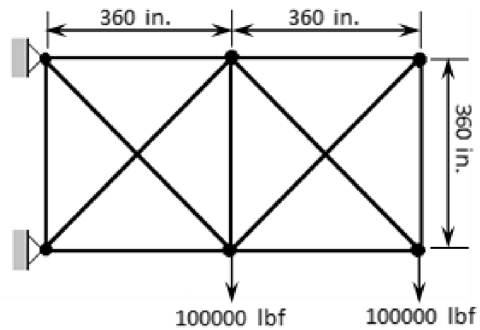



Fig. 9 Ground structure manually generated (Deb and Gulati 2001, Ringertz 1985)

The corresponding assumptions made for this example are as: The modulus of elasticity=10000 ksi, the material density=0.1 lb/in$^3$, allowable stresses in both tensile and compressive=25 ksi, allowable displacement=±2 in, minimum value of area=-35 in$^2$, maximum value of area=35in$^2$, and critical area=0.09 in$^2$.

As a first trial, the example is solved by using iPSO and GSM approach. In iPSO, the numbers of particles and maximum of iteration, respectively, are adopted as 50 and 1000. The designed truss structure at the end of the simultaneous optimization process performed with iPSO and GSM includes 7 members. The optimized structure is shown in Fig. 6, its final weight is equal to 4728.772 lb and the weight history of structure is shown in Fig. 7. It is noticeable that the GSM cannot prevent overlapping the elements (i.e. elements number 0 and 2 are overlapped). In order to recognize these elements, they are shown with little gap in Fig. 6. The history of the maximum relative (i.e., ratio of maximum available value to its allowable value) nodal displacement and stress are plot in Fig. 8. According to this figure the allowable displacement is the dominant constraint in this problem. It is notable that in Fig. 8 to make more clarity the x-axes are logarithmically scaled. The cross sections of remained elements and weight of structure comparatively are tabulated in Table 1. This table contains the results obtained by Deb and Gulati (2001) using GA with two different population sizes.

Current example is solved again, but in this case overlapping is not permitted for the structure

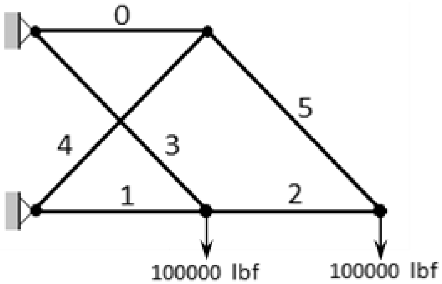



Fig. 10 Optimized structural configuration of the 11-members, 6-nodes truss system found with TUBM and iPSO

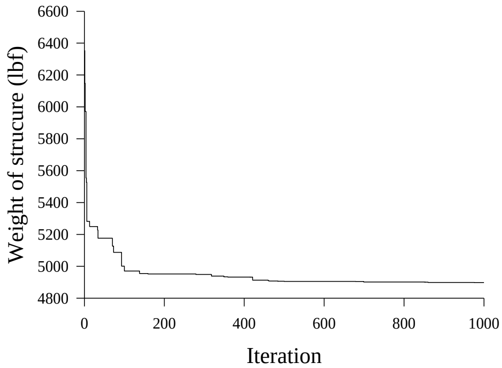



Fig. 11 Weight convergence history for size, shape and topology optimization of the 11-members, 6-nodes truss structure with TUBM and iPSO

elements. This case already has been solved by Deb and Gulati (2001), Luh and Lin (2008) and Ringertz (1985). They applied a predefined initial topology. As shown in Fig. 9, they used a manual ground structure which doesn't have any overlapped element(s). They used GA as an optimizer tool to eliminate unnecessary elements and nodes and reach to the optimum solution. But, manually defining the ground structure makes the problem very user-dependent so several potential solutions may be eliminated before starting the optimization process. However, we use the TUBM to automatically create the ground structure without any overlap. After some adjustment in this example TUBM is permitted to use 1 additional element. All parameters for adjusting TUBM are as below:

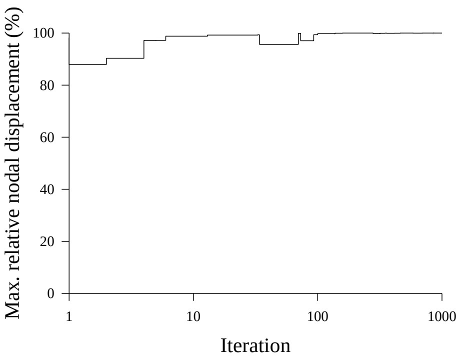
- Number of basic nodes (loads and restraints)=4
- Number of variable nodes=2
- Number of additional elements=1

All other assumption (allowable values and cross sections) are the same with those given above for this example. The swarm consists of 50 particles, the algorithm is allowed to process up to 1000 iteration. Obtained optimal solution is shown in Fig. 10. The acquired topology is similar with those obtained by Deb and Gulati (2001), Luh and Lin (2008) and Ringertz (1985) while the total weight of structure is less than both of them. Moreover, since TUBM is used rather than GSM, the ground structure is automatically generated just by entering the coordinate of the basic points.

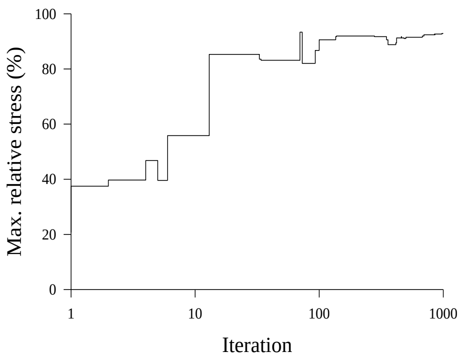
The weight history of the structure is plotted in Fig. 11. Accordingly, to assess the constraints

Table 2 Comparison of optimization results with literature for the 11-members, 6-nodes truss problem without overlaps

| Member number | Area of members($in^2$) | | | |
|---|---|---|---|---|
| | Ringertz (1985) | Deb and Gulati (2001) | Luh and Lin (2008) | This study TUBM and iPSO |
| 0 | 30.10 | 29.68 | 29.81 | 30.82 |
| 1 | 22.00 | 22.07 | 22.24 | 21.77 |
| 2 | 15.00 | 15.30 | 15.15 | 15.30 |
| 3 | 6.08 | 6.09 | 6.08 | 6.10 |
| 4 | 21.30 | 21.44 | 21.39 | 20.76 |
| 5 | 21.30 | 21.29 | 21.24 | 21.38 |
| Weight (lb.) | 4900.00 | 4899.15 | 4899.11 | 4898.26 |
| Total number of function evaluation | | 49500 | 41000 | 50000 |




(a) Maximum relative nodal displacement




(b) Maximum relative axial stress

Fig. 12 Relative deflection and stress histories for size, shape and topology optimization of the 11-members, 6-nodes truss structure with TUBM and iPSO

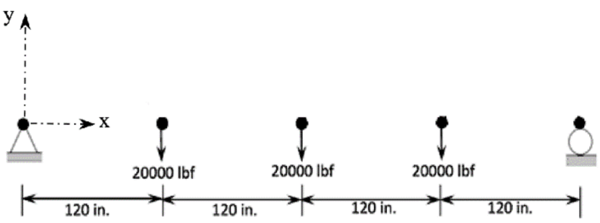



Fig. 13 A system with 5 basic points

condition during the optimization process, their relative change histories are given in Fig. 12. According to this figure, though both constraints reach near their boundary limits the displacement limitation plays more determinative role on finding optimal structure. Table 2 provides obtained optimal cross sections and weight of the structure using TUBM and iPSO. It also compares them with those available in the literature obtained by using other methods. The weight of structure is obtained to 4898.26 lb which is less than cited references.

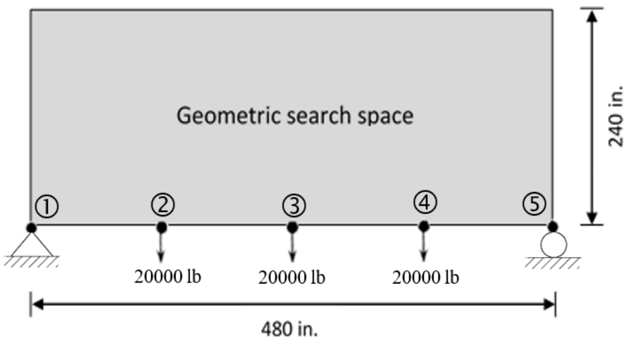



Fig. 14 Geometric search space for two-tire planar truss structure with extra 7 variable points

Table 3 Design parameters set for two-tire planar truss system problem

| Design parameters | Values |
|---|---|
| Young's modulus | 10000 ksi |
| Density | 0.1 lb/in$^3$ |
| Allowable tensile and compressive strength | 20 ksi |
| Allowable displacement | 2 in |
| $A_{min}$, $A_{max}$ | -2.25, 2.25 in$^2$ |
| Critical area ($\varepsilon$) | 0.05 in$^2$ |
| Number of basic points | 5 |
| Number of variable points | 7 |
| Number of additional elements (parameter of TUBM) | 2 |

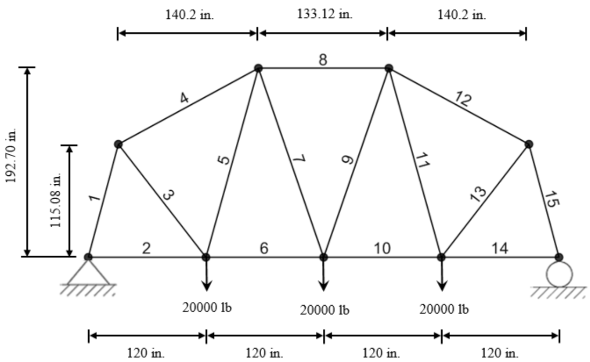



Fig. 15 Optimized structural configuration of the two-tire planar truss system found with TUBM and iPSO

### *5.2 Two-tire planar truss structure*

Two-tire planar truss consisting of 5 basic points is considered as the next example. In order to simplify the search space, symmetry is hold about $y$=240 during optimization process. 7 additional variable points are considered to find the optimum solution. The geometric boundary condition for them is shown in Fig. 14. The other fundamental assumptions necessary to model example are summarized in Table 3.

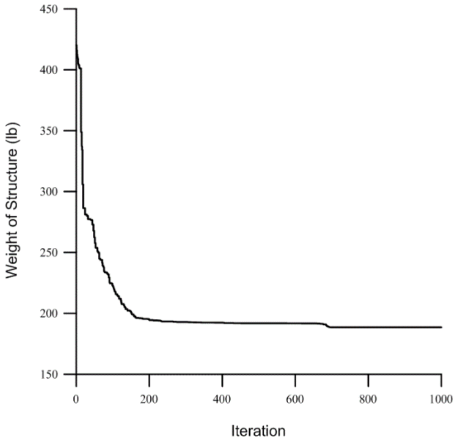
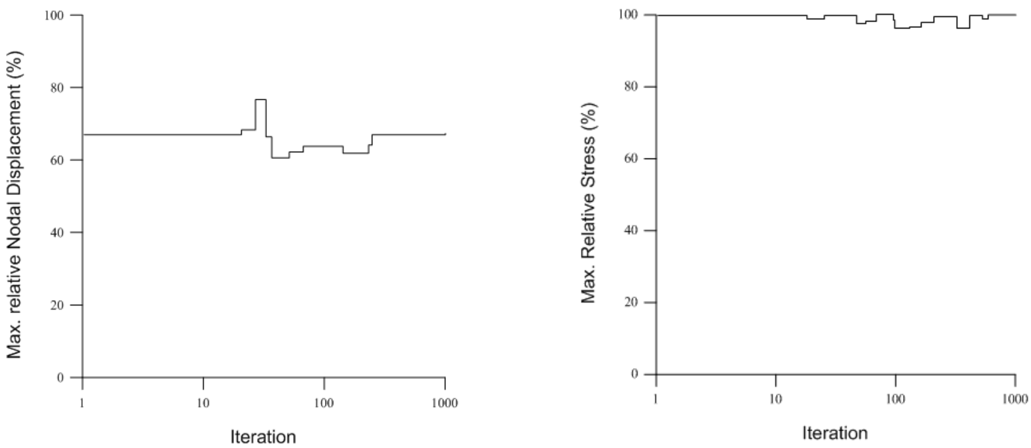



Fig. 16 Weight convergence history for size, shape and topology optimization of the two-tire planar truss system with TUBM and iPSO




(a) Maximum relative nodal displacement (b) Maximum relative axial stress

Fig. 17 Relative deflection and stress histories for size, shape and topology optimization of the two-tire planar truss system with TUBM and iPSO

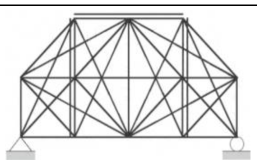
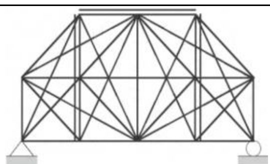
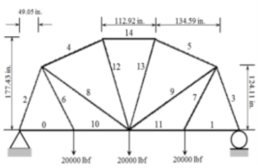
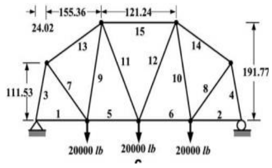
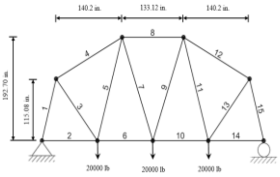
The designed structure obtained by using TUBM and iPSO is shown in Fig. 15. It can be seen that only 4 out of 7 variable nodes are remained in the structure. Also, two out of eight extra elements added by TUBM are removed during the optimization process. The weight history for optimized truss structure is plotted in Fig. 16 while obtained cross sections are presented in Table 4. Maximum relative displacement and stress of the structure are plotted in Fig. 17. In order to determine more details z-axes are logarithmically scaled in this figure. According to the results obtained, the dominant constraint for this case is the stress limitation.

The result obtained in this study is similar with the result of Deb and Gulati (2001) and Luh and Lin (2008) in accordance with the final configuration. However, optimal weight obtained with TUBM and iPSO is lighter than both of Deb and Gulati (2001) and Luh and Lin (2008). Furthermore, it can be stated that when taking into consideration of the total number of function evaluation, TUBM and iPSO requires much less evaluation process than GSM and AS which was

Table 4 Optimization results for the two-tire planar truss system problem obtained with TUBM and iPSO

| **Member number** | **Area of members($in^2$)** | | |
|---|---|---|---|
| | Deb and Gulati (2001) | Luh and Lin (2008) | This study TUBM and iPSO |
| **1, 15** | 1.615 | 1.538 | 1.553 |
| **2, 14** | 0.595 | 0.327 | 0.402 |
| **3, 13** | 1.155 | 1.221 | 1.14 |
| **4, 12** | 0.504 | 1.259 | 1.253 |
| **5, 11** | 0.051 | 0.081 | 0.103 |
| **6, 10** | 1.166 | 1.095 | 1.073 |
| **7, 9** | 1.293 | 0.525 | 0.529 |
| **8** | 1.358 | 1.256 | 1.247 |
| **Weight (lb.)** | 192.19 | 188.732 | 188.572 |
| **Total number of function evaluation** | 504000 | 453600 | 50000 |

Table 5 Comparison of optimization results with literature for the two-tire planar truss system problem

| | Deb and Gulati (2001) | Luh and Lin (2008) | This study |
|---|---|---|---|
| Optimization method | GA | AS | iPSO |
| Number of population | 1680 | 50 | 50 |
| Topology optimization method | GSM | GSM | TUBM |
| Ground structure |  |  | Automatically generated |
| Optimized structure |  |  |  |
| Weight (lb) | 192.19 | 188.732 | 188.572 |
| Total number of function evaluation | 504000 | 453600 | 50000 |

proposed by Luh and Lin (2008) (see Table 4). The differences between optimization procedure used by Deb and Gulati (2001), by Luh and Lin (2008) and this study are illustrated in Table 5. The ground structure is manually defined in Deb and Gulati (2001) and Luh and Lin (2008), however TUBM proposed in this study generates the ground structure automatically.

## 6. Conclusions

In this study, the triangular unit based method (TUMB) is proposed as a new challenge method for topology optimization of planar trusses. This method uses the advantages of stability of triangular units to decrease the size of search space by reducing the number of required elements in comparison with the traditional ground structure method (GSM). Eventually, TUMB is combined with iPSO as an optimizer tool to perform structural optimization. The iPSO is enhanced version of PSO which is strengthened with weighted particle concept and improved fly-back mechanism. It seems from the numerical investigations carried out in this study that the proposed method can be applicable for the simultaneous structural optimization process to improve the performance of the corresponding optimization process. However, in the future works, TUBM can be developed to handle simultaneous optimization for both 2D and 3D truss structures.

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