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Optimization of aircraft fuel consumption and reduction of pollutant emissions: Environmental impact assessment

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Abstract. Environmental impact of aircraft emissions can be addressed in two ways. Air quality impact occurs during landings and takeoffs while in-flight impact during climbs and cruises influences climate change, ozone and UV-radiation. The aim of this paper is to investigate airports related local emissions and fuel consumption (FC). It gives flight path optimization model linked to a dispersion model as well as numerical methods. Operational factors are considered and the cost function integrates objectives taking into account FC and induced pollutant concentrations. We have compared pollutants emitted and their reduction during LTO cycles, optimized flight path and with analysis by Döpelheuer. Pollutants appearing from incomplete and complete combustion processes have been discussed. Because of calculation difficulties, no assessment has been made for the soot, H_2O and $PM_{2.5}$. In addition, because of the low reliability of models quantifying pollutant emissions of the APU, an empirical evaluation has been done. This is based on Benson's fuel flow method. A new model, giving FC and predicting the in-flight emissions, has been developed. It fits with the Boeing FC model. We confirm that FC can be reduced by 3% for takeoffs and 27% for landings. This contributes to analyze the intelligent fuel gauge computing the in-flight fuel flow. Further research is needed to define the role of NO_x which is emitted during the combustion process derived from the ambient air, not the fuel. Models are needed for analyzing the effects of fleet composition and engine combinations on emission factors and fuel flow assessment.

Keywords: environmental impact; airports; aircraft; fuel consumption; pollutant emissions

1. Introduction

Aircraft pollutant emissions have been of concern since the beginning of commercial aviation. The continuing growth in air traffic and increasing public awareness have made environmental considerations one of the most critical aspects of commercial aviation today. This means that pollutant emissions from aviation activity are expected to grow and increase by factor 1.6 to 10, depending on the fuel use scenario (FAA 2011). Levels of air pollution still have a significant risk to the environment and to human health. Air pollution is a local, pan-European and hemispheric issue. Pollutants released in one country have deep impacts in the atmosphere, contributing in poor air quality. The EU's long-term objective is to improve air quality by acting at many levels to

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reduce pollutant impacts (legislation; cooperation, international and regional authorities, non-governmental and research organizations). EU policies aim to reduce exposure by reducing emissions and setting limits for air quality. The European Environment Agency (EEA 2013) is the European Union's air pollution data centre, supporting the implementation of EU legislation related to air emissions for different sectors. It contributes to the evaluation of EU air pollution policies for long-term strategies. The European Commission has recently launched a comprehensive review of the EU air quality legislation, based on the 2005 Thematic Strategy on Air Pollution and Clean Air for Europe (CAFE) initiatives which included the National Emission Ceilings Directive (NECD). NEC Directive imposes emission limits, in particular, for emissions of four key air pollutants (nitrogen oxides, sulphur dioxide, non-methane volatile organic compounds and ammonia). Internationally, this is considered by the United Nations Economic Commission for Europe (UNECE 2013) and its protocols. The Gothenbourg multi-pollutant protocol contains national emission limits that, for the EU Member States, are either equal to or less ambitious than those in the EU NEC Directive. The Executive Body of the Convention has adopted amendments to the Convention's 1999 Gothenburg Protocol (Gothenbourg Protocols 2003, 2005). This Convention has been extended by eight protocols identifying specific measures amended in 2012, for 2020 and beyond. Engine manufacturers have developed low-emission combustors options. These combustors have been adopted by airlines operating in European airports with strict pollutant emissions controls (Kurniawan and Khardi 2011). Environmental impact of air traffic is often mainly associated with noise nuisance, smoke and gaseous emissions of Carbon Monoxide, Unburned Hydrocarbons - also referred to as Volatile Organic Compounds, including Methane - and Nitrogen Oxides (NO_x – include Nitrogen Oxide and Nitrogen Dioxide), Sulphur Oxides in the vicinity of airports. Particles, such as Particulate Matter $PM_{2.5}$ and PM_{10}, present the most serious adverse health impacts from aircraft pollutant emissions (Henschel *et al.* 2012). These have been controlled by implementation of standards and certification of aircraft engines. International Civil Aviation Organization (ICAO) has defined reference emissions Landing and Take-off (LTO) cycle, with specific thrust settings and so-called Time in Modes (TIM) for each operating mode, which reflects all aircraft operations in the boundary layer below the so-called inversion height (usually at about 1 km) (ICAO 2007). Over the past several years, the Pollutant Emissions Indices has declined steadily. However, considerably more progress has been made with HC and CO than NO_x. Current emission regulations have focused on local air quality in the vicinity of airports. ICAO has set an environmental goal to limit and reduce the effects of aircraft pollutant emissions on Local Air Quality (LAQ) from aircraft operations (ICAO 2007). Operations of aircraft are usually divided into two main parts (Zaporozhets and Khardi 2004):

- The Landing Take-off (LTO) cycle defined by ICAO includes all activities near the airport that take place below the altitude of 3000 feet (914 m). This therefore includes taxi-in and out, take-off, climb-out and approach-landing.
- Cruise is defined as all activities that take place at altitude above 3000 feet (914 m). No upper limit altitude is given. Cruise includes climb from the end of climb-out in the LTO cycle to the cruise altitude, cruise, and descent from cruise altitudes to the start of LTO operations of landing.

Method for assessment of environmental problems of aircraft pollutant emissions have been carried out. The use of some methods will require justification and reliability that must be demonstrated and proven. The use of different and separate methodologies causes a wide variation in results and there is some lack of information. We consider the main emission products from jet

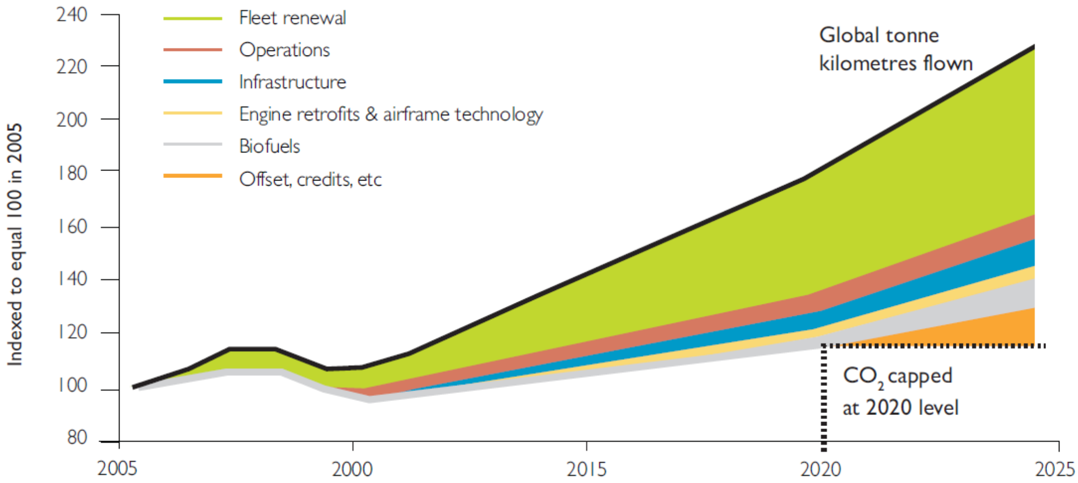

Fig. 1 Greenhouse gas emissions of the global aviation and development technology aiming to achieve carbon neutral growth by 2020 (IATA 2010a, b)

fuel combustion: Carbon Dioxide, water vapor, Nitrogen Oxides, Carbon Monoxide, Sulphur Oxides, Volatile Organic Compounds – unburned or partially combusted hydrocarbons –, Particulate Matter. It should be remembered that the main proportion of jet engine emission composition is CO_2 (Fig. 1) and H_2O produced by a complete combustion of hydrocarbon fuel.

A small subset of the VOCs and particulates are considered hazardous air pollutants (HAPs). Aircraft engine emissions are composed of about 70% CO_2, a little less than 30% H_2O, and less than 1% each of NO_x, CO, SO_x, VOC, particulates, and other trace components including HAPs. Aircraft emissions, depending on whether they occur near the ground or at altitude, are primarily considered local air quality pollutants or greenhouse gases. Water in the aircraft exhaust at altitude may have a greenhouse effect, and occasionally this water produces contrails, which also may have a greenhouse effect. About 10% of aircraft emissions of all types, except Hydrocarbons and CO, are produced during airport ground level operations and during landings and Take-offs. The bulk of aircraft emissions (90%) occur at higher altitudes. For Hydrocarbons and CO, the split is closer to 30% ground level emissions and 70 % at higher altitude. Emission from combustion processes CO_2 – Carbon Dioxide is the product of complete combustion of Hydrocarbon fuels like gasoline, jet fuel, and diesel. Carbon in fuel combines with Oxygen in the air to produce CO_2. Water Vapour is the other product of complete combustion as Hydrogen in the fuel combines with Oxygen in the air to produce H_2O. Nitrogen Oxides are produced when air passes through high temperature / high pressure combustion and Nitrogen and Oxygen present in the air combine to form NO_x. Hydrocarbons are emitted due to incomplete fuel combustion by an engine. They are also referred to as Volatile Organic Compounds. Many VOCs are also hazardous air pollutants. CO – Carbon Monoxide is formed due to the incomplete combustion of the carbon in the fuel. SO_x – Sulphur Oxides are produced when small quantities of Sulphur, present in essentially all Hydrocarbon fuels, combine with Oxygen from the air during combustion. Particulates – Small particles that form as a result of incomplete combustion, and are small enough to be inhaled, are referred to as particulates. Particulates can be solid or liquid. O_3 is not emitted directly into the air but is formed by the reaction of VOCs and NO_x in the presence of heat and sunlight. For this reason it is an important consideration in the environmental impact of aviation (ICAO 2007, 2011a, b). Compared to other sources, aviation emissions are a relatively small contributor to air quality concerns both with regard to local air quality and greenhouse gas emissions. While small, however, aviation emissions cannot be ignored. Emissions will be dependent on the fuel type, aircraft type, engine type, engine load and flying altitude. Two types of fuel are used. Gasoline is used in small piston engines aircraft only. Most aircraft run on kerosene and the bulk of fuel used for aviation is kerosene. In general, two types of engines exist; reciprocating piston engines and gas turbines (EEA-EMEP 2009). In general, a four factor in fuel consumption is reached between approaches and take-offs.

This paper presents in the first two sections methods and analysis, the third section gives the obtained results followed by a conclusion and recommendations.

2. Methods and analysis

Lyon International Airport (France) has been defined as a pilot site because of the availability of data which are regularly provided. The other reason of this choice lies in the fact that this airport is engaged for the environment within a specific framework with the help of the main environmental authorities and research laboratories. Little research concerns theoretical optimization of the CDA as a reduction of pollutant emissions. This mathematical modeling provides an alternative methodology to analyze the pollutant emissions of a variety of aircraft types. The case study for this airport yields multiple design guidelines on the development of the CDA procedures, which not only involve the arrival procedure for individual flights, but also suggest the priority of aircraft types. Comparisons of emission levels have been carried out taking into account the obtained theoretical optimized flight path and the used LTO cycles. Because LTO cycles are the main flight approach in this airport, measurements over a year around Lyon International Airport have been used in particular for SO_2, NO_x, HC, CO, PM_{10}, O_3 and CO_2 concentrations. They are compared to calculation values assuming that the aircraft performed an optimized flight path. This airport has two main parallel runways with a capacity of 9.6 million of passengers a year. It is located at 25 km East of Lyon (Fig. 2). The topography we have used in the aircraft emission modeling is:

(1) The runway features are as follow:
- Length of track A: 4000 m - Altitude of the 36L and 18R points are: 248 m and 231 m
- Length of track B: 2670 m - Altitude of the 36R and 18L points are: 250 m and 238 m
- Latitude and longitude of the 36L point are: (45° 42 ' 39.31 " N) and (5° 05 ' 24.34 " E)
- Width of each track: 45 m - outdistance between the tracks: 350 m
- Slope of the tracks compared to the North-South axis: $\alpha = 6.4°$

(2) Statistics of the traffic:
The number of movements presents a daily average of 334 in 2011 (UAF, 2012). All aircraft are considered in exception of A340, L1011, L188, B 727-200, B E3A, MD11-GE, TU154 and YAK42.

(3) Trajectories and procedures:
The general distribution of the traffic in 2006 is given in the following scheme. Because of the direction of the wind, 60% of the departures and 63% of arrivals are in the north direction.

Procedures implied a complexity, they are not straightforward, and it is necessary to follow a sequence of stages.

In this paper, the nominal used procedures are carried out and compared to optimized flight paths developed by authors: Alam *et al*. 2011, Cao *et al*. 2011, Sopjes *et al*. 2011, Khardi *et al*. 2011, Khardi 2012, Khardi 2011, Li *et al*. 2013. We have used the stabilized approach procedures by ICAO (ICAO 2011a, b, c, d). Considerations to be taken into account are given in Table 1 and parameters in Table 2. The standard takeoff procedures for some aircraft have been

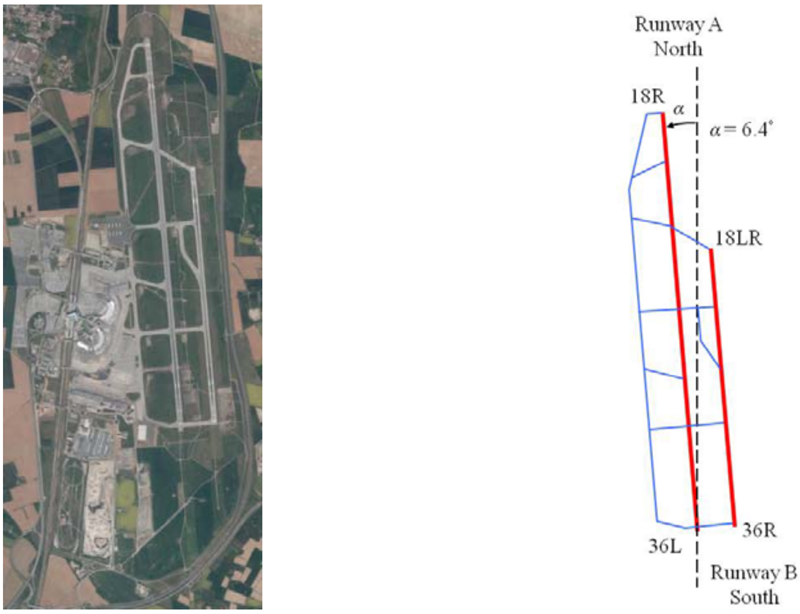

Fig. 2 Overview of Lyon International Airport (Geoportail© 2013)

Table 1 Standard and ICAO procedures

Standard procedure	ICAO A procedure	ICAO B procedure
Takeoff at full power	Takeoff at full power	Takeoff at full power
Climb to 1000 ft and pitch-over to accelerate	Cutback to climb power around 1000 feet AFE and pitch-over to accelerate	Climb to 1500 ft AFE at full power holding flaps
At full power, accelerate to clean configuration	Accelerate to clean configuration	Cutback to climb power at 1500 ft
Cutback to climb power	Climb to 3000 ft AFE	Climb to 3000 ft AFE at climb power holding flaps
Climb to 3000 ft AFE	Accelerate to 250 kts	Accelerate to clean configuration
Accelerate to 250 kts	Continued climb to 10000 ft AFE	Accelerate to 250 kts
Continued climb to 10000 ft AFE		Continued climb to 10000 ft AFE

Table 2 Input parameters (CPA: closest point of approach)

Flight step	Parameter	Input parameter
Takeoff	Weight	✓
	Speed (CAS)	
	Flaps ID	✓
Initial climb	Weight	✓
	Speed (CAS)	
	Flaps ID	✓
	Climb rate	
	Altitude at CPA	✓
Acceleration	Weight	✓
	Speed (CAS)	✓
	Flaps ID	✓
	Climb rate	✓
	Altitude at CPA	
Descent	Weight	✓
	Speed (CAS)	✓
	Flaps ID	✓
	Descent angle	✓
	Altitude at CPA	✓

Table 3 Speeds for procedure calculations (km/h) (ICAO, 2006)

Aircraft category	Vat	Range of speeds for initial approach	Range of final approach speeds	Maximum speeds for visual maneuvering
A	< 169	165/280(205)	130/185	185
B	169/223	220/335(260)	155/240	250
C	224/260	295/445	215/295	335
D	261/306	345/465	240/345	380
E	307/390	345/467	285/425	445
H	N/A	130/220	110/165	N/A

modified from an "ICAO B"-like procedure to one that applies cutback power at 1000 ft AFE. The ICAO B procedure is still retained as core standard.

Aircraft categories are referred by their letter designations as follows:

- Category A: less than 169 km/h (91 kts) indicated airspeed (IAS)
- Category B: 169 km/h (91 kts) or more but less than 224 km/h (121 kts) IAS
- Category C: 224 km/h (121 kts) or more but less than 261 km/h (141 kts) IAS
- Category D: 261 km/h (141 kts) or more but less than 307 km/h (166 kts) IAS
- Category E: 307 km/h (166 kts) or more but less than 391 km/h (211 kts) IAS

Table 4 Aircraft Rate of descent (ICAO, 2006)

Aircraft categories	Minimum	Maximum
A, B	120 m/min (394 ft/min)	200 m/min (655 ft/min)
C, D, E	180 m/min (590 ft/min)	305 m/min (1000 ft/min)

ICAO (2006) defined the adequate space for descent which is provided by establishing a maximum allowable descent gradient for each segment: the minimum/optimum descent gradient/angle in the final approach of a procedure with FAF is 5.2% / 3.0° (318 ft/NM). The maximum permissible is 6.5% / 3.7° (395 ft/NM) for A and B aircraft, 6.1% / 3.5° (370 ft/NM) for C, D and E, and 10 % / 5.7° for H. In the case of a precision approach, the operationally preferred glide path angle is 3.0°.

An ILS glide path / MLS elevation angle in excess of 3.0° is used only where alternate means available to satisfy obstacle clearance requirements are impractical. In certain cases, the maximum descent gradient of 6.5% (395 ft/NM) results in descent rates which exceed the recommended rates of descent for some aircraft (ICAO, 2006). The general recommendation of approach speeds and rate of descent are presented in the following tables.

As described by ICAO (ICAO 2006), non-standard approach procedures are those involving glide paths greater than 3.5° or any angle when the nominal rate of descent exceeds 5 m/sec (1000 ft/min). Procedure design takes into account:

(1) Increase of height loss margin
(2) Adjustment of the protection surfaces
(3) Re-survey of obstacles
(4) Application of related operational constraints

The height loss / altimeter margin should be verified by certification or flight trials to cover the effects of (ICAO 2006):

- minimum drag configuration and wind shear / control laws and handling characteristics;
- minimum power for anti-icing / GPWS modification / use of flight director / autopilot;
- engine spin-up time / Vat increase for handling considerations.

In addition, consideration should have been given to operational factors including configuration, engine out operation, maximum tailwind/minimum headwind limits, weather minima, visual aids and crew qualifications, etc.

2.1 Calculation of emission levels

ICAO Airport Local Air Quality Guidance Manual (2007) and the updated version (ICAO 2011a), can be used to assess the total pollutant emissions of CO, HC, SO_2, NO_x and CO_2. Airport Local Air Quality Study (ALAQS; Annex 1) aims to promote best practice methods for airport Local Air Quality (LAQ) analysis concerning issues such as emissions inventory, dispersion, and the data required for the calculations, including emission factors, operational data, and aircraft Landing and Take-off profiles. This methodology consists of developing Pan-European emission inventory methodology with spatial information and future application of dispersion modeling linked to GIS technologies. This objective is not achieved because of model reliability. In this paper, aircraft exhaust emissions are calculated for the following operating modes.

- Engine start / Taxi-in and taxi-out (TX, 7% thrust)
- Queuing (TX, 7% thrust) / Approach (AP, 30% thrust)
- Landing roll (AP, 30% thrust) / Take-off roll (TO, 100% thrust)
- Climb-out (CL, 85% thrust)

Other needed point is aircraft engine emissions during a particular operating mode of landing and take-off cycles which is given by the product of the Time in Mode, the fuel flow rate and the emission indices for the appropriate engine thrust setting engaged. We have used ICAO system database (aircraft-engine combination, number of engines etc.). The equation is shown below

$$ACe = FF_{mode} \times EF_{mode} \times T \times N$$

ACe is the aircraft total engine emissions for each LTO cycle. FF_{mode} is the fuel flow rate (kg/s) per engine in mode. EF_{mode} is the emission factor per engine in mode. *T* is the time in mode (sec). *N* is the number of engines. The latter is a starting point which cannot be used during optimization process. It could give us a rough idea on what is emitted in standard conditions. In this paper, we have used emission levels of pollutant expressed in Sourdine (2006)

$$EL_{seg} = \Delta T_{seg} \times \left[EF_{seg}(P_i) + \frac{P_{seg} - P_i}{P_{i+1} - P_i}\left(EF_{seg}(P_{i+1}) - EF_{seg}(P_i)\right) \right]$$

$$EF_{seg}(P_i) = EI(P_i) \times FF_{seg} \qquad \text{and} \qquad P_{seg} \frac{CNT_{seg}}{\text{Max Static Thrust}} \times 100$$

$EF_{seg}(P_i)$: the emission flow for the segment associated to power setting P_i (in g/s)

P_i: one of the tabulated engine power settings for which emission indices are provided in the data bank (7%, 30%, 85% or 100%). $EI(P_i)$: the emission indices associated to power setting P_i (in g/kg of fuel); P_{seg} = the segment-specific power setting (%). CNT_{seg}: the average corrected net thrust (lb) on the segment, calculated using the input CNT values at the two end-points of the segment. MaxStaticThrust: the engine-specific maximum sea level static thrust. EL_{seg}: the emission level of the pollutant produced on the segment (g). ΔT_{seg}: the duration (in seconds) of the flight segment.

ΔT_{seg} is calculated using the distance between the two end-points of the segment, divided by the average speed of the aircraft on the segment. P_i and P_{i+1} are the two tabulated power setting values bounding P_{seg} (%).

To calculate emission levels of different pollutants, it is necessary to have fuel flow information along the flight profiles. In this step, we used approximations by interpolations on input thrust values, as the ICAO databank provides fuel flow data associated to specific power settings. However, the ICAO – CAEP's Modeling Working Group considered that estimating fuel flow based on thrust was unsatisfactory without having a greater knowledge of individual aircraft / engine performance parameters. This point is subjected to a development of a new model of fuel consumption in the result section. As soon as optimal parameters of the flight path are obtained, they are used for calculating the pollutant levels. These assessments are carried out for the pollutants emitted on the outlet side of engines, at 1.5 m, in free-field. In addition, emission levels are implemented in a processing code of pollutant dispersion. Thus, concentrations of pollutants can be performed at any known distance around the airport. Comparisons are carried out with the empirical trajectories of the ICAO where the parameters and the procedures are known to calculate the levels of pollutants at the exit of the conduit of the engine, then to carry out calculations of dispersion (Annex 2). Another simple way consists to use the ICAO database of pollutants emitted by engines followed by dispersion calculation. This approach, performed under engine static conditions, is empirical and cannot give satisfactory results because the in-flight engine parameters are not considered.

2.2 Optimization modeling and resolution

The system of differential equations commonly employed in aircraft trajectory analysis is the following six-dimension system derived at the center of mass of the aircraft (Abdallah *et al*. 2010, Khardi *et al*. 2010, Alam *et al*. 2011, Li *et al*. 2013) and the fuel consumption given by Benson (1995)

$$
\begin{aligned}
&\dot{x} = v\cos\gamma\cos\chi \\
&\dot{y} = v\cos\gamma\sin\chi \\
&\dot{h} = v\sin\gamma \\
&\dot{v} = g(\frac{T\cos\alpha - D}{mg} - \sin\gamma) \\
&\dot{\gamma} = \frac{g}{v}(\frac{T\sin\alpha + L}{mg} co\varphi - \cos\gamma \\
&\dot{\chi} = -\frac{g}{v\cos\gamma}\left(\frac{T\sin\alpha + L}{mg}\right)\sin\varphi \\
&\dot{m} = -TSFC.T
\end{aligned}
$$

where V, γ, χ, α and μ are respectively the speed, the angle of descent, the yaw angle, the angle of attack and the roll angle. (x, y, h) is the position of the aircraft. The variables T, D, L, m and g are respectively the engine thrust, the drag force, the lift force, the aircraft mass and the aircraft weight acceleration. *TSFC* is the thrust specific fuel consumption which is depending on aircraft speed or Mach number, altitude and the net thrust per unit mass flow of the engines T_{net}, (Benson 1995, Li *et al*. 2013). This fuel consumption function is derived from the following Benson equation

$$FFT(t) = TSFC.T_{net}(t)$$

$$J\big(X(t), t_f; q\big) = \int_{t_o}^{t_f} -\dot{m}(t)dt = \big[m(t)\big]_{t_f}^{t_o} = m(t_o) - m(t_f)$$

where $m(t_0)$ and $m(t_f)$ are the initial and final aircraft mass. When $m(t_0)$ is a constant, we can write

$$\min J\big(X(t), t_f; q\big) \equiv \min\big(-m(t_f)\big) \equiv \max\big(m(t_f)\big)$$

The coupled general model can be written in the following optimization form as an optimized control problem “OCP”:

$$\begin{cases} \min\limits_{U \in \cup} J(X(t), U(t), t; q) \\ \dot{X}(t) = f(X(t), U(t), t; q) \\ \Phi_{\min} \leq \Phi(X(t_0), t_0, X(t_f), t_f; q) \leq \Phi_{\max} \\ C(X(t), U(t), t; q) \leq 0 \end{cases}$$

The objective function minimization is performed under dynamics, boundary and constraints. A set of them are collected and used as limit conditions. Optimized parameters, obtained by solving the OCP problem, are: Mach number - aircraft speed – Altitude - Throttle; net thrust / gross thrust - Fuel flow - V-exit / NPR / EPR / ETR; Engine efficiency - Flight angles describing the flight configuration.

Combination of models allows for a non-convex optimization problem. Non-convexity is raised from discreteness. The branch-and-bound scheme could be a possible way to solve the problem. The scheme operates by recursive partitioning or branching the feasible region in search of a global optimal solution. There are theoretical difficulties behind this idea. Bounds of the optimal objective values, which are based on solvable relaxation of parameters, cannot be used to decide whether to examine the branching. It is impossible for these problems to base analysis on integrality-based branching rules. It is a crucial challenge to develop for the coming years the tractable relaxation because of the semi-continuity and the guarantee of convergence. The reason we consider the problem by approximating the global maximum of a quadratic program subjected to bound and quadratic constraints transformation. To solve the OCP problem, we first consider a linear discrete time dynamical system and a time control. We optimize the system's behaviour on a finite time *T*. This makes possible a good coupling and resolution avoiding major arguments on the implicit convexity and symplecticity of our problem. Because of symplecticity, the six-dimensional properties of the previous system are not independent. Relationships among them reduce the number of degrees of freedom. Relationships are given depending on the in-flight functionalities of aircraft engines and procedures. Their forms are then described in derivations. Explicitly, the awaited behavior is modeled as a system of convex constraints on the trajectory by (Ben-Tal *et al.* 2006): $P_i + P_i w^T$ is a part of K_i. P_i is a given k_i-dimensional vector. P_i is the $k_i \times dimw^T$ matrices. K_i is sub-sets of R^{k_i}; they are given nonempty closed convex sets. We have specified the control law but not a completed state-space trajectory which depends on the control law and on inputs $d^T = (d_0, ..., d_T)$. We can write an uncertain optimization problem to solve this, similar to the one given by Ben-Tal *et al.* (2006), combining trajectory parameters and data: $\min_t\{w^t : p_i + P_i w^T\}$.

We used input data as a sequence vector. We assume that we closed the open-loop system. The control states are given by the OCP of flight dynamics. By proceeding in this manner, we can combine optimized flight path parameters, engine settings, and ICA BADA data. Quasi-relaxation techniques could be used to solve the first steps of the given problem. They are considered in particular before applying dispersion model of pollutants. Dynamical constraint assumptions are needed during this processing step. Thus, the Trust Region Sequential Quadratic Programming method has been used for the processing steps (Tenny *et al.* 2002, Alam *et al.* 2011, Cao *et al.* 2011, Khardi *et al.* 2011a, Li *et al.* 2013). It has the potential to solve complex problems of the control theory and can be generalized for air traffic. It has been tested for computational efficiency and stability. It is largely superior over conjugate gradient methods and can out-perform the quasi-Newton methods. The main objective is to diagnosis and to control, in-flight and in real-time, flight paths taking into account the FMS (flight management system) and the AMS (airspace management system) updates and to be interfaced with the Lagrangian dispersion model of pollutant emissions.

Derivatives are approximated by numerical INTLAB derivation method. Discretization is solved by SNOPT optimization algorithm. An AMPL (A Modeling Language for Mathematical Programming) (AMPL), combined with NLP solver, has been implemented for processing. Implementation has been performed under GPOPS-MATLAB®[24] software (with an Intel Core6 Quad processor). We analyze the processing speed and algorithm efficiency and their ability to be interfaced with the in-flight management system respecting airspace system constraints. Comparisons are performed stressing the computing times.

2.3 Processing inputs

Internal engine data (mass flows, temperatures and pressures, thrust, fan pressure ratio and internal engine heat cycle) are used following ICAO recommendations (ICAO 2008, 2011b) for the prediction of aircraft engine emissions. We have also used EngineSim code (2012) to predict aircraft engine emissions during operation depending on engine performance (compressor - turbine performance mapping) (Benson 1995). We considered:

- In-flight conditions / Mach number / Airspeed
- Altitude / Pressure / Temperature / Throttle and afterburner settings
- Pressure and temperature are assessed by the standard day atmospheric model
- Compressor (CPR, compressor efficiency)
- Burner (fuel, maximum temperature, efficiency, pressure ratio)
- Geometrical features of engines (size, inlet and outlet diameters)
- Variables include flight conditions, the engine features, its performance, compressor and turbine performance
- Fuel sulphur is close to 0.41 g/kg and the soot corresponds to 1.7 10^{14} particles/kg of the burned fuel.

The following features are considered for solving the coupled problem:

- Net thrust is 131.2 kN per engine (Two 262.4 kN General Electric CF6-80C2A1s)
- Max take-off 165900 kg. Operating empty 90965 kg
- Initial take-off mass m_{T0} = 140000 kg / Initial landing mass m_{LA} = 110000 kg
- T = 600 seconds
- Climb speed / Cruise speed / Descent speed: 250 kts / 300 kts / 0.78 M
- Maximum speed: CAS: 350 kts
- Stall speeds (kts, CAS):
    - Cruise (145)- Initial climb (129)
    - Take-off (118) - Approach (106) - Landing (full - 103)

In addition, area of the zone concerned with the study, around Lyon International Airport, is about 2000 m^2 centered on the aircraft touchdown point (50 km*40 km).

We have assumed that pollutants are emitted in standard atmosphere conditions which are not validated, in particular for altitudes below 3000 ft. Another limitation is due to the assumption that emission vary linearly with the thrust level. Optimized solution is achieved with KNITRO through the following optimality conditions:

- Average speedup = 43.7 / final feasibility error (abs. / rel.) = $3.3e^{-15}$ / $8.5e^{-18}$
- final optimality error (abs. / rel.) = $1e^{-13}$ / $1e^{-15}$
- Number of processors = 6 / total program time = 17738 sec
- time spent in evaluations = 9815 sec

3. Results

Local optimal solutions are obtained with an average order of feasibility error of 10^{-15}. The flight rate descent varies between 900 and 1,100 ft/mn which is close to that recommended by ICAO and practices by pilots. Two possible optimized solutions for flight paths are obtained. The first solution is a soft one-segment approach which puts the aircraft in an appropriate envelope with margins for wind uncertainties and errors. The second possible optimized flight path solution is the Shortest and Fastest Continuous Descent Approach (SF-CDA). It is a two-segment approach reducing aircraft environmental impact. Results show that this solution is well appropriated for aircraft trajectory optimization problems and could be easily implemented. The two obtained trajectories, shown in the Fig. 3, could be accepted into the airline community for a number of reasons including operational effectiveness and environmental impact reduction.

On the one hand, as observed, it is clear in our analysis that the continuous descent approach CDA starts at 8,500 ft (2,600 m) until the touchdown point of the runaway. This is called the soft descent. The flight segment confirmed only that we have the rate descent of ~1,000 ft/mn. This

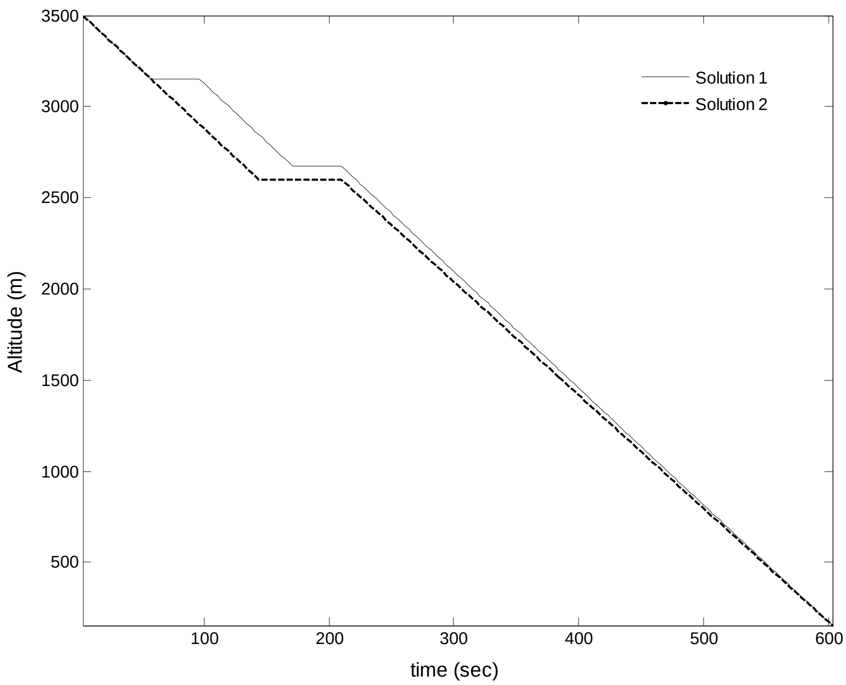

Fig. 3 Optimized flight paths for approach

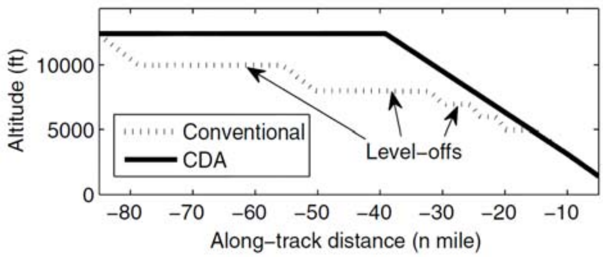

Fig. 4 Vertical profiles of a conventional approach and the CDA by Li *et al*. (2013)

CDA can reduce fuel burn, noise impact and pollutant emissions. It happens with no level flight segment at 8,500 ft. On the other hand, Shortest and Fastest Continuous Descent Approach is characterized by two segments before reaching the altitude of 9,000 ft. The second solution is slightly higher in altitude than the first one. Solutions 1 and 2 are considered as a CDA approach because no flight level after 9,000 ft and 8,500 ft respectively exists. We confirm that CDA is a continuous, idle-thrust descent without any level-offs. CDA is considered as an approach without segments under a certain altitude and others above that altitude are allowed. No definition of the CDA's speed profile has been suggested in the open literature. The CDA avoids segments at low altitude. This contributes to fuel consumption and annoyances (noise and pollutant emissions).

In this paper, the continuous descent starts at 8500 ft with the presence of a segment between 11,400 ft and 8,500 ft. This is depending on the condition limits which we have introduced during the optimization processing steps. We have considered that the end of the cruise (or the entrance point to the airport zone) happened at 11,400 ft. If we increased this condition limit to 13,000 ft or 14,000 ft, the continuous descent would obviously start earlier at a higher altitude. Li *et al*. (2013) explained that, conventionally, while descending, an aircraft flies levels till the starting point of the CDA. They are assigned by the air traffic controller allowing the aircraft to meet a variety of constraints.

Analysis by Li *et al*. (2013), given in Fig. 4, confirms our obtained results. A segment occurred before the starting point of the continuous descent. This behavior is commonly observed in optimization analysis of flight paths in a number of investigations (Alam *et al*. 2011, Cao *et al*. 2011, Li *et al*. 2013, Sopjes *et al*. 2011).

Fuel consumption model

We have used flight optimized parameters in connection with the Base of Aircraft Data (Eurocontrol 2009a, b) for building a new fuel consumption model implicitly depending on the net power thrust of engines. This improves exiting modelization's attempts. On the one hand, in-flight fuel consumption FC can be empirically written as:

$$FC(t) = N_{Ref} \sqrt{\frac{\delta_{amb}^{\gamma}}{\theta_{amb}^{3\gamma+0.24}}}$$

N_{Ref} is a normalization factor giving the fuel consumption behavior on the ground during engine tests versus the EPR (engine power settings). On the other hand, the in-flight fuel mass is expressed as

$$m_{fuel} = \frac{\delta_{amb}}{\theta_{amb}^{3\gamma+0.24} \cdot e^{0.24M^2}} m_{fuel,Ref}$$

where $\delta_{amb} = \dfrac{p(Pa)}{101325}$ and $\delta_{amb} = \dfrac{T(K)}{288.15}$.

We have empirically found that:

$m_{fuel,Ref}$ is the fuel consumption on the ground during engine tests versus the EPR and thrust setting where its behavior is easily obtained for each type of combination of aircraft-engines.

γ is called the concentration ratio of fuel consumption which is found to be in the following interval: $\gamma \in [1.02, 1.074]$.

If $1.02 \leq \gamma \leq 1.04$, FC is a similar to the model of FC performed by Boeing (Dubois and Paynter 2006). This new model gives reliable approximations of fuel consumption and emissions. This coupled model allows the quantification of aircraft emissions in order to provide their reliable inventories and their use as inputs for climate models, technological tools implementation (in-flight fuel saving), and inventories of emissions for airlines.

As shown in Fig. 5, theoretically, the use of optimized flight paths confirmed that fuel consumption can be reduced by 3% for takeoffs and 27% for landing. In 2011, 122179 aircraft movements at Lyon International Airport were recorded (UAF 2012). This corresponds to an average fuel reduction of 367 tons for takeoffs and 659 tons for landing.

Pollutant emission assessment

The flight path is segmented and the optimal fuel consumption is calculated for each trajectory segment. FC is assessed depending on optimal flight path parameters and aircraft engine functionalities (in-flight procedures). Concentrations of pollutants, called emission levels, are

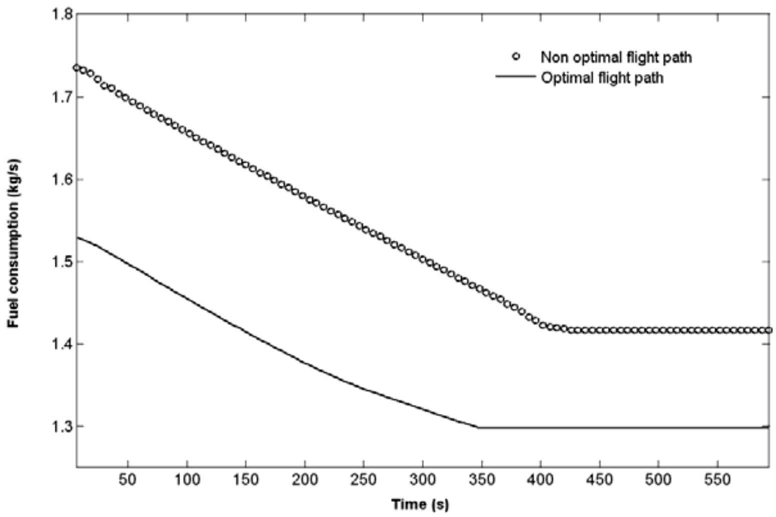

Fig. 5 Fuel consumption during approach

Table 5 Average pollutant reduction for a year

	Reduction (LTO/OFP)
CO_2	-2%
O_3	-3%
PM_{10}	-6%
CO	-6%
HC	-8%
NO_x	-23%
SO_2	-24%

estimated using inputs data of aircraft engines which are based on BADA; those emission levels are extrapolated using the aircraft dynamics and engines settings at 1.5 m. Dispersion model, describes in the appendix, is used to calculate emission levels under the flight path and at lateral distances of approximately ± 400 m of this flight projection on the ground within 50 km*40 km surface.

With the aim of carrying out comparisons showing the interest of the in-flight optimization, calculations were carried out between emission levels obtained with LTO cycles and optimized flight path (OFP). For a year measurements, average reduction is in table 5. In order of percentage, the major obtained reductions concerned SO_2, NO_x, HC, CO, PM_{10}, O_3 and CO_2.

4. Conclusions

Flight path optimization is designed for minimizing aircraft fuel consumption and environmental impacts around airports, in particular gaseous and particulate matter emissions. It fits with European and international objectives aiming to reduce levels of air pollution which have a significant risk to the environment and to human health. It is a significant contribution to the EU air quality legislation initiatives of the transportation systems, and to the protocols of the United Nations Economic Commission for Europe. Thus, this paper gives flight path optimization model linked to a Lagrangian dispersion model as well as numerical methods and algorithms. The major difficulty concerns how to select and use the best model for piloting the aircraft. Aerodynamic model, calculating external forces, is first developed in this paper. The model of the corrected net thrust of engines has also been empirically given and EngineSim code used. We solve the problem of how to fly the aircraft and which types of orders to use. We consider the real behavior of the aircraft avoiding undesirable oscillations. Neither human model nor automatic pilot is considered. We avoid this problem by using high level orders (slope, speed, attack angle) which simplify equations containing fast dynamics including moments. Operational factors including configuration, engine functionalities, weather limits and visual aids are considered. The cost function integrates the described objectives taking into account pollutant emission concentrations and fuel consumption.

Two possible optimized flight path solutions, reducing aircraft environmental impact and favoring fuel consumption saving, are used. Because computing power has increased substantially, complex problems can be solved for large variety of projects. In this paper, our coupling model offers a substantial advantage among disaggregated methods in terms of computing time, discretization complexity and result efficiency. The obtained results confirm the best formulation of this coupled problem, designed with partial empirical data, its effective resolution, and make comparisons possible with existing empirical models (EPR EngineSim and fuel consumption). They also confirm that optimized aircraft flight paths are suitable for fuel saving and emission reduction. We have also compared pollutants emitted during LTO, optimized flight paths and with analysis by Döpelheuer. In the order, the major obtained reductions between LTO and OFP cycles concern SO_2 (-24%), NO_x (-23%), HC (-8%), CO (-6%), PM_{10} (-6%), O_3(-3%) and CO_2 (-2%). It should be remembered that CO and PM appeared from an incomplete combustion process, and SO_x occurred during the combustion as sulphur is present in small quantities in hydrocarbon fuels. Comparisons with analysis by Döpelheuer indicate the following reduction: CO_2 (-13%), CO (-22%), SO_2 (-25%) and NO_x (-34%). Because of calculation difficulties and model reliability, no assessment has been made for the soot, H_2O and $PM_{2.5}$. In addition, because of the low reliability of the available models quantifying pollutant emissions of the APU (annex 3), and in spite of the difficulties of calculation, an empirical evaluation has been done. This is based on Benson's fuel flow method applied to aircraft operations on the ground around the airport. We show, using approximated and extrapolated levels from fuel consumption, that significant reduction of HC, CO, NO_x, CO_2 and SO_2 emissions can be obtained. A new model, giving fuel consumption and predicting in-flight aircraft engine emissions, is developed and coupled with flight and dispersion of pollutants models. Under some assumptions, our model can be fitted with the fuel consumption model performed by Boeing. We have confirmed that fuel consumption can be reduced by 3% for takeoffs and until 27% for landing. For a year movements at Lyon International Airport and using OFP, fuel reduction is about 367 tons for takeoffs and 659 tons for landing. This finding contributes to analyze the coming intelligent fuel gauge computing the in-flight aircraft fuel flow. This can be able to provide accurate details on fuel remaining, trip fuel, total fuel used, fuel consumption rate and the remaining time of flight versus the flow rate.

To conclude, this model allows the quantification of aircraft emissions in order to provide their reliable inventories, their use as inputs for climate models, technological tools implementation, inventories of emissions for airlines, and aircraft impacts on the health of population around airports. Further research is needed for incoming alternative fuels producing less particulate matters and SO_x. It is also needed in order to validate dispersion models existing in the open literature. Connection between models (flight path optimization - emissions – dispersion) has also to be improved. It will also be necessary to precisely define the role of NO_x which are emitted during the combustion process derived from the ambient air, not the fuel itself containing only trace amounts of fuel-bound nitrogen (because of storage stability problems, NO_x is quite absent). Models are needed for analyzing the effects of fleet composition in terms of aircraft types and engine combinations on emission factors, fuel flow assessment using performance and operational modes. Development of a new concept of an optimized APU reducing the ground pollutant emission reduction is necessary.

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EC

Annex

Annex 1 (Estimation of aircraft emissions)

Hourly calculations by Henschel *et al.* (2012), in μg/m^3, of SO_2, O_3, PM_{10}, NO_2, NO, Benzene, Toluene as well $PM_{2.5}$, Black Smoke, NO_x, CO, and Black Carbon are used. Method for the estimation of aircraft emissions affecting ground level air quality is based on the ICAO methodology. The ICAO Airport Local Air Quality Guidance Manual (2007) used three approaches – simple, advanced and sophisticated, quantifying aircraft engine emissions. In the simple approach, NO_x, HC, CO, SO_2 and CO_2 emissions and fuel consumption can be calculated as follows (ICAO 2007).

$$EX = \sum_{\text{All Aircraft}} (\text{Number of } LTOs_{\text{of Aircraft } Y}) \times (\text{Emission Factor}_{f \text{ or Species } X})$$

EX (kg) is emission of species X (kg). $EX = \sum_{\text{All Aircrt}} (N\_LTO_Y) \times (\text{Fuel Consumption})$

FC: fuel consumption (kg). N_LTO_Y is the number of LTO of the aircraft Y.

The previous formula simply recalls how to calculate the pollutant masses. To not overburden this appendix, we did not give all intermediate expressions which are necessary to lead to this expression of EX. Details can be found in the literature review given in this paper.

Annex 2 (Dispersion model of pollutants)

A Lagrangian dispersion model has been used (Van Ulden 1978, Ferrero and Anfossi 1998, Vilhena *et al.* 1998, Carvalho *et al.* 2007) considering optimized flight parameters obtained in the previous section. We have solved the Langevin equation based on Picard's Iterative Method which is suggested, in particular, by Carvalho *et al.* (2005). Langevin equation for inhomogeneous turbulence and in a stationary atmosphere has been written versus the turbulent velocity as

$$\frac{du_i}{dt} = a_i(x_i, u_i)dt + b_i(x_i, u_i)\mu_i(t)$$

u_i the turbulent velocity, $a_i(x_i, u_i)\,dt$ is the deterministic term, $b_i(x_i, u_i)\,\mu_i(t)$ is the stochastic term and μ_i is a normally distributed random increment. Each pollutant movement is described as: $dx_i = (U_i + u_i)\,dt$. As described by Carvalho *et al.* (2005), the deterministic coefficient a_i depends on the Eulerian PDF of the turbulent velocity and it assessed using Fokker–Planck equation given by Rodean (1996). Series of Hermite polynomials is associated with the Gram–Charlier PDF truncated to the fourth order can be expressed as (Carvalho *et al.* 2005)

$$P(r_i) = \frac{e^{-\left(\frac{r_i^2}{2}\right)}}{\sqrt{2\pi}}\left[1 + C_3H_3(r_i) + C_4H_4(r_i)\right]$$

r_i: turbulent velocity standard deviation. H_3 and H_4: Hermite polynomials. C_3 and C_4: Hermite polynomial coefficients obtained by

$$C_i = \frac{1}{i}\int_{-\infty}^{+\infty} P(r)H_i(r)dr$$

Gaussian turbulence induces that $P(r)$ is considered as a normal distribution, considering C_3 and C_4 equal to zero. The third order Gram–Charlier PDF is obtained with $C_4 = 0$. Carvalho *et al.* (2005) give detailed theoretical considerations and application of the Picard's Iterative Method and assumptions solving the presented and applied problem. Turbulence parameterization schemes are performed introducing the wind velocity variances and the Lagrangian decorrelation time scales. The applied classical statistical diffusion theory allows obtaining spectral properties of the turbulent mechanisms.

Annex 3 (APU emissions assessment)

Assessment of auxiliary power unit (APU) emissions before taking-off, after the touchdown point of the aircraft, and during taxiing, is not regulated by any certification standards (Henschel *et al*. 2012). It has been empirically carried out by the ICAO simple approach (ICAO 2011). There is no existing APU emission database unlike for aircraft engine emissions. Because of the low reliability of the available models quantifying pollutant emissions of the APU, and in spite of the difficulties of calculation, an empirical evaluation has been done. We used Benson fuel flow method applied to aircraft operations on the ground (in the airport) coupled to the ICAO simple for calculating only the total APU emissions for CO, NO_x and HC (*X* pollutant) for a year of air traffic at Lyon International Airport

$$E_X = \left(N\_LTO_{sh} \times EV_{XY} * 1000\right) + \left(N\_LTO_{lh} \times EV_{XZ} * 1000\right)$$

E_X: APU emissions [kg] of pollutant *X* produced by aircraft types (a year). EV_{XY}: emission value of pollutant *X* [g/LTO] for short flight. EV_{XZ}: emission value of pollutant *x* [g/LTO] for long-haul flight. N_LTO_{sh}: number of *LTO* (short-haul). N_LTO_{lh}: number of *LTO* (long-haul). On the one hand, VOC emissions due to aircraft re-fuelling, aircraft de-icing, fuel farms, hydrant systems, vehicle refuelling stations, fire training and aircraft / airport maintenance facilities are not considered (Henschel *et al*. 2012).