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Identification of flutter derivatives of bridge decks using stochastic search technique

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Abstract. A more applicable optimization model for extracting flutter derivatives of bridge decks is presented, which is suitable for time-varying weights for fitting errors and different lengths of vertical bending and torsional free vibration data. A stochastic search technique for searching the optimal solution of optimization problem is developed, which is more convenient in understanding and programming than the alternate iteration technique, and testified to be a valid and efficient method using two numerical examples. On the basis of the section model test of Sutong Bridge deck, the flutter derivatives are extracted by the stochastic search technique, and compared with the identification results using the modified least-square method. The Empirical Mode Decomposition method is employed to eliminate noise, trends and zero excursion of the collected free vibration data of vertical bending and torsional motion, by which the identification precision of flutter derivatives is improved.

Keywords: flutter derivatives extraction; parameter identification; empirical mode decomposition; stochastic search technique; section model.

1. Introduction

Self-excited forces of bridge decks may be expressed as the linearized functions of small amplitude sinusoidal displacements, velocities and flutter derivatives (Scanlan and Tomko 1971). In this work, a free vibration method of identifying flutter derivatives of bridge decks was discussed in detail. The uncoupled and coupled terms are obtained from pure vertical bending and torsional free oscillations and coupled oscillation data individually. Such system identification method as extended Kalman filtering (Yamada, *et al.* 1992) has been employed to extract all the 8 flutter derivatives that giving the lift force and torsional moment due to torsional rotations/velocities and vertical displacements/velocities from coupled oscillation data of bridge decks. In this method, the time histories of the displacement and velocity as well as the information of the initial condition are both required, which can simplify the experimental procedure significantly. Combining control theory and system identification technique, flutter derivatives for the Great Belt East Bridge were extracted

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(Poulsen, *et al*. 1992). Modified Ibrahim Time-domain (MITD in short) method was developed to extract all the direct and cross derivatives simultaneously from noisy displacement time histories (Sarkar and Scanlan 1994). This method requires selection of the time shifts N_1 and N_2, and the parameters play an important role in computation precision and convergence. Sarkar and Scanlan found an effective way to select the two time shifts close to optimal values. Combining with the extended Kalman filter, a method of simultaneous identification of all the 8 flutter derivatives from free oscillation data was proposed (Iwamoto and Fujino 1995). In this method, the uniqueness problem in reducing the identified quantities to non-dimensionalized flutter derivatives was discussed. In addition, some difficulties in identifying aerodynamic unsteady coefficients at high wind speed near the flutter onset speed were discussed. Such conclusion as the increase of mass and inertial moment of the section model leads to better identification precision at higher wind speed was drawn. The method for the determination of flutter derivatives by employing conversion of buffeting response data to the response covariance function estimate was proposed (Jokobsen and Hansen 1995). In this article, ambient vibration data were analyzed by a system identification method valid for a linear structure driven by a linearly filterd white noise loading process. Covariance block Hankel matrix (CBHM in short) method was used for parameter extraction of a 2-DOF system (Brownjohn and Jakobsen 2001). The Eigensystem Realization Algorithm (ERA in short) was developed for the identification of flutter derivatives from free vibration histories of section model of bridge decks (Ma, *et al*. 2006). Combining with the Random Decrement Technique (RDT in short), the flutter derivatives in turbulent flows can be extracted using ERA. The Covariance-driven Stochastic Subspace Identification (CSSI in short) technique was presented for the estimation of 8 flutter derivatives from the responses of bridge decks (Qin, *et al*. 2004). This technique can be applied to signals collected in both smooth and turbulent flow.

The unifying least-squares method was presented for extracting 8 flutter derivatives from the coupled free vibration data of 2-DOF model (Gu, *et al*. 2000). In this method, a unified error function which is linearly composed of two error components of vertical bending and torsional motions is defined as the objective function to optimize the flutter derivatives. Nevertheless, if distinct difference exists in quantity between the two error components, unsatisfactory identification precision may occur. In order to improve the precision, the modified least-square method for adding weights to error components was proposed subsequently (Ding, *et al*. 2001). In addition, the weighting ensemble least-square method was developed to extract 8 flutter derivatives of bridge decks (Li, *et al*. 2003). In this method, several vibration records at the same wind speed are regarded as an ensemble. It is simultaneously fitted to identify the mode parameters by nonlinear least-square method in the sense of minimizing the total error function.

The above researches all focused on 8 flutter derivatives identification of section model. Identification of all 18 flutter derivatives of streamlined bridge deck was first attempted by Singh (Singh, *et al*. 1995). The general least-square theory and experimental system of bridge deck sections was developed for identifying 18 flutter derivatives (Chen, *et al*. 2002). The identification precision may be improved by adding weights to error components of vertical bending, lateral bending and torsional vibration data. The iterative least squares approach was presented to identify all 18 flutter derivatives for a streamlined bridge deck and an airfoil section model (Chowdhury and Sarkar 2004). Without output covariance estimation, a stochastic subspace approach (SSA in short) identifying 18 flutter derivatives of section and aeroelastic models of bridges was developed (Xu 2006). This algorithm is superior in computation time saving to the covariance-driven stochastic subspace identification technique. The free vibration method for extracting flutter derivatives has been widely used for its simplicity and less expenditure.

The above least-square method and its generalized ones commonly apply alternate iteration technique to obtain solutions, and the same lengths of the vertical bending and torsional vibration histories is necessary. Nevertheless, the solution precision of alternate iteration technique is closely relevant with and sensitive to the initial selected values of mode parameters, and the solution can not be convergent sometimes. In addition, vertical bending vibration decays rapidly at high wind speed, and the ratio of noise to signal tend to be higher for the latter signal. In such cases, it is reasonable to use the former part of vertical bending vibration signal and relatively longer torsional signal to improve identification precision, i.e., different lengths of vertical bending and torsional vibration signals are preferable. But the existent model is only applicable to the case of the same length of vertical bending and torsional signals.

Undoubtedly, the collected signals are inevitably contaminated by many sorts of noises. In most cases, it is assumed that the measurement noises are Gaussian zero-mean processes. Nevertheless, such an assumption is not the verity, but a kind of approximation. Especially, at higher wind speed, the noise of vibration signal is very strong and the vertical bending vibration signal is very short due to high damping. Long-period trends exist occasionally in the vibration responses. Empirical Mode Decomposition (EMD in short) method (Huang, *et al.* 1998) is an effective way to filter and reduce the noises and eliminate the trends and zero excursion of the collected data.

Then, in this work, a more applicable optimization model that is suitable for time-varying weights for fitting errors and different lengths of torsional and vertical bending vibration data is presented, in which the demand of the same lengths of vertical bending and torsional time-series relaxes. Basing on this optimization model, a stochastic search technique for extracting flutter derivatives of bridge decks is subsequently developed. Compared with the alternate iteration technique, the stochastic search technique is more convenient for both understanding and programming. Before the extraction of flutter derivatives from the collected data, the EMD method is carried out beforehand to filter and reduce the noises and eliminate the trends and zero excursion mingled with the true signals, by which the identification precision can be improved.

2. Application of Empirical Mode Decomposition to flutter derivatives extraction

The essence of EMD is to identify and extract the intrinsic different oscillatory modes by their respective characteristic time scales in the data. Such modes represent for the oscillation imbedded in the data and may be defined as Intrinsic Mode Functions (IMFs in short). In this study, the EMD method is employed to process the free vibration signals and improve the identification precision of flutter derivatives of bridge decks. The detailed procedure of EMD is not discussed in this work, one may see Huang, *et al.* (1998).

Certain vertical bending and torsional free vibration signals are shown in Fig. 1 and Fig. 2 respectively, in which the IMFs components acquired using the EMD method are also given. In Fig. 1 and Fig. 2, $c_i(i=1\sim6)$ are the IMFs, and r is the residue that can be also regarded as a special IMF. It can be seen that c_3, c_4 in Fig. 1 are respectively similar to c_3, c_5 in Fig. 2 with different amplitude. In some sense, they respectively correspond to torsional and vertical bending components contained in the coupled vibration signals. There is an assumption that the coupled vertical-torsional free vibration signals for extracting flutter derivatives only consist of two types of frequency data sets. Nevertheless, the signals collected from a large amount of wind tunnel tests contain more than two types of frequency components for the reason of mingled noises. On the basis of the EMD

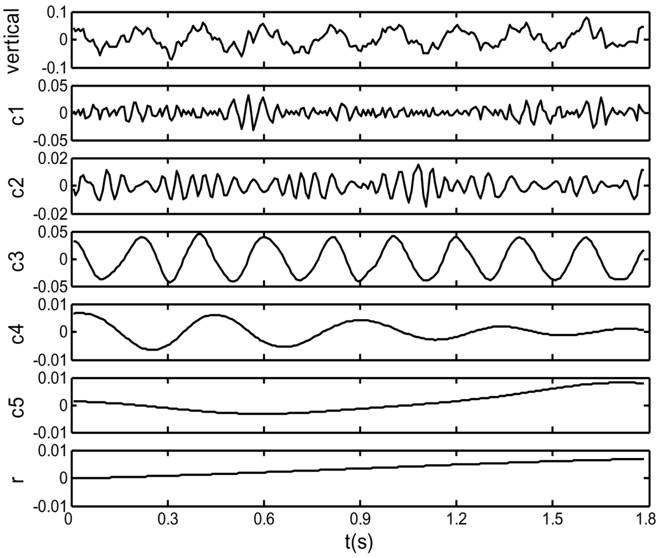

Fig. 1 Vertical bending signal and its IMF components

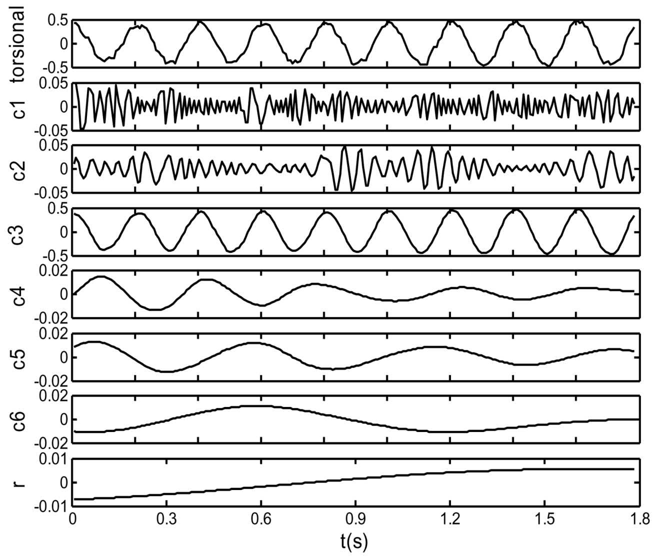

Fig. 2 Torsional signal and its IMF components

characteristics, IMFs c_1, c_2 in Fig. 1 and Fig. 2 may be regarded as the noise, and residue r may be regarded as the trend. If the noise and trend are eliminated, the identification precision can be improved, which will be investigated detailedly in the following paper. It can be seen that the amplitudes of the vertical bending components are close to those of noises mixed in both vertical bending and torsional signals. This is one of the main reasons that result in the large errors of mode parameters of vertical bending components.

3. Mathematical model for modal parameters identification

Under the action of self-excited forces, the mathematical model for vertical bending and torsional motion of section model can be expressed in the form of coupled differential equations

$$\begin{aligned} m(\ddot{h}+2\xi_h\omega_h\dot{h}+\omega_h^2h) &= \rho U^2B\left[KH_1^*(K)\frac{\dot{h}}{U}+KH_2^*(K)\frac{B\dot{\alpha}}{U}+K^2H_3^*(K)\alpha+K^2H_4^*(K)\frac{h}{B}\right] \\ I(\ddot{\alpha}+2\xi_\alpha\omega_\alpha\dot{\alpha}+\omega_\alpha^2\alpha) &= \rho U^2B^2\left[KA_1^*(K)\frac{\dot{h}}{U}+KA_2^*(K)\frac{B\dot{\alpha}}{U}+K^2A_3^*(K)\alpha+K^2A_4^*(K)\frac{h}{B}\right] \end{aligned} \tag{1}$$

where m and I are the model mass and mass inertia moment per unit length respectively; ξ_h and ξ_α are the mechanical damping ratios in vertical bending and torsion respectively; ω_h and ω_α are the corresponding natural mechanical frequencies respectively; $h, \alpha, \dot{h}, \dot{\alpha}, \ddot{h}, \ddot{\alpha}$ are displacements, velocities and accerleration of vertical bending, torsional movement respectively; ρ is the air density; flutter derivatives H_1^* and A_1^* $(i=1, 2, 3, 4)$ are functions of the reduced frequency K; K is the product of frequency of the motion ω and the model width B scaled by the mean oncoming wind speed U.

If the displacement vector is given as $x(t)=[h(t)\ \ \alpha(t)]^T$, then Eq. (1) may be rewritten compactly in the matrix form

$$\ddot{x}+C^e\dot{x}+K^ex = 0 \tag{2}$$

where

$$C^e = \begin{bmatrix} 2\xi_h\omega_h-\frac{\rho B^2\omega}{m}H_1^*(K) & -\frac{\rho B^3\omega}{m}H_2^*(K) \\ -\frac{\rho B^3\omega}{I}A_2^*(K) & 2\xi_\alpha\omega_\alpha-\frac{\rho B^4\omega}{I}A_2^*(K) \end{bmatrix},$$

$$K^e = \begin{bmatrix} \omega_h^2-\frac{\rho B^2\omega^2}{m}H_4^*(K) & -\frac{\rho B^3\omega^2}{m}H_3^*(K) \\ -\frac{\rho B^3\omega^2}{I}A_4^*(K) & \omega_\alpha^2-\frac{\rho B^4\omega^2}{I}A_3^*(K) \end{bmatrix}$$

C^e and K^e are the damping and stiffness matrices of the wind-model system.

Eq. (2) can be rewritten as

$$\dot{Y} = AY \tag{3}$$

where $Y = [x, \dot{x}]^T$ is the state vector, $A = \begin{bmatrix} 0 & E \\ -K^e & -C^e \end{bmatrix}$ is the eigenmatrix, E denotes identity matrix.

On the basis of complex mode theory, Eq. (3) can be uncoupled in the complex mode coordinates. In the physical coordinates, the free vibration histories may be described as

$$x(t) = \sum_{i=1}^{2} (\alpha_i \psi_i e^{\lambda_i t} + \alpha_i^* \psi_i^* e^{\lambda_i^* t}) \tag{4}$$

where λ_1, λ_2, λ_1^*, λ_2^* are the eigenvalues of the eigenmatrix A; ψ_1', ψ_2', $\psi_1'^*$, $\psi_2'^*$ are the eigenvectors of A; α_1, α_1^*, α_2, α_2^* are constants, depending on the original recorded signals.

When complex mode parameters are determined according to the data sets, eigenmatrix A can be acquired. All 8 flutter derivatives at a certain wind speed can be extracted from the difference of stiffness and damping of model system with those of no-wind condition.

For bridge deck section model, the estimated values of the free vibration response of vertical bending and torsinal motions can be constructed as

$$\begin{Bmatrix} \hat{h}(t) \\ \hat{\alpha}(t) \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{2} e^{a_i t}(c_i \cos b_i t + d_i \sin b_i t) \\ \sum_{i=1}^{2} e^{a_i t}(e_i \cos b_i t + f_i \sin b_i t) \end{Bmatrix} \tag{5}$$

Let m_h and m_α denote the signal lengths of h and α respectively. The error vectors of h and α between the measured and estimated values may be individually expressed as

$$\begin{aligned} \{e_h\}^T &= \{h_1 - \hat{h}_1,\ \ h_2 - \hat{h}_2,\ \ \ldots,\ \ h_{m_h} - \hat{h}_{m_h}\} \\ \{e_\alpha\}^T &= \{\alpha_1 - \hat{\alpha}_1,\ \ \alpha_2 - \hat{\alpha}_2,\ \ \ldots,\ \ \alpha_{m_\alpha} - \alpha_{m_\alpha}\} \end{aligned} \tag{6}$$

Correspondingly, the time-varying weights of the errors may be written as

$$\begin{aligned} \{w_h\} &= \{w_{h1},\ \ w_{h2},\ \ \ldots,\ \ w_{hm_h}\} \\ \{w_\alpha\} &= \{w_{\alpha 1},\ \ w_{\alpha 2},\ \ \ldots,\ \ w_{\alpha m_\alpha}\} \end{aligned} \tag{7}$$

The individual error functions are defined as the sum of weighting errors square

$$\begin{aligned} J_h &= [\{w_h\} \cdot \{e_h\}^T][\{w_h\} \cdot \{e_h\}] \\ J_\alpha &= [\{w_\alpha\} \cdot \{e_\alpha\}^T][\{w_\alpha\} \cdot \{e_\alpha\}] \end{aligned} \tag{8}$$

Eq. (8) can be rewritten as

$$\begin{aligned} J_h &= \{w_h\} \cdot [\{h\} - ([C^h]\{c\} + [S^h]\{d\})]^T \cdot \{w_h\} \cdot [\{h\} - ([C^h]\{c\} + [S^h]\{d\})] \\ J_\alpha &= \{w_\alpha\} \cdot [\{\alpha\} - ([C^\alpha]\{e\} + [S^\alpha]\{f\})]^T \cdot \{w_\alpha\} \cdot [\{\alpha\} - ([C^\alpha]\{e\} + [S^\alpha]\{f\})] \end{aligned} \tag{9}$$

where $C_{im_h}^h = e^{a_i m_h \Delta t} \cos(b_i m_h \Delta t)$, $\ \ S_{im_h}^h = e^{a_i m_h \Delta t} \sin(b_i m_h \Delta t)$,

$C_{im_\alpha}^\alpha = e^{a_i m_\alpha \Delta t} \cos(b_i m_\alpha \Delta t)$, $\ \ S_{im_\alpha}^\alpha = e^{a_i m_\alpha \Delta t} \sin(b_i m_\alpha \Delta t)$

Let $\dfrac{\partial J}{\partial\{c\}} = 0,\quad \dfrac{\partial J}{\partial\{d\}} = 0,\quad \dfrac{\partial J}{\partial\{e\}} = 0,\quad \dfrac{\partial J}{\partial\{f\}} = 0$

The coupled equations in matrix style can be obtained as follows

$$\begin{bmatrix} A_h & D_h \\ D_h^T & B_h \end{bmatrix} \begin{Bmatrix} c \\ d \end{Bmatrix} = \begin{Bmatrix} X_h \\ Y_h \end{Bmatrix} \tag{10}$$

$$\begin{bmatrix} A_\alpha & D_\alpha \\ D_\alpha^T & B_\alpha \end{bmatrix} \begin{Bmatrix} e \\ f \end{Bmatrix} = \begin{Bmatrix} X_\alpha \\ Y_\alpha \end{Bmatrix} \tag{11}$$

where $A_h = [C^h]^T[C^h],\ \ B_h = [S^h]^T[S^h],\ \ D_h = [C^h]^T[S^h]$

$A_\alpha = [C^\alpha]^T[C^\alpha],\ \ B_\alpha = [S^\alpha]^T[S^\alpha],\ \ D_\alpha = [C^\alpha]^T[S^\alpha]$

$X_h = [C^h]^T[h],\ \ Y_h = [S^h]^T[h],\ \ X_\alpha = [C^\alpha]^T[\alpha],\ \ Y_\alpha = [S^\alpha]^T[\alpha]$

The total sum of errors is $J=J_h+J_\alpha$.
Displacements, velocities and accelerations may be respectively expressed as

$$\begin{Bmatrix} h(t) \\ \alpha(t) \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{2} e^{a_i t}(c_i\cos b_i t + d_i \sin b_i t) \\ \sum_{i=1}^{2} e^{a_i t}(e_i\cos b_i t + f_i \sin b_i t) \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} \dot{h}(t) \\ \dot{\alpha}(t) \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{2} e^{a_i t}(\bar{c}_i\cos b_i t + \bar{d}_i \sin b_i t) \\ \sum_{i=1}^{2} e^{a_i t}(\bar{e}_i\cos b_i t + \bar{f}_i \sin b_i t) \end{Bmatrix} \tag{13}$$

$$\begin{Bmatrix} \ddot{h}(t) \\ \ddot{\alpha}(t) \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{2} e^{a_i t}(\tilde{c}_i\cos b_i t + \tilde{d}_i \sin b_i t) \\ \sum_{i=1}^{2} e^{a_i t}(\tilde{e}_i\cos b_i t + \tilde{f}_i \sin b_i t) \end{Bmatrix} \tag{14}$$

where $\bar{c}_i = a_i c_i + b_i d_i,\ \ \bar{d}_i = -b_i c_i + a_i d_i,\ \ \bar{e}_i = a_i e_i + b_i f_i,\ \ \bar{f}_i = -b_i e_i + a_i f_i,$

$\tilde{c}_i = (a_i^2 - b_i^2)c_i + 2a_i b_i d_i,\ \ \tilde{d}_i = (a_i^2 - b_i^2)d_i - 2a_i b_i c_i,$

$\tilde{e}_i = (a_i^2 - b_i^2)e_i + 2a_i b_i f_i,\ \ \tilde{f}_i = (a_i^2 - b_i^2)f_i - 2a_i b_i e_i.$

Y and $\dot{Y}$ in Eq. (3) can be written as

$$Y = \{h, \alpha, \dot{h}, \dot{\alpha}\}^T = PZ, \quad \dot{Y} = \{\dot{h}, \dot{\alpha}, \ddot{h}, \ddot{\alpha}\}^T = QZ$$

where $Z = \{e^{a_1 t}\cos b_1 t \quad e^{a_1 t}\sin b_1 t \quad e^{a_2 t}\cos b_2 t \quad e^{a_2 t}\sin b_2 t\}$

Eigenmatrix A can be expressed as $A = QP^{-1}$.

Up to now, the flutter derivatives can be extracted easily according to the obtained damping matrix C^e and stiffness matrix K^e.

4. Stochastic search technique for flutter derivatives extraction

For traditional methods, 12 parameters a_i, b_i, c_i, d_i, e_i, f_i, $(i = 1, 2)$ in Eq. (12) can be acquired by alternate iteration solving approach with J as the objective function. In essence, since it is an optimization problem, a stochastic search technique will be developed to search for the optimal solution which minimizes the objective function J_h, J_α or J. The stochastic search technique may be summarized as the following general procedure.

(1) Select the initial mode parameters $\omega_i^{(0)}$, $\xi_i^{(0)}$ $(i = 1, 2)$ and determine the appropriate weights $\{w_h\}$, $\{w_\alpha\}$. Identifying results at the last level wind speed can be regarded as the initial values of mode parameters for current wind speed. Let $J_0 \to +\infty$;

(2) Compute $\{a\}$, $\{b\}$, $C^h_{im_h}$, $S^h_{im_h}$, $C^\alpha_{im_\alpha}$, $S^\alpha_{im_\alpha}$;

(3) Compute A_h, B_h, D_h, A_α, B_α, D_α, X_h, Y_h, X_α, Y_α;

(4) Compute $\{c\}$, $\{d\}$, $\{e\}$, $\{f\}$;

(5) Compute the unifying sum of weighting errors square J;

(6) If $J < J_0$, then let $J_0 = J$, and the mode parameters should be replaced correspondingly; else, keep J_0 and mode parameters unchanged;

(7) Fluctuate and alter the values of mode parameters ω_i, ξ_i $(i = 1, 2)$ in such a way as $\omega_i = \omega_i + [0.5 - rand(0, 1)] \cdot \omega_i^{(0)} \cdot T_{\omega_i}$, $\xi_i = \xi_i + [0.5 - rand(0, 1)] \cdot \xi_i^{(0)} \cdot T_{\xi_i}$, where $rand$ (0, 1) is an uniformly distributed random number in interval(0,1), T_{ω_i}, T_{ξ_i} are parameters determining the amplitude of fluctuation. In general, it is sensible to range them from 0.05 to 0.2;

(8) Procedure (2) to (7) are repeated until the fitting error is less than the tolerance or enough fluctuation times (Generally, 1000~3000 may be appropriate) is reached and determines all the 12 parameters a_i, b_i, c_i, d_i, e_i, f_i, $(i = 1, 2)$;

(9) Construct the system eigenmatrix A on the basis of the 12 parameters and other known relevant parameters, extract the flutter derivatives for this wind speed level.

If the original collected signals are processed and filtered by EMD, the identification procedure can be continued likewise and the identification precision will be improved. As to the partial vertical bending and torsional errors' least square, it is enough to substitute J_h or J_α for J. Identification results prove that satisfactory precision can be obtained when the fluctuation search times attain 3000, and the common CPU time is within 5 seconds.

The above procedure can not only serve for section model data, but also may be employed to data recorded in taut strip and full bridge aeroelastic model tests. Identification of 18 flutter derivatives

Table 1 Identification results for different cases

σ_N/σ_S	p_1, p_2	w_h, w_α		ω_h	ω_α	ξ_h	ξ_α
0	1, 1	1, 1		5.0106	10.0018	0.1493	0.0101
	1, 10	1, 1		6.0041	10.0010	0.0526	0.0098
	1, 10	10, 1		4.9682	9.9946	0.1405	0.0099
0.05	1, 1	1, 1	NO EMD	5.0156	10.0021	0.1431	0.0102
			EMD	5.0112	9.9983	0.1486	0.0099
0.1	1, 1	1, 1	NO EMD	4.9746	10.0032	0.1742	0.0097
			EMD	4.9885	10.0026	0.1557	0.0101

based on similar procedure can also be performed.

5. Numerical examples

Before exploiting the present stochastic search technique, simulation data have been tested first in order to check the validity and performance of this technique.

Example 1 Two exponential decaying cosinusoidal motion functions may be expressed as

$$\begin{aligned} h(t) &= p_1 \cdot \exp(-\xi_h \omega_h t / \sqrt{1-\xi_h^2}) \cdot \cos[\omega_h t + 2\pi \cdot rand(0,1)] \\ \alpha(t) &= p_2 \cdot \exp(-\xi_\alpha \omega_\alpha t / \sqrt{1-\xi_\alpha^2}) \cdot \cos[\omega_\alpha t + 2\pi \cdot rand(0,1)] \end{aligned} \tag{15}$$

The theoretical values of ω_h, ω_α, ξ_h, ξ_α are 5, 10, 0.15, 0.01, respectively. The initial search values of mode parameters are 4, 12, 0.1, 0.05, T_{ω_i}, $T_{\xi_i}(i=1,2)$ are all 0.1, and the fluctuation times is 3000. Identification results for different cases are listed in Table 1. σ_N, σ_S are respectively standard deviations of noise and original signal. Identification results corresponding to "NO EMD" and "EMD" are on the basis of original contaminated signals and preprocessed and filtered signals using EMD, respectively.

It can be seen that the identification precision is satisfactory if the amplitudes of two data sets are identical. But if data set amplitude increases by ten times, the identification error of the other data set increases sharply simultaneously. It is testified that more difference of amplitudes cause more significant identification error of 'weak' signals. For such cases, the identification precision may be improved by appending appropriate weights to the errors between the original signals and their estimated ones. If the selected initial values of mode parameters are closer to their theoretical values or the fluctuation time is ample, the identification errors will tend to be zero. It goes without saying that the parameter errors increase with increasing noise level. If the original contaminated signal is preprocessed and eliminate the noises using EMD, the identification precision will be improved.

For example, a coupled exponential decaying cosinusoidal signal is contaminated by white noise, and the curve of the signal and noise are respectively plotted in the first and second subplots in Fig. 3. The noise data are also plotted in the second subplot. Using the EMD method, the signal can be decomposed into several intrinsic mode functions. Since the noise is 'white', there is no trend in its IMFs. According to the curves of all IMFs, components c_1~c_4 can be regarded as noise, as are shown in Fig. 3. The sifting error difference between the true noise and the sifted one (sum of

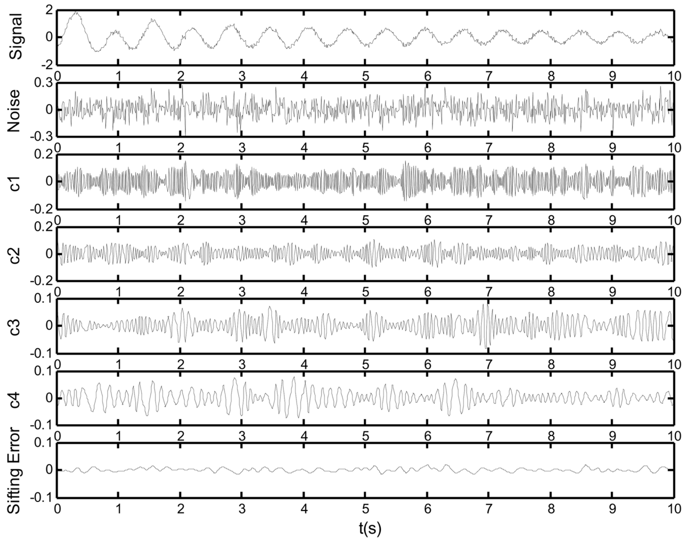

Fig. 3 Signal filtering using EMD

c_1~c_4) is quite minor, as can be seen from the last subplot in Fig. 3.

Example 2 The motion equation of a 2 DOF system is given by

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 0 \tag{16}$$

Based on mass, damping and stiffness matrices, the frequencies and damping ratios can be calculated easily. In order to check the validity of programs, the initial displacements and velocities are all considered as $10*[rand(0, 1) - 0.5]$ randomly. The free vibration responses with initial conditions can be obtained by middle integration method with time interval 0.001s, and the number of data is 5000. Identification results for different cases are listed in Table 2. The meanings of “NO EMD” and “EMD” are the same as those in Table 1.

As for case 1, the initial mode parameters are assumed to be the theoretical values. The identified

Table 2 Determinations of parameters for different cases

Parameters		c_{11}	c_{12}	c_{21}	c_{22}	k_{11}	k_{12}	k_{21}	k_{22}
Original values		1	-0.2	-0.2	1	50	-5	-5	20
Case1		1.0000	-0.1999	-0.2001	1.0000	50.0002	-5.0002	-4.9999	20.0000
Case2		1.0002	-0.2000	-0.2004	0.9977	50.0016	-5.0023	-5.0159	20.0176
Case3	NO EMD	0.9939	-0.1988	-0.1977	1.0015	50.0074	-4.9829	-5.0075	20.0278
	EMD	0.9996	-0.1997	-0.1989	1.0008	50.0017	-4.9951	-5.0016	20.0039
Case4	NO EMD	0.9970	-0.2158	-0.1911	0.9974	50.0346	-4.8620	-4.9836	19.9454
	EMD	0.9989	-0.2024	-0.1976	1.0015	50.0085	-4.9927	-4.9959	20.0186

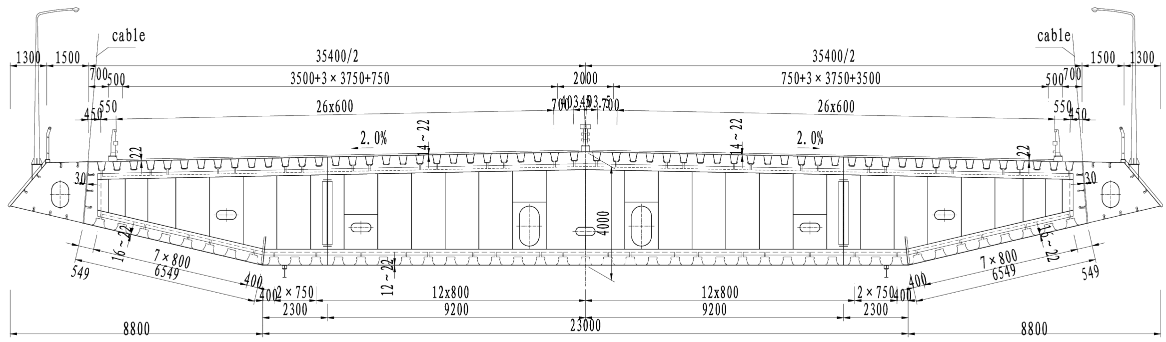

Fig. 4 Cross section layout of Sutong Bridge deck

results show that the programs are right and valid. As for case 2, the initial mode parameters are assumed to be 0.9 times of the theoretical values. It is verified that the stochastic search technique is effective and practical. As for case 3 and case 4, there are additional noises mingled with the responses with the intensity of 5% and 10% respectively. Obviously, the stronger noise produces the higher identification error. Similarly, the identification precision may be improved by means of EMD.

The above results may be taken as an encouragement to proceed with genuine recordings consisting of the simultaneous vertical bending and torsional vibration responses.

6. Flutter derivatives Identification of Sutong Bridge deck

The Sutong Bridge over the Yangtze River is a cable-stayed bridge spanning 1088 m, which is under construction presently in China. This bridge has a steel girder deck with a cross section layout as is shown in Fig. 4.

According to the parameters of the bridge and the dimensions of the working section and the wind speed range of the wind tunnel, the geometry scale of the section model and the wind speed scale were selected to be 1/70 and 1/6.75, respectively. The length, width and the height of the section model are 1800 mm, 580 mm and 57 mm, respectively. The equivalent mass and inertial mass moment per unit length of the section model are 6.53 kg and 0.379 $kg \cdot m^2$. In still air, the vertical bending frequency f_h is 1.91 Hz, while torsional frequency f_α is 5.50 Hz. The flutter derivatives of Sutong Bridge deck model at the attack angle of 0 are extracted, as shown in Fig. 5.

There are four curves named as MLSM, vertical & torsional, vertical, torsional individually in each plot, which represent the identified results using the modified least-square method (MLSM in short) [Ding 2001], the stochastic search technique based on the unifying (i.e. vertical & torsional), vertical, torsional least-square principle, respectively. It is worth mentioning that the results corresponding to MLSM are the mean identification values from several free vibration data at the same wind speed, and the other results are acquired from one of sets of collected data for the reason of comparisons of different least-square criterion.

The value of A_4^* has been proven to be sensitive to noises, and A_4^* itself is unimportant and may be negligible, therefore the results of A_4^* are not shown here. It can be observed that:

(1) Largest scatter among the flutter derivatives appear for the coupling damping term H_2^*. Although there are obvious differences among 4 sets of results at high reduced wind speed,

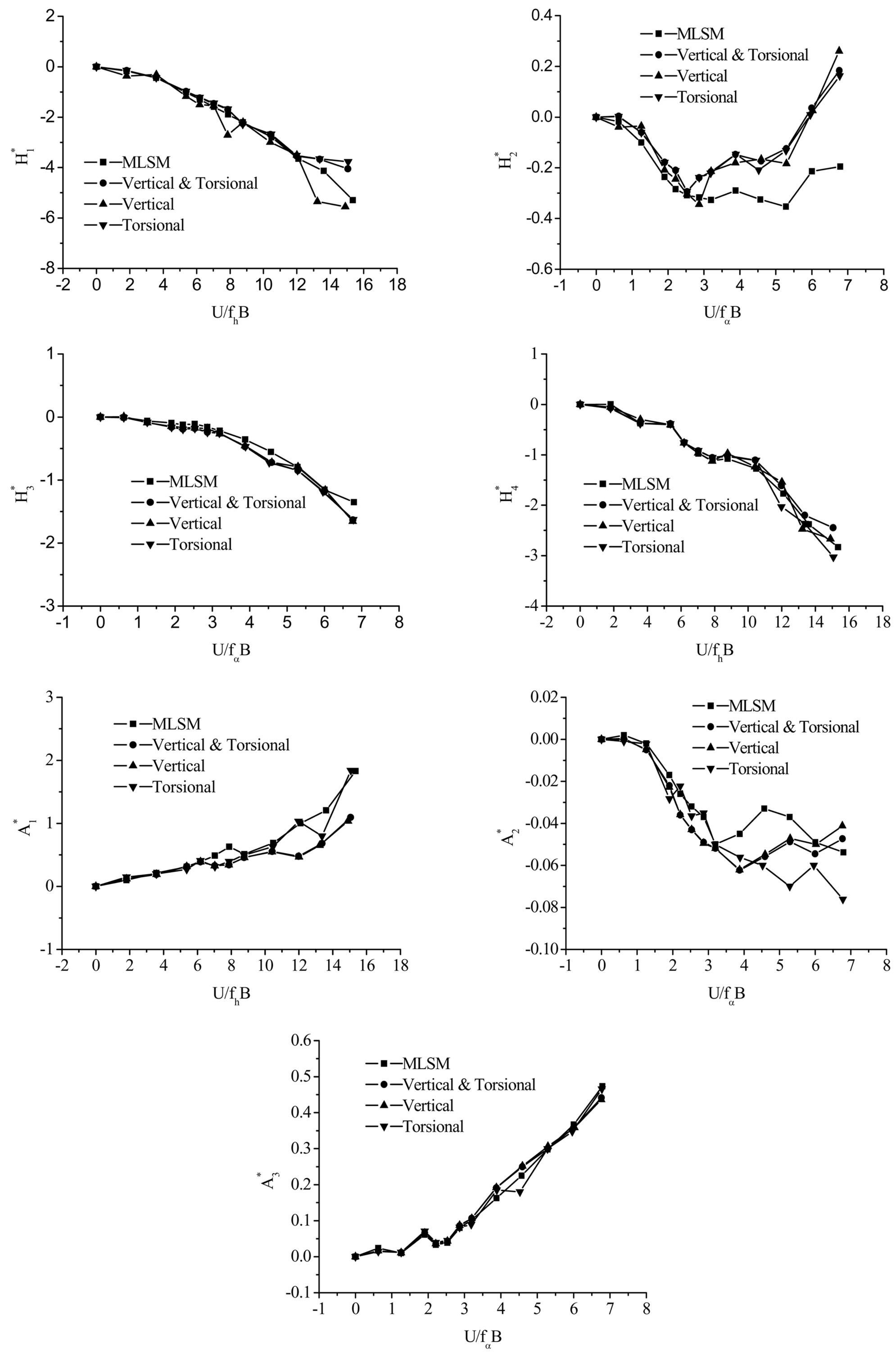

Fig. 5 Comparisons of flutter derivatives of Sutong Bridge deck

results acquired using the stochastic search technique are close to each other comparatively.

(2) As for the coupling damping term A_1^*, at high reduced wind speed, there are certain differences among different cases.

(3) As for the torsional damping term A_2^*, apparent differences exist among different cases at high reduced wind speed.

(4) Since the vertical bending frequency term H_4^* and torsional frequency term A_3^* are related to the stiffness matrix diagonal elements, the identification results are very stable.

(5) The results corresponding to stochastic search technique indicate that the unifying least-square can not improve the identification precision of flutter derivatives significantly compared with the vertical bending, torsional partial least-square.

The derivatives shown in Fig. 5 stem from response in smooth flow at mean wind speeds up to the flutter speed. If the derivatives are used for checking the flutter speed from the flutter equations, short extrapolation of the flutter derivatives related to the torsional degree of freedom will commonly be needed.

In order to testify the effectiveness of EMD method in identification precision improvement, the collected vertical-torsional coupled free vibration signals at different wind speeds are decomposed into intrinsic mode functions by virtue of EMD. Some higher and lower frequency components may be regarded as noises or trends and be eliminated. According to the original and preprocessed and filtered signals, two sets of flutter derivatives can be extracted by the stochastic search technique respectively, which are both based on the unifying least-square principle. The two sets of fitting errors are calculated before determining flutter derivatives. The ratios of the fitting error corresponding to the preprocessed signals (E_P in short) to those of original signals (E_O in short) can be readily computed as E_P/E_O subsequently. Similarly, the ratios of the two sets of flutter derivatives (FD in short) can be written as F_P/F_O. The curves of E_P/E_O and FD_P/FD_O versus wind speed are plotted in Fig. 6 and Fig. 7, respectively.

It can be seen from Fig. 6 that the fitting errors decay evidently if the signals are filtered by EMD in advance, and the identification precision is improved. There are distinct differences between the flutter derivatives that correspond to original and preprocessed signals, especially for H_2^* and A_1^*, as are observed from Fig. 7. The influence and significance of EMD is demonstrated intuitively, by

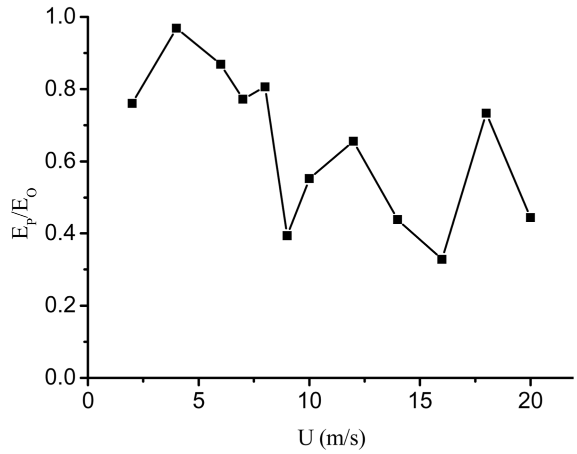

Fig. 6 Fitting error ratio versus wind speed

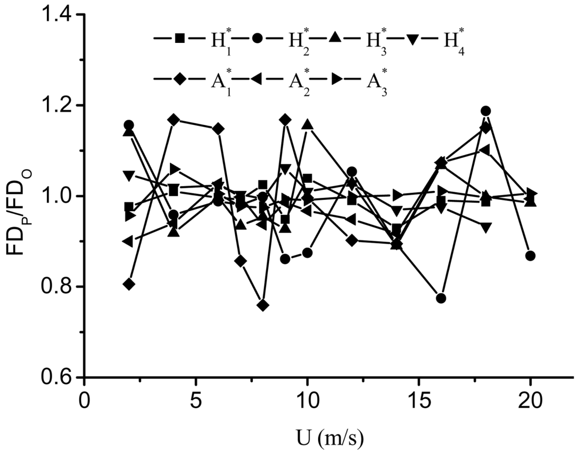

Fig. 7 Flutter derivatives ratio versus wind speed

which EMD is proven to be an effective tool to improve the identification precision of flutter derivatives of bridge decks.

7. Conclusions

The main conclusions in this study are summarized as follows:

(1) A more applicable optimization model is presented, which is suitable for flutter derivatives identification with time-varying weights and different lengths of vertical bending and torsional free vibration data. So the scope of mathematical model for flutter derivatives identification is broadened.

(2) Noises in collected vibration signals in wind tunnel can be filtered and reduced by Empirical Mode Decomposition method. On the basis of the processed signals, the identification precision of parameters and flutter derivatives can be improved, which has been verified by two simulation examples and a testing example of the Sutong Bridge. So EMD method is an effective implement to improve the precision of parameter identification.

(3) A stochastic search technique for searching the optimal solution of optimization problem is developed, which is more convenient in understanding and programming than that of the alternate iteration technique, and testified to be a valid and efficient method by two simulation examples and the extraction of flutter derivatives of Sutong Bridge deck using free vibration history collected in wind tunnel tests.

(4) The unifying least-square can not improve the identification precision of flutter derivatives significantly compared with the vertical bending, torsional partial least-square. The quality of signals plays a more important role than identification technique on the accuracy of flutter derivatives extraction.

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