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A multilevel framework for decomposition-based reliability shape and size optimization

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Abstract. A method for decoupling reliability based design optimization problem into a set of deterministic optimization and performing a reliability analysis is described. The inner reliability analysis and the outer optimization are performed separately in a sequential manner. Since the outer optimizer must perform a large number of iterations to find the optimized shape and size of structure, the computational cost is very high. Therefore, during the course of this research, new multilevel reliability optimization methods are developed that divide the design domain into two sub-spaces to be employed in an iterative procedure: one of the shape design variables, and the other of the size design variables. In each iteration, the probability constraints are converted into equivalent deterministic constraints using reliability analysis and then implemented in the deterministic optimization problem. The framework is first tested on a short column with cross-sectional properties as design variables, the applied loads and the yield stress as random variables. In addition, two cases of curvilinearly stiffened panels subjected to uniform shear and compression in-plane loads, and two cases of curvilinearly stiffened panels subjected to shear and compression loads that vary in linear and quadratic manner are presented.

Keywords: reliability analysis; shape optimization; reliability-based design optimization; stiffened panels

1. Introduction

In the future, the use of probabilistic methods both in structural design and during the certification process is expected to undergo a significant increase. There is an increasing realization in many quarters that designing a structure considering various uncertainties is a more rational approach than the current approach of using safety factors (Mohaghegh 2005). Therefore, several investigations have been performed using an uncertainty based approach.

The conventional approach to formulate the Reliability Based Design Optimization (RBDO) is to utilize a double-loop optimization and uncertainty analysis which are nested one into another to

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the notion of probabilistic sufficiency factor (Qu and Haftka 2004), which has a high correlation with the target reliability. A probabilistic sufficiency factor of 1.0 represents that the achieved probability of failure is equal to the target one. If a probabilistic sufficiency factor is less than one, the probability of failure exceeds the target and the design is not safe, and a probabilistic sufficiency factor larger than one means that the probability of failure is less than the target probability. Qu and Haftka (2004) compared the performance of response surface approximations fitted for three approaches of describing failure in a non-deterministic manner, namely: (a) the probability of failure, (b) the safety index, and (c) the probabilistic sufficiency factor. They showed that the response surface approximation can have better accuracy when it is fitted to the probabilistic sufficiency factor than to either of the other two. Furthermore, they showed that the probabilistic sufficiency factor provides more information in regions of low probability than either of the other two.

Wu *et al.* (2001) also proposed a safety factor based RBDO by converting reliability constraints to the equivalent deterministic constraint with safety factor in the optimization cycle. Du and Chen (2004) developed a sequential RBDO methodology. In their framework, the optimization is conducted by first using the MPP of the previous design point and then performing a reliability analysis to update the MPP. The combined optimization and reliability analysis cycle is repeated until the objective convergence and the reliability requirement is achieved. Ba-abbad *et al.* (2006) improved the Du and Chen technique to distribute the reliability of the system over its components in an optimal way. In Ba-abbad *et al.* (2006)'s technique, at each iteration, the first-order reliability analysis is carried out to check if this design has an acceptable reliability. Then, the performance measure analysis is performed to calculate the MPPs of the various failure modes. Finally, the approximate deterministic optimization is conducted to find the optimum design and measure the maximum of the safety indices.

Another approach for improveing the efficiency of RBDO is to use surrogate models such as response surfaces. Lopez *et al.* (2015) developed a methodology based on the response surface method and Firefly Algorithm to perform design optimization of truss structures. The response surface method is responsible for the reduction of the computational cost corresponding to the evaluation of the probabilistic constraints and the Firefly Algorithm addresses the issues related to the non-convexity and mixed-variables of the optimization problem.

As was discussed earlier, most of the reported single-loop RBDOs in the literature utilize an iterative procedure which finds the design variables while updating the MPPs and replacing the probabilistic constraints by equivalent deterministic constraints. Recently, some single-loop RBDO techniques have emerged that do not involve the iterative cycle. Shan and Wang (2008) proposed an approach by finding the feasible and reliable space of solutions in the first step and then performing an optimization in that space. Although the developed method eliminates the reliability analysis in the RBDO process, using the design point to calculate the gradient vector instead of using the MPPs may lead to an accuracy issue in some engineering applications (Li *et al.* 2013).

The robustness and efficiency problems in reliability based shape design optimization result in its limited range of applications (Yao *et al.* 2011). This research proposes a multilevel approach for decomposition-based reliability shape and size optimization. A sequential optimization and reliability analysis methodology is developed that utilizes the shape and size variables as design variables, and the applied loads and the Young's modulus as random variables. The proposed approach, first, conducts the reliability analysis to find MPPs and the probability of satisfying the given constraints. Next, each probability constraint in the multilevel shape and size optimization is

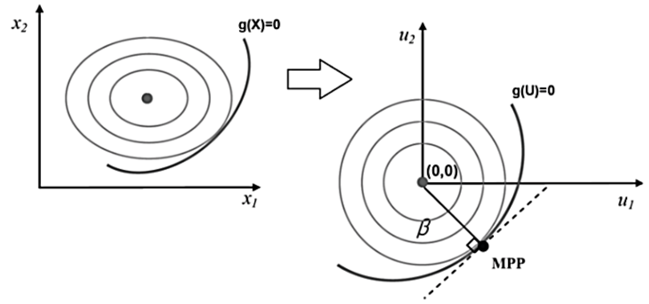

Fig. 2 Transformation and MPP (Choi *et al.* 2006)

analysis framework, first the double-loop RBDO, Fig. 1, is presented

$$\begin{cases} find\ d \\ \min f(d) \\ s.t. P[g_i(d, X) \leq 0] \geq R \quad i = 1, 2, \ldots, m \\ d^L \leq d \leq d^U \end{cases} \tag{1}$$

where d is the design variables' vector and d^L and d^U are the design variables' lower and upper bounds. Here, X is the random variables vector, f is the objective function, g_i is the *ith* constraint function, m is the number of constraints, $Prob[g_i(d, X) \leq 0]$ is the probability of satisfying the *ith* constraint, and R is the specified reliability for the constraints. The RBDO goal is to minimize the structural weight subjected to given probabilistic constraints on buckling, von Mises stress, and crippling (g_i, m=3). The buckling factor is considered for the buckling constraint and is defined as the inverse of the fundamental eigenvalue. The von Mises stresses for all elements were aggregated using the Kreisselmeier and Steinhauser (KS) function and the crippling constraint is calculated as the ratio of maximum negative principal stress and the maximum allowable stress in the stiffener. The structure responses including the first buckling eigenvalue, von Mises stress, and the principal stress vector are calculated using finite element analysis.

In order to perform the RBDO efficiently rather than utilizing a double-loop framework (Eq. (1)) the deterministic optimization and the uncertainty quantification are decoupled from one another. Various techniques have been developed to decouple the optimization and the reliability analysis. One of these techniques is sequential optimization and reliability analysis (SORA). The main idea behind SORA is to perform optimization by applying the equivalent deterministic constraints, instead of using the reliability constraints. The constraints on the probability of satisfying constraints of a structure can be converted to the equivalent deterministic constraints by using the MPPs at the desired level of safety.

For MPP based sequential RBDO, the random variables are replaced with their MPPs; therefore, the deterministic constraints are shifted to meet the desired reliability level. The calculated MPP is improved after each iteration to provide an accurate MPP for the deterministic optimization. A multilevel MPP RBDO framework is developed utilizing the sequential optimization and reliability analysis (see Fig. 3). In the proposed approach, first the reliability analysis of initial design (d^0) is conducted to find the MPP corresponding to the desired reliability (R). MPP is the design point that has most significant contribution to the Probability of failure (Choi *et al.* 2006). The MPP is defined in a standardized and independent coordinate system. In the transformation procedure, the design vector X is transformed into the vector of standardized,

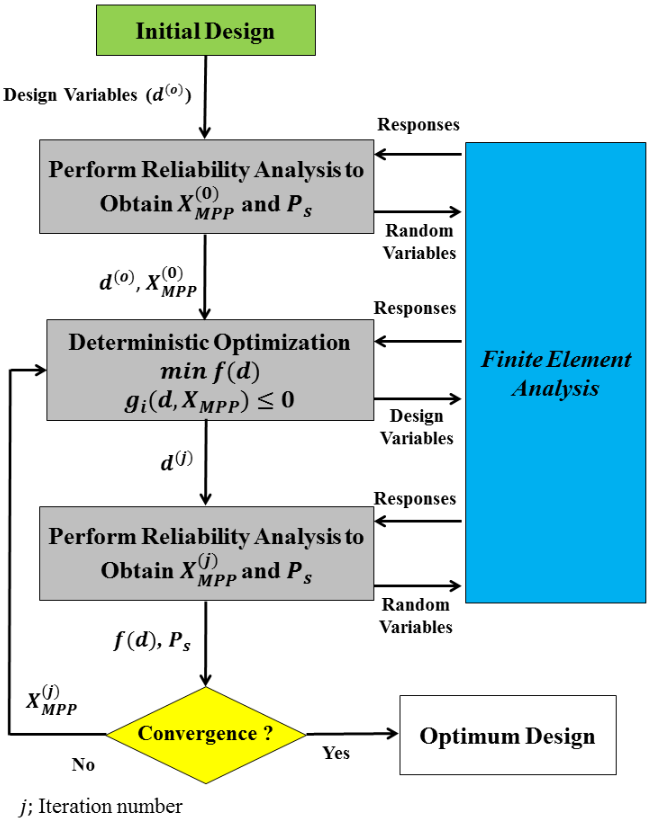

Fig. 3 The sequential framework for RBDO

MSC.PATRAN® and MSC.NASTRAN® to perform FEA on a panel with curvilinear, blade-type stiffeners and returns the mass of the panel and constraints on yielding, buckling, and crippling or local failure of the panel. In this framework, the user needs to create an initial Patran database using either EBF3PanelOpt or utilizing Patran directly.

After giving the input variables, *EBF3PanelOpt* writes a Patran session file in Python, launches Patran and executes this session file to update geometry of the stiffened panel. After successful execution of the session file, it writes the input file (bdf) for Nastran. The successful execution of this Nastran (bdf) file, the Nastran response file (f06) is read by Python; responses like bucking factor, von Mises stress and the crippling stress for a stiffener are calculated by Python. During the execution of Patran and Nastran, if any error occurs or if it takes more time than the allocated time, these processes will be terminated and default responses will be sent with “pass/fail” as an active constraint. The pass/fail response indicates a failure so that the design can be disregarded without discontinuities being introduced into the design space. During analysis, completion of bdf and session files closure is monitored for successful execution.

A key feature of *EBF3PanelOpt* is the ability to specify the geometry in a parametric fashion such that the optimizer fully specifies the panel shape and size. Sizing quantities such as panel thickness, stiffener thickness, and stiffener height are used as design variables to define the geometry. The stiffener curve is represented using a third order uniform rational B-spline using two end-points and a control point so the stiffener always remains in the panel area. The stiffener

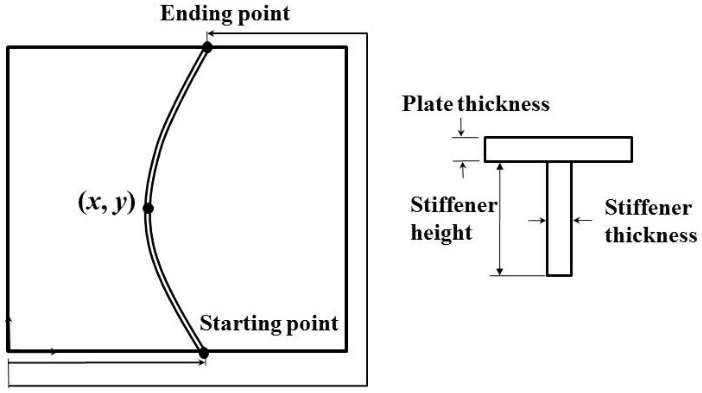

Fig. 4 Description of the design variables

constraints on buckling parameter, crippling, and von Mises stress. Next, with the size parameters fixed as obtained in size optimization, the shape optimization is performed. In shape optimization, the optimal shape (layout) is obtained which is best for resisting critical constraints with fixed values of size variables. These two sub-optimizations must be iterated, as size parameters obtained in size optimization may not be optimal for the shape variables obtained in shape optimization. For this iterative optimization procedure, the error in the mass of two successive iterations is observed to terminate the optimization procedure.

The RBDO problem can be formulated mathematically as follows

$$\begin{cases} find\ d\ (size\ and\ shape\ design\ variables) \\ \min mass(d) \\ s.t. P[\lambda(d,X) \le 1] \ge R \\ s.t. P[KS(d,X) \le 1] \ge R \\ s.t. P[(\sigma_{cc}(d,X) \le 1] \ge R \\ d^L \le d \le d^U \end{cases} \tag{3}$$

where λ, KS, and σ_{cc} are buckling parameter (1/buckling eigenvalue), Kreisselmeier and Steinhauser (KS) stress, and the crippling, respectively. The shape and size design variables (d) are listed in Table 1 and shown in Fig. 4.

Using the sequential RBDO, as explained in Fig. 3, the optimization and reliability assessment are separated, and the optimization is defined as

$$\begin{cases} find\ d\ (size\ and\ shape\ design\ variables) \\ \min mass(d) \\ \lambda(d\ , X_{MPP1}) \le 1 \\ KS(d\ , X_{MPP2}) \le 1 \\ \sigma_{cc}(d\ , X_{MPP3}) \le 1 \\ d^L \le d \le d^U \end{cases} \tag{4}$$

The optimization problem, Eq. (4), is divided into two step optimization. The two-step optimization formulation of layout and size of structure can be expressed as follows. First, perform size optimization for fixed values of shape design variables. In this step, the optimizer optimizes the mass using size design variables while satisfying the buckling, stress, and crippling constraints

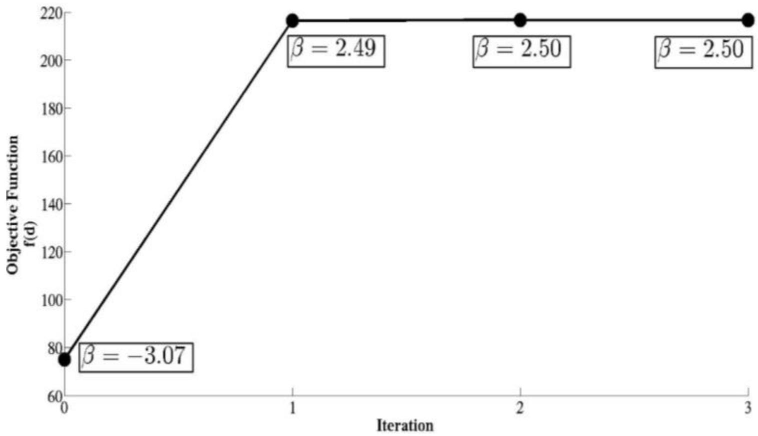

Fig. 5 Iteration history of the sequential optimization and reliability analysis of short column

Finally, the application of developed RBDO framework for curvilinearly stiffened panels is demonstrated. The design variables include the shape and size variable of stiffened panels, and the in-plane loads and young modulus are defined as random variables. To determine the effect of in-plane load variations on the optimal mass of the panel, four sets of in-plane load distributions are considered. The computations are performed on a computer with dual four-core 3.00 GHz Intel Xeon processors with 20 GB of RAM. Parallel processing is used for all cases, resulting in eight simultaneous analyses for each iteration of the optimization process.

6. Short column

This test problem involves the plastic analysis and design of a short column with rectangular cross section (width b and depth h) having uncertain material properties (yield stress *Y*) and subject to uncertain loads (bending moment *M* and axial force *F*) (Kuschel and Rackwitz 1997). The objective and limit state functions are defined as

$$\begin{cases} f(d) = b.h \\ G(d,X) = 1 - \frac{4M}{bh^2Y} - \frac{F^2}{(bhY)^2} \end{cases} \tag{8}$$

The distributions for *F*, *M*, and *Y* are normal (500, 100), normal (2000, 400), and lognormal (5, 0.5), respectively, with a correlation coefficient of 0.5 between *F* and *M*. An objective function of cross-sectional area and a target reliability index of 2.5 (cumulative failure probability P_f≤0.00621) are used in the design problem

$$\begin{cases} min\, f(d) \\ s.t\, \beta \geq 2.5 \\ 5.0 \leq b \leq 15.0 \\ 15.0 \leq h \leq 25.0 \end{cases} \tag{9}$$

First, the reliability analysis of the initial design is performed to find the MPP corresponding to the desired target reliability index of 2.5 (see iteration zero in Fig. 5). Starting from the initial

Table 3 Material properties of curvilinearly stiffened plate

Modulus of Elasticity	73×10^{9} *Pa*
Density	2795 kg/m^3
Poisson's Ratio	0.33
Yield stress	427.4 MPa

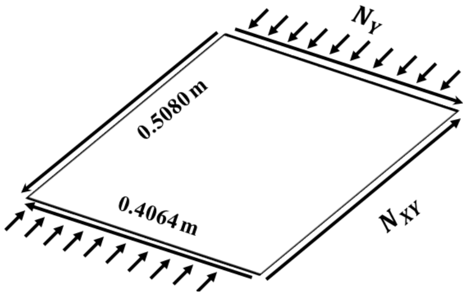

Fig. 6 Panel dimensions and loading conditions

for all cases is 0.9998.

The first load case (L1) studied is a stiffened panel under combined shear and compression with dominant compression (N_Y/N_{XY}=4.36), as shown in Fig. 6. N_Y and N_{XY} are normally distributed and uncorrelated with a mean of 308 kN/m and 71 kN/m, respectively and 15% coefficient of variation (COV). The distribution for E is lognormal with a mean of 73 GPa and 1% COV. The second load case (L2) has smaller ratio of shear and compressive load magnitudes (N_Y/N_{XY}=1.13). In this load case, N_Y and N_{XY} are normally distributed and uncorrelated with a mean of 152 kN/m and 134 kN/m, respectively, and 15% COV. The distribution, mean, and covariance for E are similar to the previous case.

Following the sequential RBDO scheme presented in the previous section, the iteration histories for load case one are shown in Fig. 7 (L1). It is shown that both the optimization and reliability analysis converges after two iterations. The panel's mass is 2.031 kg, which is slightly lighter than the deterministic optimum of 2.032 kg. The applied loads for deterministic optimization are obtained after applying a factor of safety of 1.5 to the limit loads and the panels are designed for that loads. The deterministic optimum configuration is shown in Fig. 8 (L1). The optimum objective, probability of safety, and shape and sizing design variable values for RBDO and deterministic optimization are shown in Table 4 (L1). A few observations are of interest here. First, since in the studied case the compression is the dominant load, using an appropriate safety factor (here it is 1.5) can give the desired probability of failure, as can be seen in Table 4. When there is only one important random variable, the safety factors can be directly expressed by the required reliability levels. However, in many cases, there does not exist a relationship between the safety factor and reliability levels. Furthermore, the buckling constraint is active for both configurations, which yield closely optimal results.

The optimum configuration and iteration histories for the second load case are shown in Fig. 7 (L2). The shape and sizing design variable values obtained from the sequential RBDO and the related final optimum mass, and the probability of safety for three constraints are shown in Table 4 (L2). The deterministic optimization result for the second load case using a factor of safety of 1.5 is presented in Fig. 8 (L2). The optimum mass of the panel is 1.746 kg, which is lighter than the

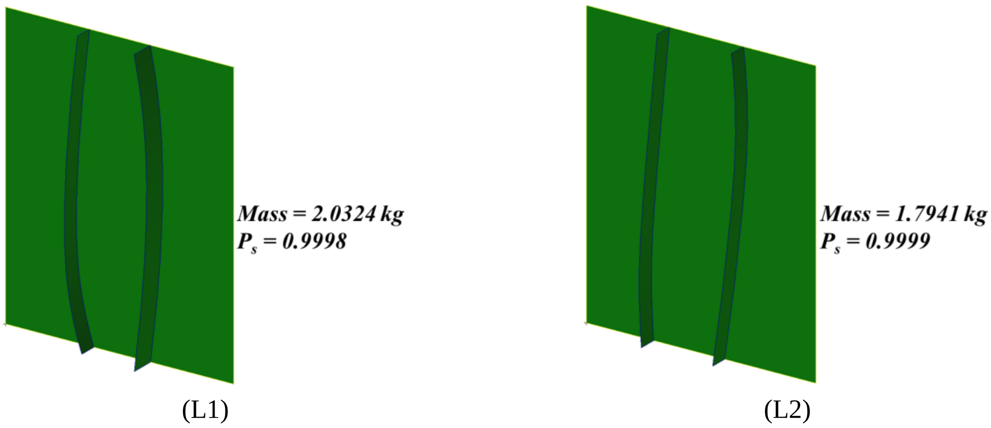

Fig. 8 Deterministic optimum configuration (L1) N_Y/N_{XY}=4.36 (L2) N_Y/N_{XY}=1.13

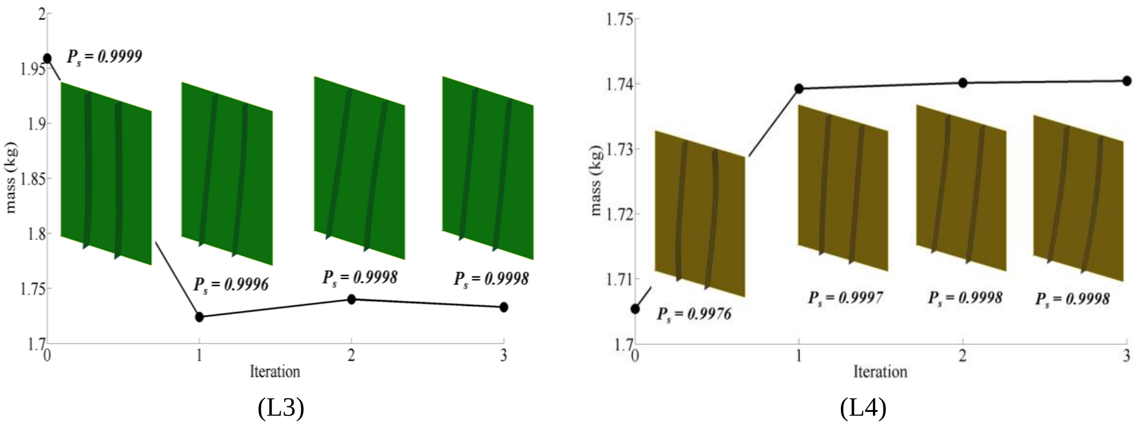

Fig. 9 Iteration history of mass and probability of safety for linearly (L3) and parabolically (L4) varying load cases

safety factor did not yield the RBDO final configuration. It is important to note that changing the safety factor would change the shear and compression loads, but it does not change their ratio, and consequently it only changes the size variables while having no effect on the shape variables. The final configuration of curvilinear stiffeners is governed by the ratio of the shear and compression loads, rather than their magnitudes.

8. Curvilinearly stiffened panels subjected to non-uniform shear and compression in-plane loads

It is seen from the results shown in the previous subsection that the influence of the ratio of shear and compression loads on the final results is substantial. Furthermore, it is also important to understand the influence of the additional random variables, such as the linearly and parabolic varying loads, and to study their effect on the optimal mass and probability of safety. In this subsection, the curvilinearly stiffened panels under shear and compression loads with linearly and

Table 6 Optimum mass, constraint and design variable obtained for panel with four curvilinear stiffeners

Variable/Response		Variable/Response		Variable/Response	
x_1	0.0896	x_{11}	0.4527	x_{21}, *m*	0.0020
x_2	0.1980	x_{12}	0.0259	x_{22}, *m*	0.0022
x_3	0.0095	x_{13}	0.5827	x_{23}, *m*	0.0020
x_4	0.6652	x_{14}	0.3191	x_{24}, *m*	0.0020
x_5	0.1721	x_{15}	0.5516	x_{25}, *m*	0.0021
x_6	0.4365	x_{16}	0.1167	*Mass, kg*	1.4501
x_7	0.8158	x_{17}, *m*	0.0233	$Prob[\lambda(d,X)\leq 1]$	0.9999
x_8	0.5238	x_{18}, *m*	0.0220	$Prob[KS(d,X)\leq 1]$	0.9999
x_9	0.6117	x_{19}, *m*	0.0237	$Prob[(\sigma\_cc\ (d,X)\leq 1]$	0.9999
x_{10}	0.4443	x_{20}, *m*	0.0228	Number of evaluations	62040

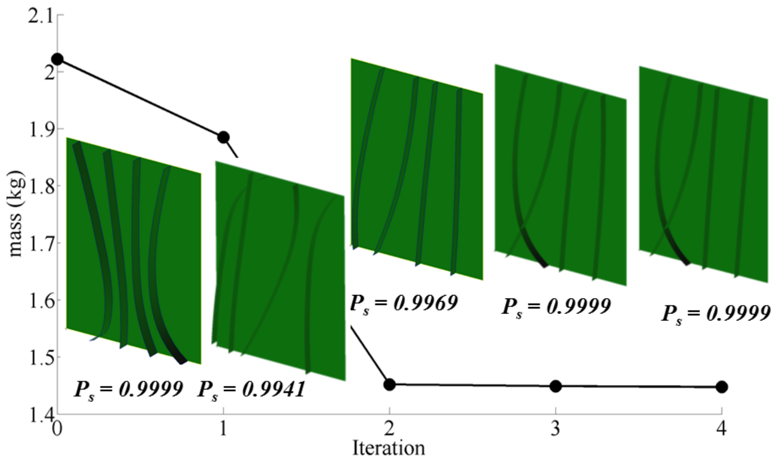

Fig. 10 Iteration history of mass and probability of safety for panel with four curvilinear stiffeners

As for the parabolic load distribution (Eq. (10)), three random variables are defined for each of the shear and compression loads. Both N_{Y1} and N_{XY1} are normally distributed and uncorrelated with a mean of 152 kN/m and 134 kN/m and 15% COV. Similarly, N_{Y2} and N_{XY2} are also normally distributed and uncorrelated with a mean of zero, and the standard deviation is 5% of means of N_{Y1} and N_{XY1}, and N_{Y3} and N_{XY3} are normally distributed and uncorrelated with a mean of zero, and the standard deviation is 2% of the mean values of N_{Y1} and N_{XY1}.

Table 5 compares the optimal design variables, the objective function, the probability constraints, and the number of iterations of two load cases (L3 and L4), all achieved by using sequential RBDO. Note that, as expected, the parabolically varying load case has objective function values larger than linearly varying load case, due to the effect of third variable in the parabolic load distribution. However as shown in Fig. 9, the stiffener layouts obtained for two load cases are similar to each other. This can be explained by the fact that the first variable in the compression and shear loads (N_{Y1} and N_{XY1}) are dominant, and they appear to be governing the final optimal layout while the other parameters (N_{Y2}, N_{XY2}, N_{Y3}, and N_{XY3}) change the size variables and consequently the weight of the stiffened panel.

requires imposing various safety factors for shear and compression loads. This makes the requirement for an RBDO further evident. The stiffened panel test cases include up to 25 shape and size design variables, which makes them complex engineering designs and demanding computational problems. The successful convergence of the proposed methodology after a few iterations of deterministic optimization clearly shows its capability in solving complex engineering uncertainty design problems.

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