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Accumulation of wind induced damage on bilinear SDOF systems

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Abstract. The evaluation of the accumulation of permanent set for inelastic structures due to wind action is important in establishing a criterion to select a reduced design wind load and in incorporating the beneficial ductile behaviour in wind engineering. A parametric study of the accumulation of the permanent set as well as the ductility demand for bilinear single-degree-of-freedom (SDOF) systems is presented in the present study. The dynamic analysis of the inelastic SDOF system is carried out using the method of Newmark for artificially generated time history of wind speed. Simulation results indicate that the mean of the normalized damage rate is highly dependent on the natural frequency of vibration. This mean value is relatively insensitive to the damping ratio if the damping ratio is larger than 5%. The scatter associated with the accumulation of the permanent set is very significant. The consideration of the postyield stiffness can significantly reduce the accumulation of the permanent set if the ratio of the yield strength to the expected peak response is small. The results also show that the ductility demand due to the wind action over a period of one hour for flexible structures can be much less than that for rigid structures or structures with larger damping ratio if the SDOF systems are designed with a reduced peak response caused by the fluctuating wind.

Keywords: inelastic behaviour; ductility; wind loads; permanent set.

1. Introduction

The accumulation of wind induced damage for structures has been investigated by Vickery (1970), Wyatt and May (1971), Tshcanz (1983) and Chen and Davenport (2000). The damage is measured using the accumulation of permanent set or plastic deformation. The study of Vickery (1970) was based on the random vibration of an elastoplastic single-degree-of-freedom (SDOF) system, and considered that the permanent set due to wind load on a lightly damped SDOF elastoplastic system can be estimated from the analysis of an equivalent linear elastic SDOF system. Simple analytical equations were derived for evaluating the mean and coefficient of variation of the permanent set. The derivation of these equations was based on the assumption that only one maximum exists between successive upcrossings. Therefore, possible clumping effect was not considered. Further, it was indicated that the derived equations are accurate if the permanent set due to each upcrossing is not very significant and the ratio of the force induced by the fluctuating wind to the difference between the yield force and the force associated with the mean wind speed is relatively small. The

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approach for evaluating the permanent set given by Wyatt and May (1971) is similar to that of Vickery (1970). Wyatt and May (1971) indicated that the strain-hardening can be important in reducing the permanent set, however, details on how to estimate such an effect were not provided. They also indicated that by considering the ductility of structures more economical design could be achieved. Recently, Chen and Davenport (2000) carried out a study for structures subjected to hurricanes. Since the mean wind speed for hurricanes varies in time, they divided the wind actions in blocks to evaluate the accumulation of the permanent set. For each block, the mean wind speed is considered to be a constant, and the permanent set was evaluated using the approach develop by Vickery (1970). They showed that by considering the ductility capacity savings could be achieved in designing structures under wind actions.

In the present study, a numerical simulation analysis was carried out for evaluating the accumulation of the permanent set considering bilinear inelastic SDOF systems. The analysis is aimed at validating and assessing the accuracy of the results available in the literature for bilinear inelastic SDOF systems. For the simulation of the fluctuating wind, the longitudinal turbulent spectrum given by Davenport (1961) was employed and the time history of the wind speed was generated using an algorithm developed by Shinozuka (1972, 1987). The method of Newmark (Chopra 2000) was employed to evaluate the accumulation of the inelastic deformation of SDOF bilinear hysteretic systems. The simulation results for elastoplastic SDOF systems subjected to the wind load over a period were used to assess the mean and the coefficient of variation of the accumulation of the permanent set as well as the ductility demand. The results were compared with those given by Vickery (1970). Also, simulation results were provided for bilinear SDOF systems to illustrate the impact of the post yield stiffness on the accumulation of the permanent set and the ductility demand.

2. Peak response of bilinear SDOF systems

Consider an SDOF system shown in Fig. 1 with x_y representing the yield displacement, F_{xy} denoting the yield force and γ representing the ratio of the post yield stiffness to the initial stiffness k_0. The system is subjected to the wind load that is usually expressed as the superposition of the

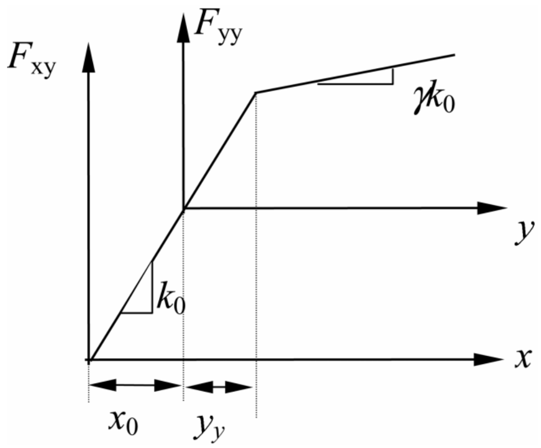

Fig. 1 Schematic of bilinear system

mean and fluctuating components. Let U and $u(t)$ denote the mean and fluctuating components of the wind speed. The wind force $F(t)$ can be approximated by Simiu and Scanlan (1996),

$$F(t) = \frac{1}{2}\rho C_D A U^2 + \rho C_D A U u(t) \tag{1}$$

where ρ is the density of air, C_D is the drag coefficient, A is the projected area normal to the flow. The deflection caused by the mean wind load, x_0, equals $(\rho C_D A U^2/2)/k_0$. By removing this displacement, the deflection due to the fluctuating wind force, y, $y = x - x_0$, satisfies the following governing equation:

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2(k_y(y, \dot{y})/k_0)y = \rho C_D A U u(t)/m \tag{2}$$

where ξ is the damping ratio, ω_n is the natural vibration frequency, m is the mass, and $k_y(y, \dot{y})$ is the stiffness. The yield displacement, y_y, equals $x_y - x_0$.

The fluctuating wind speed $u(t)$ is usually treated as a stationary process that can be characterized by a power spectrum density (PSD) function $S_u(\omega)$. For the analysis to be carried out in this study, we adopt the PSD function of the fluctuating wind developed by Davenport (1961) (see also Simiu and Scanlan 1996),

$$S_u(\omega, U(10)) = \frac{u_*^2}{\omega}\frac{4(1200\omega/(2\pi U(10)))^2}{(1 + (1200\omega/(2\pi U(10)))^2)^{4/3}} \tag{3a}$$

where u_* is the shear friction velocity, $U(10)$ (m/s) is the mean wind speed at a height of 10 meters from the ground. Note that if $w(t)$ equals $u(t)/u_*$, the PSD function of $w(t)$, $S_w(\omega, U(10))$, is given by

$$S_w(\omega, U(10)) = S_u(\omega, U(10))/u_*^2 = \frac{1}{\omega}\frac{4(1200\omega/(2\pi U(10)))^2}{(1 + (1200\omega/(2\pi U(10)))^2)^{4/3}} \tag{3b}$$

Therefore, by letting

$$z = y/B \tag{4}$$

and the yield displacement $z_y = y_y/B$, Eq. (2) can be re-written as,

$$\ddot{z} + 2\xi\omega_n\dot{z} + \omega_n^2(k_z(z, \dot{z})/k_0)z = w(t) \tag{5}$$

where $B = \rho C_D A U u_*/m$. Using this equation rather than Eq. (2) simplifies the parametric analysis since z is independent of U, u_*, A, and C_D.

If the nonlinear behaviour is ignored in Eq. (5) (i.e., $k_z(z, \dot{z})/k_0 = 1$), the peak displacement due to the fluctuating wind over a period T can be evaluated using the method developed by Davenport (1964). For nonlinear systems, however, the evaluation of the peak displacement is usually based on the time step integration methods such as the method of Newmark (Chopra 2000). By using this method, the time history of the fluctuating wind or $w(t)$ in this case is required. For the generation of the artificial time history of $w(t)$ based on $S_w(\omega, U(10))$, one can use the method given by

Shinozuka (1972, 1987) leading to,

$$w(t) = \sqrt{2}\sum_{i=1}^{n} A_i \cos(\omega_i t + \phi_i) \tag{6}$$

where $\omega_i = (i-1/2)\Delta\omega$, $A_i = \sqrt{S_w(\omega_i, U(10))\Delta\omega}$, $\Delta\omega$ is a selected constant frequency interval, and ϕ_i are uniformly distributed random variables between 0 and 2π.

Note that since the PSD function of $w(t)$ depends on the velocity $U(10)$, the response z depends on $U(10)$ as well. To further simply the analysis one can select an arbitrary "standard" value of $U(10)$, say, 30 (m/sec). By defining $\alpha = U(10)/30$, $z = \zeta/\alpha^2$, $t = \tau/\alpha$, $\omega_n = \alpha\tilde{\omega}_n$, and $\omega_i = a\tilde{\omega}_i$, and substituting Eq. (6) into Eq. (5) results in,

$$\ddot{\zeta} + 2\xi\tilde{\omega}_n\dot{\zeta} + \tilde{\omega}_n^2(k_\zeta(\zeta, \dot{\zeta}))\zeta = \sqrt{2}\sum_{i=1}^{n}\sqrt{S_w(\tilde{\omega}_i, 30)\Delta\tilde{\omega}_i}\cos(\tilde{\omega}_i\tau + \phi_i) \tag{7}$$

where $\dot{\zeta}$ and $\ddot{\zeta}$ represent the first- and second-order derivatives of ζ with respect to τ. This equation shows that the response ζ depends only on $\tilde{\omega}_n$ and ξ but independent of $U(10)$ and α. It is noteworthy that by ignoring the nonlinear behaviour (i.e., $k_\zeta(\zeta, \dot{\zeta})/k_0 = 1$), the peak displacement due to the fluctuating wind over a period T_τ depends on the zero-upcrossing rate $v^+_{\zeta 0}(\tilde{\omega}_n)$ and the standard deviation $\sigma_\zeta(\tilde{\omega}_n)$ which are given by Davenport (1964),

$$v^+_{\zeta 0}(\tilde{\omega}_n) = \sqrt{\lambda_{\zeta 2}(\tilde{\omega}_n)/\lambda_{\zeta 0}(\tilde{\omega}_n)}/2\pi \tag{8}$$

and

$$\sigma_\zeta(\tilde{\omega}_n) = \sqrt{\lambda_{\zeta 0}(\tilde{\omega}_n)} \tag{9}$$

where

$$\lambda_{\zeta i}(\tilde{\omega}_n) = \int_0^\infty \tilde{\omega}^i S_w(\tilde{\omega}, 30)\frac{1}{(\tilde{\omega}_n^2 - \tilde{\omega}^2)^2 + (2\xi\tilde{\omega}_n\tilde{\omega})^2}d\tilde{\omega}, \quad i = 0,2 \tag{10}$$

Let ζ_I denote the permanent displacement of the system after the system has sustained the fluctuating wind and, ζ_y, $\zeta_y = \alpha^2 z_y$, denote the yield displacement. Given the values of $\tilde{\omega}_n$, ξ, and the duration T_τ, the numerical assessment of the normalized damage rate per each zero-upcrossing (i.e., $(\zeta_I - \zeta_y)/(v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau\sigma_\zeta(\tilde{\omega}_n))$, can be carried out as follows:

(1) Evaluate $v^+_{\zeta 0}(\tilde{\omega}_n)$ and $\sigma_\zeta(\tilde{\omega}_n)$ according to Eqs. (8) to (10);
(2) Generate the values of the random variables ϕ_i;
(3) Select a value of $\zeta_y/\sigma_\zeta(\tilde{\omega}_n)$, and calculate ζ_y;
(4) Solve Eq. (5) using the method of Newmark, and find ζ_I;

(5) Repeat Step 3) and Step 4) for a set of selected values of $\zeta_y/\sigma_\zeta(\tilde{\omega}_n)$; and
(6) Repeat Steps 2) to 5) sufficient times to obtain the statistics of ζ_I, or the normalized damage rate per each zero-upcrossing.
The obtained simulation results can be used in establishing the relationship between $(\zeta_I - \zeta_y)/(v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau\sigma_\zeta(\tilde{\omega}_n))$ and $\zeta_y/\sigma_\zeta(\tilde{\omega}_n)$ which will be discussed in the following.

If the nonlinear behaviour in Eq. (5) is ignored, it can be shown that for a mean wind speed at a height of 10 m, $U(10)$, the zero-upcrossing rate for the response z, $v^+_{z0}(\omega_n, U(10))$, and the standard deviation of z, $\sigma_z(\omega_n, U(10))$, are given by,

$$v^+_{z0}(\omega_n, U(10)) = \alpha v^+_{\zeta 0}(\tilde{\omega}_n) \tag{11}$$

and

$$\sigma_z(\omega_n, U(10)) = \sigma_\zeta(\tilde{\omega}_n)/\alpha^2 \tag{12}$$

Similarly, it can be shown that the zero-upcrossing rate for the response y, $v^+_{y0}(\omega_n, U(10))$ equals $v^+_{z0}(\omega_n, U(10))$, and that the standard deviation of y, $\sigma_y(\omega_n, U(10))$, equals $B\sigma_z(\omega_n, U(10))$. Based on the above, we have,

$$\frac{x_I - x_y}{v^+_{y0}(\omega_n, U(10))T\sigma_y(\omega_n, U(10))} = \frac{\zeta_I - \zeta_y}{v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau\sigma_\zeta(\tilde{\omega}_n)} \tag{13}$$

and

$$\frac{x_y - x_0}{\sigma_y(\omega_n, U(10))} = \frac{\zeta_y}{\sigma_\zeta(\tilde{\omega}_n)} \tag{14}$$

where the definition of x_I is similar to that of ζ_I except that it is in terms of the response x.

This shows that the relation between $(\zeta_I - \zeta_y)/(v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau\sigma_\zeta(\tilde{\omega}_n))$ and $\zeta_y/\sigma_\zeta(\tilde{\omega}_n)$ is equivalent to the relation between $(x_I - x_y)/(v^+_{y0}(\omega_n, U(10))T\sigma_y(\omega_n, U(10)))$ and $(x_y - x_0)/(\sigma_y(\omega_n, U(10)))$ which describes the original nonlinear SDOF system. In particular, it is expected that if the excursions into the yield region are rare events, this relationship developed based on the simulation results should follow closely to the one given by Vickery (1970),

$$\frac{E(x_I - x_y)}{v^+_{y0}(\omega_n, U(10))T\sigma_y(\omega_n, U(10))} = \sqrt{2\pi}\Phi\left(-\frac{x_y - x_0}{\sigma_y(\omega_n, U(10))}\right) \tag{15}$$

where E( ) denotes the expectation. Eq. (15) can be further approximated by (Wyatt and May 1971, Chen and Davenport 2000),

$$\frac{E(x_I - x_y)}{v^+_{y0}(\omega_n, U(10))T\sigma_y(\omega_n, U(10))} = \frac{\sigma_y(\omega_n, U(10))}{x_y - x_0}\exp\left(-\frac{1}{2}\left(\frac{x_y - x_0}{\sigma_y(\omega_n, U(10))}\right)^2\right) \tag{16}$$

A simple plot of Eqs. (15) and (16) will show that Eq. (16) provides slightly higher predicted normalized damage rate per each zero-uprossing than Eq. (15) for a given value of $(x_y - x_0)/\sigma_y(\omega_n, U(10))$.

The coefficient of variation (cov) of the normalized damage rate average over a period T, v_c, (i.e., the cov of the accumulation of the permanent set ($x_I - x_y$) over a period T) given in Vickery (1970) for the response based on the original elastoplastic SDOF system is,

$$v_c = \left(2\left(\frac{1}{d^2}\exp\left(-\frac{1}{2}\left(\frac{x_y - x_0}{\sigma_y(\omega_n, U(10))}\right)^2\right) - \frac{1}{d}\frac{x_y - x_0}{\sigma_y(\omega_n, U(10))}\right) - 1\right)^{1/2} / \sqrt{v^+_{y0}(\omega_n, U(10))T} \quad (17a)$$

where $d = \sqrt{2\pi}\Phi(-(x_y - x_0)/\sigma_y(\omega_n, U(10)))$. By noting that d is equal to $\sqrt{2\pi}\Phi(-\zeta_y/\sigma_\zeta(\tilde{\omega}_n))$, and $v^+_{y0}(\omega_n, U(10))T$ is equal to $v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau$, Eq. (17a) can be rewritten for the response based on ζ as follows:

$$v_c = \left(2\left(\frac{1}{d^2}\exp\left(-\frac{1}{2}\left(\frac{\zeta_y}{\sigma_\zeta(\tilde{\omega}_n)}\right)^2\right) - \frac{1}{d}\frac{\zeta_y}{\sigma_\zeta(\tilde{\omega}_n)}\right) - 1\right)^{1/2} / \sqrt{v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau} \quad (17b)$$

which represents the cov of the accumulation of the permanent set ($\zeta_I - \zeta_y$) over a period T_τ. Based on the above, statistics of the accumulation of the permanent set obtained from simulation results based on the response ζ can be used to characterize the statistics of the accumulation of the permanent set for the original system whose response is represented by x.

3. Simulation results

3.1. Elastoplastic SDOF systems

The numerical results presented in this section are for the elastoplastic hysteretic SDOF systems only. Consider that $\tilde{\omega}_n = 618$, $\xi = 1\%$, and $T_\tau = 3600$ (sec). The mean of the normalized damage rate, $E(\zeta_I - \zeta_y)/(v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau\sigma_\zeta(\tilde{\omega}_n))$, and the cov of the accumulation of the permanent set ($\zeta_I - \zeta_y$) over a period T_τ, v_c, (see Eq. (17)) obtained using the aforementioned analysis procedure with the number

Table 1 Comparison of the statistics of the accumulation of damage rate

	Mean			Coefficient of variation		
$\zeta_y/\sigma_\zeta(\tilde{\omega}_n)$	$E(\zeta_I - \zeta_y)/(v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau\sigma_\zeta(\tilde{\omega}_n))$			v_c		
	j=250	j=500	Eq. (15)	j=250	j=500	Eq. (17)
0.2	1.51E+01	1.51E+01	1.05E+00	0.053	0.053	0.011
0.25	1.18E+01	1.17E+01	1.01E+00	0.054	0.055	0.011
0.5	3.01E+00	3.00E+00	7.73E-01	0.157	0.163	0.014
1	2.04E-01	2.03E-01	3.98E-01	0.399	0.383	0.022
1.25	8.84E-02	8.93E-02	2.65E-01	0.209	0.197	0.028
1.5	5.04E-02	5.09E-02	1.67E-01	0.138	0.136	0.036
1.75	3.05E-02	3.08E-02	1.00E-01	0.148	0.148	0.046
2	1.80E-02	1.82E-02	5.70E-02	0.216	0.211	0.061
2.25	9.83E-03	9.92E-03	3.06E-02	0.375	0.366	0.081
2.5	4.75E-03	4.79E-03	1.56E-02	0.692	0.662	0.111
3	8.10E-04	7.34E-04	3.38E-03	1.818	1.888	0.226
3.5	5.85E-05	4.75E-05	5.83E-04	6.344	6.563	0.513
4	7.08E-06	3.54E-06	7.94E-05	15.811	22.361	1.313

Table 2 Zero-upcrossing rate and standard deviation

ξ	$\tilde{\omega}_n=10\pi$		$\tilde{\omega}_n=2\pi$		$\tilde{\omega}_n=\pi$		$\tilde{\omega}_n=0.4\pi$		$\tilde{\omega}_n=0.2\pi$	
	$v^+_{\zeta 0}(\tilde{\omega}_n)$	$\sigma_\zeta(\tilde{\omega}_n)$	$v^+_{\zeta 0}(\tilde{\omega}_n)$	$\sigma_\zeta(\tilde{\omega}_n)$	$v^+_{\zeta 0}(\tilde{\omega}_n)$	$\sigma_\zeta(\tilde{\omega}_n)$	$v^+_{\zeta 0}(\tilde{\omega}_n)$	$\sigma_\zeta(\tilde{\omega}_n)$	$v^+_{\zeta 0}(\tilde{\omega}_n)$	$\sigma_\zeta(\tilde{\omega}_n)$
1%	3.8530	0.0039	0.9073	0.1442	0.4704	0.7004	0.1938	5.7159	0.0983	27.5494
2%	3.2057	0.0032	0.8358	0.1103	0.4455	0.5218	0.1882	4.1517	0.0966	19.7453
5%	2.2288	0.0028	0.6910	0.0836	0.3889	0.3755	0.1738	2.8205	0.0921	12.9616
10%	1.5763	0.0026	0.5554	0.0724	0.3284	0.3111	0.1558	2.1973	0.0859	9.6637
20%	1.0975	0.0025	0.4192	0.0658	0.2606	0.2716	0.1319	1.7908	0.0766	7.4152

of simulation cycles, j, equal to 250 were presented in Table 1. By using the same parameters except that the simulation cycles equals 500, the simulation was carried out again. The mean of the normalized damage rate and the cov of the accumulation of the permanent set obtained for j = 500 were also shown in Table 1 and compared with those obtained based on 250 simulation cycles and with the ones given by Eqs. (15) and (17). The table shows that for $\zeta_y/\sigma_\zeta(\tilde{\omega}_n)$ less than 1.0 the mean obtained by the simulation is less than about 50% of the value given by Eq. (15), and that the cov values obtained from simulation differ significantly from those given by Eq. (17). It also indicates that use of j = 250 provides results that are almost identical to those obtained with j = 500. Therefore, in all the following numerical analyses T_τ = 3600(s) and j = 250 were employed. It must emphasized that to describe the mean of the normalized damage rate per each zero-upcrossing and the cov of the accumulation of the permanent set based on the response x, it is sufficient to replace $E(\zeta_I-\zeta_y)/(v^+_{\zeta 0}(\tilde{\omega}_n)T_\tau\sigma_\zeta(\tilde{\omega}_n))$ by $E(x_I-x_y)/(v^+_{y0}(\omega_n, U(10))T\sigma_y(\omega_n, U(10)))$ and, $\zeta_y/\sigma_\zeta(\tilde{\omega}_n)$ by $(x_y-x_0)/(\sigma_y(\omega_n, U(10)))$ in Table 1. Further note that the frequency $\tilde{\omega}_n$ is related to ω_n through $\omega_n = \tilde{\omega}_n U(10)/30$.

For sets of values of ξ and $\tilde{\omega}_n$, the zero-uprossing rate $v^+_{\zeta 0}(\tilde{\omega}_n)$ and the standard deviation $\sigma_\zeta(\tilde{\omega}_n)$ were evaluated and presented in Table 2. Based on Eqs. (11) and (12), these values can be used to evaluate the zero-upcrossing rate $v^+_{y0}(\omega_n, U(10))$,

$$v^+_{y0}(\omega_n, U(10)) = (U(10)/30) \times v^+_{\zeta 0}(30\omega_n/U(10)) \tag{18}$$

and the standard deviation $\sigma_y(\omega_n, U(10))$,

$$\sigma_y(\omega_n, U(10)) = B(30/U(10))^2 \times \sigma_\zeta(30\omega_n/U(10)) \tag{19}$$

where B is defined previously.

Simulation analyses for the combinations of ξ and $\tilde{\omega}_n$ shown in Table 2 were carried out for elastoplastic hysteretic SDOF systems (i.e., with $\gamma = 0$). The obtained simulation results that can be used to establish the relation between the mean of the normalized damage rate per each zero-upcrossing and $(x_y-x_0)/(\sigma_y(\omega_n, U(10))$ expressed as,

$$E(x_I-x_y)/(v^+_{y0}(\omega_n, U(10))T\sigma_y(\omega_n, U(10))) = f_m((x_y-x_0)/\sigma_y(\omega_n, U(10)), \xi, \gamma) \tag{20}$$

were shown in Fig. 2. In Eq. (20), f_m ( ) denotes a function that can be determined based on the results presented in Fig. 2. Similarly, the results obtained from the simulation analyses that can be

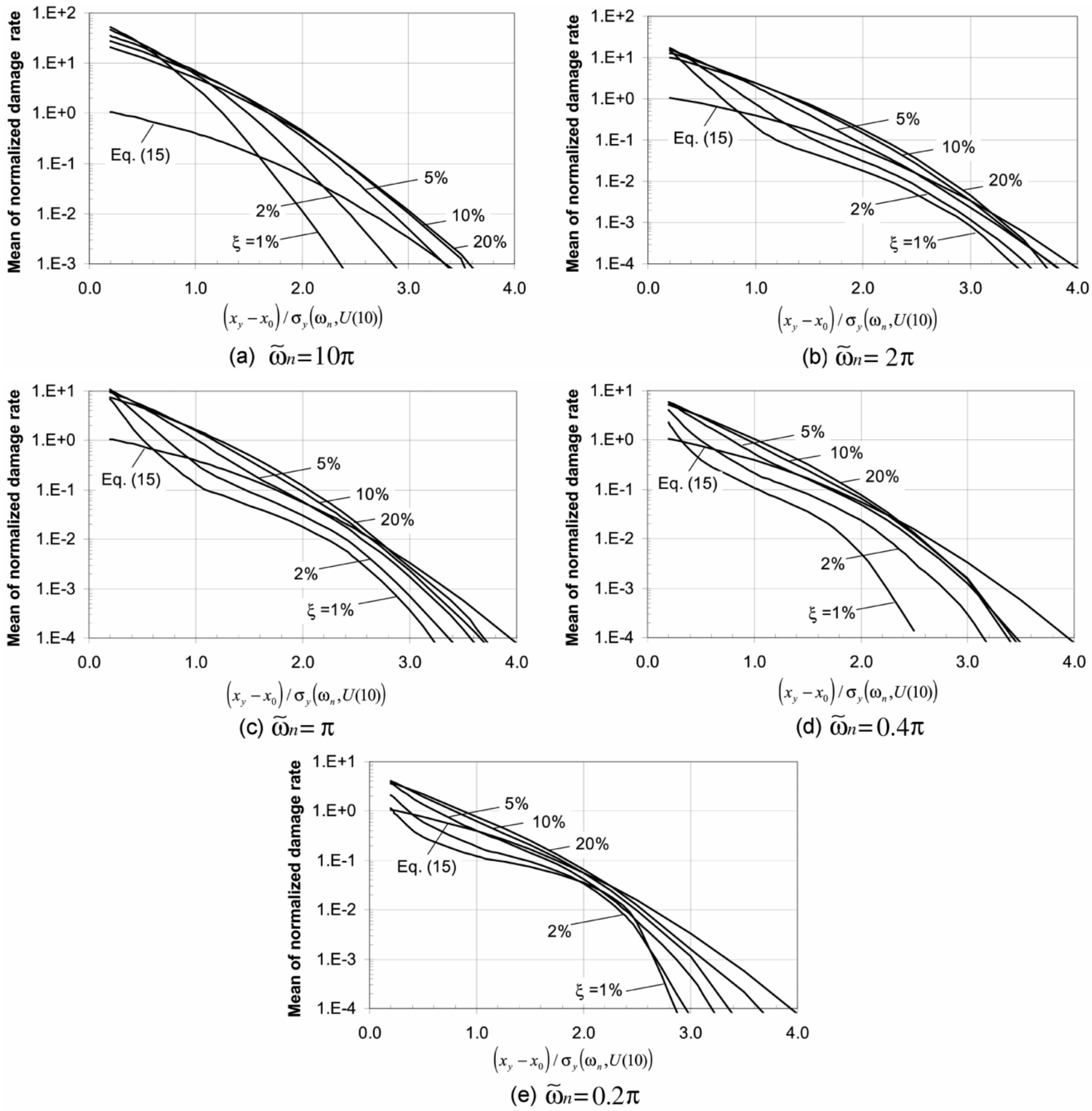

Fig. 2 Mean of the normalized damage rate for elastoplastic systems ($\omega_n=\tilde{\omega}_n\,U(10)/30$)

employed to establish the relation between the cov of the accumulation of the permanent set, v_c, and $(x_y-x_0)/(\sigma_y(\omega_n, U(10))$ expressed as,

$$v_c=f_c((x_y-x_0)/\sigma_y(\omega_n,\ U(10)),\ \xi,\ \gamma) \tag{21}$$

were presented in Fig. 3. The function f_c ( ) relates the quantity on the left hand side of Eq. (21) to the quantities on the right hand side of Eq. (21). This relation may be empirically determined from the results depicted in Fig. 3.

Fig. 2 shows that the mean of the normalized damage rate increases as the damping ratio increases. However, for the damping ratio greater than 10% the increase in the mean of the normalized damage rate as compared to that of the damping ratio equal to 5% is insignificant. It must be noted that the same mean of the normalized damage rate for different damping ratios does not necessary lead to the same accumulation of the permanent set because the latter depends not only on the damage rate but also on the upcrossing rate v_{y0}^{+} (ω_n, U(10)) and the standard deviation $\sigma_y(\omega_n$, U(10)). The results shown in the figure also indicate that Eq. (15) serves as a conservative estimate for the mean of the normalized damage rate if $(x_y-x_0)/\sigma_y(\omega_n$, U(10)) is greater than about 2 and $\tilde{\omega}_n$ is small. Eq. (15) becomes increasingly inaccurate as $(x_y-x_0)/\sigma_y(\omega_n$, U(10)), decreases and/or $\tilde{\omega}_n$ increases.

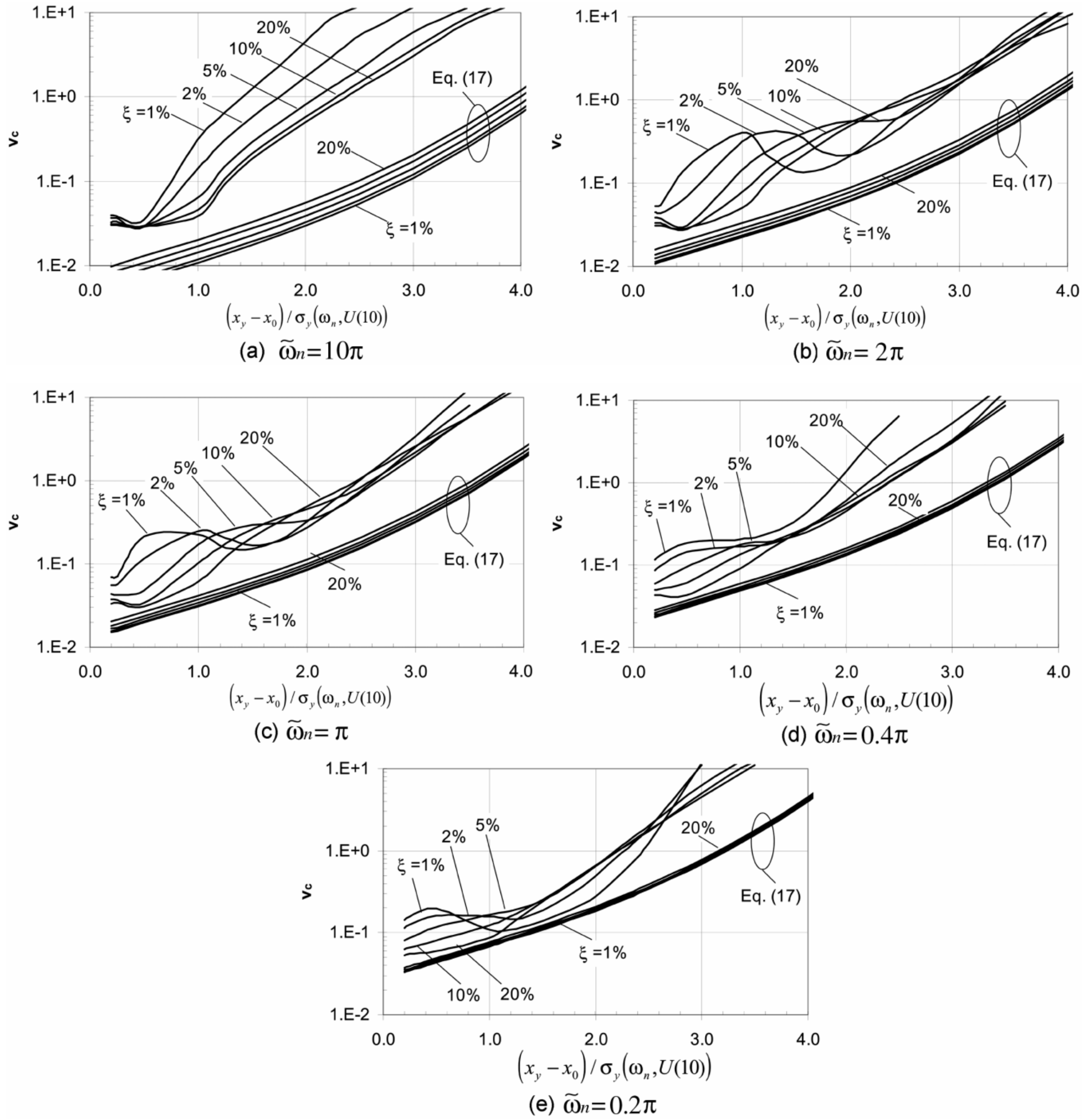

Fig. 3 Coefficient of variation of the normalized damage rate for elastoplastic systems ($\omega_n = \tilde{\omega}_n\, U(10)/30$)

The cov values presented in Fig. 3 are greater than those given by Eq. (17). The former in some cases can be orders of magnitude higher than the latter. The differences between the former and the latter decrease as the damping ratio ξ increases and/or the frequency $\tilde{\omega}_n$ decreases (i.e., the structure becomes very flexible). This indicates that the analytical equation given by Vickery (1970), which was derived by approximating an elastoplastic hysteretic SDOF system by a linear elastic SDOF system, is unconservative in estimating the cov of the damage accumulation.

3.2. Bilinear SDOF systems

To investigate the effect of the (isotropic) strain-hardening on the mean of the normalized damage rate per each zero-upcrossing, simulation analyses for ξ = 1%, 2%, 5%, 10% and 20% and $\tilde{\omega}_n$ equal to 0.4π were carried out for γ = 0.05 and 0.10. The obtained results were shown in Figs. 4 and 5, respectively. Comparison of the results shown in Figs. 2(d) and 4 indicates that the strain-hardening reduces the mean of the normalized damage rate. The reduction depends on the ratio of the post yield stiffness to the initial stiffness, γ, and is most significant for $(x_y-x_0)/\sigma_y(\omega_n, U(10))$ less than 2.0.

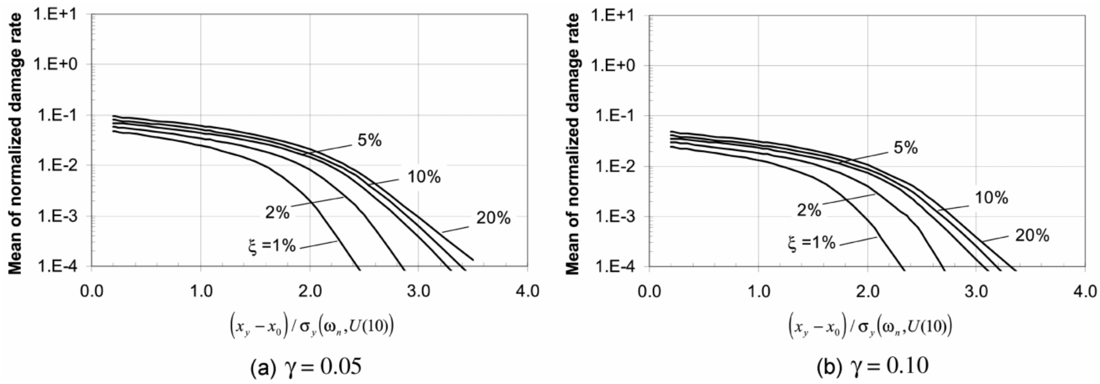

Fig. 4 Mean of the normalized damage rate for bilinear systems with $\tilde{\omega}_n = 0.4\pi$ and different damping ratio

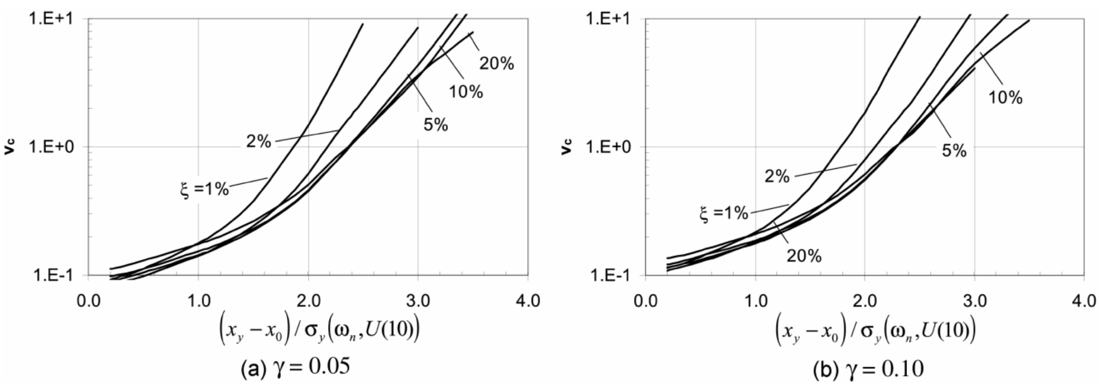

Fig. 5 Coefficient of variation of the normalized damage rate for bilinear systems with $\tilde{\omega}_n = 0.4\pi$ and different damping ratio

Results shown in Figs. 3(d) and 5 suggest that the variation of v_c for different values of the damping ratio ξ is decreased if the strain-hardening effect is considered and $(x_y-x_0)/\sigma_y(\omega_n, U(10))$ is less than about 1.0. However, for $(x_y-x_0)/\sigma_y(\omega_n, U(10))$ greater than about 1.0 the values of v_c are similar for the cases considering or ignoring the strain-hardening.

4. Ductility demand

While the philosophy in seismic design is to allow the structures subjected to severe earthquake excitations to undergo a permanent inelastic deformation, the design for wind actions requires the structures to remain in the elastic range. To investigate the ductility demand due to wind actions on inelastic SDOF systems, consider that for a specified wind speed $U(10)$ equal to u_n, that corresponds to the 50-year return period value, the elastoplastic hysteretic SDOF systems are designed such that the yield displacement, x_y, is given by,

$$x_y = \alpha_d(x_0 + R_g P_{gn}\sigma_{yn}) \tag{22}$$

where α_d is the design wind load factor, x_0 is the displacement due to the design mean wind speed P_{gn}, $p_{gn} = \sqrt{2\ln(v^+_{y0n}T)} + 05772/\sqrt{2\ln(v^+_{y0n}T)}$, represents the peak factor for the design wind speed of u_n, $v^+_{y0n} = v^+_{y0}(\omega_n, u_n)$, $\sigma_{yn} = \sigma_y(\omega_n, u_n)$, and R_g is a (reduction) factor applied to the response due to the fluctuating wind. Let R_0, $R_0 = x_0/(P_{gn}\sigma_{yn})$, denote the ratio of x_0 to the mean peak displacement due to the fluctuating wind, and let μ, $\mu = x_I/x_y$, denote the ductility demand due to the wind actions. Based on the above and Eqs. (20) and (21), it can be shown that the mean of μ, m_μ, and the cov of μ, v_μ, due to the specified wind speed over a period T are given by,

$$m_\mu = 1 + \frac{1}{\alpha_d(R_0 P_{gn} + R_g P_{gn})} v^+_{y0d} T f_m((\alpha_d - 1)R_0 P_{gn} + \alpha_d R_g P_{gn}, \xi, \gamma) \tag{23}$$

and

$$v_\mu = f_c((\alpha_d - 1)R_0 P_{gn} + \alpha_d R_g P_{gn}, \xi, \gamma) \times \frac{v^+_{y0n} T f_m((\alpha_d - 1)R_0 P_{gn} + \alpha_d R_g P_{gn}, \xi, \gamma)}{\alpha_d(R_0 P_{gn} + R_g P_{gn}) + v^+_{y0n} T f_m((\alpha_d - 1)R_0 P_{gn} + \alpha_d R_g P_{gn}, \xi, \gamma)} \tag{24}$$

By adopting that the duration of the wind storm is 3600 (s) and the mean wind speed is constant during this period (Vickery 1970), the values of m_μ and v_μ are calculated using the above equations and some of the simulation results presented in Figs. 2 and 3 for $u_n = 30$ (m/s) and $\alpha_d = 1.0$. The calculated results were depicted in Figs. 6 and 7. Fig. 6 shows the results for $\xi = 0.05$ while Fig. 7 for $\omega_n = 0.4\pi$.

The results presented in Figs. 6(a) and 7(a) indicate that if the elastoplastic systems designed based on the mean peak response (i.e., $R_g = 1.0$), the expected ductility demand m_μ equals 1.0 (i.e., on average the structures under the specified wind action behave elastically). In general, the ductility demand increases as the vibration frequency and/or damping ratio increases. The ductility demand increases as the static component (i.e., R_0) becomes smaller. For a wide range of natural vibration frequencies, the expected ductility demand as shown in Fig. 6(a) is less than about 2.0 for $R_g = 0.8$. This implies that if the ductile behaviour of the designed structures with a ductility

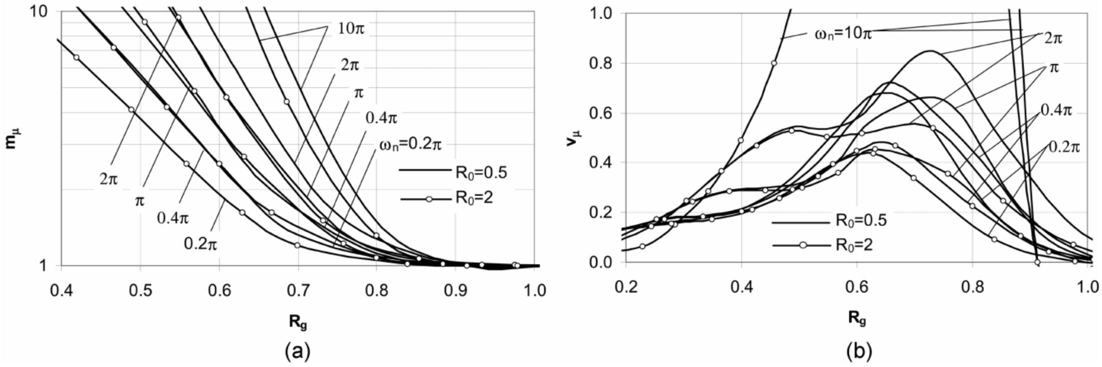

Fig. 6 Ductility demand for SDOF systems with ξ = 0.05

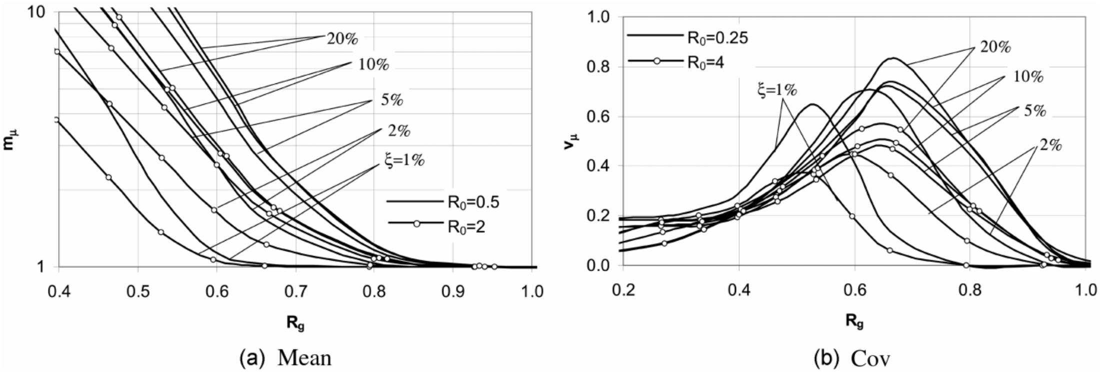

Fig. 7 Ductility demand for SDOF systems with ω_n = 0.4π

capacity of 2.0 is considered, a value of R_g less than 0.8 could be used. The value of this reduction factor can be smaller for flexible structures with a higher ductility capacity. For example, if the natural frequency of vibration of the structure equals 0.4π (i.e., natural vibration period equal to 5 (sec)), and the ductility capacity equals 5, a value of the reduction factor less than 0.6 may be considered (see Fig. 7(a)). The cov of the ductility demand varies significantly as shown in Figs. 6(b) and 7(b). It attains highest value for R_g between about 0.4 to 0.8 for the considered SDOF systems.

Similarly, for the bilinear systems under the same design condition as above, the obtained mean and cov of the ductility demand for γ = 0.05 were shown in Fig. 8. Comparison of results shown in Fig. 7 to those presented in Fig. 8 indicates that the expected value and the scatter of the ductility demand are largely decreased if the strainhardening is considered.

Note that if one repeats the above analysis by maintaining the wind action equal to the specified wind action used for design but varying α_d up to 1.4 which takes into account that the design wind load effect is higher than the specified wind load effect, the obtained results are similar to those shown in Figs. 6 to 8 except that for the same ductility levels a smaller value of R_g as compared to the case with α_d = 1.0 may be considered. This implies that the reduction factor could be further reduced as α_d is increased. However, a design code calibration must be carried out for providing a definite recommendation on the values of R_g.

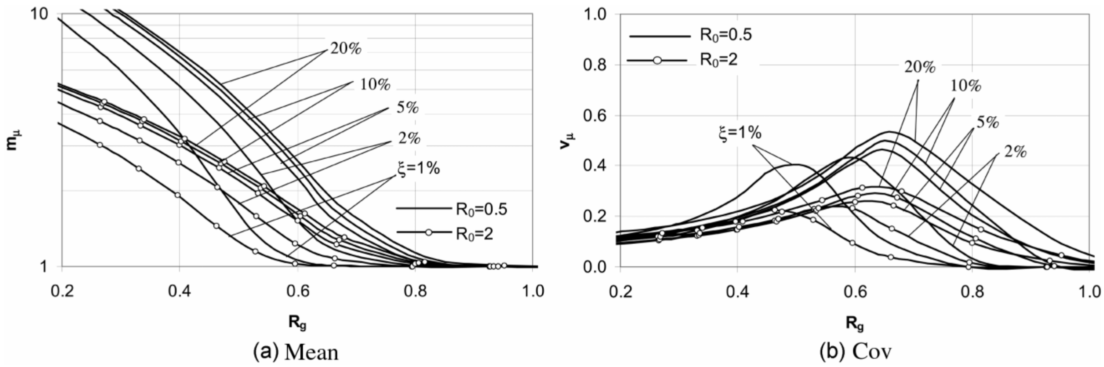

Fig. 8 Ductility demand for bilinear SDOF systems with $\gamma = 0.05$, and $\omega_n = 0.4\pi$

5. Conclusions

A numerical analysis is carried out for evaluating the accumulation of permanent set considering bilinear inelastic single-degree-of-freedom systems. The results indicate that the mean of the normalized damage rate is sensitive to the natural vibration frequency. The mean for structures with a damping ratio larger than 5% is similar to that obtained for structures with a damping ratio of 5%.

The ductility demand due to the wind action over a period of one hour for flexible structures could be much less than that for rigid structures. The strain-hardening reduces significantly the ductility demand. However, in all cases, the scatter associated with the ductility demand can be very higher.

A reduction that applies directly to the peak factor could be obtained from the provided results. A value of this factor less than 0.8 may be employed to design the structures to limit the ductility demand less than 2.0, and less than 0.6 for ductility demand less than 5. It must be emphasized that this conclusion should be limited to the case that the wind action is equal to that used for design and the mean wind speed is a constant during the wind storm. Therefore, to provide definite recommendation on the values of R_g for codified design, a code calibration analysis must be carried out by considering, among other uncertainty quantities, the uncertainty in the ductility demand, ductility capacity, the wind speed and the duration of the wind.

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