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Bi-modal spectral method for evaluation of along-wind induced fatigue damage

S. Gomathinayagam†, P. Harikrishna‡, A. Abraham‡†, and N. Lakshmanan‡‡

Structural Engineering Research Centre, CSIR, Chennai 600 113, India
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Abstract. Several analytical procedures available in literature, for the evaluation of wind induced fatigue damage of structures, either assume the wide band random stress variations as narrow band random process or use correction factors along with narrow band assumption. This paper compares the correction factors obtained using the Rainflow Cycle (RFC) counting of the measured stress time histories on a lamp mast and a lattice tower, with those evaluated using different frequency domain methods available in literature. A Bi-modal spectral method has been formulated by idealising the single spectral moment method into two modes of background and resonant components, as considered in the gust response factor, for the evaluation of fatigue of slender structures subjected to "along-wind vibrations". A closed form approximation for the effective frequency of the background component has been developed. The simplicity and the accuracy of the new method have been illustrated through a case study by simulating stress time histories at the base of an urban light pole for different mean wind speeds. The correction factors obtained by the Bi-modal spectral method have been compared with those obtained from the simulated stress time histories using RFC counting method. The developed Bi-modal method is observed to be a simple and easy to use alternative to detailed time and frequency domain fatigue analyses without considerable computational and experimental efforts.

Keywords: along-wind; background; Bi-modal; closed form approximation; fatigue damage; Gaussian process; measurements; narrow band; rainflow cycles; resonant; simulations; spectral moments; wide band; wind.

1. Introduction

The slender steel and metal structures are prone to wind induced vibrations at normal as well as high wind speeds causing cycles of stress with varying amplitudes leading to fatigue cracks. Studies in wind induced fatigue of slender structures are receiving particular attention in recent years because of the occurrence of premature fatigue damage to tubular steel lighting columns and lattice towers. This has prompted investigators, (see Robertson, *et al.* 1999, Gilani and Whittaker 2000, 2000a, Repetto and Solari 2001, Peil and Behrens 2002, Robertson, *et al.* 2004 and Solari and

† Assistant Director, Corresponding Author, E-mail: goms@sercm.csir.res.in
‡ Assistant Director
‡† Scientist
‡‡ Director

Repetto 2004), to derive more reliable procedures to estimate life of these masts and to safeguard against wind induced fatigue failure of these life line structures.

Several analytical methods which are available provide semi-empirical solutions for the evaluation of wind induced fatigue damage of structures. Wyatt (1984) assessed the sensitivity of lattice towers to wind gust induced fatigue. Dionne and Davenport (1988) introduced a simple relationship between the along-wind gust response factor and the fatigue damage. Holmes (2002) developed a Closed Form Approximation (CFA) for the fatigue damage and the fatigue life of structures subjected to along-wind vibrations, based on a semi-empirical formula of the stress state using probabilistic approach. Repetto and Solari (2001) embedded a CFA into refined stochastic procedures by making the calculation of the along-wind and cross-wind induced fatigue for dynamically wind sensitive slender vertical structures, conceptually attractive and simple to apply. These CFA procedures either assume the stress variations as narrow band random process or represent the stress variations with equivalent narrow band approximation of the fatigue under wide band random process based on the expressions derived by Wirsching and Light (1980), which were widely used for offshore structures (Wirsching 1984, and Gomathinayagam, *et al.* 1990). However, Lutes, *et al.* (1984) found that different spectra having the same irregularity factor sometimes have very different fatigue damage predictions by rain flow analysis and thus the approximation suggested by Wirsching and Light (1980) has been found to be inadequate in those situations. Ortiz and Chen (1987) have also suggested a more accurate correction factor using five spectral moments of the process under consideration. Later a new approximation has been introduced by Lutes and Larsen (1990) based on a single spectral moment. Even though these frequency domain analyses provide accurate solutions, they need detailed stochastic analysis to arrive at the spectra under consideration. Based on probabilistic analysis, Jiao and Moan (1990) presented a theoretical model for estimating fatigue damage under stationary Gaussian processes with well-separated bimodal spectral density functions. Based on this model, Solari and Repetto (2004) have formulated a more refined counting method for estimating the wind-induced fatigue using background and resonant response components.

This paper studies the wind induced fatigue damage obtained by the Rainflow Cycle (RFC) counting of the measured stress time histories from a lamp mast and a lattice tower, and compares the correction factors obtained from measurements with those evaluated using different methods available in the literature. A Bi-modal spectral method has been formulated by idealising the single spectral moment method into two modes of background and resonant components, as considered in the gust response factor, for the evaluation of fatigue of slender structures subjected to "along-wind vibrations". A case study as presented by Repetto and Solari (2001) has been used along with the derivations for along wind response spectra to simulate stress time histories for different mean wind speeds and to evaluate fatigue damage using RFC counting method. In this paper, a CFA has been developed for the evaluation of effective frequency of background component. This has been used along with other CFA already developed by Repetto and Solari (2001), for the evaluation of background and resonant components of response, to calculate the correction factor based on Bi-modal spectral method. The correction factors obtained by the Bi-modal spectral method have been compared with those obtained from the simulated stress time histories using RFC counting method.

2. Review of fatigue damage models for broad band random loading in frequency domain

A criterion well established in the design practice for cumulative fatigue damage under repeated loading, with a range of variable amplitudes is Miner's rule:

$$D = \sum_{i:} (n_i / N_i) \tag{1}$$

where n_i is the number of stress cycles at an amplitude (stress range), S_i, for which N_i cycles are required to cause failure of the material with a specific detail.

The results of constant amplitude fatigue tests are usually expressed in the form of an *S-N* curve, where *S* is the stress amplitude (stress range), and *N* is the number of cycles until failure. For many materials, the *S-N* curve is well approximated by a straight line when log *S* is plotted against log *N*, and would have the following form,

$$N_i = K\, S_i^{-m} \tag{2}$$

where '*K*' is a constant which depends on the material and type of structural detail and the exponent '*m*' which represents the slope of the *S-N* curve on log-log plot and varies in the range of 3 to 10.

By substituting Eq. (2) in Eq. (1),

$$D = (1 / K) \sum_{i:} (n_i S_i^m) \tag{3}$$

For a stationary, Gaussian and narrow band stress process, the cumulative fatigue damage as per Rayleigh approximation is given as (Wirsching and Light 1980),

$$D_{NB} = (1/K)\,(f_o\, T)\,(2\sqrt{2}\sigma_s)^m\, \Gamma[1 + (m/2)] \tag{4}$$

where f_o is the mean zero up-crossing frequency estimated using the spectral moments of the random stress process, T is the duration, and σ_s is the standard deviation of the process and $\Gamma[.]$ is the gamma function.

Wirsching and Light (1980) have developed an approach of equivalent narrow band approximation for predicting fatigue damage under a wide band stationary random stress process using a correction factor, λ_w, based on numerical simulations as given below,

$$D = \lambda_W\, D_{NB} \tag{5}$$

$$\lambda_W = a + [1 - a]\,(1 - \varepsilon)^b \tag{6}$$

where

$a = 0.926 - 0.033\, m$; $b = 1.587\, m - 2.323$ and

$$\varepsilon = \text{spectral width parameter} = \sqrt{[1 - \alpha^2]} \tag{7}$$

$$\alpha = \text{irregularity factor} = f_o / n_o \tag{8}$$

$$f_o = \text{zero crossing frequency} = \sqrt{[M_2/M_0]} \tag{9}$$

and $$n_o = \text{rate of peaks} = \sqrt{[M_4/M_2]} \tag{10}$$

where $$M_k = k^{\text{th}} \text{ spectral moment} = \int f^k\, S(f)\, df \tag{11}$$

where f is frequency (Hz) and $S(f)$ is spectral density function of the random stress process.

By substituting, $\sigma_s = M_0^{1/2}$ and using Eq. (9), D_{NB} (Eq. 4) can be expressed in the form of spectral moments as,

$$D_{NB} = (1/K)\ (T)\ (2\sqrt{2})^m\ \Gamma[1 + (m/2)]\ M_0^{(m-1)/2}\ M_2^{1/2} \tag{12}$$

However, Lutes, *et al.* (1984) found that different spectra having the same irregularity factor sometimes have very different fatigue damage predictions by rain flow analysis and thus the approximation suggested by Wirsching and Light (1980) has been found to be inadequate in those situations. Hence, Lutes, *et al.* (1984) have suggested a correction factor based on a band width parameter, α_b, using spectral moments, M_0, M_b, and M_{2b} with b dependent on the value of m.

Ortiz and Chen (1987) proposed another procedure for evaluating fatigue due to broadband Gaussian processes based on the spectral moments, M_2, M_4, M_b, and M_{2+b}, with $b = (2/m)$ as given below,

$$D_{OC} = (1/K)\ (T)\ (2\sqrt{2})^m\ \Gamma[1 + (m/2)]\ \{M_{2/m}{}^{m/2}\ M_2^{(m-1)/2}\ M_4^{1/2}\}/\{M_{2+2/m}{}^{m/2}\} \tag{13}$$

The corresponding correction factor can be expressed by dividing Eq. (13) by Eq. (12) as,

$$\lambda_{OC} = \{M_{2/m}{}^{m/2}\ M_2^{(m-1)/2}\ M_4^{1/2}\}/\{M_0^{(m-1)/2}\ M_2^{1/2}\ M_{2+2/m}{}^{m/2}\} \tag{14}$$

This formulation does appear to offer significant improvements in accuracy by the use of five spectral moments. But it is less simple to calculate.

A new approximation introduced by Lutes and Larsen (1990) estimates the fatigue damage due to broad band Gaussian processes based on single spectral moment as given below,

$$D_{SM} = (1/K)\ (T)\ (2\sqrt{2})^m\ \Gamma[1 + (m/2)]\ M_{2/m}{}^{m/2} \tag{15}$$

The corresponding correction factor can be expressed by dividing Eq. (15) by Eq. (12) as,

$$\lambda_{SM} = \{M_{2/m}{}^{m/2}\}/\{M_0^{(m-1)/2}\ M_2^{1/2}\} \tag{16}$$

3. Comparison with measured data

In present study, the strain time histories measured at the bottom of a 7.35 m tall lamp mast (Lakshmanan, *et al.* 2003) and at the bottom leg members of a 52 m tall lattice tower (Harikrishna, *et al.* 1999) have been analysed to obtain the cumulative fatigue damage for measured duration of 15 minutes, which is between the accepted measurement durations of 10 minutes and 1 hour in field experiments to obtain mean and spectral characteristics of wind and response under natural wind conditions (Cook 1984). Fig. 1 shows measured strain time history for a typical data obtained at the bottom of the lamp mast. The method of instrumentation and measurements were detailed elsewhere (Lakshmanan, *et al.* 2003). Fig. 2 shows the corresponding spectra of the typical measured strain which indicates the strain process as a wide band process. Rainflow cycle counting method (Matsuishi and Endo 1968), which is recommended for wide band random loading (Kumar, *et al.* 2003), has been used to obtain the number of cycles for different stress ranges from the evaluated stress time histories.

Using the Rainflow counted number of cycles, n_i, measured stress ranges, S_i, and standard deviation of stress, σ_s, a ratio of measured in-service fatigue (Eq. 3) to the predicted damage assuming narrow band excitation (Eq. 4) is obtained as, λ_E, given below

$$\lambda_E = \{\sum_i (n_i S_i^m)\}/\{(f_o\ T)\ (2\sqrt{2}\sigma_s)^m\ \Gamma[1 + (m/2)]\} \tag{17}$$

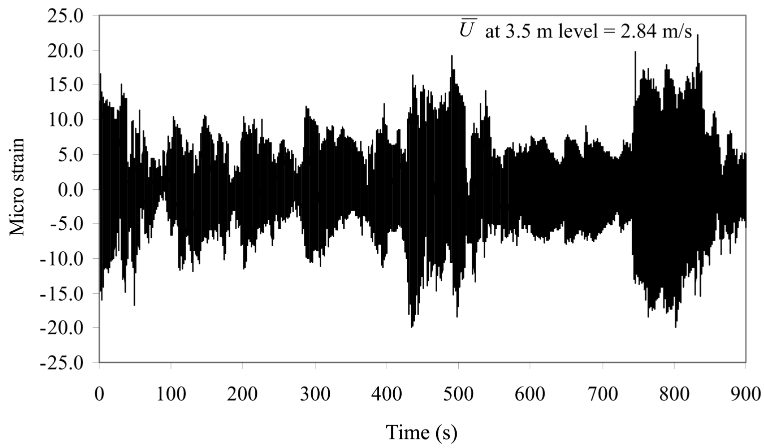


Fig. 1 Typical time history of measured strain

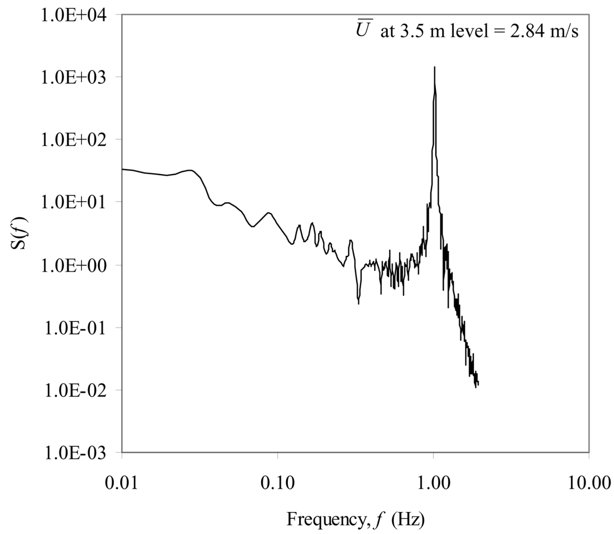


Fig. 2 Typical spectra of measured strain

Measured stress time histories, which are almost Gaussian process (i.e. skewness ≈ 0 and kurtosis ≈ 3), were considered to study the variation of λ_E values with variation in α values. Figs. 3 and 4 show the comparison of the λ_E values (Eq. 17) with the λ_W (Eq. 6), λ_{OC} (Eq. 14), λ_{SM} (Eq. 16) for all the stress data with respect to Irregularity factors (α) of individual stress time histories measured at the bottom of the lamp mast (Lakshmanan, *et al.* 2003) and at the bottom leg members of the lattice tower (Harikrishna, *et al.* 1999), for two regions of stress ranges in the *S-N* curve corresponding to $m = 3$ and 5, respectively. It clearly shows that the λ_w values based on Wirsching and Light method (Eq. 6) are observed to be over predicting the fatigue damage at low values of α. However, the λ_{OC} values based on Ortiz and Chen method (Eq. 14) and the λ_{SM} values based on Lutes and Larsen method (Eq. 16) are observed to compare well with λ_E values obtained from the rainflow counting of the measured stress time histories (Lakshmanan, *et al.* 2003 and Harikrishna, *et al.* 1999).

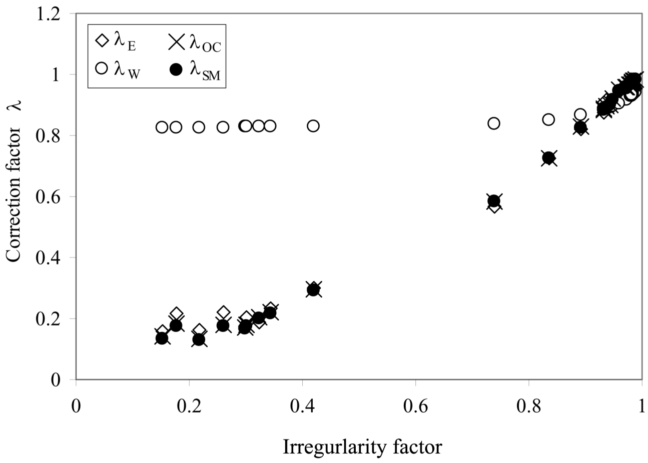


Fig. 3 Comparison of correction factors for $m = 3$

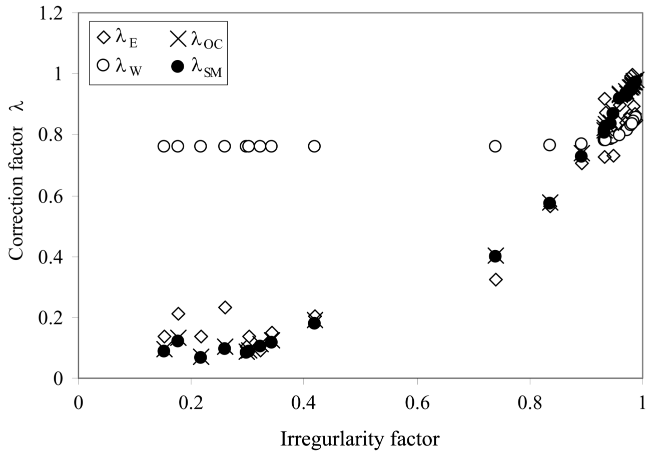


Fig. 4 Comparison of correction factors for $m = 5$

4. Bi-modal spectral method

The method suggested by Lutes and Larsen (1990), which requires single spectral moment, is observed to be simpler than the method suggested by Ortiz and Chen (1987). Even though Lutes and Larsen method is based on single spectral moment (Eq. 15), it requires detailed stochastic analysis to obtain the dynamic structural response spectra. In the field of wind engineering, the dynamic response spectra has been idealized as background and resonant response components for practical design. Further Dionne and Davenport (1988) showed the simplest relationship between the along wind gust response factor and fatigue damage by using the standard deviation of dynamic response obtained from gust response factor and by using the fundamental natural frequency of the structure. Repetto and Solari (2001) embedded CFA into refined stochastic procedures by using the standard deviation of dynamic response and zero-crossing frequency which were obtained from the closed form expressions for the background and resonant response components. However, narrow

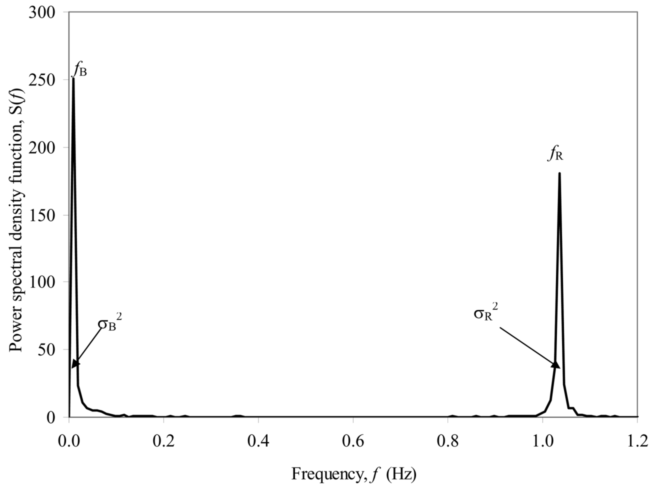


Fig. 5 Typical wind induced dynamic response spectrum

band assumption was used in both of these methods for fatigue damage evaluation using Rayleigh's approximation. Recently the probabilistic model presented by Jiao and Moan (1990) has been applied by Solari and Repetto (2004) in formulating a bi-modal counting method for estimating the wind-induced fatigue using the background and resonant response components.

In the present study, an attempt has been made to derive a simple relationship in obtaining the correction factor as per single spectral moment method for broadband process by idealizing the wind induced dynamic structural response spectra as bi-modal, based on the concept of distinctly separable background and resonant components of response. A typical wind induced dynamic structural response spectrum has been shown in Fig. 5 as linear-linear plot. The response spectrum can be viewed as bi-modal process (i) one mode predominantly in low frequency region at an effective frequency f_B and with a dynamic response component of σ_B^2 and (ii) another mode predominantly at an effective frequency f_R and with a dynamic response component of σ_R^2.

The spectral moment used in the single spectral moment method (Eq. 15) can be expressed as,

$$M_{2/m} = \int f^{2/m}\, S(f)\, df \approx f_B^{2/m}\, \sigma_B^2 + f_R^{2/m}\, \sigma_R^2 \tag{18}$$

Now by substituting Eq. (18) in Eq. (15),

$$D'_{SM} = (1/K)\,(T)\,(2\sqrt{2})^m\, \Gamma[1+(m/2)]\, \{f_B^{2/m}\, \sigma_B^2 + f_R^{2/m}\, \sigma_R^2\}^{m/2} \tag{19}$$

Using $B' = (\sigma_B^2/\sigma_s^2)$ and $R' = (\sigma_R^2/\sigma_s^2)$ and by introducing the fundamental natural frequency of the structure, f_1, Eq. (19) can be rewritten as,

$$D'_{SM} = (1/K)\,(f_1\, T)\,(2\sqrt{2}\sigma_s)^m\, \Gamma[1+(m/2)]\, \{(f_B/f_1)^{2/m}\, B' + (f_R/f_1)^{2/m}\, R'\}^{m/2} \tag{20}$$

where $\sigma_s = \sqrt{M_0} = \sqrt{\int S(f)df} \approx \sqrt{\sigma_B^2 + \sigma_R^2}$

Let
$$D'_{NB} = (1/K)\,(f_1\, T)\,(2\sqrt{2}\sigma_s)^m\, \Gamma[1+(m/2)] \tag{21}$$

Using Eq. (21), Eq. (20) can be rewritten as,

$$D'_{SM} = \lambda'_{Bi} D'_{NB} \tag{22}$$

where
$$\lambda'_{Bi} = \{(f_B/f_1)^{2/m} B' + (f_R/f_1)^{2/m} R'\}^{m/2} \tag{23}$$

In the present study, f_R has been assumed to be equal to f_1, because of the narrow banded nature of the resonant component. Hence Eq. (23) can be written as,

$$\lambda'_{Bi} = \{(f_B/f_1)^{2/m} B' + R'\}^{m/2} \tag{23a}$$

Alternately, B' and R' can be evaluated using the expressions of background and resonant components, B and R, respectively, of the gust response factor ($G = 1 + g\sqrt{[B + R]}$) as,

$$B' = B/(B + R) \text{ and } R' = R/(B + R) \tag{24}$$

5. Closed form approximation for f_B

In the present study, the effective frequency of background component, f_B, has been derived in line with Solari (1993) as given below:

$$f_B = \Phi\frac{\bar{U}(h)}{L_u(h)} \tag{25}$$

where
$$\Phi = \left(\frac{\Psi_{2/m}}{\Psi_0}\right)^{m/2} \tag{26}$$

$$\Psi_k = \int_0^{\infty}\left[\frac{fL_u(h)}{\bar{U}(h)}\right]^k\left(\frac{S_u(h;f)}{\sigma_u^2(h)}\right)\chi\left\{0.4\frac{fC_xb}{\bar{U}(h)}\right\}\chi\left\{0.4\frac{fC_zH}{\bar{U}(h)}\right\}df \tag{27}$$

H = total height of structure
h = reference height of the structure = $0.6H$
b = reference size of the structure orthogonal to wind direction
$\bar{U}(h)$ = mean wind speed at level 'h' above ground level
$L_u(h)$ = length scale of turbulence at level 'h' above ground level (Solari 1993)
C_z = exponential decay coefficient in vertical direction
C_x = exponential decay coefficient in direction normal to wind

$$\frac{S_u(h;f)}{\sigma_u^2(h)} = \frac{6.868L_u(h)/\bar{U}(h)}{(1 + 10.302fL_u(h)/\bar{U}(h))^{5/3}} \tag{28}$$

and
$$\chi\{\eta\} = \frac{1}{\eta} - \frac{1}{2\eta^2}(1 - e^{-2\eta}) \qquad (\chi = 1 \text{ for } \eta = 0) \tag{29}$$

A CFA for the derived effective frequency of background component, f_B, has been developed as given below:

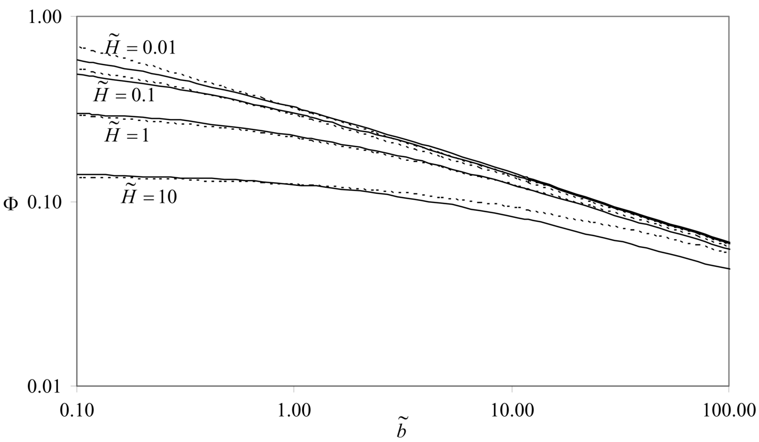
$$f_B(\tilde{b}, \tilde{H}, h) = \Phi(\tilde{b}, \tilde{H})\frac{\bar{U}(h)}{L_u(h)} \tag{30}$$

where
$$\Phi(\tilde{b}, \tilde{H}) = \frac{0.08 + \dfrac{0.75}{m}}{(\tilde{b} + \tilde{H} + \sqrt{\tilde{b}\tilde{H}})^{0.3 + \frac{2}{9m}}} \tag{31}$$

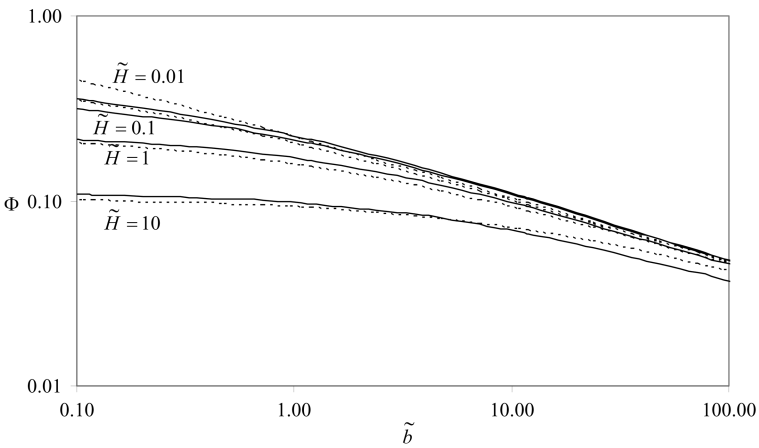
$$\tilde{b} = \frac{C_x b}{L_u(h)} \tag{32}$$

and
$$\tilde{H} = \frac{C_z H}{L_u(h)} \tag{33}$$

The CFA for Φ compared well with the derived values for different values of m. Fig. 6 shows the comparison of derived values with the CFA for Φ for $m = 3$ and 5.



(a) m = 3



(b) m = 5

Fig. 6 Comparison of CFA for Φ

6. Case study

The fatigue behaviour of an urban light pole already examined by Repetto and Solari (2001) has been chosen to demonstrate the accuracy and efficiency of the proposed method of Bi-modal spectral method. The height of the pole, H_0, is 14 m. The diameters of circumference are 0.28 m and 0.08 m at base and top of the pole, respectively, with a reference size, b_0, of 0.164 m. The size of lighting device at top are $b_1 = 1.85$ m and $H_1 = 1.8$ m with mass of 145 kg. Using the derivations for power spectrum of the fluctuating stress process as given below (Repetto and Solari 2001), spectra of stress at the base have been derived for reference mean wind speeds, $\overline{U}_{ref}$, ranging from 1 to 30 m/s.

$$S_S(z,f) = [2I_u(h_0)\bar{S}(z)]^2 \frac{|H_1(f)|^2}{\left(\sum_{k=0}^{M} \overline{K}_k\right)^2} \sum_{l=0}^{M} \sum_{k=0}^{M} \Omega_{lk}(f) \tag{34}$$

where $\bar{S}(z)$ = mean stress at level 'z' above ground level

$I_u(h_0)$ = turbulence intensity at level 'h' above ground level ($h_0 = 0.6H_0$)

$H_1(f)$ = mechanical admittance function of the 1st mode of the structure (see Appendix I)

$\overline{K}_k$ = non dimensional coefficients as given in Repetto and Solari (2001) (see Appendix I)

$\Omega_{lk}(f)$ = closed form spectral terms as given in Repetto and Solari (2001) (see Appendix I)

and M = number of localized masses

Fig. 7 shows the derived stress spectrum at the base of the light pole ($z = 0$) for reference mean wind speed of 10 m/s. Stress time histories, assumed as Gaussian random stationary process, were simulated over 15 minutes with a time step of 0.05s, from the derived stress spectra by using inverse FFT procedure (Kumar 2000). Fig. 8 shows the typical simulated stress time history for

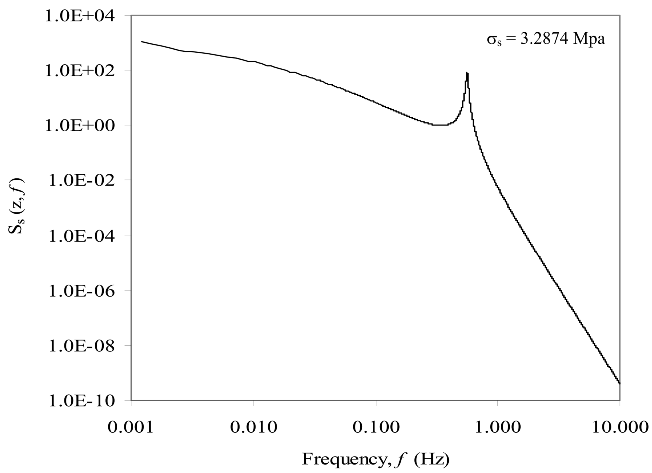


Fig. 7 Typical derived spectrum of stress at base

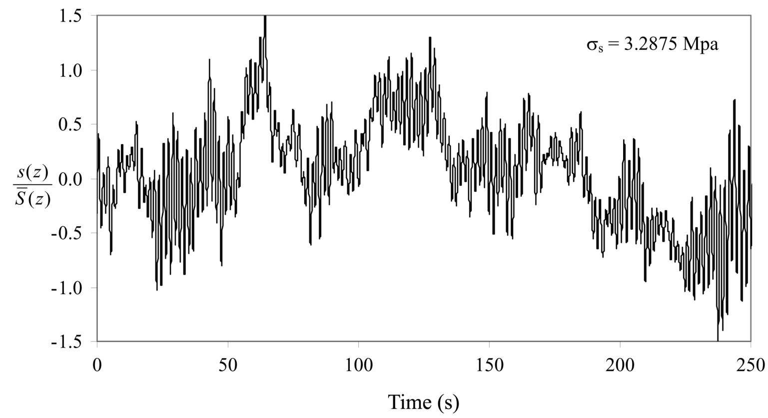


Fig. 8 Simulated time history of normalized stress at base

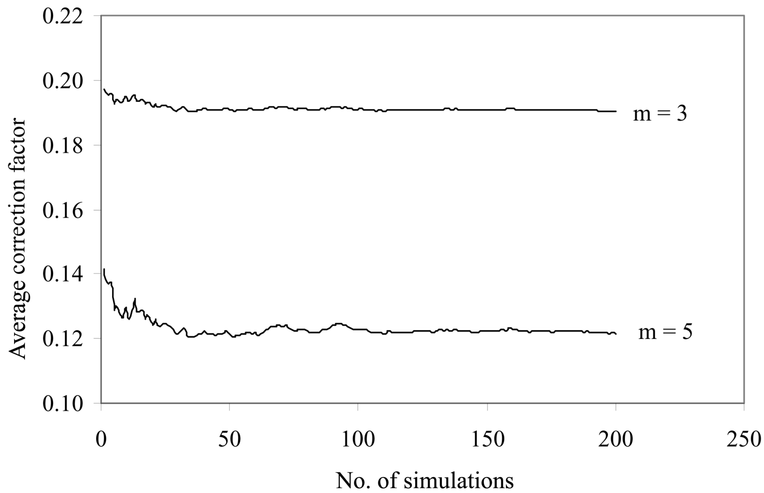


Fig. 9 Average correction factor from simulated time histories

reference mean wind speed of 10 m/s. Each stress time history has been analysed using RFC counting method to obtain the fraction of fatigue damage per hour using Eq. (3). Using the Rainflow counted number of cycles, measured stress ranges and standard deviation of stresses, a ratio of measured in-service fatigue (Eq. 3) to the predicted damage assuming narrow band excitation (Eq. 21) are obtained as, λ'_{RFC}, given below

$$\lambda'_{RFC} = \{\Sigma n_i\, S_i^m\}/\{(f_1\, T)\, (2\sqrt{2}\sigma_s)^m\, \Gamma[1 + (m/2)]\} \tag{35}$$

Fig. 9 shows the good convergence of the average correction factor for $m = 3$ and 5 obtained over 200 simulations for a specific reference mean wind speed of 10 m/s.

To obtain the fraction of fatigue damage as per the Bi-modal spectral method using Eq. (22), the factors B' and R' have been obtained (Eq. 24) using the CFA for the background and resonant components, B and R, respectively, as given below (Repetto and Solari 2001):

$$B = \left(\sum_{k=0}^{M} \bar{K}_k\right)^{-2} \left(\sum_{k=0}^{M} K'_k \sqrt{B_k}\right)^2 \tag{36}$$

$$R = \left(\sum_{k=0}^{M} \bar{K}_k\right)^{-2} \frac{\pi}{4\xi_1} \left(\sum_{l=0}^{M} \sum_{k=0}^{M} K'_l K'_k \sqrt{R_l}\sqrt{R_k}\, Coh(z_l, z_k; \theta_{lk} f_1)\right) \tag{37}$$

where K'_k, B_k, R_k are non dimensional quantities as given in Repetto and Solari (2001)

ξ_1 = total damping = $\xi_{s1} + \xi_{a1}$

ξ_{s1} = structural damping = 0.01

ξ_{a1} = aerodynamic damping as given in Repetto and Solari (2001)

$Coh(z_l, z_k; \theta_{lk} f_1)$ = coherence function as given in Repetto and Solari (2001)

and $\theta_{lk} = 1$ for l and $k \neq 0$
$= 0.5$ for l or $k \neq 0$

The standard deviation of fluctuating stress, σ_s, is obtained as given below:

$$\sigma_s(z) = \{2I_u(h_0)\sqrt{B+R}\}\bar{S}(z) \tag{38}$$

The effective frequency of background component, f_B, for the lighting pole is calculated using Eq. (30), as given below:

$$f_B = f_B(\tilde{b}_0, \tilde{H}_0, h_0) \tag{39}$$

where $\tilde{b}_0 = \dfrac{C_x b_0}{L_u(h_0)}$ and $\tilde{H}_0 = \dfrac{C_z H_0}{L_u(h_0)}$

with $C_x = C_z = 11.5$

As per the Bi-modal spectral method, the fraction of fatigue damage for different reference mean wind speeds have been calculated using Eq. (22) by evaluating B, R, σ_s, and f_B, using Eqs. (36), (37), (38), and (39), respectively and by substituting them in Eqs. (23a) and (24). The correction factors, λ'_{Bi}, obtained from the Bi-modal spectral method (Eq. 23a) have been compared with the λ'_{RFC} values obtained from the simulated stress time histories using RFC counting method (Eq. 35) in Fig. 10 for $m = 3$ and 5. The correction factors obtained from the Bi-modal spectral method are observed to compare well with those obtained using RFC counting method factors at low mean wind speeds. However, the Bi-modal spectral method is observed to be predicting the correction factor conservatively upto 10% difference with respect to the values obtained using RFC counting method in high mean wind speeds, which is acceptable from the design point of view since high mean wind speeds have lower probability of occurrence in the design life. The contributions of background (B') and resonant (R') components in the correction factor, λ'_{Bi} are studied by considering the resonant component (R') alone in Eq. (23a) and comparing it with the total correction factor, λ'_{Bi} for $m = 3$ and 5 (Fig. 11). For $m = 3$, the contribution of resonant component varies from 53% to 68% and for $m = 5$, the same varies from 12.5% to 40% for the various

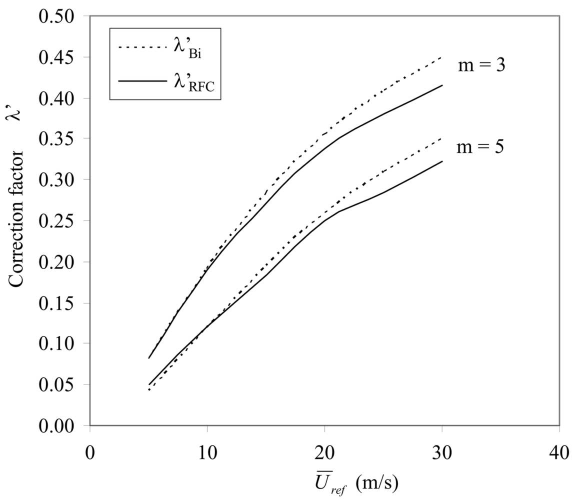


Fig. 10 Comparison of correction factors

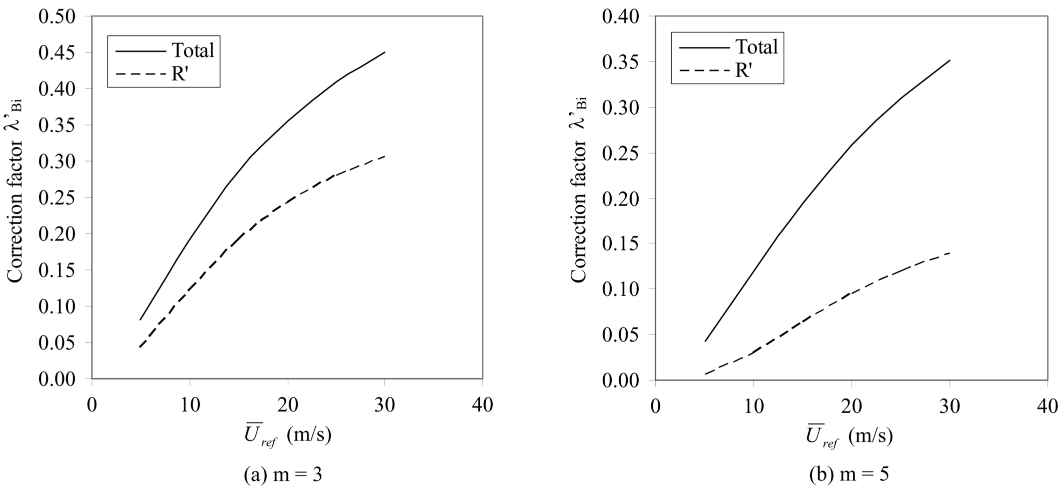


Fig. 11 Contribution of resonant component in correction factor

reference mean wind speeds. This indicates that the contribution of background component is also significant in the correction factor, λ'_{Bi}, and more so in the case of higher *m* values.

7. Conclusions

In this study the experimentally measured stress cycles have been counted using Rainflow cycle counting method. The ratios of fractional fatigue damage obtained from Rainflow cycle counting to that obtained using narrow band assumption have been compared with the correction factors calculated using the expressions suggested by Wirsching and Light (1980), Ortiz and Chen (1987) and Lutes and Larsen (1990). For irregularity factors less than 0.9, the correction factors suggested by Wirsching and Light (1980) have been observed to be over predicting. The correction factors suggested by Ortiz and Chen (1987) and Lutes and Larsen (1990) have been observed to be comparing well with those obtained from Rainflow counting method. Ortiz and Chen method requires multiple spectral moments of the measured response spectra as compared to Lutes and Larsen method which requires only single spectral moment. A Bi-modal spectral method has been formulated by idealizing the single spectral moment method using background and resonant components as considered in gust response factor. A closed form approximation for the effective frequency for the background component has been developed. The simplicity and the accuracy of the new method has been illustrated through a case study by choosing an urban light pole studied earlier by Repetto and Solari (2001), along with the closed form approximations for background and resonant components. The correction factors evaluated based on the Bi-modal spectral method compared well with those calculated from the simulated time histories using RFC counting method for low mean wind speeds. For high mean wind speeds, the Bi-modal spectral method observed to predict the correction factors more by about 10% which is conservative for fatigue design. The developed Bi-modal method is observed to be a simple and easy to use alternative to detailed time and frequency domain fatigue analyses without considerable computational and experimental efforts.

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Appendix-I

Various key expressions required for the evaluation of response spectra as given in Repetto and Solari (2001) are given below.

$$\bar{K}_0 = \frac{1}{Hb_0 C_{d0}\phi_1(h)}\int_0^H b(z)C_d(z)\Delta^2(z)\phi_1(z)dz \tag{A.1}$$

$$\bar{K}_k = \frac{A_k C_{dk}\Delta^2(z_k)\phi_1(z_k)}{Hb_0 C_{d0}\phi_1(h)} \qquad (k = 1,\ldots, M) \tag{A.2}$$

$$\Omega_{00}(f) = K'^2_0 \frac{S_u(z, f)}{\sigma_u^2}\chi(k_{0z}\tau_{0z} f) \tag{A.3}$$

$$\Omega_{lk}(f) = K'_l K'_k \sqrt{\frac{S_u(z_l, f)}{\sigma_u^2}\chi(0.4\,\tau_{lz}f)\chi(0.4\,\tau_{ly}f)}\sqrt{\frac{S_u(z_k, n)}{\sigma_u^2}\chi(0.4\,\tau_{kz}f)\chi(0.4\,\tau_{ky}f)}Coh(z_l, z_k, f)$$
$$(l,m = 1,\ldots, M) \tag{A.4}$$

$$\Omega_{0k}(f) = K'_0 K'_k \sqrt{\frac{S_u(h, f)}{\sigma_u^2}\chi(k_{0z}\tau_{0z}f)}\sqrt{\frac{S_u(z_k, f)}{\sigma_u^2}\chi(0.4\,\tau_{kz}f)\chi(0.4\,\tau_{ky}f)}Coh(z_l, z_k, 0.5f)$$
$$(l,m = 1,\ldots, M) \tag{A.5}$$

$$\frac{S_u(h;f)}{\sigma_u^2(h)} = \frac{6.868 L_u(h)/\bar{U}(h)}{(1 + 10.302 f L_u(h)/\bar{U}(h))^{5/3}} \tag{A.6}$$

$$\chi\{\eta\} = \frac{1}{\eta} - \frac{1}{2\eta^2}(1 - e^{-2\eta}) \qquad (\chi = 1 \text{ for } \eta = 0) \tag{A.7}$$

$$Coh(z_l, z_k, f) = \exp\left\{-\frac{2fC_{uz}|z_l - z_k|}{U(z_l) + U(z_k)}\right\} \tag{A.16}$$

$$H_1(f) = \frac{1}{1 - (f/f_1)^2 + 2i\xi_1(f/f_1)} \tag{A.17}$$

$$\xi_1 = \xi_{s1} + \xi_{a1} \tag{A.18}$$

where M = no. of localized masses on the structure
H = total height of structure
h = reference height of the structure = $0.6H$
z = height above ground level
z_k = height of the k^{th} mass above ground level
$b(z)$ = size of the structure orthogonal to wind direction at level z
$C_d(z)$ = drag coefficient of the structure at level z
$\Delta(z)$ = $\bar{U}(z)/\bar{U}(h)$
$\phi_1(z)$ = first mode shape value at level z
b_0 = reference size of the structure orthogonal to wind direction
C_{d0} = reference value of drag coefficient of the structure
A_k = surface projection of k^{th} mass
C_{dk} = drag coefficient for the k^{th} mass
f_1 = natural frequency of the first mode of the structure
ξ_{s1} = structural damping ratio
ξ_{a1} = aerodynamic damping as given in Repetto and Solari (2001)
C_{uz} = exponential decay coefficient in vertical direction
and C_{uy} = exponential decay coefficient in direction normal to wind

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