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Optimal dimensioning for the corner combined footings

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Abstract. This paper shows optimal dimensioning for the corner combined footings to obtain the most economical contact surface on the soil (optimal area), due to an axial load, moment around of the axis "X" and moment around of the axis "Y" applied to each column. The proposed model considers soil real pressure, i.e., the pressure varies linearly. The classical model is developed by trial and error, i.e., a dimension is proposed, and after, using the equation of the biaxial bending is obtained the stress acting on each vertex of the corner combined footing, which must meet the conditions following: 1) Minimum stress should be equal or greater than zero, because the soil is not withstand tensile. 2) Maximum stress must be equal or less than the allowable capacity that can be capable of withstand the soil. Numerical examples are presented to illustrate the validity of the optimization techniques to obtain the minimum area of corner combined footings under an axial load and moments in two directions applied to each column.

Keywords: corners combined footings; optimization techniques; contact surface; more economical dimension; optimal area

1. Introduction

Footings are structural elements that transmit column or wall loads to the underlying soil below the structure. Footings are designed to transmit these loads to the soil without exceeding its safe bearing capacity, to prevent excessive settlement of the structure to a tolerable limit, to minimize differential settlement, and to prevent sliding and overturning. The choice of suitable type of footing depends on the depth at which the bearing stratum is localized, the soil condition and the type of superstructure. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems (Bowles 2001, Das *et al.* 2006).

The design of superficial solution is done for the following load cases: 1) the footings subjected to concentric axial load, 2) the footings subjected to axial load and moment in one direction (uniaxial bending), 3) the footings subjected to axial load and moment in two directions (biaxial bending) (Bowles 2001, Das *et al.* 2006, Calabera 2000, Tomlinson 2008, McCormac and Brown

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The papers for optimal design of reinforced concrete foundations are: flexural strength of square spread footing (Jiang 1983); Closure to "Flexural strength of square spread footing" by Da Hua Jiang (Jiang 1984); Flexural limit design of column footing (Hans 1985); Economic design optimization of foundation (Wang and Kulhawy 2008); Reliability-Based Economic design optimization of spread foundation (Wang 2009); Structural cost of optimized reinforced concrete isolated footing (Al-Ansari 2013); Multi-objective optimization of foundation using global-local gravitational search algorithm (Khajehzadeh *et al.* 2014).

Some papers presenting the equations to obtain the dimension of footings are: A mathematical model for dimensioning of footings rectangular (Luévanos-Rojas 2013); A mathematical model for dimensioning of footings square (Luévanos-Rojas 2012a); A mathematical model for the dimensioning of circular footings (Luévanos-Rojas 2012b); A new mathematical model for dimensioning of the boundary trapezoidal combined footings (Luévanos-Rojas 2015); A mathematical model for the dimensioning of combined footings of rectangular shape (Luévanos-Rojas 2016b).

This paper shows optimal dimensioning for the corner combined footings to obtain the most economical contact surface on the soil (optimal area), due to an axial load, moment around of the axis "X" and moment around of the axis "Y" applied to each column. The proposed model considers soil real pressure, i.e., the pressure varies linearly. The classical model is developed by trial and error, i.e., a dimension is proposed, and after, using the equation of the biaxial bending is obtained the stress acting on each vertex of the corner combined footing, which must meet the conditions following: 1) Minimum stress should be equal or greater than zero, because the soil is not withstand tensile. 2) Maximum stress must be equal or less than the allowable capacity that can be capable of withstand the soil. The paper presents numerical examples for two property lines adjacent to illustrate the validity of the optimization techniques to obtain the minimum area of the corner combined footings under an axial load and moments in two directions applied to each column.

2. Formulation of the proposed model

The general equation for any type of footings subjected to bidirectional bending (Luévanos-Rojas 2012a, b, 2013, 2015, 2016b, Gere and Goodno 2009)

$$\sigma = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y y}{I_y} \tag{1}$$

Where: σ is the stress exerted by the soil on the footing (soil pressure), A is the contact area of the footing, P is the axial load applied at the center of gravity of the footing, M_x is the moment around the axis "X", M_y is the moment around the axis "Y", x is the distance in the direction "X" measured from the axis "Y" to the fiber under study, y is the distance in direction "Y" measured from the axis "X" to the farthest under study, I_y is the moment of inertia around the axis "Y" and I_x is the moment of inertia around the axis "X".

Fig. 1 shows a corner combined footing under axial load and moment in two directions (biaxial bending) in each column, the pressure below the footing vary linearly (Luévanos-Rojas 2012a, b, 2013, 2015, 2016b).

Fig. 2 presents the pressure diagram below the corner combined footing, and also the stresses in each vertex are shown.

$$\sigma_6 = \frac{R}{A} - \frac{M_{xT} y_b}{I_x} + \frac{M_{yT}(x_b - b_2)}{I_y} \tag{7}$$

Where R is resultant force, M_{xT} is resultant moment around the axis "X" and M_{yT} is resultant moment around the axis "Y" are obtained

$$R = P_1 + P_2 + P_3 \tag{8}$$

$$M_{xT} = M_{x1} + M_{x2} + M_{x3} + P_1\left(y_b - \frac{c_3}{2}\right) + P_2\left(y_b - \frac{c_3}{2}\right) - P_3\left(L_2 + \frac{c_3}{2} - y_b\right) \tag{9}$$

$$M_{yT} = M_{y1} + M_{y2} + M_{y3} + P_1\left(x_b - \frac{c_1}{2}\right) - P_2\left(L_1 + \frac{c_1}{2} - x_b\right) + P_3\left(x_b - \frac{c_1}{2}\right) \tag{10}$$

The geometric properties of section are

$$A = (a - b_2)b_1 + bb_2 \tag{11}$$

$$y_b = \frac{(a - b_2){b_1}^2 + b^2 b_2}{2[(a - b_2)b_1 + bb_2]} \tag{12}$$

$$y_i = \frac{(2b - b_1)(a - b_2)b_1 + b^2 b_2}{2[(a - b_2)b_1 + bb_2]} \tag{13}$$

$$I_x = \frac{a^2 {b_1}^4 + 2ab_1 b_2 (b - b_1)(2b^2 - bb_1 + {b_1}^2) + {b_2}^2 (b - b_1)^4}{12[(a - b_2)b_1 + bb_2]} \tag{14}$$

$$x_b = \frac{a^2 b_1 + (b - b_1){b_2}^2}{2[(a - b_2)b_1 + bb_2]} \tag{15}$$

$$x_i = \frac{a^2 b_1 + (2a - b_2)(b - b_1)b_2}{2[(a - b_2)b_1 + bb_2]} \tag{16}$$

$$I_y = \frac{b^2 {b_2}^4 + 2bb_1 b_2 (a - b_2)(2a^2 - ab_2 + {b_2}^2) + {b_1}^2 (a - b_2)^4}{12[(a - b_2)b_1 + bb_2]} \tag{17}$$

Geometry conditions are

$$a \geq \frac{c_1}{2} + L_1 + \frac{c_2}{2} \tag{18}$$

$$a = x_b + x_i \tag{19}$$

The stresses generated by soil on contact surface of the combined footing must meet the following conditions: 1) The minimum stress should be equal or greater than zero; 2) The maximum stress must be equal or less than the soil allowable load capacity "σ_{adm}" (Bowles 2001, Das *et al.* 2006, McCormac and Brown 2013, González-Cuevas and Robles-Fernandez-Villegas 2005).

Now the objective function to minimize the total area of the contact surface "A_t" is

$$A_t = (a - b_2)b_1 + bb_2 \tag{30}$$

Constraint functions for dimensioning of the corner combined footings are

$$R = P_1 + P_2 + P_3 \tag{31}$$

$$M_{xT} = \frac{R\left[(a - b_2)b_1{}^2 + b^2 b_2\right]}{2[(a - b_2)b_1 + bb_2]} + M_{x1} + M_{x2} + M_{x3} - \frac{Rc_3}{2} - P_3 L_2 \tag{32}$$

$$M_{yT} = \frac{R\left[a^2 b_1 + (b - b_1)b_2{}^2\right]}{2[(a - b_2)b_1 + bb_2]} + M_{y1} + M_{y2} + M_{y3} - \frac{Rc_1}{2} - P_2 L_1 \tag{33}$$

$$\begin{aligned}\sigma_1 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2)b_1{}^2 + b^2 b_2\right]}{a^2 b_1{}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + b_1{}^2\right) + b_2{}^2(b - b_1)^4} \\ & + \frac{6M_{yT}\left[a^2 b_1 + (b - b_1)b_2{}^2\right]}{b^2 b_2{}^4 + 2bb_1 b_2(a - b_2)\left(2a^2 - ab_2 + b_2{}^2\right) + b_1{}^2(a - b_2)^4}\end{aligned} \tag{34}$$

$$\begin{aligned}\sigma_2 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(a - b_2)b_1{}^2 + b^2 b_2\right]}{a^2 b_1{}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + b_1{}^2\right) + b_2{}^2(b - b_1)^4} \\ & - \frac{6M_{yT}[a^2 b_1 + (2a - b_2)(b - b_1)b_2]}{b^2 b_2{}^4 + 2bb_1 b_2(a - b_2)\left(2a^2 - ab_2 + b_2{}^2\right) + b_1{}^2(a - b_2)^4}\end{aligned} \tag{35}$$

$$\begin{aligned}\sigma_3 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - ab_1{}^2\right]}{a^2 b_1{}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + b_1{}^2\right) + b_2{}^2(b - b_1)^4} \\ & + \frac{6M_{yT}\left[(a - b_2)^2 b_1 - bb_2{}^2\right]}{b^2 b_2{}^4 + 2bb_1 b_2(a - b_2)\left(2a^2 - ab_2 + b_2{}^2\right) + b_1{}^2(a - b_2)^4}\end{aligned} \tag{36}$$

$$\begin{aligned}\sigma_4 = {} & \frac{R}{(a - b_2)b_1 + bb_2} + \frac{6M_{xT}\left[(b - b_1)^2 b_2 - ab_1{}^2\right]}{a^2 b_1{}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + b_1{}^2\right) + b_2{}^2(b - b_1)^4} \\ & - \frac{6M_{yT}[a^2 b_1 + (2a - b_2)(b - b_1)b_2]}{b^2 b_2{}^4 + 2bb_1 b_2(a - b_2)\left(2a^2 - ab_2 + b_2{}^2\right) + b_1{}^2(a - b_2)^4}\end{aligned} \tag{37}$$

$$\begin{aligned}\sigma_5 = {} & \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2 b_1{}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + b_1{}^2\right) + b_2{}^2(b - b_1)^4} \\ & + \frac{6M_{yT}\left[a^2 b_1 + (b - b_1)b_2{}^2\right]}{b^2 b_2{}^4 + 2bb_1 b_2(a - b_2)\left(2a^2 - ab_2 + b_2{}^2\right) + b_1{}^2(a - b_2)^4}\end{aligned} \tag{38}$$

$$\sigma_6 = \frac{R}{(a - b_2)b_1 + bb_2} - \frac{6M_{xT}[(2b - b_1)(a - b_2)b_1 + b^2 b_2]}{a^2 b_1{}^4 + 2ab_1 b_2(b - b_1)\left(2b^2 - bb_1 + b_1{}^2\right) + b_2{}^2(b - b_1)^4} \tag{39}$$

considerations: R=2400 kN, M_{xT} and M_{yT} are not constrained, 5.40 m=a, 6.40 m=b, 0≤b_1, 0≤b_2, A_t is objective function, $0 \leq \sigma_1 \leq \sigma_{adm}$, $0 \leq \sigma_2 \leq \sigma_{adm}$, $0 \leq \sigma_3 \leq \sigma_{adm}$, $0 \leq \sigma_4 \leq \sigma_{adm}$, $0 \leq \sigma_5 \leq \sigma_{adm}$, $0 \leq \sigma_6 \leq \sigma_{adm}$.

Tables 2, 3 and 4 are shown in the appendix.

4. Results

Table 2 shows the following results (dimensions a, b, b_1 and b_2 are assumed nonnegative): 1) The minimum area is the same for the four types in each case; 2) If the soil allowable load capacity decreases, the minimum area increases; 3) The resultant mechanical elements are R=2400 kN, M_{xT}=0, M_{yT}=0 for all cases; 4) The stresses generated by loads in each vertex are the same to σ_{adm} in each case. It means that the resultant force is located in the center of gravity of the contact area of the footing with soil, i.e., the total eccentricity of the resultant force “R” in the two directions is zero.

Table 3 presents the following results (dimensions a and b are assumed nonnegative, and b_1 and b_2 are greater than or equal to one): 1) The minimum area is deferent for the four types in each case; 2) If the soil allowable load capacity decreases, the minimum area increases; 3) The resultant mechanical elements are R=2400 kN, M_{xT} and M_{yT} are equal or less than zero for all cases; 4) The stresses generated by loads in each vertex are equal or less than σ_{adm} and greater than zero in each case (meets the conditions indicated by the stresses). It means that the resultant force is located in the center of gravity of the contact area of the footing with soil for the case 4 of the type 1 and case 5 of the types 1, 2, 3, because the total moments are M_{xT}=0, M_{yT}=0 (the stresses generated by loads in each vertex are the same to σ_{adm}), and for the other cases the resultant force is not located in the center of gravity of the contact area of the footing with soil, and therefore the resultant force “R” has an eccentricity.

Table 4 shows the following results (dimensions b_1 and b_2 are assumed nonnegative. and a=5.40 m and b=6.40 m): 1) The minimum area is deferent for the four types in each case; 2) If the soil allowable load capacity decreases, the minimum area increases; 3) The resultant mechanical elements are R=2400 kN, M_{xT} and M_{yT} are equal or less than zero for all cases; 4) The stresses generated by loads in each vertex are equal or less than σ_{adm} and greater than zero in each case (meets the conditions indicated by the stresses). It means that the resultant force is not located in the center of gravity of the contact area of the footing with soil, and therefore the resultant force “R” has an eccentricity for all cases.

Table 2 shows the optimal area for the dimensions $a \geq 0$, $b \geq 0$, $b_1 \geq 0$ and $b_2 \geq 0$. Table 3 presents the optimal area for the dimensions $a \geq 0$, $b \geq 0$, $b_1 \geq 1$ and $b_2 \geq 1$. Table 4 shows the optimal area for the dimensions a=5.40, b=6.40, $b_1 \geq 0$ and $b_2 \geq 0$. If the three tables are compared for the same cases and types in function of the optimal area: Table 2 is smaller than Table 3, but Table 3 is smaller than Table 4.

5. Conclusions

The foundation is an essential part of a structure that transmits column or wall loads to the underlying soil below the structure.

The mathematical approach suggested in this paper produces results that have a tangible accuracy for all problems, main part of this research to find the more economical dimensions of

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Appendix

Table 2 Results obtained by software for $b_1 \geq 0$ and $b_2 \geq 0$

Type	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex					
	R kN	M_{xT} kN-m	M_{yT} kN-m	A_t m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2
Case 1 (σ_{adm}=250.00 kN/m^2)														
1	2400	0.00	0.00	9.60	6.12	9.13	0.99	0.43	250.00	250.00	250.00	250.00	250.00	250.00
2	2400	0.00	0.00	9.60	8.11	8.99	0.54	0.62	250.00	250.00	250.00	250.00	250.00	250.00
3	2400	0.00	0.00	9.60	10.44	7.15	0.41	0.79	250.00	250.00	250.00	250.00	250.00	250.00
4	2400	0.00	0.00	9.60	9.19	10.46	0.56	0.45	250.00	250.00	250.00	250.00	250.00	250.00
Case 2 (σ_{adm}=225.00 kN/m^2)														
1	2400	0.00	0.00	10.67	7.34	7.15	0.71	0.85	225.00	225.00	225.00	225.00	225.00	225.00
2	2400	0.00	0.00	10.67	7.98	8.90	0.61	0.70	225.00	225.00	225.00	225.00	225.00	225.00
3	2400	0.00	0.00	10.67	9.67	7.44	0.54	0.79	225.00	225.00	225.00	225.00	225.00	225.00
4	2400	0.00	0.00	10.67	10.25	9.26	0.48	0.65	225.00	225.00	225.00	225.00	225.00	225.00
Case 3 (σ_{adm}=200.00 kN/m^2)														
1	2400	0.00	0.00	12.00	6.46	7.69	1.06	0.78	200.00	200.00	200.00	200.00	200.00	200.00
2	2400	0.00	0.00	12.00	7.82	8.76	0.70	0.81	200.00	200.00	200.00	200.00	200.00	200.00
3	2400	0.00	0.00	12.00	8.21	8.53	0.86	0.64	200.00	200.00	200.00	200.00	200.00	200.00
4	2400	0.00	0.00	12.00	8.73	10.56	0.77	0.54	200.00	200.00	200.00	200.00	200.00	200.00
Case 4 (σ_{adm}=175.00 kN/m^2)														
1	2400	0.00	0.00	13.71	6.28	7.32	1.26	0.96	175.00	175.00	175.00	175.00	175.00	175.00
2	2400	0.00	0.00	13.71	6.88	7.37	1.06	0.77	175.00	175.00	175.00	175.00	175.00	175.00
3	2400	0.00	0.00	13.71	7.91	8.47	1.06	0.72	175.00	175.00	175.00	175.00	175.00	175.00
4	2400	0.00	0.00	13.71	10.47	8.64	0.57	0.96	175.00	175.00	175.00	175.00	175.00	175.00
Case 5 (σ_{adm}=150.00 kN/m^2)														
1	2400	0.00	0.00	16.00	5.72	7.04	1.77	1.12	150.00	150.00	150.00	150.00	150.00	150.00
2	2400	0.00	0.00	16.00	7.37	8.20	1.00	1.20	150.00	150.00	150.00	150.00	150.00	150.00
3	2400	0.00	0.00	16.00	7.40	8.51	1.41	0.78	150.00	150.00	150.00	150.00	150.00	150.00
4	2400	0.00	0.00	16.00	8.70	9.73	1.00	0.84	150.00	150.00	150.00	150.00	150.00	150.00

Table 3 Results obtained by software for $b_1 \geq 1$ and $b_2 \geq 1$

Type	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex					
	R kN	M_{xT} kN-m	M_{yT} kN-m	A_t m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2
Case 1 (σ_{adm}=250.00 kN/m^2)														
1	2400	-404.93	-436.54	11.44	6.04	6.40	1.00	1.00	169.13	240.54	190.41	250.00	229.65	241.47
2	2400	-862.14	-665.20	12.34	6.04	7.30	1.00	1.00	131.87	236.18	162.95	250.00	232.74	250.00
3	2400	-693.66	-837.18	12.43	7.03	6.40	1.00	1.00	131.64	234.58	161.70	250.00	230.33	250.00
4	2400	-1176.81	-1114.65	13.35	7.03	7.32	1.00	1.00	101.13	232.20	137.57	250.00	231.37	250.00
Case 2 (σ_{adm}=225.00 kN/m^2)														
1	2400	-300.82	-281.14	11.87	6.28	6.59	1.00	1.00	176.30	218.64	189.40	225.00	218.26	225.00
2	2400	-667.06	-515.91	12.91	6.31	7.60	1.00	1.00	142.00	215.61	163.05	225.00	213.33	225.00
3	2400	-565.96	-647.79	12.95	7.32	6.63	1.00	1.00	140.74	213.81	161.92	225.00	215.01	225.00
4	2400	-982.82	-931.38	13.98	7.34	7.63	1.00	1.00	112.30	212.01	138.87	225.00	211.42	225.00
Case 3 (σ_{adm}=200.00 kN/m^2)														
1	2400	-111.86	-104.62	12.48	6.58	6.90	1.00	1.00	183.73	197.96	187.94	200.00	197.84	200.00
2	2400	-439.64	-340.97	13.57	6.62	7.95	1.00	1.00	150.86	194.65	162.82	200.00	193.38	200.00
3	2400	-377.74	-431.73	13.62	7.66	6.96	1.00	1.00	149.59	193.58	161.75	200.00	194.26	200.00
4	2400	-756.01	-716.86	14.71	7.70	8.00	1.00	1.00	122.28	191.41	139.85	200.00	191.03	200.00
Case 4 (σ_{adm}=175.00 kN/m^2)														
1	2400	0.00	0.00	13.71	6.37	7.20	1.21	1.00	175.00	175.00	175.00	175.00	175.00	175.00
2	2400	-169.55	-131.93	14.37	6.99	8.38	1.00	1.00	158.20	173.25	162.10	175.00	172.85	175.00
3	2400	-153.17	-174.75	14.42	8.08	7.34	1.00	1.00	156.93	172.81	161.09	175.00	173.03	175.00
4	2400	-485.70	-460.87	15.58	8.13	8.45	1.00	1.00	130.86	170.35	140.36	175.00	170.14	175.00
Case 5 (σ_{adm}=150.00 kN/m^2)														
1	2400	0.00	0.00	16.00	5.72	7.04	1.77	1.12	150.00	150.00	150.00	150.00	150.00	150.00
2	2400	0.00	0.00	16.00	7.06	8.50	1.11	1.10	150.00	150.00	150.00	150.00	150.00	150.00
3	2400	0.00	0.00	16.00	8.31	7.39	1.07	1.13	150.00	150.00	150.00	150.00	150.00	150.00
4	2400	-155.49	-147.67	16.65	8.66	8.99	1.00	1.00	137.73	148.78	140.23	150.00	148.72	150.00

Table 4 Results obtained by software for a=5.40 and b=6.40

Type	Resultant mechanical elements			Optimal area	Dimension of the footing				Stresses generated by loads in each vertex					
	R kN	M_{xT} kN-m	M_{yT} kN-m	A_t m^2	a m	b m	b_1 m	b_2 m	σ_1 kN/m^2	σ_2 kN/m^2	σ_3 kN/m^2	σ_4 kN/m^2	σ_5 kN/m^2	σ_6 kN/m^2
Case 1 (σ_{adm}=250.00 kN/m^2)														
1	2400	-562.10	-502.51	11.89	5.40	6.40	1.32	0.94	146.52	231.78	179.59	250.00	235.15	250.00
2	2400	-1073.17	-830.21	13.91	5.40	6.40	1.25	1.39	81.98	224.41	144.27	250.00	213.30	250.00
3	2400	-927.87	-985.36	14.50	5.40	6.40	2.04	0.80	68.49	198.43	139.42	250.00	230.64	250.00
4	2400	-1491.83	-1344.31	16.46	5.40	6.40	2.00	1.28	21.35	191.08	120.64	250.00	209.63	250.00
Case 2 (σ_{adm}=225.00 kN/m^2)														
1	2400	-471.98	-421.88	12.85	5.40	6.40	1.44	1.02	142.35	209.24	170.77	225.00	212.34	225.00
2	2400	-952.61	-738.92	15.05	5.40	6.40	1.35	1.54	82.17	202.04	139.21	225.00	190.92	225.00
3	2400	-817.15	-861.13	15.68	5.40	6.40	2.26	0.84	70.30	176.14	135.59	225.00	208.57	225.00
4	2400	-1358.36	-1213.51	17.84	5.40	6.40	2.23	1.39	25.65	168.33	119.05	225.00	188.27	225.00
Case 3 (σ_{adm}=200.00 kN/m^2)														
1	2400	-366.72	-327.49	13.96	5.40	6.40	1.59	1.12	138.98	187.32	161.66	200.00	190.00	200.00
2	2400	-810.79	-632.51	16.36	5.40	6.40	1.47	1.71	83.32	180.27	133.73	200.00	169.32	200.00
3	2400	-689.05	-713.66	17.01	5.40	6.40	2.53	0.86	73.43	155.09	131.40	200.00	187.02	200.00
4	2400	-1203.77	-1057.16	19.41	5.40	6.40	2.51	1.50	31.13	146.27	116.89	200.00	167.97	200.00
Case 4 (σ_{adm}=175.00 kN/m^2)														
1	2400	-242.80	-216.32	15.24	5.40	6.40	1.77	1.23	136.65	166.25	152.12	175.00	168.27	175.00
2	2400	-643.34	-507.74	17.85	5.40	6.40	1.60	1.92	85.71	159.25	127.63	175.00	148.83	175.00
3	2400	-540.85	-540.38	18.48	5.40	6.40	2.86	0.85	78.39	135.83	126.63	175.00	165.93	175.00
4	2400	-1024.52	-868.40	21.19	5.40	6.40	2.87	1.61	38.14	125.21	113.89	175.00	149.04	175.00
Case 5 (σ_{adm}=150.00 kN/m^2)														
1	2400	-95.82	-84.93	16.73	5.40	6.40	1.99	1.36	135.64	146.37	141.96	150.00	147.30	150.00
2	2400	-446.25	-360.53	19.56	5.40	6.40	1.73	2.18	89.70	139.07	120.60	150.00	130.02	150.00
3	2400	-370.32	-344.97	20.04	5.40	6.40	3.24	0.80	85.73	120.00	120.81	150.00	144.93	150.00
4	2400	-816.14	-639.89	23.19	5.40	6.40	3.34	1.68	47.20	105.87	109.61	150.00	131.72	150.00