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Testing of tuned liquid damper with screens and development of equivalent TMD model

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Abstract. The tuned liquid damper (TLD) is increasingly being used as an economical and effective vibration absorber. It consists of a water tank having the fundamental sloshing fluid frequency tuned to the natural frequency of the structure. In order to perform efficiently, the TLD must possess a certain amount of inherent damping. This can be achieved by placing screens inside the tank. The current study experimentally investigates the behaviour of a TLD equipped with damping screens. A series of shake table tests are conducted in order to assess the effect of the screens on the free surface motion, the base shear forces and the amount of energy dissipated. The variation of these parameters with the level of excitation is also studied. Finally, an amplitude dependent equivalent tuned mass damper (TMD), representing the TLD, is determined based on the experimental results. The dynamic characteristics of this equivalent TMD, in terms of mass, stiffness and damping parameters are determined by energy equivalence. The above parameters are expressed in terms of the base excitation amplitude. The parameters are compared to those obtained using linear small amplitude wave theory. The validity of this nonlinear model is examined in the companion paper.

Keywords: tuned liquid damper; TLD; vibration absorber; tuned mass damper; sloshing; nonlinear; control; dynamic, damping.

1. Introduction

An economical solution to mitigate building motions to acceptable levels under dynamic loading is to provide additional effective damping by means of a dynamic vibration absorber. One passive dynamic vibration absorber (DVA) shown to be effective is the tuned liquid damper (TLD), which consists of a rigid tank partially filled with a liquid, usually water. When a structure fitted with a properly tuned TLD begins to sway during a dynamic loading event, it causes a fluid sloshing motion inside the tank. The fluid sloshing motion imparts inertia forces approximately anti-phase to the dynamic forces exciting the structure, thereby reducing structural motion. Therefore, this device modifies the frequency response of the structure and reduces the magnitude of the response in a way comparable to an increase in the effective damping.

The sloshing action must dissipate a certain amount of energy in order for the TLD to operate

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effectively. Unfortunately, the inherent damping of the sloshing fluid is usually significantly lower than the value required for the TLD to perform efficiently. An increase in the inherent damping can be achieved by attaching a number of screens inside the tank (Noji, *et al*. 1988, Fediw 1995, Warnitchai and Pinkaew 1998 and Kaneko and Ishikawa 1999).

The first part of this paper reports on the results of an experimental program conducted on a rectangular tank partially filled with water to be used as a TLD. Two screens consisting of horizontal slats are attached inside the tank. The experimental program involves shake table testing of the TLD in order to determine its dynamic characteristics. The variations of the free surface, the base shear force, and energy dissipated by the sloshing fluid are studied at several excitation amplitudes with the screens installed. The purpose is to assess the amplitude dependency of the above parameters. Additionally, a single test is conducted on the liquid filled tank without screens for comparative purposes.

The second part of the paper focuses on evaluating the properties of an equivalent amplitude dependent tuned mass damper (TMD) having equal energy dissipation as the TLD. Graham and Rodriguez (1952) introduced the concept of a simple mechanical model that produces the equivalent forces developed by the sloshing fluid. This model is based on the assumption of potential flow with linearized boundary conditions. This equivalent mechanical model has been used to represent a TLD; however, the above model has limitations. It is unable to capture the non-linear amplitude dependent properties of the TLD, and it does not account for the effect of internal damping devices such as baffles or screens.

More recently, semi-empirical models of an equivalent TMD that reproduce the nonlinear behaviour of a TLD have been introduced (Chaiseri 1990, Sun, *et al*. 1995, Yu, *et al*. 1999 and Yalla 2001). Sun, *et al*. (1995) used the concept of virtual mass and virtual damping to determine an equivalent TMD by matching these two parameters simultaneously. Yu, *et al*. (1999) experimentally evaluated equivalent TMD stiffness and damping properties to represent a TLD using the concept of equivalent energy dissipation. The model assumed full participation of the fluid mass, which is valid for large excitation amplitudes or shallow water depths. No additional energy dissipating mechanisms such as screens were present in the tank.

In the current study experimental results are used to evaluate the properties of an equivalent amplitude dependent TMD, having equal energy dissipation as a TLD equipped with damping screens. In this model, the percentage of fluid mass participating in the sloshing motion is considered as a variable parameter. Therefore, the assumption of full participation of the fluid mass is not made. Validation of the amplitude dependent TMD model is provided in the companion paper (Tait, *et al*. 2004b).

2. Description of tuned liquid damper tested

Fig. 1 shows the dimensions of the tank that is used in the experimental study. The dimensions L, b, and h represent the tank length (in the direction of excitation), the tank width (perpendicular to excitation) and the water depth, respectively. The fundamental sloshing frequency, f_w, for the water inside this tank can be estimated using linear wave theory according to the following equation (Lamb 1932)

$$f_w = \frac{1}{2\pi}\sqrt{\frac{\pi g}{L}\tanh\left(\frac{\pi h}{L}\right)} \tag{1}$$

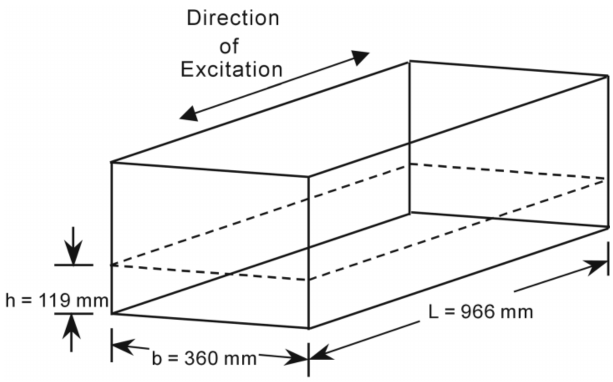


Fig. 1 Schematic of a 1-D TLD tank dimensions

where g is the acceleration of gravity. Substituting L and h from Fig. 1 into the above equation, leads to a value of $f_w \approx 0.545$ Hz.

This TLD represents a 1:10 scale model of one of the prototype multiple tanks designed to be used in a full-scale building that has a fundamental frequency, f_s, of approximately 0.172 Hz. According to dimensional analysis theory, the fundamental fluid sloshing frequency of the prototype tank is estimated to be equal to 0.172 Hz. This means that the sloshing frequency of the prototype tank is approximately equal to the fundamental frequency of the building. The tuning ratio, Ω, is an important parameter that influences the performance of a TLD, and is given by

$$\Omega = \frac{f_{TLD}}{f_s} \tag{2}$$

where f_{TLD} is the natural frequency of the TLD absorber attached to the primary structure. In this study the ratio between the natural frequency of the TLD, f_{TLD}, and the fluid sloshing frequency, f_w, is approximately unity for small sloshing fluid response amplitudes.

A second parameter affecting the response of the structure-TLD system is the mass ratio, μ, which is defined as

$$\mu = \frac{\phi^2 m_{TLD}}{M^*} \tag{3}$$

where M^* is the generalized mass of the structure in the specific vibration mode to be suppressed, m_{TLD} is the portion of the fluid mass contributing to the fundamental sloshing mode of the TLD and ϕ is the normalized modal amplitude value at the damper location. In this study it is assumed that the TLD is located at the top of the building and the modal amplitude, ϕ, has a value of unity.

An estimate of m_{TLD} can be made by assuming $m_{TLD} \approx m_1$. The value of m_1 can be calculated from potential flow theory using the following equation (Graham and Rodriquez 1952)

$$m_n = \frac{8\tanh\left((2n+1)\pi\frac{h}{L}\right)}{\pi^3(2n+1)^3\left(\frac{h}{L}\right)} m_w \tag{4}$$

where m_w is the total mass of the water inside the tank. Substituting the values of the tank length, L, and water depth, h, into Eq. (4) and setting $n=0$, a value of $m_1=0.77m_w$ is obtained. The value of m_w can be varied by changing the width of the tank, b, or the number of tanks used. For this study a target value of $\mu \approx 1.3\%$ is chosen assuming $m_{TLD}=m_1$.

A third parameter that influences the performance of a structure-TLD system is the inherent damping ratio, ζ_{TLD}, of the sloshing fluid inside the tank. Warburton (1982) has established an expression for determining the optimum inherent damping value for a linear TMD as a function of the mass ratio, μ. Due to the analogy existing between the TMD and TLD devices, Warburton's approach is used, and accordingly an initial target value of $\zeta_{TLD}=5.7\%$ is estimated. The main source of inherent damping within the tank arises from viscous dissipation in the boundary layers at the solid boundaries of the tank and from free surface contamination. Assuming linear wave theory, ζ_{TLD} of sloshing liquid inside a rectangular tank, without additional devices can be estimated by Sun (1991)

$$\zeta_{TLD} = \frac{1}{2h}\sqrt{\frac{\nu_w}{\pi f_w}}\left(1+\frac{h}{b}\right) \tag{5}$$

where ν_w is the kinematic viscosity of water. According to Eq. (5), the inherent damping for the TLD used is estimated to be approximately 0.45%. This is significantly less than the target value. Inspection of Eq. (5) suggests that either increasing the fluid viscosity or decreasing the fluid depth would provide additional damping. Often the TLD is used as a water storage tank preventing the use of a higher viscosity liquid. Yu, *et al.* (1999) have found experimentally higher inherent damping values can be achieved using shallow water TLDs that dissipate a significant amount of energy as a result of wave breaking. However, using extremely shallow water depths requires the use of multiple tanks in order to achieve the desired mass ratio, resulting in larger space requirements. Also the nonlinear response characteristics of a TLD increase as the water depth to tank length ratio is reduced, making the TLD response less predictable.

The damping device chosen to increase ζ_{TLD} is shown in Fig. 2. Horizontal slats are spaced apart in order to form screens. This particular screen arrangement can be easily installed and allows the solidity ratio, S, of the damping screens to be altered, by adjusting the space between the slats, in order to obtain the required value ζ_{TLD}. The model scale individual slats are approximately 5 mm wide, and are spaced 7 mm apart, resulting in a solidity ratio of $S=0.42$. Fig. 3(b) shows a

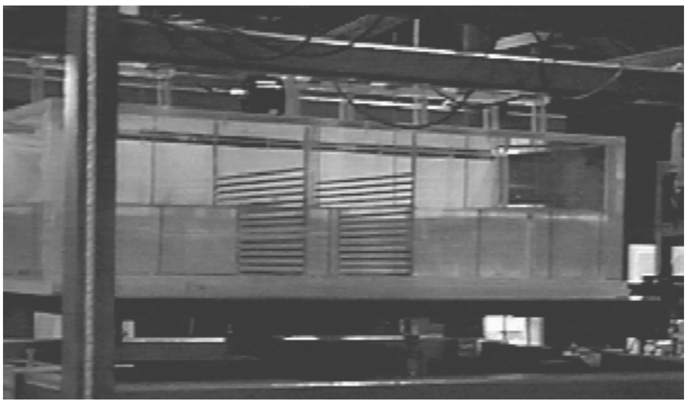


Fig. 2 Photo of TLD equipped with internal damping screens

schematic of a tank equipped with the slat screens used in this study. The screens are located at distances of 0.4*L* and 0.6*L* from the end wall of the tank.

3. Experimental set-up and procedure

A schematic of the experimental set-up is shown in Fig. 3. The set-up includes a rigid support

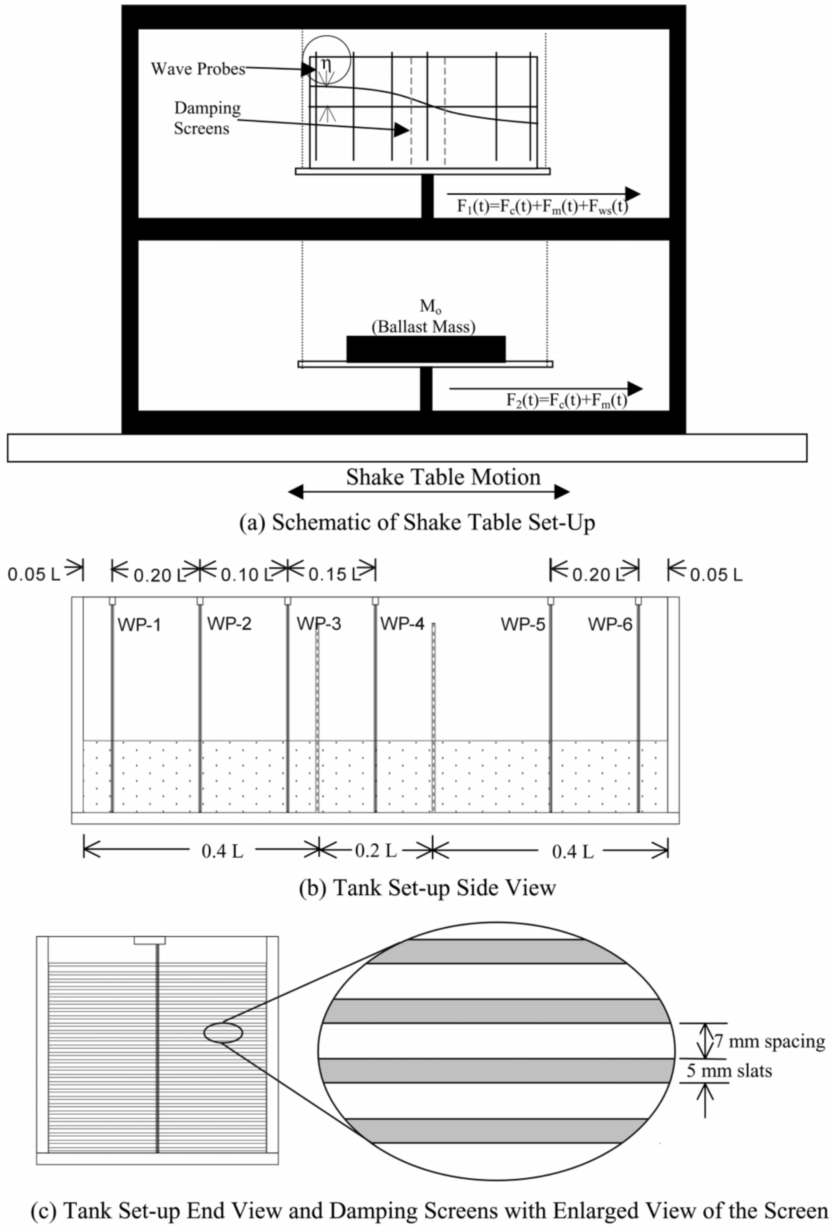


Fig. 3 Test frame set-up of TLD equipped with damping screen

frame attached to a shake table. As shown in the figure, the tank is attached to the top section of the frame using four cables that provided vertical support. The lateral support of the tank is provided by two load cells that connect the bottom of the tank to a rigid intermediate frame member. In the bottom section, a similar attachment is made for the ballast mass, M_o, equal to the mass of the tank and the water contained inside. The data recorded include the shake table displacement, the base shear forces developed by both the TLD and ballast mass and the position of the temporal free surface at six locations along the tank length.

The base shear force F_1, measured by the load cells that connect the tank to the frame can be separated into three components. These are the inertial force due to the tank itself, F_c, the inertial force due to the mass of the contained fluid, F_m, and the additional forces due to the sloshing fluid, F_{sw}. The base shear force F_2, measured by the load cells connecting the ballast mass to the frame represents the summation of F_c and F_m. The liquid sloshing forces, $F_{sw}(t)$, can be evaluated by subtracting the outputs of the two base shear forces, i.e.,

$$F_{sw}(t) = F_1(t) - F_2(t) \tag{6}$$

The total base shear forces, F_w, due to the inertial and sloshing forces of the fluid, can be obtained by

$$F_w(t) = F_1(t) - F_c(t) \tag{7}$$

The frame assembly is subjected to harmonic excitations with the excitation amplitude ranging from 2.5 mm to 40.0 mm. The forcing frequency ratio, β_w, is defined as the ratio between the excitation frequency, f, and the fundamental sloshing frequency, f_w, and is given by

$$\beta_w = \frac{f}{f_w} \tag{8}$$

The wave probes consist of capacitance-type bow-string sensors comprising of a taught loop of wire connected to an amplifier designed to convert the changes of capacitance to a measurable change in voltage. The locations of the six wave probes used to measure the free surface motion, η, are shown in Fig. 3. The normalized free surface motion, η', is expressed as

$$\eta' = \frac{\eta}{h} \tag{9}$$

Discrete frequency sweep tests are conducted by varying the ratio β_w in discrete increments in the range of $0.65<\beta_w<1.50$. The data are recorded after the fluid sloshing motion reaches a steady state condition. A description of the tests conducted is provided in Table 1.

Table 1 Data for shake table tests

Test Identification	Excitation Amplitude	
	A (mm)	*A/L*
NS-2.5 (no screens)	2.5	0.0026
S-2.5 (screens)	2.5	0.0026
S-5.0 (screens)	5.0	0.0052
S-7.5 (screens)	7.5	0.0078
S-10.0 (screens)	10.0	0.0100
S-12.5 (screens)	12.5	0.0129
S-15.0 (screens)	15.0	0.0155
S-20.0 (screens)	20.0	0.0207
S-30.0 (screens)	30.0	0.0311
S-40.0 (screens)	40.0	0.0414

4. Experimental results and discussion

Results obtained from the free surface motion and the base shear measurements are presented below. In addition, the energy dissipated by the TLD is calculated from the experimental data and is presented for various tests.

4.1. Free surface motion

The nonlinear response of this particular TLD is found to exhibit a hardening type behaviour, characterized by an increase in the sloshing resonant frequency value with an increase in the excitation amplitude. A characteristic of such a nonlinear type dynamic system is that higher harmonics or superharmonics are present in the sloshing fluid response (Chester 1968). The presence of superharmonics in the response are demonstrated by the three dimensional plots of the free surface given in Figs. 4 to 6. The plots show the variation, in time, of the free surface height along the centreline of the tank. Figs. 4, 5 and 6 describe the free surface wave motion that occurred during the NS-2.5, S-2.5 and S-12.5 tests at an excitation frequency ratio of β_w=1.07, 1.00, and 1.04, respectively. The maximum wave height measured in these tests occurred at these specific β_w values. Unfiltered plots of the free surface motion of the three tests are provided by Figs. 4(a), 5(a) and 6. The time histories of the free surface motions for the 2.5 mm tests are band-pass filtered to show the contribution of the various harmonics of the response motion. Figs. 4(b) and 5(b) show the contribution of the fundamental response harmonic that matches the excitation frequency and occurs normally when a linear system is dynamically excited. Figs. 4(c) and 5(c) show the contribution of a second harmonic, which is oscillating at a frequency two times that of the excitation frequency. Figs. 4(d) and 5(d) show the contribution of a third harmonic, which is oscillating at a frequency three times that of the excitation frequency. Upon examining the wave forms, it is apparent that the second and third harmonics are similar to the second and third sloshing modes, respectively. A comparison between Figs. 4 and 5 indicates that the introduction of the screens leads to a substantial reduction in the maximum wave amplitude. Consequently, the increase in ζ_{TLD} developed by the screens reduces the degree of nonlinearity and the presence of superharmonics. As a result, the TLD response is near linear with the screens present, particularly for the S-2.5 test.

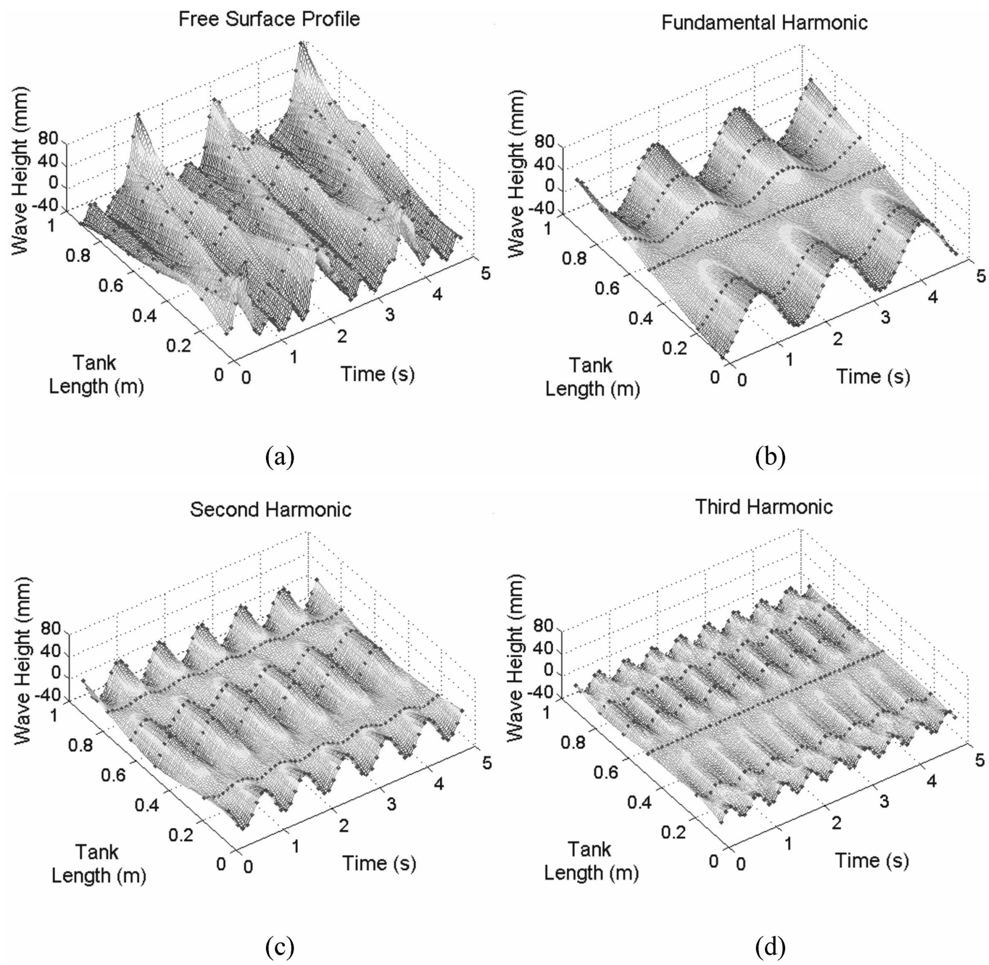


Fig. 4 Free surface motion of TLD with no damping screens (NS-2.5, β_w=1.07)

The presence of higher harmonics in the response of the sloshing fluid motion results in the excitation of higher sloshing modes. The higher sloshing modes are excited at excitation frequencies that are integer multiple values less than their calculated values (Shimizu and Hayama 1987). The β_w values at which the first three sloshing modes will be excited have been calculated and are presented in Table 2. It can be seen that the second and third sloshing modes are prone to excitation by superharmonics in the range of excitation frequencies used in this experiment. This will be evident by the following example.

Frequency response curves of wave probe WP-1, located near the wall of the tank, for tests NS-2.5, S-2.5 and S-12.5 are presented in Figs. 7(a), 7(b) and 7(c), respectively. In these figures, the variation of the normalized wave height η' is plotted versus the forcing frequency ratio, β_w. In interpreting the plots provided in Fig. 7, it should be noted that for a certain mode, n, and for a forcing frequency ratio, β_w, the fluid responds at frequency ratio values equal to $\beta_w \cdot n$, when the TLD is excited at frequencies near the fundamental sloshing frequency. For example, consider test S-2.5, shown in Fig. 7(b), the maximum response for the first mode occurred at β_w=1.0. The second

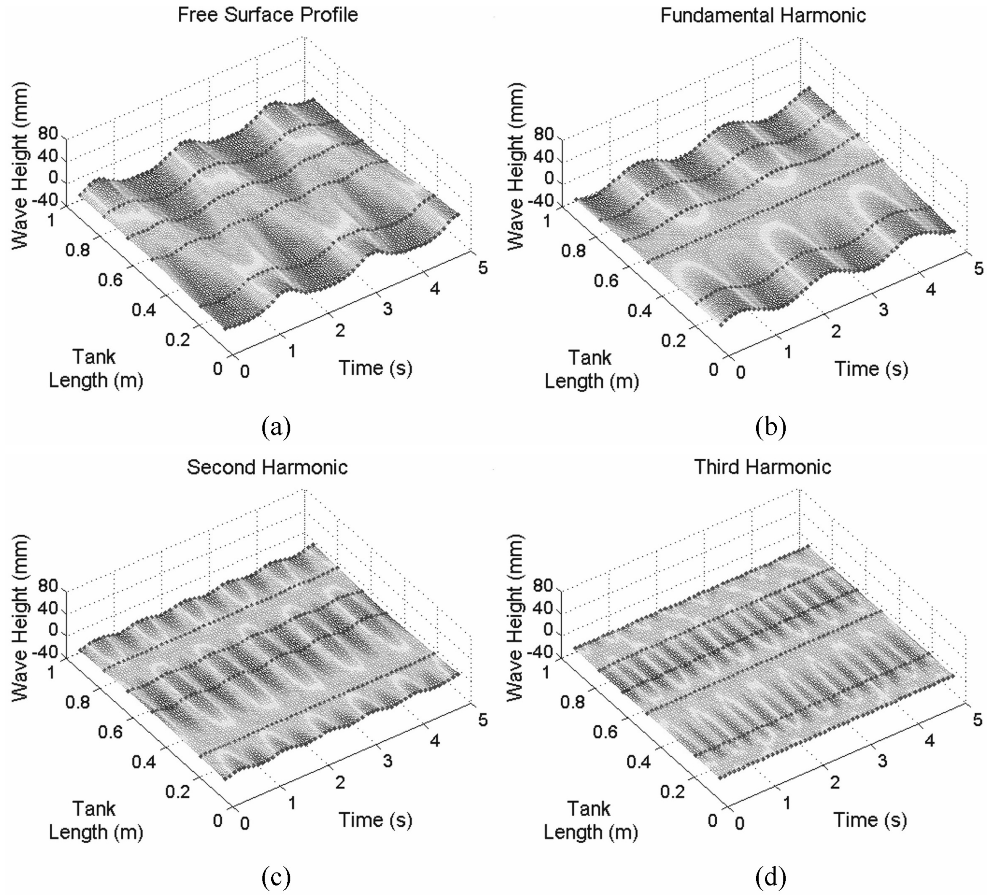


Fig. 5 Free surface motion of TLD with damping screens (S−2.5, β_w=1.0)

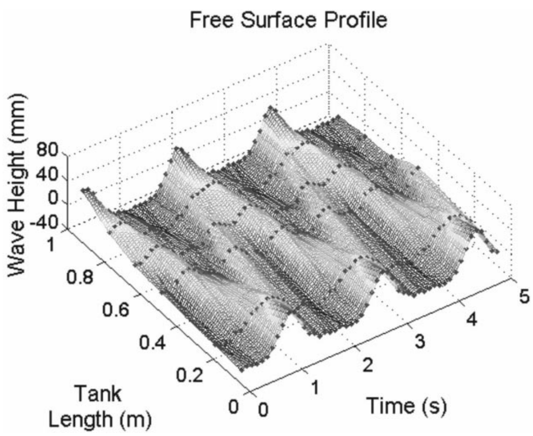
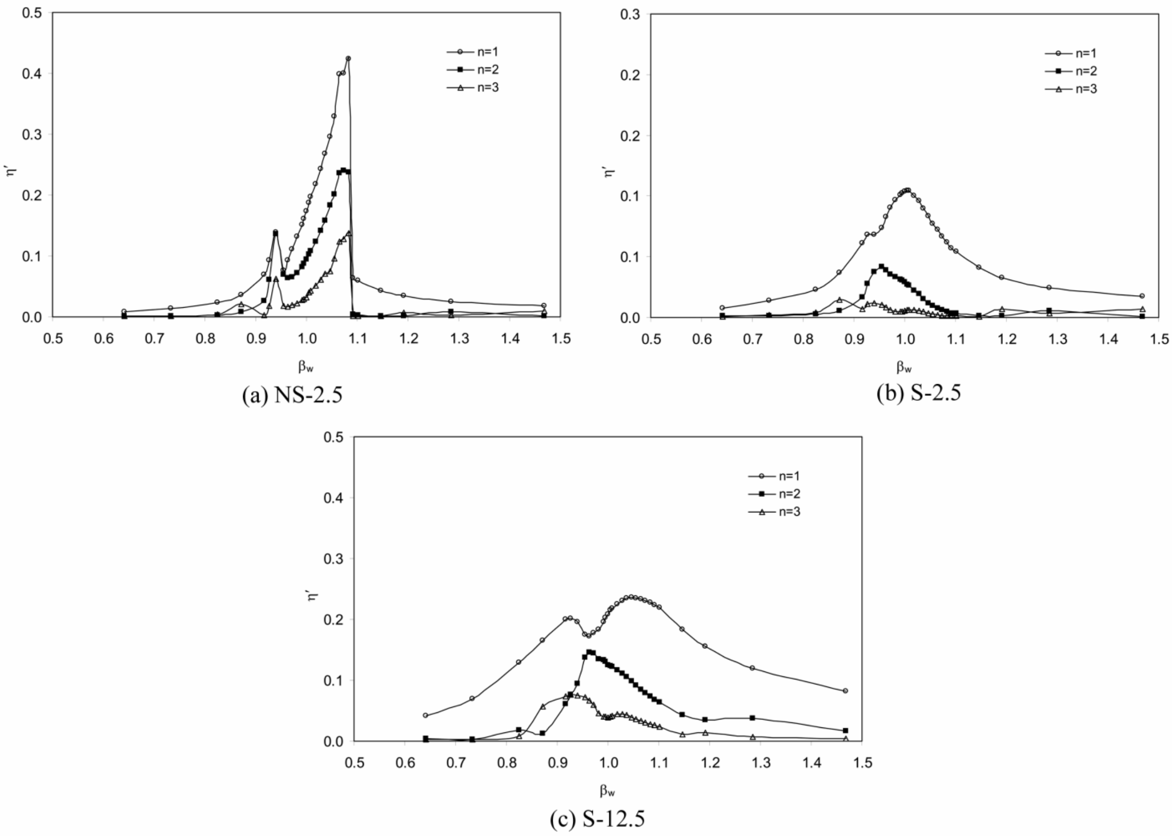


Fig. 6 Free surface motion of TLD with damping screens (S−12.5, β_w=1.04)

harmonic had a maximum response at β_w=0.939. The value of $\beta_w \cdot 2 \cdot f_w$ is 1.024 Hz, which corresponds to the natural frequency of the second sloshing mode. Similarly, the maximum response

Table 2 Excitation frequency ratio values relating superharmonics to the excitation of higher sloshing modes

MODE (n)	f_w (Hz)	β_w	n
1	0.545	1.000	1
2	1.024	0.939	2
3	1.411	0.863	3



(a) NS-2.5 (b) S-2.5

(c) S-12.5

Fig. 7 Frequency response of fundamental, second and third harmonics for tests

of the third harmonic occurred at $\beta_w = 0.863$ resulting in $\beta_w \cdot 3 \cdot f_w$ matching the natural frequency of the third sloshing mode.

A number of observations can be drawn from the free surface motion responses curves:

- The presence of screens has significantly reduced the maximum normalized wave height, η'. A comparison between Figs. 7(a) and 7(b) indicates that under the same level of excitation (2.5 mm), the maximum free surface response elevation has been reduced by a factor of approximately 4. These results are in agreement with those found by Fediw, *et al.* (1995) for a TLD equipped with lattice damping screens.
- The resonant frequency values corresponding to the S-2.5 test show good agreement with values calculated using linear wave theory and reported in Table 2. This is true for both the fundamental and higher sloshing modes. This implies that under low excitation levels the presence of screens

has limited the sloshing response to “near” linear type behaviour.

- Results of the no-screen test NS-2.5 indicate a higher value of natural sloshing frequency compared to that predicted by linear wave theory. This exhibited nonlinear behaviour is that of a ‘hardening’ type system and results in an increase in the resonant frequency with increased free surface motion response amplitude. Another characteristic of this type of system is the discontinuity that occurs at the ‘jump frequency’ near resonance, which can be seen in Fig. 7(a). This discontinuity is no longer present with the addition of screens for the amplitudes tested, which has been observed in previous experiments on TLDs equipped with damping screens (Fediw, *et al.* 1995).
- The superharmonics affect the shape of the fundamental frequency response curve resulting in a multi-peak response curve. This interference on the frequency response curve, which results in multiple peaks, is a characteristic of nonlinear dynamic systems (Szemplinska-Stupnicka 1968, Chester 1968).

4.2. Base shear force measurements and energy dissipation characteristics

The horizontal interaction forces that occur between the TLD and the shake table are reported in this section. Base shear forces develop as a direct result of the fluid sloshing motion. Using potential flow and assuming linearized boundary conditions, an estimate of the total base shear force, F_w, can be made from the amplitude of the free surface elevation measured experimentally. The relationship between the difference in the free surface amplitudes at the two end walls of the tank, $\eta_{x=0}$ and $\eta_{x=L}$ and the total base shear force, F_w is given by Dean and Dalrymple (1984)

$$F_w = \frac{1}{2}\rho g \frac{(k\eta_{x=0}^2 + 2\tanh(kh)\eta_{x=0} - k\eta_{x=L}^2 + 2\tanh(kh)\eta_{x=L})}{k} \tag{10}$$

where ρ_w is the density of water and k is the wave number which is equal to

$$k = \frac{\pi}{L} \tag{11}$$

for the fundamental sloshing mode.

Time histories of the free surface motions and the base shear forces are digitally filtered in order to remove the contribution of the higher harmonics. Comparisons are made between measured and calculated total base shear forces using Eq. (10). Results of this comparison are provided in Fig. 8 for tests S-2.5 and S-12.5, respectively. Fig. 8 shows excellent agreement between the measured and calculated total base shear forces for the S-2.5 case. This indicates that although the screens may locally alter the kinematics of the liquid sloshing from those estimated from linear wave theory, an estimate of the total base shear forces based on the free surface amplitude, while neglecting the effect of viscosity, is still applicable at small response amplitudes. The calculated forces underestimate the measured base shear forces, when the level of excitation was increased in the S-12.5 test. The presence of screens significantly increases the range of base excitation amplitudes for linear wave theory validity by reducing the response amplitude of the free surface motion.

Figs. 9(a) to 9(c) show experimentally measured base shear frequency response values obtained using Eqs. (6) and (7) for the fundamental sloshing mode. Both the non-dimensional total, F'_w, and sloshing, F'_{sw}, base shear force frequency response curves have been plotted for the NS-2.5, S-2.5 and S-12.5 cases, respectively. The forces have been normalized by the product, $m_w(2\pi f)^2A$, which

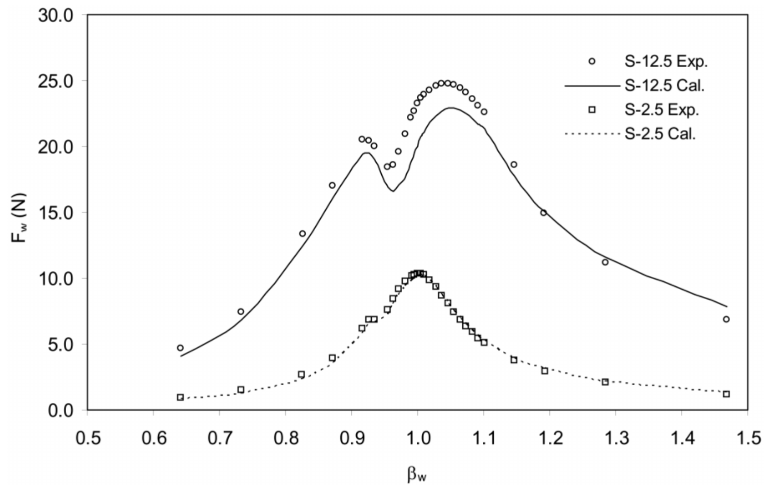
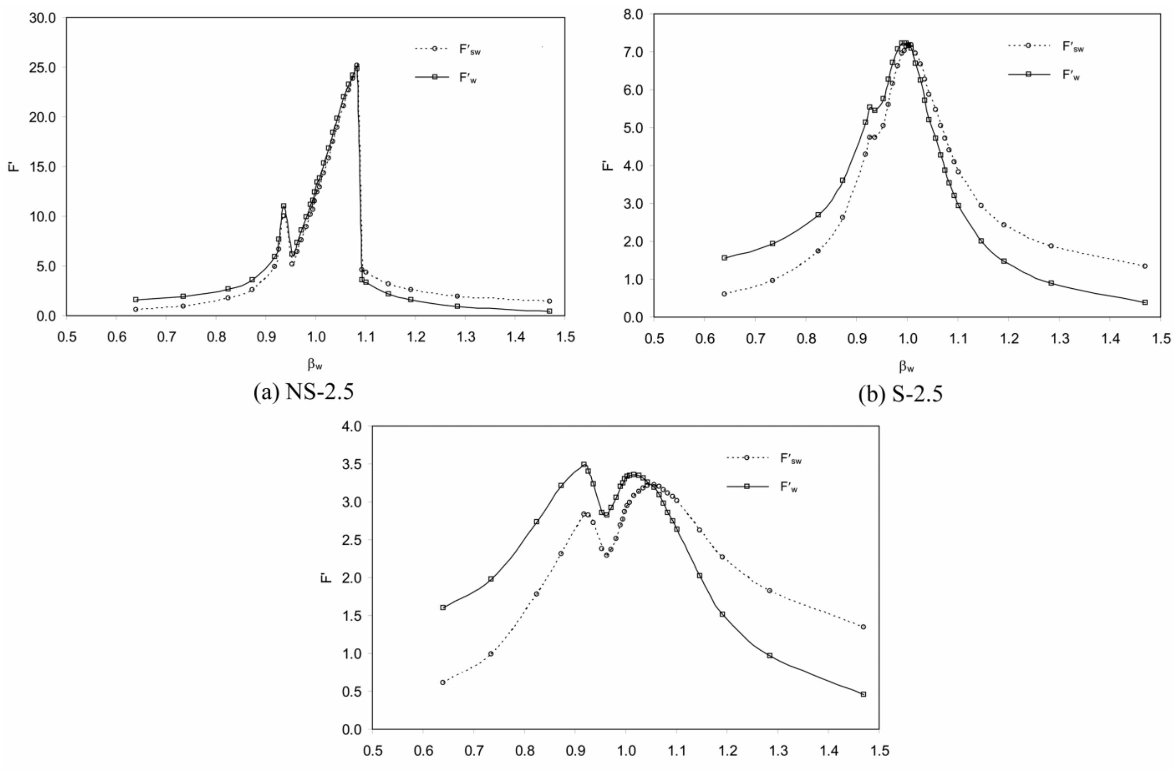


Fig. 8 Comparsion of measured and calculated total base shear forces using potential flow theory



(a) NS-2.5 (b) S-2.5 (c) S-12.5

Fig. 9 Effect of damping screens and excitation amplitude on total and sloshing fluid forces for tests

represents the inertia force of the water if it is treated as a solid mass. The following observations can be drawn from these figures:

- The presence of the screens has significantly reduced both the total and sloshing forces. A reduction factor of approximately 3.5 in the resonant response results from the addition of the screens, under excitation amplitude of 2.5 mm.
- The plots of the sloshing and total forces are similar for the no-screen case, indicating the nearly the entire fluid mass contributed in the sloshing mode of vibration.
- With increased amplitude of excitation, the non-dimensional magnitude of the response is found to decrease. This indicates that the additional damping added to the TLD is dependent on the excitation amplitude.

4.3. Energy dissipation of sloshing fluid

An estimate of the energy dissipation within the TLD can be obtained by means of the measured base shear forces and the corresponding shake table displacements. The area enclosed by the base shear force versus shake table displacement loop represents the energy dissipated per cycle, E_w, given by

$$E_w = \int_T F dx \tag{12}$$

where T is the period of the shake table motion and x is the shaking table displacement. The force displacement loops obtained for the S-2.5 test are given in Fig. 10. In order to discuss the variation

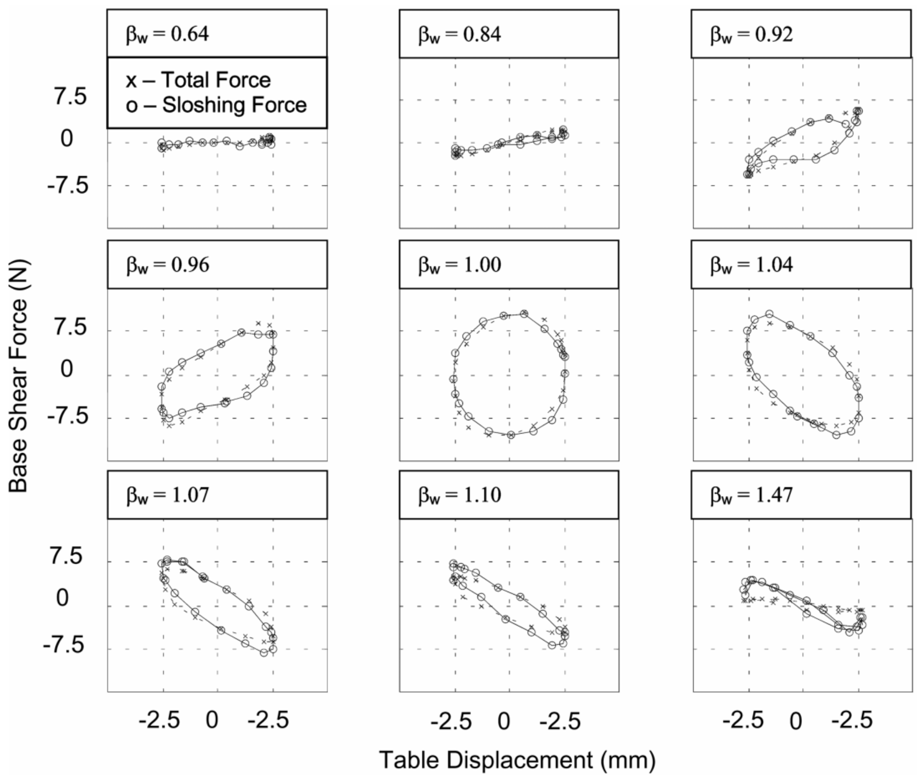


Fig. 10 Energy dissipation curves of base shear force versus shake table displacement test S-2.5

of the energy dissipation with the excitation frequency, the figure contains the response plots resulting from nine different values of β_w. The plots contain both the total and sloshing base shear forces versus shake table displacement. The area inside the plotted loops for both forces is found to be equal. The contribution of the non-participating mass is included in the total base shear forces. However, this non-participating mass produces additional conservative forces but dissipates no additional energy. It only changes the orientation of the loops. Therefore, the energy dissipated per cycle is a unique value, independent on the definition of the base shear force, i.e., total or sloshing base shear forces.

For each β_w value, the energy dissipated per cycle, E_w, is calculated using Eq. (12). A dimensionless parameter, E'_w, is then evaluated using the following relationship

$$E'_w = \frac{E_w}{\frac{1}{2}m_w(2\pi fA)^2} \tag{13}$$

where the denominator represents the maximum kinetic energy of the contained fluid when treated as a solid. The variation of E'_w with the excitation frequency ratio, β_w, is provided in Fig. 11 for the NS-2.5, S-2.5 and S-12.5 tests. The following information can be extracted from this figure:

- A comparison between the curves obtained for the tests conducted with the presence of damping screens indicates that as the amplitude of excitation increases, the maximum value of E'_w decreases.
- A large amount of energy dissipation is found to occur only in a narrow frequency band near resonance during the no screen test. It should be noted that splashing of the water against the lid of the TLD occurred during this test increasing the amount of energy dissipated. The amplitude of the resonant response of the sloshing fluid with no screens present is multi-valued. As such, it should not be expected that the amplitude of the response curve near resonance, given in Fig. 11, would be maintained under more general type loading conditions. The energy dissipated by a TLD (with no screens) sinusoidally excited is also dependent on the direction of the frequency sweep. For increasing values of excitation frequency (forward sweep), a hardening system will

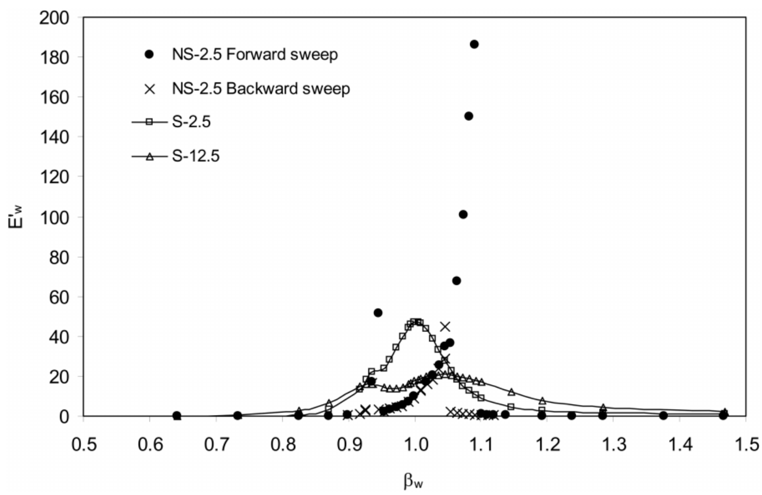


Fig. 11 Variation of energy dissipation with excitation frequency

jump to a lower amplitude value. For a decreasing excitation frequency (backward sweep), a hardening system will jump to higher amplitude. Fig. 11 indicates that the maximum energy dissipated at a particular excitation frequency can reduce by as much as a factor of 4, if the TLD is excited by a backward discrete frequency sweep.

To identify the influence of the screens on the energy dissipated, comparisons are made at an equivalent response level, i.e., at an equivalent value for η, corresponding to the fundamental sloshing response amplitude, with and without the screens inside the tank. Based on the response curves given in Fig. 7, such a comparison can be conducted between NS-2.5 and S-12.5 tests at $\beta_w = 1.018$. For these two tests, the free surface response amplitude values of η are near equal at 26.0 mm and 26.8 mm, respectively. The corresponding maximum measured base shear forces for the tests are equal to 21.1 N and 22.5 N, respectively. This shows that the screens do not reduce the base shear sloshing forces for a given fundamental free surface response amplitude.

5. Equivalent TMD analogy

In this section, an equivalent amplitude dependent TMD model is reported on. The parameters for the model are obtained by matching the energy dissipated by a partially fluid filled tank containing screens (TLD) to an equivalent single degree of freedom (TMD). This model is based on previous work conducted by Yu, *et al*. (1999) but does not make the assumption that all the fluid mass participates, i.e., $m_{TLD} \neq m_w$. This leads to the evaluation of the damper mass, m_{TLD}, the natural frequency of the damper, f_{TLD}, and the inherent damping, ζ_{TLD}, as a function of the excitation amplitude.

5.1. Energy dissipation matching

The energy dissipated by the equivalent TMD, E_d, shown in Fig. 12, can be expressed in terms of the amplitude of the excitation, A as

$$E_d = m_{TLD}(2\pi f)^2 A^2 \pi |H_{z/x}(f)| \left(\frac{f}{f_{TLD}}\right)^2 \sin(\theta_{z/x}) \tag{14}$$

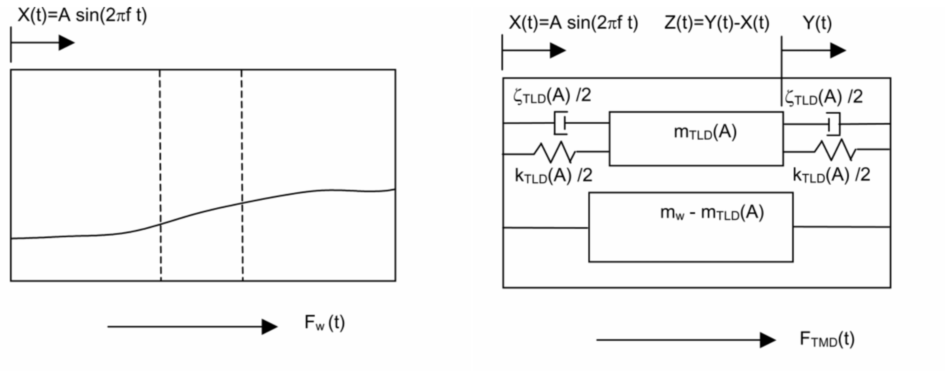
Normalizing this expression by $1/2\, m_w(A2\pi f)^2$, results in

$$E'_d = \frac{m_{TLD}}{m_w} |H_{z/x}(f)| \left(\frac{f}{f_{TLD}}\right) 2\pi \sin(\theta_{z/x}) \tag{15}$$

where the frequency response function, $|H_{z/x}(f)|$, also referred to as the modulus of the mechanical admittance function, between the TMD relative response motion and the table input motion is given by

$$|H_{z/x}(f)| = \frac{1}{\sqrt{\left(1-\left(\frac{f}{f_{TLD}}\right)^2\right)^2 + \left(2\zeta_{TLD}\left(\frac{f}{f_{TLD}}\right)\right)^2}} \tag{16}$$

and the corresponding phase angle, $\theta_{z/x}$, by



(a) TLD Equipped with Damping Screens (b) Equivalent Amplitude Dependent TMD Model

Fig. 12 TLD and equivalent TMD model

$$\theta_{z/x} = \tan^{-1}\left(\frac{2\zeta_{TLD}\left(\frac{f}{f_{TLD}}\right)}{1-\left(\frac{f}{f_{TLD}}\right)^2}\right) \tag{17}$$

A least squares curve fitting procedure with constraints forcing the theoretical expression E'_d to match both the maximum value of the energy dissipated and the total energy dissipated over the range of frequencies tested is applied. This procedure is used to estimate the equivalent TMD parameters m_{TLD}, f_{TLD}, and ζ_{TLD} for all amplitudes of excitation tested.

5.2. Equivalent amplitude dependent TMD parameters

Fig. 13 compares the experimentally obtained energy dissipation curves, E'_w, with the fitted equivalent, E'_d, TMD curves determined using regression analysis. Excellent agreement is found between the two curves for the S-2.5 case, indicating that under low sloshing response amplitudes the energy dissipation characteristics of the TLD can be matched with an equivalent SDOF system. As a result of the nonlinear response characteristics that increase with larger sloshing response amplitudes, the equivalent TMD fit for the S-12.5 energy dissipation curve is not as good as that of the S-2.5 case. Fig. 14 compares the normalized measured base shear sloshing forces, F'_{sw}, with the estimated normalized base shear forces, F'_d, obtained using the equivalent TMD. Again for the S-2.5 the agreement between the TLD and equivalent TMD is good, while the fit for the S-12.5 is in reasonable agreement.

The influence of the excitation amplitude on the estimated equivalent mass, damping and natural frequency parameters are shown in Figs. 15(a) to 15(c), respectively. Trend lines are fitted to each of the parameters in order to estimate the parameter values between data points.

Fig. 15(a) shows the mass, m_{TLD}, normalized by m_1 as a function of the normalized excitation amplitude. The value of m_1 from potential flow provides a first estimate for m_{TLD}, with the presence of screens. Furthermore, m_{TLD} approaches m_1 under larger excitation amplitudes. An increasing value in m_{TLD} with increased excitation amplitude agrees with the findings of others (Sun, *et al.* 1995). The value of m_{TLD} exceeds m_1 at the larger excitation amplitudes tested. At higher excitation amplitudes nonlinear response phenomena including the onset of slamming of the liquid against the

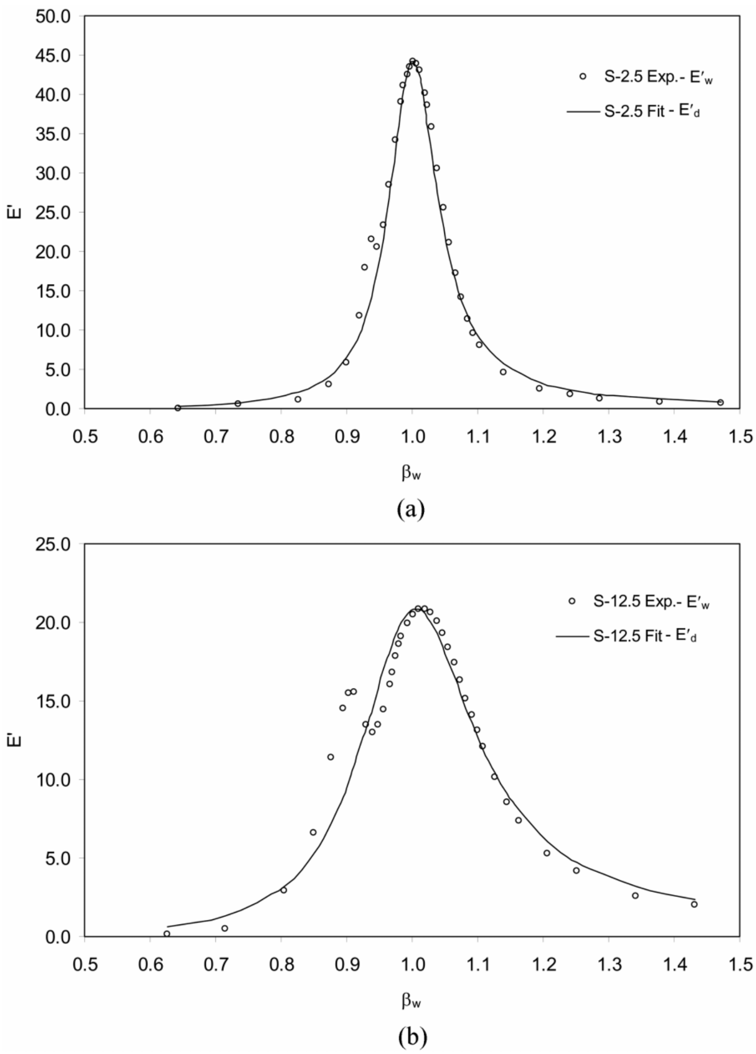


Fig. 13 Comparison of normalized energy dissipation frequency response curves obtained experimentally and evaluated based on equivalent TMD

tank wall can result in a change in the estimated effective mass. This type of impact action has been modelled by Yalla (2001) for a TLD without screens. However, as a result of the screens this model may not be directly applicable.

The estimated damping ratio value of 0.45% (from Eq. (5)) resulting from the water is negligible compared to ζ_{TLD} determined for the TLD with screens. The inherent damping of the TLD without screens is determined using the fitting procedure outlined for the TLD equipped with damping screens and is found to be between approximately 1.0-1.5% depending on the frequency sweep direction. Although this is larger than the value estimated using Eq. (5) it remains less than the target inherent damping value. Fig. 15(b) indicates that ζ_{TLD} increases nonlinearly with increased excitation amplitude. This strong dependence is a result of the pressure loss across the screens and

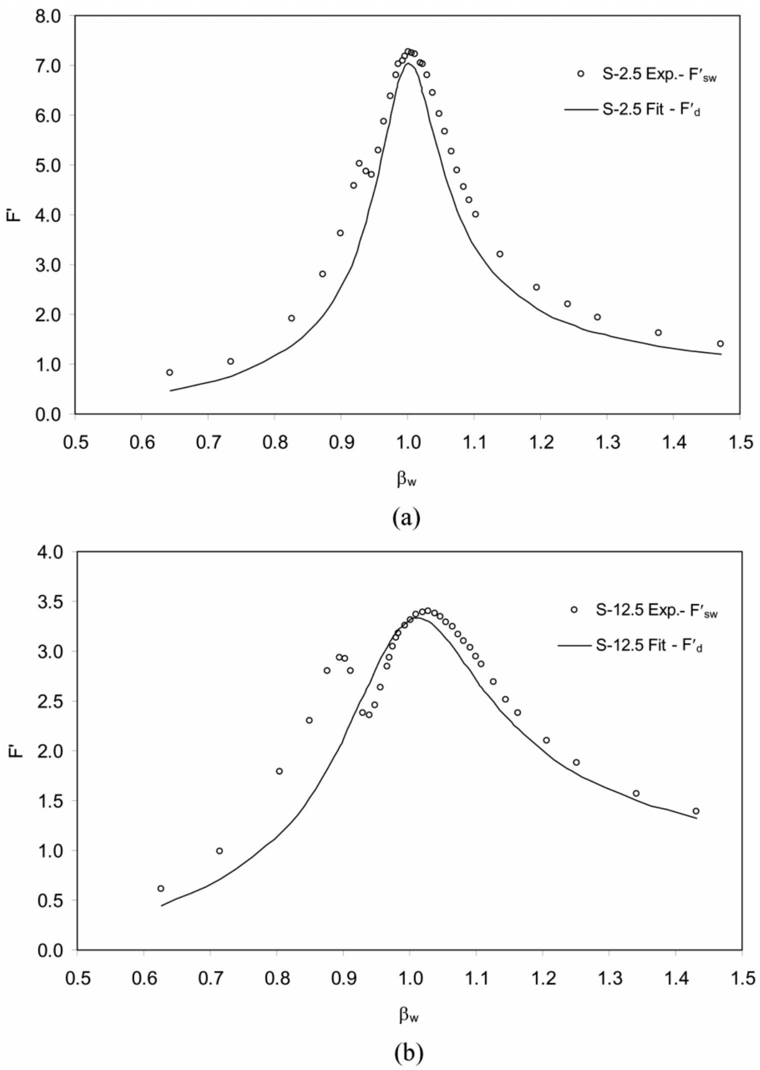


Fig. 14 Comparison of normalized sloshing base shear force frequency response curves obtained experimentally and evaluated based on equivalent TMD

hence the amount of energy dissipated being related to the water velocity squared at the screen location. The value of ζ_{TLD} increases above the calculated optimal value of 5.6% for large normalized excitation amplitudes. Fortunately, the efficiency of a vibration absorber is not significantly reduced if ζ_{TLD} moderately exceeds the optimal value.

Fig. 15(c) shows an increase in f_{TLD} with increased normalized excitation amplitude, indicating a hardening type response. The screens have prevented the discontinuity at the jump frequency that is found to occur without the damping screens. This is significant as the effectiveness of a TLD is strongly dependent on the tuning ratio. By maintaining a near constant natural sloshing frequency, the TLD will remain more efficient over a larger range of excitation amplitudes. This results in a more robust TLD.

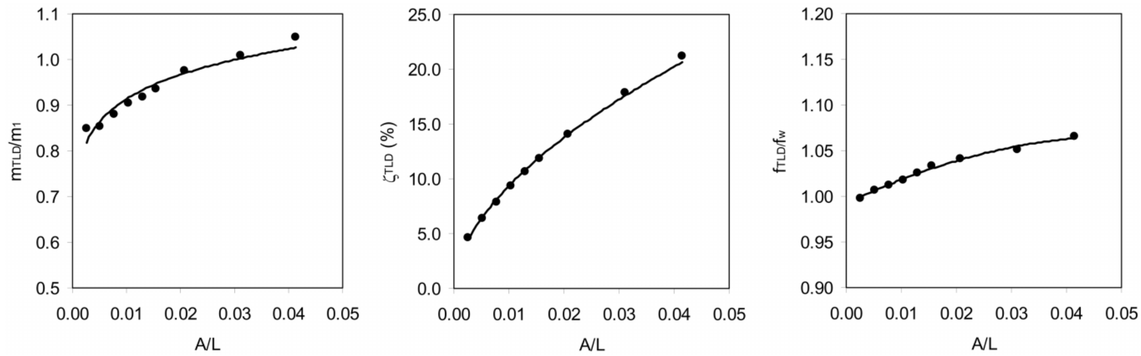


Fig. 15 Amplitude dependent equivalent TMD parameters

6. Conclusions

The influence of screens on the liquid motion, base shear force, and energy dissipation characteristics of a tuned liquid damper, subjected to several base excitation amplitudes, are studied experimentally. Comparisons are made between the response of the sloshing fluid with and without screens installed in the tank. It is observed that the damping screens reduce the hardening type response behaviour substantially by adding inherent damping to the TLD, thereby reducing the free surface response amplitude. Furthermore, response discontinuity at the jump frequency which occurs in a TLD when the response amplitudes are large is not found to occur for any tests conducted in this study. The inherent damping, ζ_{TLD}, is found to be insufficient with no additional energy dissipating mechanisms placed in the tank. These findings are in agreement with those found for a TLD equipped with lattice screens (Fediw, *et al.* 1995). The screens provide the required amount of damping at low excitation amplitudes. However, the optimum damping ratio is exceeded at increased amplitudes of excitation.

The experimentally obtained data are used to determine the amplitude dependent parameters, m_{TLD}, ζ_{TLD}, and f_{TLD}. This allows the nonlinear TLD to be modelled as an amplitude dependent TMD. All three parameters are found to increase in value as the base excitation amplitude is increased. This nonlinear amplitude dependent model will be validated using the results of the structure-TLD tests reported on in Tait, *et al.* (2004b).

The dynamic properties of the TLD obtained from the experimental study are compared to values obtained using linear wave theory. For the range of excitation amplitudes tested the following observations are made:

- For small excitation amplitudes, the natural sloshing frequency, f_{TLD}, estimated using linear wave theory is in good agreement with experimental results, however, the effective mass is less than the value calculated using linear wave theory.
- For large excitation amplitudes, the effective mass is in good agreement with calculated values, however the calculated sloshing frequency is found to be lower than the experimentally observed values.

The dynamic properties, of a TLD containing screens, estimated using linear wave theory appear to provide a good basis for the initial design. The possible use of constant values for the participating mass and natural sloshing frequency predicted by linear potential flow theory, along with a simple model to estimate the inherent damping, is an attractive possibility and needs to be investigated.

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