

Finite element based dynamic analysis of multilayer fibre composite sandwich plates with interlayer delaminations

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Abstract. Although the aircraft industry was the first to use fibre composites, now they are increasingly used in a range of structural applications such as flooring, decking, platforms and roofs. Interlayer delamination is a major failure mode which threatens the reliability of composite structures. Delamination can grow in size under increasing loads with time and hence leads to severe loss of structural integrity and stiffness reduction. Delamination reduces the natural frequency and as a consequence may result in resonance. Hence, the study of the effects of delamination on the free vibration behaviour of multilayer composite structures is imperative. The focus of this paper is to develop a 3D FE model and investigate the free vibration behaviour of fibre composite multilayer sandwich panels with interlayer delaminations. A series of parametric studies are conducted to assess the influence of various parameters of concern, using a commercially available finite element package. Additionally, selected points in the delaminated region are connected appropriately to simulate bolting as a remedial measure to fasten the delamination region in the aim of reducing the effects of delamination. First order shear deformation theory based plate elements have been used to model each sandwich layer. The findings suggest that the delamination size and the end fixity of the plate are the most important factors responsible for stiffness reduction due to delamination damage in composite laminates. It is also revealed that bolting the delaminated region can significantly reduce the natural frequency variation due to delamination thereby improving the dynamic performance.

Keywords: fibre composite multilayer plates; three dimensional modelling; dynamic analysis; free vibration behaviour; delamination

1. Introduction

Multi-layered structures are increasingly used in aerospace, automotive and ship vehicles, the most common and best-known examples being composite sandwich panels (Carrera 2002). The main advantage of using the composite sandwich concept is that the resulting structural element has high bending stiffness and high strength to weight ratio. Compared with traditional concrete or steel structures, their low self-weight is considered a remarkable advantage of FRP composite structures, although it raises concerns with respect to their dynamic response (Frostig 2002).

Fibre-reinforced composites are made up of a combination of fibres in a matrix material. Composite materials are increasingly being used in aircraft primary structures because of their

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superior strength properties over metallic materials (Diamanti and Soutis 2010). In recent years, advanced composites have been replacing traditional structural materials in primary load carrying aircraft structures to a significant extent (Katnam *et al.* 2013). Glass fibre reinforced polymer composites are two phase materials with glass fibre acting as dispersed phase and polymer as the matrix. Composite sandwiches are fabricated by attaching two stiff glass fibre skins to a thick core, which can be used for plate or slab type structural applications. A structural composite multilayer plate can be manufactured by gluing two or more composite sandwiches together to form a laminated composite. Although the aircraft industry was the first to use fibre composites, now they are increasingly used in a range of structural applications such as flooring, decking, platforms and roofs. Although sandwich construction has some prominent benefits it also has some weaknesses such as a wide variety of failure modes and complexity of analysis. Detection of failure modes and their effects remain a challenge for the optimum use of fibre composites in structural applications. In general, damage to composite components needs to be assessed by using appropriate non-destructive methods to determine the extent and location of the damage as a first step to decide whether to repair or replace the damaged component (Katnam *et al.* 2013).

Delamination or interlayer debonding is a predominant form of damage phenomenon in laminated composites, which can often be pre-existing or can take place during service life (Kim *et al.* 2003). It occurs in a multilayer laminated sandwich material, leading to split-up of the sandwich layers, which in turn can severely affect the structural integrity of the laminated multilayer composite panel. Delamination can considerably reduce the structural stiffness and strength of composite components (Katnam *et al.* 2013). The natural frequencies of a delaminated composite plate will reduce because of the loss in stiffness caused by the presence (Ju *et al.* 1995) of delamination. Since the natural frequencies and vibration modes change considerably with respect to the undamaged material, they can indeed be used to develop methods for damage detection and evaluation (Gallego *et al.* 2013). Hence it is vital to predict the changes in natural frequencies and mode shapes due to delamination with respect to developing damage detecting techniques.

Several effective theories have been proposed for the accurate modeling of multilayered structures, beams, plates and shells, subjected to static and dynamic loading and various restraints, and they are currently used by the engineering community through both analytical and numerical solution methods (Roberta and Francesca 2014). Modeling of delaminations in composites plates has been investigated frequently with classical lamination theory (CLT) and first order shear deformation theory. The CLT is found to be suitable for most applications where the thickness of the laminate is small by two orders of magnitude compared to the in plane dimensions (Reddy and Miravete 1995). According to Agarwal *et al.* (2006), in classical lamination theory, the bond between two laminae in a laminate is assumed to be perfect, infinitesimally thin and not shear deformable, yet for real structures this analysis can only be used when the laminate is subjected to constant in-plane forces and moments. However practical laminates are often subjected to transverse loads. Shear deformation laminate theories do not neglect these out-of-plane stresses (Bunsell and Renard 2005). Frostig (2002) presented a review of some of the classical analytical methods and compared them with a high-order approach. The need to focus attention on the layer level in order to study interply stresses, has led to Layerwise theories (Gallego *et al.* 2013). Numerous structural theories and numerical models anticipated in evaluating the characteristics of composite laminates have been comprehensively reviewed by Carrera (2002), and more recently by Della and shu (2007).

Kulkarani and Frederick (1971) were among the earliest researchers to study the delamination

damage in laminated composite structures. Ramkumar *et al.* (1979) modelled a beam with a full width delamination assuming four Timoshenko beams connected at delamination ends. The anticipated natural frequencies obtained with the use of their model were constantly lower than the results reported in experimental studies. The authors suggested that this disagreement is due to the contact between the delaminated 'free' surfaces during vibrations and proposed that the presence of the effect of contact might improve the analytical results. Wang *et al.* (1982) presented an analytical model, referred to as 'free model' which is also known as 'without-contact model', consisting of four separate Euler-Bernoulli beam segments joined together with appropriate boundary and continuity conditions to get the response of the beam. In their model, it was assumed that delaminated layers deform freely and have different transverse deformations. Although their numerical results were in reasonable agreement with experimental results, they included physically unreal overlapping at the delamination. Mujumdar and Suryanarayan (1988) supposed that the appearance of the open modes in a dynamic response is not possible because of probable overlap between the delaminated sublaminates. To avoid this incompatibility, two delaminated segments of the beam were constrained to have transverse displacements along the whole length of the beam under consideration. Their model was called 'the constrained model', which is also referred to as 'with-contact model' or 'contact model' in the literature. Their analytical results were in very good agreement with experimental results, and it was shown that the contact model is simple and accurate for analysing vibration characteristics of delaminated composites. Later, a comparable analytical model was proposed by Tracey and Pardo (1992) to study the effects of delamination. Duggan and Ochoa (1992) suggested that the natural frequencies are sensitive to the size, location, and shape of delamination in structural components. The contact model was later extended by Hu and Hwu (1995) for sandwich beams by including the effects of transverse shear deformations and rotary inertia, and by Shu and Fan (1996) for bi-material beams. The model proposed by Mujumdar and Suryanarayan (1988) was extended by Grouve *et al.* (2008) for anisotropic laminated composite beams, to study effect of delamination on the resonance frequencies. Della and Shu (2005, 2007) studied on free vibration of beams with double delaminations and also on delaminated bimaterial beams. Kwon and Lannamann (2002) used a finite element analysis with surface-to-surface contact model to predict the dynamic behaviour of a debonded cantilever sandwich beam subjected to an impact load at the free end. Schwarts *et al.* (2008) presented a high-order analytical approach for the free vibration analysis of fully bonded and debonded unidirectional sandwich panels with a transversely flexible core. In their method, compressibility and shear deformability of the core, as well as 'with' and 'without' contact conditions at the debonding between skin and core were taken into account. Same authors reported a modified Galerkin method to tackle the same problem (Schwartz *et al.* 2007). They verified their results against those from finite element analysis with ANSYS code.

Mendelsohn (2006) studied the progressive failure of debonding in a sandwich plate by using the Dugdale-Barenblatt cohesive zone model. Chakrabarti and Sheikh (2009) did a dynamic analysis of a debonded sandwich plate by using a linear spring model in the interfacial region. Burlayenko and Sadowski (2010) investigated influence of debonding on free vibration behaviour of foam and honeycomb cored sandwich plates using finite element (FE) package ABAQUS and found that core types of the sandwich plates strongly affect their dynamic response. Moreover the same authors (Burlayenko and Sadowski 2011) reported their findings on dynamic characteristics of honeycomb and PVC foam core sandwich plates containing skin/core debonding by FE modelling with ABAQUS, and revealed that natural frequencies are poorly sensitive to the number of debonding zones. Furthermore the same two authors (Burlayenko and Sadowski 2012)

developed a finite element model for analysing the dynamic response of sandwich plates with partially damaged face sheet and core using three-dimensional FE modelling using the software package ABAQUS. It was revealed from the study that, for an accurate simulation of dynamics of debonded sandwich plates, the contact phenomenon within the debonded region need be taken into account. Newly, the same authors (Burlayenko and Sadowski 2014) carried out a nonlinear dynamic analysis of a rectangular simply supported sandwich plate with a central penny shaped debonded zone under harmonic loading using FE analysis within the ABAQUS code. The predictions performed by them showed that the finite element model they applied would be useful for non-destructive evaluation of defects in composite sandwich plates.

A new generation composite sandwich made up of E glass fibre reinforced polymer skins and high strength modified phenolic core material have been developed in Australia. According to Ven Erp and Rodgers (2008), using a new plant based resin technology for both the skins and the core, the panel offers unprecedented performance at a price that is comparable to traditional building materials. Awad *et al.* (2012) did an experimental study and finite element analysis using ABAQUS code to investigate the free vibration behaviour of fully bonded GFRP sandwich panels with different sizes and end conditions. The only reported study on the free vibration behaviour of debonded novel sandwich plates is by Karunasena (2010). The author examined the deviations in natural frequency due to various amounts of debonding along the glue line in a four layer laminated fibre composite sandwich plate structure using 2D finite element modelling with Strand7.

The versatility of the finite element method (FEM) for resolving complex topological and multi-physical problems has made it a popular technique in investigations of debonded sandwich panels (Burlayenko and Sadowski 2014). Dynamic analysis of three-dimensional models of structures enables more realistic assessment of their free vibration as well as forced vibration behaviour. While the dynamic behavior of undamaged sandwich panels is the subject of extensive studies, papers reported on the dynamic behavior of sandwich panels with debonding are less presented in the literature (Burliyanko and Sadowski 2014). This paper presents the study of dynamic behaviour of multilayer composite sandwich slab panels with interlayer delamination using finite element based numerical modelling with Strand7 using 3D models to precisely simulate the real behaviour. This study deals with the behaviour of multi-layer composite slabs with pre-existing delaminated regions at different locations of interest.

According to Hosur *et al.* (2003), repair procedures for fibre reinforced composite laminates have been utilizing prepregs and wet layup techniques in conjunction with bonding, bolting and flush patching, while bolted and bonded repair are the two major repair techniques. Composite repairs are generally either mechanically fastened or adhesively bonded patches (Katnam *et al.* 2013). Present analysis examines the efficiency of repairing the delaminated region using mechanical fasteners. In this context, selected points in the delaminated region are connected appropriately to simulate fastening the delaminated section using bolting as a corrective measure to reduce the adverse effects due to delamination.

2 Methodologies

2.1 Finite element modelling

Finite element method is particularly versatile and effective for the analysis of complex

structural behaviour of the composite laminated structures. The formation of delamination is a complex process and the problem is three dimensional in nature (Senthil *et al.* 2013). In this context, numerical simulations are carried out using FE code Strand7 (2010), using 3D finite element models of the structures to enable more realistic assessment of their free vibration behaviour. Note here that only static conditions of delamination (delaminations are pre-existing and their sizes stay constant during vibration) are considered here, since the purpose is to analyse existing delaminations through a FEM based model.

Table 1 reports the effective mechanical properties used by Awad *et al.* (2012) for the glass fibre skin and the phenolic core

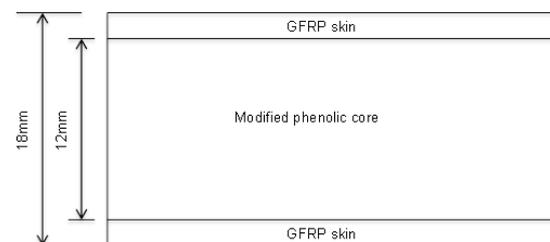


Fig. 1 Cross-sectional view of the single layer GFRP composite sandwich used for validation

2.2 Model development and application

The major steps in developing the model are explained below.

(i) Linear elastic orthotropic top and bottom thin skins are modelled using 4-noded rectangular plate elements. The plate element mesh for each skin lies at the horizontal plane at the mid-thickness level of the relevant skin.

(ii) Orthotropic 3D brick elements are used to model the thick core. The brick elements take care of any shear deformations in the thick core.

(iii) Core 3D Finite Element mesh is created by extruding the 4-noded plate element mesh using the 'extrude' command in strand7. This procedure warrants that a vertical line through corresponding plate nodes in the top and bottom skins will pass through corresponding brick nodes in the core.

(iv) In the fully bonded portions of the plate, structural integrity between each skin and core is assured by connecting plate nodes with corresponding brick nodes at the top and bottom surface levels of the core through vertical 'rigid link' elements. Rigid link provides restraints to the nodal rotations, in addition to the translational displacements (Strand7 2010).

(v) In multilayer laminated composite sandwich panels, each composite layer (see Fig. 2) is connected to the neighbouring layer by connecting adjacent plate nodes of each composite layer using rigid links to ensure compatibility each layer.

(vi) Delaminated regions are assumed to be in contact vertically but can slide in the horizontal plane (similar to contact or constrained model).

(vii) The Finite element model for the delaminated section of the two layer plate is obtained by converting the rigid links within the delaminated region to 'master slave links' in Strand7, innovatively assigning the proper degrees of freedom to simulate the situation explained in (vi) above.

2.3 Model verification

The developed numerical model has been verified and validated for capability and accuracy through comparison of the results with published experimental and numerical results for a similar problem reported by Awad *et al.* (2012). The novel composite sandwich panel reported in Awad *et al.* (2012) has a middle core of thickness 12 mm and 3 mm thick GFRP skin in the top and bottom faces as shown in Fig. 1. Four different sizes of square one way slabs were used for the verification, namely 400, 600, 800 and 1000 mm. The boundary conditions considered here were fixed restraints for both ends of the slabs.

Table 1 Effective mechanical properties used for model verification (Awad *et al.* 2012)

Property	Skin	Core
Young's modulus along long direction (MPa)	12360	1350
Young's modulus in transverse direction (MPa)	10920	1350
Poisson's ratio	0.3	0.2
Density (kg/m ³)	1425	950



Fig. 2 Two layer laminated composite sandwich used and the positions of delamination considered

2.4 Selection of parameters for the analysis

Plate size used for the parametric analysis is 1000 mm square slab with two composite sandwich layers bonded to form a laminated composite as shown in Fig. 2(a). Each sandwich plate or layer consists of top and bottom skins of thickness 3 mm each and a middle core of 12 mm thickness as illustrated in Fig. 1.

As shown in Fig. 2(a), interlayer delamination between the two bonded composite sandwich lamina in the 1000 mm two layer square plate is considered. Three different positions of delamination as seen Fig. 2(b) namely positions 1, 2 and 3, and four different percentages of delamination (0.5%, 1%, 5% and 10% by area of the plate) are chosen for analysis. In addition, 20% delamination by area of the plate have been simulated only in the case of C-C-C-C plate with position 3 delamination, to examine the efficiency of repairing the delaminated region using mechanical fasteners.

Four different types of end conditions for the slab are selected for analysis. They are, one way simply supported (S-S-F-F), two way simply supported (S-S-S-S), one way clamped (C-C-F-F) and two way clamped (C-C-C-C). The mechanical properties of the skin and core of the novel composite multilayer slab panel used for the present analysis are shown in Table 2 below.

Table 2 Mechanical properties of fibre composite skin and core for present analysis

Property	Skin	Core
Young's modulus along long direction (MPa)	15380	1150
Young's modulus in transverse direction (MPa)	12631	1150
Poisson's ratio	0.25	0.30
Density (kg/m ³)	1366	855

2.5 Fastening the delaminated regions as a remedial measure

According to (Chutima and Blackie 1996), mechanical fasteners such as bolts and rivets are commonly utilized in composite structures to provide a convenient means of assembly or to enable disassembling of sections where regular inspection and/or repair is required. Such fasteners are extensively used mainly because they are easy to assemble or disassemble (Ju *et al.* 1995). As such, the possibility and effectiveness of using bolts to fasten the delaminated regions of the multilayer composite plates as a remedial measure to reduce the effects of delamination are considered here. The fastening of the regions of delamination is simulated by converting 'master slave links to rigid links at the intended locations of the bolts.

3. Results and discussion

3.1 Model verification

The results of the verification study are presented in Table 3, where experimental and numerical results reported by Awad *et al.* (2012) for fully bonded plates are compared with the

Table 3 Comparison of natural frequency results for model verification

Slab size	Support type	Restraint type	Mode number	Experimental results	FEA with ABAQUS	Present analysis
				(Awad <i>et al.</i> 2012) Frequency in Hz	(Awad <i>et al.</i> 2012) Frequency in Hz	with Strand7 Frequency in Hz
400×400	One-way	Glue	1	193	194	195
400×400	One-way	Glue	2	230	226	234
600×600	One-way	Glue	1	95	96	90
600×600	One-way	Glue	2	123	114	121
800×800	One-way	Glue	1	49	51	52
800×800	One-way	Glue	2	70	64	70
1000×1000	One-way	Glue	1	28	29	34
1000×1000	One-way	Glue	2	41	37	45

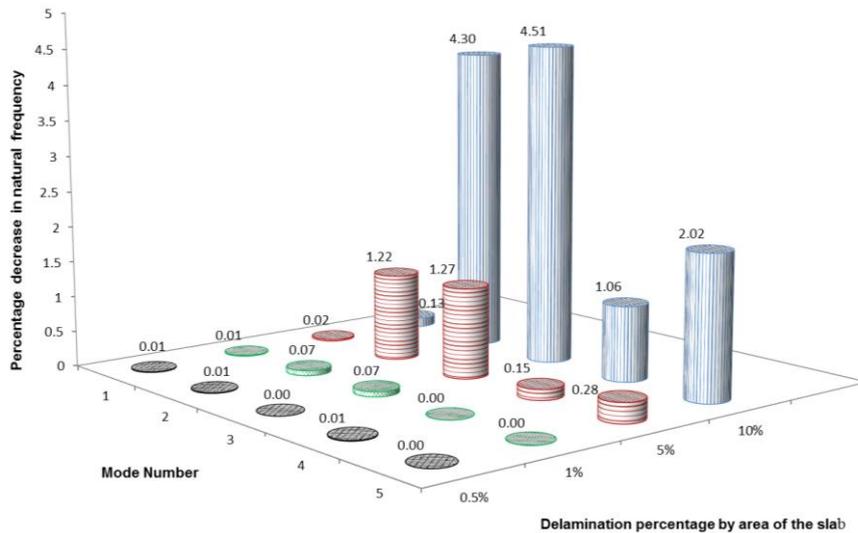


Fig. 4 Shift in natural frequency for two way simply supported panel: delamination position 3

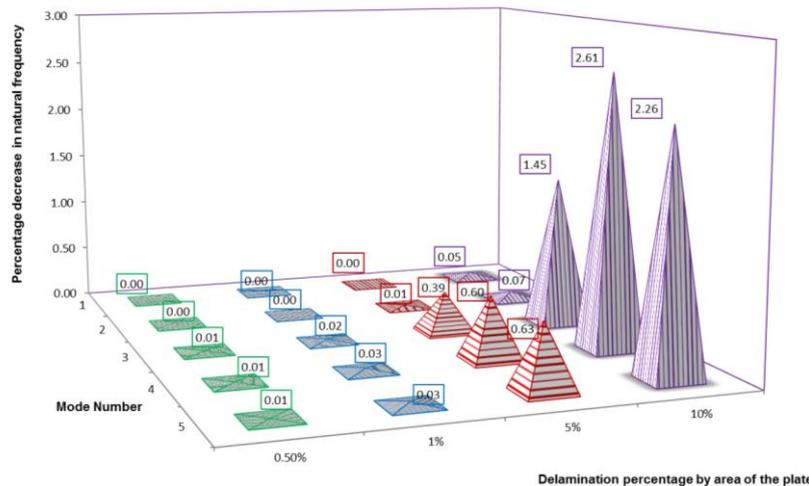


Fig. 5 Shift in natural frequency for one way clamped panel: delamination position 2

position for FF two way case (C-C-C-C) as described above. The extent of natural frequency reduction for different extents of delamination in the case of (S-S-S-S) end condition for position 3 is illustrated in Fig. 4. It is interesting to observe from Figs. 3 and 4 that the percentage reduction in natural frequency is less for S-S-S-S case when compared to C-C-C-C case for all the delamination percentages in general. This is confirmed true in the comparisons for other two positions as well. Thus it is revealed that when the panel is more restrained the influence on natural frequency reduction is more.

Comparison of percentage reduction in natural frequency for one way fixed case for the three positions of delamination reveals that the most critical position for this particular case is position 1. Fig. 5 shows shows the variation of percentage reduction in natural frequency due to various

percentages of interlayer delaminations in 1000 mm square slab panel with C-C-F-F end condition for delamination position 1, which is the worst location for this end condition. Furthermore it is of special interest to observe from all the comparisons explained above that delamination size plays a major role in reducing natural frequency thus leading to significant stiffness reduction for larger delamination size.

In contrast to C-C one way case, for SS one way case (S-S-F-F), the most critical position is position 2. The variation in natural frequency for this particular case for (S-S-F-F) end condition is shown below in Fig. 6. Careful observation of Figs. 3, 4, 5 and 6 confirms that the percentage of natural frequency decrease with respect to delamination follows different trends depending on the boundary condition, extent of delamination and location of the delamination.

Table 4 reports the natural frequency values in Hz for the first five modes of vibration for the four end conditions in the case of fully bonded plate as well as delaminated plates. Note here that

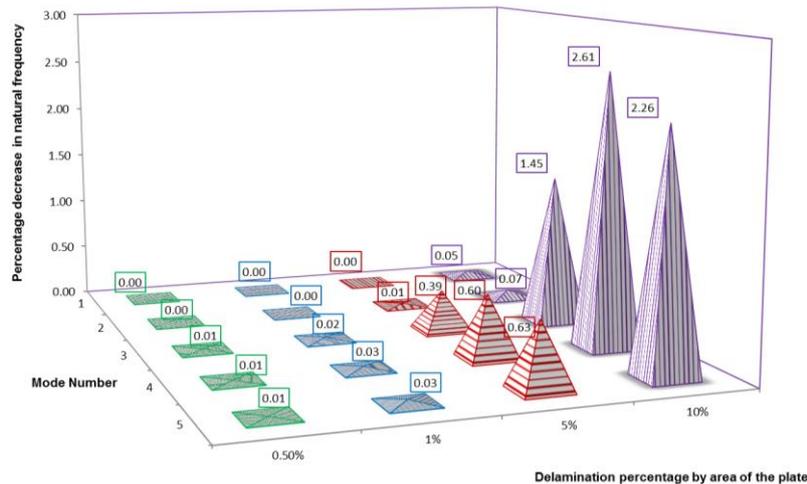


Fig. 6 Shift in natural frequency for one way simply supported panel: delamination position 2

Table 4 Natural frequency values in Hz for the position 3 delamination

Percentage delamination by total area of plate	Mode Number	C-C-C-C end condition	S-S-S-S end condition	C-C-F-F end condition	S-S-F-F end condition
0% (Fully bonded)	1	142.02	102.21	88.73	57.91
	2	274.00	198.46	98.34	69.72
	3	288.11	207.83	160.61	143.17
	4	392.65	311.21	235.67	154.84
	5	473.98	390.52	248.51	181.40
0.5% of plate area	1	142.02	102.21	88.73	57.91
	2	273.93	198.45	98.34	69.72
	3	288.03	207.82	160.61	143.17
	4	392.64	311.17	235.64	154.80
	5	473.98	390.51	248.51	181.40

Table 4 Continued

1% of plate area	1	142.02	102.21	88.73	57.91
	2	273.72	198.32	98.33	69.72
	3	287.80	207.68	160.61	143.17
	4	392.63	311.20	235.58	154.80
	5	473.96	390.51	248.51	181.40
5% of plate area	1	141.97	102.19	88.72	57.91
	2	269.13	196.04	98.28	69.68
	3	282.80	205.19	160.56	143.14
	4	391.66	310.74	234.05	154.32
	5	471.84	389.44	248.45	181.37
10% of plate area	1	141.60	102.08	88.65	57.89
	2	258.18	189.92	98.11	69.57
	3	270.82	198.45	160.23	142.91
	4	386.07	307.90	229.57	154.25
	5	459.11	382.62	248.05	181.14

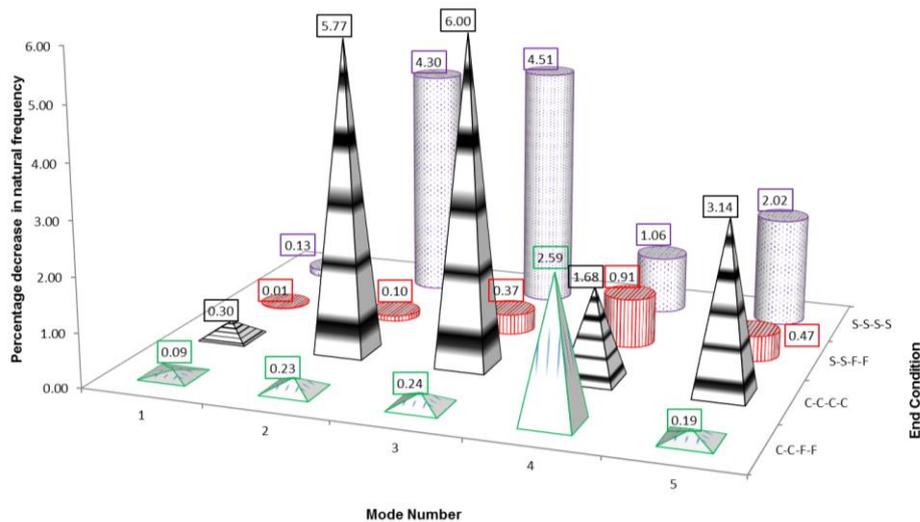


Fig. 7 Comparison of the shift in natural frequency for the four end conditions

the position considered is delamination position 3. It is obvious from Table 4 that the plate with C-C-C-C end condition, which has the highest end fixity, gives highest natural frequency variation. Close examination of Table 4 reveals that the fundamental natural frequencies remains the same when the percentage of delamination is not greater than 1%, for all four end conditions considered in the analysis. This leads to a major reflection that variations of free vibration characteristics are negligible for small delaminations, in the order of 1% of the plate area.

Fig. 7 illustrates the comparison of the variation in natural frequency with regards to the four

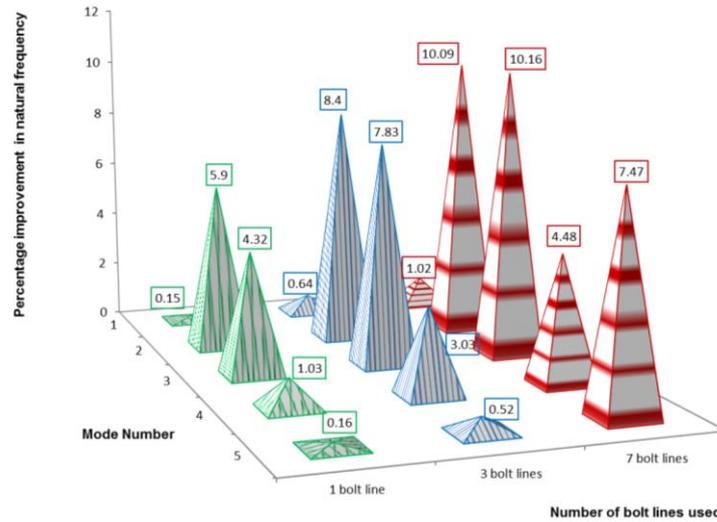


Fig. 8 Natural frequency improvement due to fastening the delaminated region in C-C-C-C plate

end conditions used in the analysis for similar conditions of location and extent of delamination. Here the location of interest is position 3 and the percentage of delamination considered is 10% by area of the plate. It is interesting to reveal that C-C-C-C end condition which has the highest end fixity exhibits the highest variation out of the four conditions considered in the simulation. Comparison between the percentage natural frequency decrease for end conditions C-C-F-F and C-C-C-C (see Fig. 7) reveals that higher end fixity attributes to drastic variations in natural frequency for identical sizes and locations of delamination.

3.3 Results for simulations with fastening the delaminated regions as a corrective measure

Here the plate with most adverse end condition and the corresponding delamination position have been selected to predict the efficiency of fastening (or some other form of joining) to improve the dynamic performance of a delaminated double layer panel. As such, the end condition considered here is two way clamped (C-C-C-C) with position 3 and 20% delamination by area of the panel.

Fig. 8 shows the percentage improvement in natural frequency (when compared with delaminated plate frequency) for the above mentioned panel with single bolt line, three bolt lines and seven bolt lines connected through the delamination region. It is witnessed that effectively bolting the delaminated region gives significant improvement of natural frequency when compared to the initially delaminated panel.

4. Conclusions

When the extent of delamination is small, its effect on natural frequency reduction is negligible. Although generally there is a trend that the extent of natural frequency variation with respect to

interlayer delamination increases with the mode number, this does not show the same trend for all cases and follows different trends depending on the boundary condition, extent of delamination and location of the delamination. Provided that the extents and locations of delamination are the same, the effect of delamination on natural frequencies appears prominently dependent on the end conditions of the panel, giving larger reduction in natural frequency when the panels are more restrained. The results further illustrated that the more the supports are restrained, the more the effect on free vibration behaviour due to delamination, especially in terms of natural frequency reduction. The findings reveal that the delamination size and the end fixity of the plate are the most important factors responsible for stiffness reduction due to delamination damage in composite laminates. Higher end fixity attributes to drastic variations in natural frequency for identical sizes and locations of delamination. Interestingly, the free vibration characteristics are negligible for small delaminations, in the order of 1% of the plate area, for the plates considered in the analysis. The results also suggest that fastening the delamination region is an effective corrective measure in decreasing the natural frequency variation hence improving its dynamic performance compared to the delaminated panel. Finally the results demonstrate the feasibility of non-destructive methods to detect delamination damage in practical composite structures.

References

- Agarwal, B.D., Broutman, L.J. and Chandrashekhara, K. (2006), *Analysis and Performance of Fiber Composites*, 3rd Edition, Wiley and sons Inc., Hoboken, New Jersey.
- Awad, Z.K., Aravinthan T. and Yan, Z. (2012), "Investigation of the free vibration behaviour of an innovative GFRP sandwich floor panel", *Constr. Build. Mater.*, **37**, 209-219.
- Burlayenko, V.N. and Sadowski, T. (2010), "Influence of skin/core debonding on free vibration behavior of foam and honeycomb cored sandwich plates", *Int. J. Nonlin. Mech.*, **45**(10), 959-968.
- Bunsell, A.R. and Renard, J. (2005), *Fundamentals of Fibre Reinforced Composite Materials*, IOP Publishing Ltd, Bristol, UK.
- Burlayenko, V.N. and Sadowski, T. (2011), "Dynamic behaviour of sandwich plates containing single/multiple debonding", *Comput. Mater. Sci.*, **50**(4), 1263-1268.
- Burlayenko, V.N. and Sadowski, T. (2012), "Finite element nonlinear dynamic analysis of sandwich plates with partially detached face sheet and core", *Finite Elem. Anal. Des.*, **62**, 49-64.
- Burlayenko, V.N. and Sadowski, T. (2014), "Nonlinear dynamic analysis of harmonically excited debonded Sandwich plates using finite element modelling", *Compos. Struct.*, **108**, 354-366.
- Carrera, E. (2002), "Theories and finite elements for multilayered, anisotropic, composite plates and shells", *Arch. Comput. Meth. Eng.*, **9**(2), 87-140.
- Chakrabarti, A. and Sheikh, A.H. (2009), "Vibration and buckling of sandwich laminates having interfacial imperfections", *J. Sandw. Struct. Mater.*, **11**, 313-328.
- Chutima, S. and Blackie, A.P. (1996), "Effect of pitch distance, row spacing, end distance and bolt diameter on multi-fastened composite joints", *Compos. Part A*, **27**(A), 105-110.
- Della, C.N. and Shu, D. (2005), "Vibration of beams with double delaminations", *J. Sound Vib.*, **282**, 919-935.
- Della, C.N. and Shu, D. (2007), "Free vibration analysis of delaminated bimaterial beams", *Compos. Struct.*, **80**, 212-220.
- Diamanti, K. and Soutis, C. (2010), "Structural health monitoring techniques for aircraft composite structures", *Prog. Aerosp. Sci.*, **46**(8), 342-352.
- Duggan, M. and Ochoa, O. (1992), "Natural frequency behaviour of damaged composite materials", *J. Sound Vib.*, **158**, 545-551.
- Frostig Y. (2002), "Classical and high-order computational models in the analysis of modern sandwich

- panels”, *Compos. Part B: Eng.*, **34**(11), 83-100.
- Gallego, A., Moreno-García, P. and Casanova, C.F. (2013) “Modal analysis of delaminated composite plates using the finite element method and damage detection via combined Ritz/2D-wavelet analysis”, *J. Sound Vib.*, **332**, 2971-2983.
- Grouve, W.J.B., Warnet, L., De Boer, A., Akkerman, R. and Vlekken, J. (2008), “Delamination detection with fibre bragg gratings based on dynamic behaviour”, *Compos. Sci. Tech.*, **68**, 2418-2424.
- Hosur, M.V., Vaidya, U.K., Myers, D. and Jeelani, S. (2003), “Studies on the repair of ballistic impact damaged S2-glass/vinyl ester laminates”, *Compos. Struct.*, **61**, 281-290.
- Hu, J.S. and Hwu, C. (1995), “Free vibration of delaminated composite sandwich beams”, *AIAA J.*, **33**(10), 1911-1918.
- Ju, F., Lee, H.P. and Lee, K.H. (1995), “Free vibration of composite plates with delamination around cutouts”, *Compos. Struct.*, **31**, 177-183.
- Karunasena, W. (2010), “The effect of debonding on the natural frequencies of laminated fibre composite sandwich plates”, *Proceedings of 6th Australasian Congress on Applied Mechanics, ACAM 6*, Perth, Australia, December.
- Katnam, K.B., Da Silva, L.F.M. and Young, T.M. (2013), “Bonded repair of composite aircraft structures: A review of scientific challenges and opportunities”, *Prog. Aerosp. Sci.*, **61**, 26-42.
- Kim, H.S., Chattopadhyay, A. and Ghoshal, A. (2003), “Characterisation of delamination effect on composite laminates using a new generalised layerwise approach”, *Comput. Struct.*, **81**, 1555-1566.
- Kulkarni, S.V. and Fredericks, D. (1971), “Frequency as a parameter in delamination problem-A preliminary investigation”, *J. Compos. Mater.*, **5**, 112-119.
- Kwon, Y.W. and Lannamann, D.L. (2002), “Dynamic numerical modeling and simulation of interfacial cracks in sandwich structures for damage detection”, *J. Sandw. Struct. Mater.*, **4**(2), 175-199.
- Massabò, R. and Campi, F. (2014), “An efficient approach for multilayered beams and wide plates with imperfect interfaces and delaminations”, *Compos. Struct.*, **116**, 311-324.
- Mendelsohn, D.A. (2006), “Free vibration of an edge-cracked beam with a Dugdale-Barenblatt cohesive zone”, *J. Sound Vib.*, **292**, 59-81.
- Mujumdar, P.M. and Sunyanarayan, S. (1988), “Flexural vibration of beams with delaminations”, *J. Sound Vib.*, **25**(3), 447-461.
- Ramkumar, R.L., Kulkarni, S.V. and Pipes, R.B. (1979), “Free Vibration Frequencies of a Delaminated Beam”, *34th Annual Technical Conference Proceedings*, Reinforced/Composite Institute, Society of Plastics Industry, Section 22-E, 1-5.
- Reddy, J.N. and Miravete, A. (1995), *Practical Analysis of Composite Laminates*, CRC Press, Boca Raton, Fla.
- Schwartz, G.H., Rabinovich, O. and Frostig, Y. (2008), “Free vibration of delaminated unidirectional sandwich panels with a transversely flexible core and general boundary conditions-A high-order approach”, *J. Sandw. Struct. Mater.*, **10**, 99-131.
- Schwartz, G.H., Rabinovich, O. and Frostig, Y. (2008), “Free vibrations of delaminated unidirectional sandwich panels with a transversely flexible core-a modified Galerkin approach”, *Journal of Sound and Vibration*, **301**, 253-277.
- Senthil, K., Arockiarajan, A., Palaninathan, R., Santhosh, B. and Usha, K.M. (2013), “Defects in composite structures: Its effects and prediction methods-A comprehensive review”, *Compos. Struct.*, **106**, 139-149.
- Shu, D. and Fan, H. (1996), “Free vibration of a bimaterial split beam”, *Compos. Part B*, **27**(1), 79-84.
- Strand7 (2010), *Strand7 finite element analysis FEA software*, Release 2.4.1, Sydney, Australia. (website: www.strand7.com)
- Tracy, J.J. and Pardo, G.C. (1992), “Effect of delamination on the natural frequencies of composite laminates”, *J. Compos. Mater.*, **23**(12), 1200-1215.
- Van Erp, G. and Rogers, D. (2008), “A highly sustainable fibre composite building panel”, *Proceedings of the International Workshop on Fibre Composites in Civil Infrastructure-Past, Present and Future*, University of Southern Queensland, Toowoomba, Queensland, Australia, December.
- Wang, J.T.S., Lin, Y.Y. and Gibby, J.A. (1982), “Vibration of split beams”, *J. Sound Vib.*, **84**(4), 491-520.