

## Bending analysis of exponentially varied FG plates using trigonometric shear and normal deformation theory

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**Abstract.** In this paper, bending analysis of exponentially varying functionally graded (FG) plate is presented using trigonometric shear deformation theory (TSDT) considering both transverse shear and normal deformation effects. The in-plane displacement field consists of sinusoidal functions in thickness direction to include transverse shear strains and transverse displacement include the effect of transverse normal strain using the cosine function in thickness coordinate. The governing equations and boundary conditions of the theory are derived using the virtual work principle. System of governing equations, for simply supported conditions, Navier's solution technique is used to obtain results. Plate material properties vary across thickness direction according to exponential distribution law. In the current theory, transverse shear stresses are distributed accurately through the plate thickness, hence obviates the need for a shear correction factor. TSDT results are compared with those from other theories to ensure the accuracy and effectiveness of the present theory. The current theory is in excellent agreement with the semi-analytical theory.

**Keywords:** trigonometric shear deformation theory, exponential law, functionally graded plate

### 1. Introduction

The FGM is an enhanced composite where material properties vary gradually in the desired direction from one layer to another. The volume fractions of ingredient materials are paired continuously and smoothly due to location along a given domain direction of structure to obtain the ideal functioning and properties. The primary benefit of FG material is the removal of delamination failure at interfaces that occur with conventional laminated components when material properties change abruptly. Therefore, the FGM has numerous benefits compared to monolithic material and conventional laminated composites and sandwiches see Pradhan *et al.* (2019). Over the years, researchers have developed several theories to analyze the structural components of the FGM under different loading conditions such as mechanical, thermal and electrical loadings etc. Khan *et al.* (2019), Saleh *et al.* (2020) provided brief review of FGM and its analysis.

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## Appendix

The elements of stiffness matrix  $[K]$  in Eq. (19) are given as follows

$$\begin{aligned}
 K_{11} &= A_{11}\alpha^2 + A_{66}\beta^2, K_{12} = (A_{12} + A_{66})\alpha\beta, \\
 K_{13} &= -[B_{11}\alpha^3 + (B_{12} + 2B_{66})\alpha\beta^2], \\
 K_{14} &= A_{S11}\alpha^2 + A_{S66}\beta^2, \\
 K_{15} &= (A_{S12} + A_{S66})\alpha\beta, K_{16} = \left(\frac{\pi}{h}\right) A_{S13}\alpha, \\
 K_{22} &= A_{66}\alpha^2 + A_{22}\beta^2, \\
 K_{23} &= -[B_{22}\beta^3 + (B_{12} + 2B_{66})\alpha^2\beta], \\
 K_{24} &= (A_{S12} + A_{S66})\alpha\beta, \\
 K_{25} &= A_{S66}\alpha^2 + A_{S22}\beta^2, \\
 K_{26} &= \frac{\pi}{h} A_{S23}\beta, \\
 K_{33} &= D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4, \\
 K_{34} &= -[B_{S11}\alpha^3 + (B_{S12} + 2B_{S66})\alpha\beta^2], \\
 K_{35} &= -[B_{S22}\beta^3 + (B_{S12} + 2B_{S66})\alpha^2\beta], \\
 K_{36} &= -\left(\frac{\pi}{h}\right) (B_{S13}\alpha^2 + B_{S23}\beta^2), \\
 K_{44} &= (A_{SS11}\alpha^2 + A_{SS66}\beta^2 + Acc_{55}), \\
 K_{45} &= (A_{SS12} + A_{SS66})\alpha\beta, \\
 K_{46} &= \left(\frac{\pi}{h}\right) A_{SS13} + \left(\frac{h}{\pi}\right) Acc_{55} \alpha, \\
 K_{55} &= (A_{SS66}\alpha^2 + A_{SS11}\beta^2 + Acc_{44}), \\
 K_{56} &= \left(\frac{\pi}{h}\right) A_{SS23} + \left(\frac{h}{\pi}\right) Acc_{44} \beta, \\
 K_{66} &= \left(\frac{h}{\pi}\right)^2 Acc_{55}\alpha^2 + \left(\frac{h}{\pi}\right)^2 Acc_{44}\beta^2 + \left(\frac{\pi}{h}\right)^2 A_{SS33}
 \end{aligned}$$