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Tuned mass dampers for torsionally coupled systems

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Abstract. The steady state response of a torsionally coupled system with tuned mass dampers (TMDs) to external wind-induced harmonic excitation is presented. The torsionally coupled system is considered as one-way eccentric system. The eccentricity considered in the system is accidental eccentricity only. The performance of single tuned mass damper (TMD) optimally designed without considering the torsion is investigated for the torsionally coupled system and found that the effectiveness of a single TMD is significantly reduced due to torsion in the system. However, the design of TMD system without considering the torsion is only justified for torsionally stiff systems. Further, the optimum parameters of a single TMD considering the accidental eccentricity are obtained using numerical searching technique for different values of uncoupled torsional to lateral frequency ratio and aspect ratio of the system. The optimally designed single TMD system is found to be less effective for torsionally coupled system in comparison to uncoupled system. This is due to the fact that a torsionally coupled system has two natural frequencies of vibration, as a result, at least two TMDs are required which can control both lateral and torsional response of the system. The optimum damper parameters of different alternate arrangements such as (i) two identical TMDs placed at opposite corners, (ii) two independent TMDs and (iii) four TMDs are evaluated for minimum response of the system. The comparative performance of the above TMDs arrangements is also studied for both torsionally coupled and uncoupled systems. It is found that four TMDs arrangement is quite effective solution for vibration control of torsionally coupled system.

Key words: vibration control; wind excitation; harmonic; TMDs; torsional coupling; robustness; accidental eccentricity.

1. Introduction

Tuned mass damper (TMD) is a classical engineering device consisting of a mass, a spring and a viscous damper attached to a vibrating main system in order to attenuate any undesirable vibration. The natural frequency of the damper system is tuned to a frequency near to the natural frequency of the main system. Thus, the vibration of the main system (especially due to wind-induced) causes the damper to vibrate in resonance, as a result, the vibration energy is dissipated through the damping in the TMD. The solution for determining the optimum tuning frequency and the optimum damping of a tuned mass damper for undamped main system subjected to harmonic external force over a broad band of forcing frequencies is described by Brock (1946) and Den Hartog (1956). Using Den Hartog’s procedure Warburton and Ayorinde (1980) have derived the optimum damper parameters for the undamped main system subjected to the

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harmonic support motion where the acceleration amplitude is fixed for all input frequencies. The explicit formulae for the optimum parameters of a TMD and its effectiveness are available under different combinations of system response and excitation (Warburton 1982, Tsai and Lin 1994, Thompson 1981, Fujino and Abe 1993).

The phenomenon of building vibrations caused by vortex shedding, galloping and flutter is random in nature and depends heavily on the building's geometry and dynamic characteristics and the local climatological factors. As a result, quantifying the design wind load is a complex process and is usually not readily amenable to closed form solutions. The TMD have often been used by tall building designers as a reliable vibration control mechanism that is not sensitive to wind load variations (Weisher 1979, Kwok 1984, Fur *et al*. 1996). A number of TMDs have been installed in tall buildings, bridges, towers and smoke stacks for response control against primarily wind-induced external loads (Housner *et al*. 1997). The first structure in which a TMD was installed appears to be Centerpoint Tower in Sydney. There are two buildings in the United States equipped with TMDs namely the Citicorp Center in New York City and the John Hancock Tower in Boston. In Japan, the first TMD was installed in Chiba Port Tower, followed by installations in Funade Bridge Tower, Osaka and in steel stacks, Kimitsu City. These examples show that the success of TMD is now well established for control of wind-induced vibration in structure.

The main disadvantage of a single TMD is its sensitivity of the effectiveness to the error in the natural frequency of the structure and/or that in the damping ratio of the TMD. The mistuning or off-optimum damping significantly reduces the effectiveness of a TMD. As a result, the use of more than one tuned mass dampers with different dynamic characteristics has been proposed in order to improve the effectiveness. Iwanami and Seto (1984) had shown that two tuned mass dampers are more effective than a single TMD. However, the improvement on the effectiveness was not significant. Recently, multiple tuned mass dampers (MTMD) with distributed natural frequencies were proposed by Xu and Igusa (1992) and also studied by Yamaguchi and Harnpornchai (1993), Abe and Fujino (1994), Jangid (1995, 1999). It is shown that the MTMD is more effective for vibration control as compared to the single TMD. In addition, the effectiveness of the MTMD system is not much influenced by the change or estimation error in the natural frequency of the structure. The above review shows that a lot of work has been done on the use of TMD and MTMD for a system with symmetric in plan under different types of excitation. Since an accidental eccentricity in the system always exists, therefore, it will be interesting to study the performance of tuned mass dampers for controlling the coupled lateral-torsional response of the system.

In the present study, the steady-state response of a torsionally coupled system with tuned mass dampers to wind-induced external harmonic excitation is investigated. The specific objectives of the study are (i) to study the performance of a single TMD system designed without considering the torsional effects for controlling the coupled lateral and torsional response of a torsionally coupled system, (ii) to obtain the optimum parameters of a single TMD for effective vibration control of a torsionally coupled system, (iii) to explore alternative effective TMDs arrangements (i.e. two identical TMDs placed at opposite corners, two independent TMDs, four TMDs etc.) for vibration control of a torsionally coupled system and (iv) to obtain the optimum parameters of various TMDs arrangements for torsionally coupled system using numerical searching technique and study their comparative performance.

2. Structural model

The system configuration consists of a main system on which n numbers of tuned mass dampers with different dynamic characteristics are attached as shown in Fig. 1. The main system consists of rectangular deck supported on columns. The width of the deck is b and breadth as d. The centre of resistance (CR) of the main system does not coincide with the centre of mass (CM). As a result, the main system undergoes to torsional vibration when excited in the lateral direction. The TMDs are placed at a distance of y_1, y_2, ..., y_n from the CM of the main system to control the vibration of the system. The system is excited by wind-induced harmonic external force applied at the CM of the system. The main system has two degrees-of-freedom and the combined system will have total n+2 degrees-of-freedom.

Two uncoupled frequencies of the main system are defined as

$$\omega_s = \sqrt{\frac{k_s}{m_s}} \tag{1}$$

$$\omega_\theta = \sqrt{\frac{k_\theta}{m_s r_s^2}} \tag{2}$$

where k_s (i.e., $k_{s1}+k_{s2}$) is the lateral stiffness of the main system; k_θ (i.e., $k_{s1}y_{s1}^2 + k_{s2}y_{s2}^2$) is the torsional stiffness of the system about the CM of the system; k_{s1} and k_{s2} are the lateral stiffness of the columns of the main system located at the distance y_{s1} and y_{s2} from the CM of the system, respectively; m_s is the mass of the main system; and r_s is the radius gyration of the system.

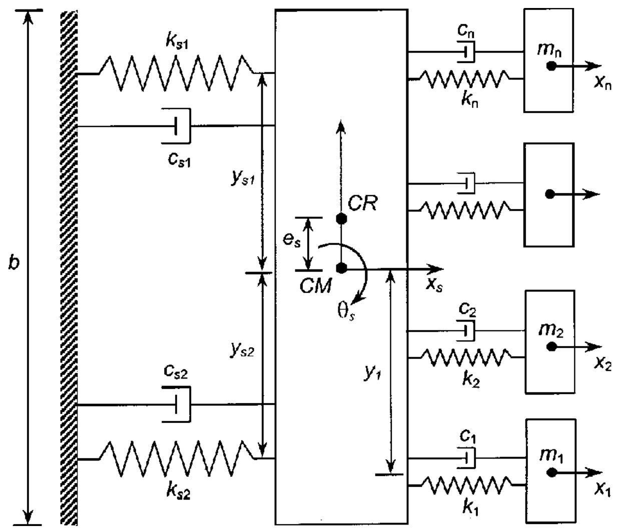

Fig. 1 Schematic sketch of torsionally coupled system with TMDs

The eccentricity between the CM and the CR of the main system is given by

$$e_s = \frac{k_{s1}y_{s1} - k_{s2}y_{s2}}{k_s} \tag{3}$$

The frequencies ω_s and ω_θ can be interpreted as the natural frequencies of the main system if it were torsionally uncoupled system, i.e. the system with $e_s = 0$; but m_s, the mass of the main system, k_s and k_θ are the same as those in the coupled system. The parameters $k_{s1,}$ $k_{s2,}$ y_{s1} and y_{s2} can be adjusted to provide the desired values of the parameters ω_s, ω_θ and e_s. Further, it is to be noted that the simplified model considered in Fig. 1 can also be used for evaluating the response of a multi-storey building or tower using the modal analysis. The frequencies ω_s and ω_θ shall be adjusted to the natural lateral and torsional frequency of the building in which the vibrations of the structure are to be controlled, respectively.

The stiffness and damping of the i^{th} TMD are given by

$$k_i = m_i \omega_i^2 \tag{4}$$

$$c_i = 2\xi_i m_i \omega_i \tag{5}$$

where m_i, c_i and k_i are the mass, damping and stiffness of the i^{th} TMD, respectively; and ω_i and ξ_i are the natural frequency and damping ratio of the i^{th} TMD, respectively.

The tuning frequency ratio, f_i for the i^{th} TMD is defined as

$$f_i = \frac{\omega_i}{\omega_s} \tag{6}$$

The mass ratio, μ_i for the i^{th} TMD is defined as

$$\mu_i = \frac{m_i}{m_s} \tag{7}$$

The total mass ratio of the TMDs is defined as

$$\mu = \sum_{i=1}^{n} \mu_i \tag{8}$$

2.1. Equations of motions

The equations of motion of the combined system subjected to wind-induced external excitation at the main system are expressed in the following matrix form

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{1\}f(t) \tag{9}$$

where $\{X\}=\{x_s, \theta_s, x_1, \cdots x_n\}^T$ is the displacement vector of the structural system; x_s and θ_s are the lateral and torsional displacement of the main system, respectively; and x_i is the lateral displacement of the i^{th} TMD; $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrix of the structural system, respectively; $\{1\}=\{1, 0, 0, \cdots, 0\}^T$ and $f(t)$ is the external wind force applied at the CM of the main system.

The matrices $[M]$, $[C]$ and $[K]$ are expressed by

$$[M] = \begin{bmatrix} m_s & 0 & 0 & 0 \\ 0 & m_s r_s^2 & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ & & & \ddots \\ 0 & 0 & 0 & m_n \end{bmatrix} \tag{10}$$

$$[C] = \begin{bmatrix} c_s + \sum c_j & c_{s\theta} + \sum c_i y_i & -c_1 & -c_2 & \cdots & -c_n \\ c_{s\theta} + \sum c_i y_i & c_\theta + \sum c_i y_i^2 & -c_1 y_1 & -c_2 y_2 & \cdots & -c_n y_n \\ -c_1 & -c_1 y_1 & c_1 & 0 & \cdots & 0 \\ -c_2 & -c_2 y_2 & & c_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -c_n & 0 & 0 & 0 & \cdots & c_n \end{bmatrix} \tag{11}$$

$$[K] = \begin{bmatrix} k_s + \sum k_j & k_{s\theta} + \sum k_i y_i & -k_1 & -k_2 & \cdots & -k_n \\ k_{s\theta} + \sum k_i y_i & k_\theta + \sum k_i y_i^2 & -k_1 y_1 & -k_2 y_2 & \cdots & -k_n y_n \\ -k_1 & -k_1 y_1 & k_1 & 0 & \cdots & 0 \\ -k_2 & -k_2 y_2 & 0 & k_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_n & -k_n y_n & 0 & 0 & \cdots & k_n \end{bmatrix} \tag{12}$$

where c_s, $c_{s\theta}$ and c_θ are the elements of the damping matrix of the main system without TMDs which are obtained by assuming a modal damping ratio ξ_s, in both modes of vibration; $k_{s\theta}$ $(=k_s e_s)$ is the coupling term between the lateral and torsional degrees-of-freedom of the main system; and y_i is the distance of i^{th} TMD from the CM of main system.

For the present study, the external excitation force acting at the main system is modeled by harmonic force expressed as $f(t)=f_0 e^{i\omega t}$ (in which f_0 is the amplitude of excitation; ω is the circular frequency; t denote the time; and $i = \sqrt{-1}$). The harmonic excitation had been widely used in the past for the vibration control of system using the TMDs (Warburton and Ayorinde 1980, Warburton 1982, Tsai and Lin 1994, Thompson 1981, Fujino and Abe 1993, Yamaguchi and Harnpornchai

1993, Abe and Fujino 1994, Jangid 1995, 1999). This is due to fact that the response of any system to harmonic frequencies gives considerable insight into the dynamic characteristics of the system, which may be helpful in interpreting the response to the other type of excitation (including the wind force). In addition, it is also possible to express any time varying load as a summation of several sinusoidal motions through Fourier transform.

The corresponding steady-state harmonic response of the system to the harmonic excitation will be $\{X\}=X(\omega)e^{i\omega t}$. The $X(\omega)$ indicates the amplitude vector of the steady-state response of the combined system which is expressed by

$$X(\omega) = (-\omega^2[M] + i\omega[C] + [K])^{-1}\{1\}f_0 \tag{13}$$

The first two elements of the vector, $X(\omega)$ are the amplitudes of lateral and torsional displacement of the main system. The corner displacements are calculated as

$$x_{c1,\,c2} = x_s \pm \theta_s b/2 \tag{14}$$

where x_{c1} and x_{c2} are the displacement of the corner of the main system; and b is the lateral dimension of the main system in the direction of eccentricity.

The displacement of the main system at different locations is normalized by corresponding static displacement of the main system without eccentricity. The normalized lateral displacement of the main system at different locations (such as at the center and corners) is expressed by the ratio R defined as

$$R = \frac{\textit{Dynamic displacement of main system}}{\delta_{st}} \tag{15}$$

where $\delta_{st}=f_0/k_s$ is the static displacement of the CM of the main system without torsional coupling. Note that the R denotes the corresponding dynamic magnification factor of lateral displacement for a torsionally uncoupled system.

3. Numerical study

The main system is characterized by the uncoupled natural frequency, ω_s, the aspect ratio b/d, the ratio of uncoupled torsional to lateral frequency, ω_θ/ω_s and the damping ratio of the main system, ξ_s. The damping of the main system is assumed to be 2 per cent. The eccentricity in the system is considered to be 5 per cent of the dimension b, as specified by the UBC Code (1997). This implies that the main structure as designed is symmetric and the torsion arises only due to the accidental eccentricity in the system. The each TMD system is characterized by damping ratio, ξ_i, tuning frequency ratio, f_i, mass ratio of the TMD, μ_i and placement of the TMD, y_i. The steady-state displacement of the torsionally coupled system at different locations expressed by the ratio R is evaluated to study the effects of system eccentricity on the effectiveness tuned mass dampers. However, it was found that the maximum amplitude of displacement of a torsionally coupled

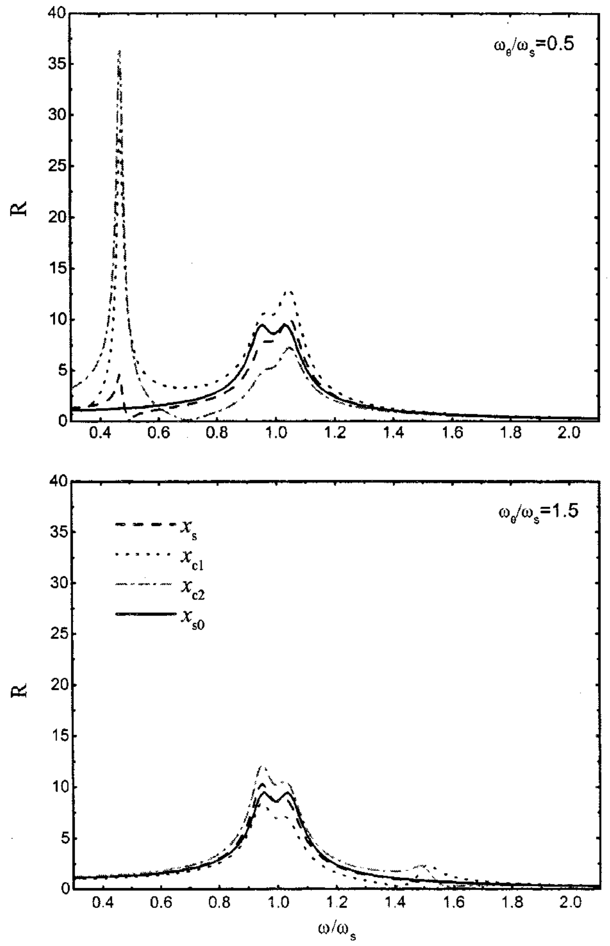

Fig. 2 Effectiveness of optimum single TMD designed without considering torsion in a torsionally coupled system (b/d = 2)

system occurs only at the corners. Thus, the normalized displacement R indicated without any specific displacement implies the maximum displacement of the main system.

3.1. Single tuned mass damper

Firstly, the effectiveness of an optimum single TMD, designed without considering the torsion, is studied. In Fig. 2, variation of the normalized displacement for x_s, x_{c1}, x_{c2} and x_{s0} of torsionally coupled system with single TMD is plotted against harmonic excitation frequency for different values of ω_θ / ω_s ratios. The x_{s0} denotes the corresponding lateral displacement, x_s of the main system without any eccentricity (i.e., e_s=0) and under such condition the lateral displacement at different locations of the main system becomes identical.

The parameters considered for the TMD system are μ=μ_1=0.01, f_1=0.9869, ξ_1=0.0646 and y_1=0 (i.e., TMD is placed at the CM) which are the optimum parameters of a single TMD system for

Table 1 Optimum parameters for single and divided TMD attached to torsionally coupled system ($\xi_s = 0.02$)

Main system				Single TMD							
				$\mu = 0.01$				$\mu = 0.02$			
ω_θ/ω_s	b/d	$\frac{\varpi_1}{\omega_s}$	$\frac{\varpi_2}{\omega_s}$	f_1	ξ_1	y_1	R	f_1	ξ_1	y_1	R
0.5	1	0.480	1.010	0.750	0.3706	$b/2$	20.60	0.680	0.3897	$b/2$	17.39
0.5	2	0.468	1.015	0.507	0.3959	$b/2$	24.78	0.532	0.4125	$b/2$	20.27
0.5	3	0.464	1.017	0.489	0.3566	$b/2$	25.55	0.515	0.3916	$b/2$	20.88
1	1	0.937	1.059	0.958	0.0766	0	19.01	0.958	0.0891	0	15.05
1	2	0.919	1.075	0.977	0.0441	$-b/2$	20.93	0.934	0.0387	$-b/2$	17.47
1	3	0.914	1.079	0.986	0.0525	$-b/2$	20.80	0.968	0.0487	$-b/2$	17.02
1.5	1	0.994	1.504	0.980	0.0725	$-b/2$	9.90	0.963	0.1000	$-b/2$	7.755
1.5	2	0.990	1.506	0.975	0.0750	$-b/2$	10.12	0.962	0.1031	$-b/2$	7.934
1.5	3	0.989	1.507	0.974	0.0762	$-b/2$	10.21	0.973	0.1044	$-b/2$	7.992

Divided single TMD (y_1=$-b/2$, y_2=$b/2$, f_2=f_1, ξ_2=ξ_1)						Un-controlled
$\mu = 0.01$			$\mu = 0.02$			
f_1	ξ_1	R	f_1	ξ_1	R	R
0.801	0.309	21.43	0.726	0.368	18.70	28.466
0.551	0.441	25.87	0.575	0.493	21.95	36.294
0.525	0.407	26.70	0.545	0.498	22.77	41.757
0.953	0.096	13.76	0.941	0.118	10.19	31.435
0.937	0.104	14.49	0.922	0.130	10.34	37.448
0.932	0.111	14.66	0.916	0.136	10.37	39.239
0.981	0.064	10.57	0.970	0.089	8.32	27.951
0.976	0.064	11.18	0.966	0.089	8.78	29.795
0.975	0.066	11.37	0.965	0.091	8.93	30.352

(ϖ_1 and ϖ_2 are the natural frequencies of torsionally coupled system)

torsionally uncoupled system with 2 per cent damping (Tsai and Lin 1994). From the Fig. 2, it is observed that due to the torsional coupling the displacements x_s, x_{c1} and x_{c2} are significantly increased in comparison to the displacement, x_{s0} implying the loss of effectiveness of TMD system for vibration control. Thus, an optimally designed single TMD system without considering torsion is not at all effective in controlling the response of the system if it is either a torsionally flexible ($\omega_\theta/\omega_s = 0.5$) or strong torsionally coupled system ($\omega_\theta/\omega_s = 1$). However, for torsionally stiff system ($\omega_\theta/\omega_s > 1.5$), the design of TMD system by ignoring the torsion is justified because the lateral displacement of the main system is not much influenced due to torsional coupling.

Since the effectiveness of a single TMD designed based on symmetric system for a torsionally coupled system is significantly reduced due to eccentricity in the main system. As a result, there is a need to find out the optimum parameters of a single TMD (i.e., f_1, ξ_1 and y_1) for a torsionally coupled system which minimize the lateral displacement at corners. These parameters are obtained

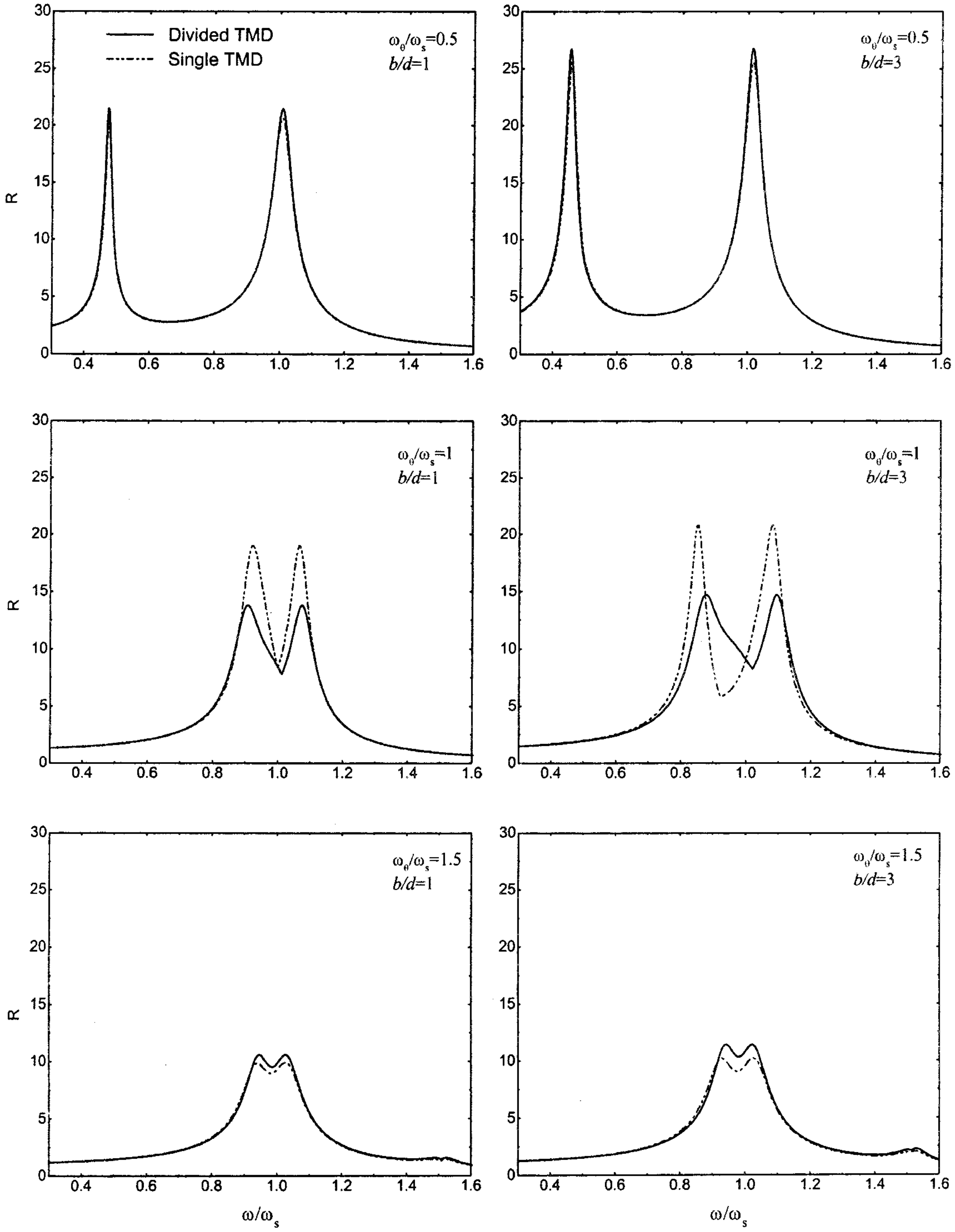

Fig. 3 Comparison between single TMD and divided TMD for response reduction of torsionally coupled system

using the numerical searching technique in which the values of the parameters f_1, ξ_1 and y_1 is varied in the admissible range and the maximum lateral displacement of the main system is evaluated (Tsai and Lin 1994). The optimum parameters are then selected which provide the minimum value of the maximum lateral displacement at any location of the main system. These parameters are tabulated in Table 1 for different values of ω_θ/ω_s, b/d and μ. From the Table 1, it is observed that the value of optimum TMD damping ratio for a torsionally flexible system is much higher due to the presence of torsional mode of vibration. The high optimum damping value is required to minimize the peak displacement occurring in the vicinity of torsional frequency (refer the Fig. 2). In addition, the optimum parameters of single TMD for torsionally stiff system (i.e., $\omega_\theta/\omega_s = 1.5$) are quite close to the optimum parameters for uncoupled system (i.e., f_1=0.9869 and ξ_1=0.0646 from reference (Tsai and Lin 1994)). In Table 1, the corresponding value of R for an uncontrolled (i.e., without TMD) main system is also shown to study the effectiveness of the TMD system. It is observed that there is reduction in the response of the main system due to TMD implying that such device is effective for vibration control of the torsionally coupled system. Further, the effectiveness of a single TMD for vibration control of a torsionally flexible system or strong torsionally coupled system is less in comparison to torsionally stiff or uncoupled main system.

Table 2 Optimum parameters for two TMDs system (ξ_s=0.02)

Main System				TMD System							
				TMD 1 (y_1=-b/2)			TMD 2 (y_2=b/2)			R	
$\frac{\omega_\theta}{\omega_s}$	b/d	$\frac{\varpi_1}{\omega_s}$	$\frac{\varpi_2}{\omega_s}$	f_1	μ_1	ξ_1	f_2	μ_2	ξ_2	e_s=0.05b	e_s=0
μ=0.01											
0.5	1	0.480	1.010	0.4714	0.0028	0.0453	0.9939	0.0072	0.0647	10.8	12.4
0.5	2	0.468	1.015	0.4526	0.0053	0.0744	1.0017	0.0047	0.0597	12.5	14.9
0.5	3	0.464	1.017	0.4468	0.0059	0.0819	1.0052	0.0041	0.0535	13.1	15.8
1.0	1	0.937	1.059	0.9220	0.0067	0.0825	1.0420	0.0033	0.0560	9.98	16.7
1.0	2	0.919	1.075	0.9006	0.0071	0.1018	1.0516	0.0029	0.0662	10.5	18.6
1.0	3	0.914	1.079	0.9857	0.0071	0.1013	1.0523	0.0029	0.0625	10.6	18.9
1.5	1	0.994	1.504	0.9691	0.0087	0.0625	1.0313	0.0013	0.0248	9.47	10.2
1.5	2	0.991	1.506	0.9632	0.0079	0.0625	1.0278	0.0021	0.0304	9.77	10.7
1.5	3	0.989	1.507	0.9623	0.0083	0.0650	1.0280	0.0017	0.0272	9.86	11.1
μ=0.02											
0.5	1	0.480	1.010	0.4628	0.0056	0.0625	0.9811	0.0144	0.0912	8.50	10.3
0.5	2	0.468	1.015	0.4416	0.0102	0.0987	0.9892	0.0098	0.0844	9.76	12.4
0.5	3	0.464	1.017	0.4345	0.0114	0.1181	0.9943	0.0086	0.0750	10.2	13.2
1.0	1	0.937	1.059	0.9091	0.0130	0.1162	1.0280	0.0070	0.0837	7.84	12.0
1.0	2	0.919	1.075	0.8882	0.0136	0.1375	1.0281	0.0064	0.0894	8.17	12.5
1.0	3	0.914	1.079	0.8777	0.0136	0.1500	1.0273	0.0064	0.0919	8.28	12.3
1.5	1	0.994	1.504	0.9500	0.0164	0.0837	1.0390	0.0036	0.0398	7.32	8.22
1.5	2	0.991	1.506	0.9505	0.0176	0.0925	1.0373	0.0024	0.0360	7.56	9.06
1.5	3	0.989	1.507	0.9497	0.0180	0.0968	1.0359	0.0020	0.0335	7.64	9.34

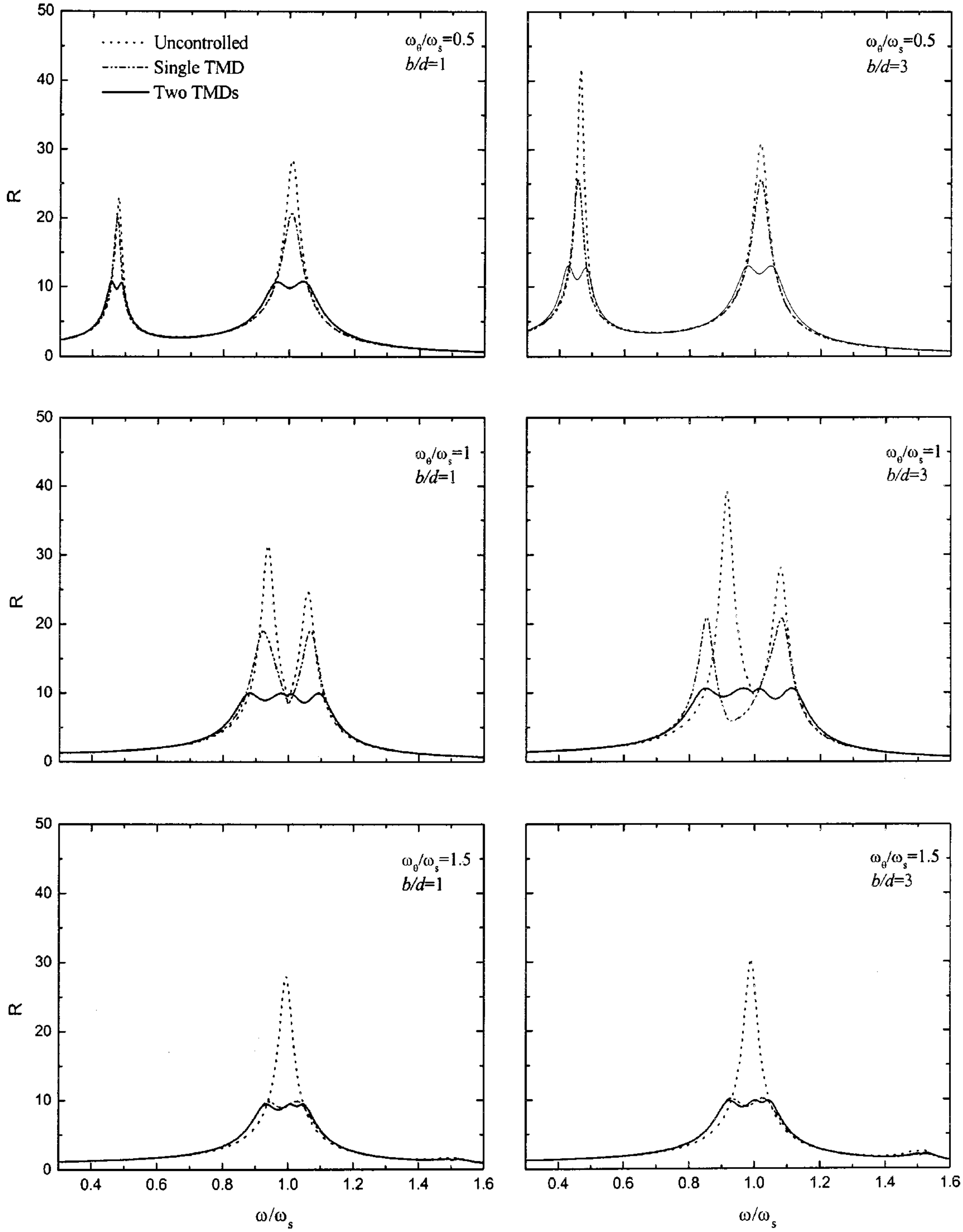

Fig. 4 Effectiveness of optimum two TMD system against optimum single TMD system for torsionally coupled system

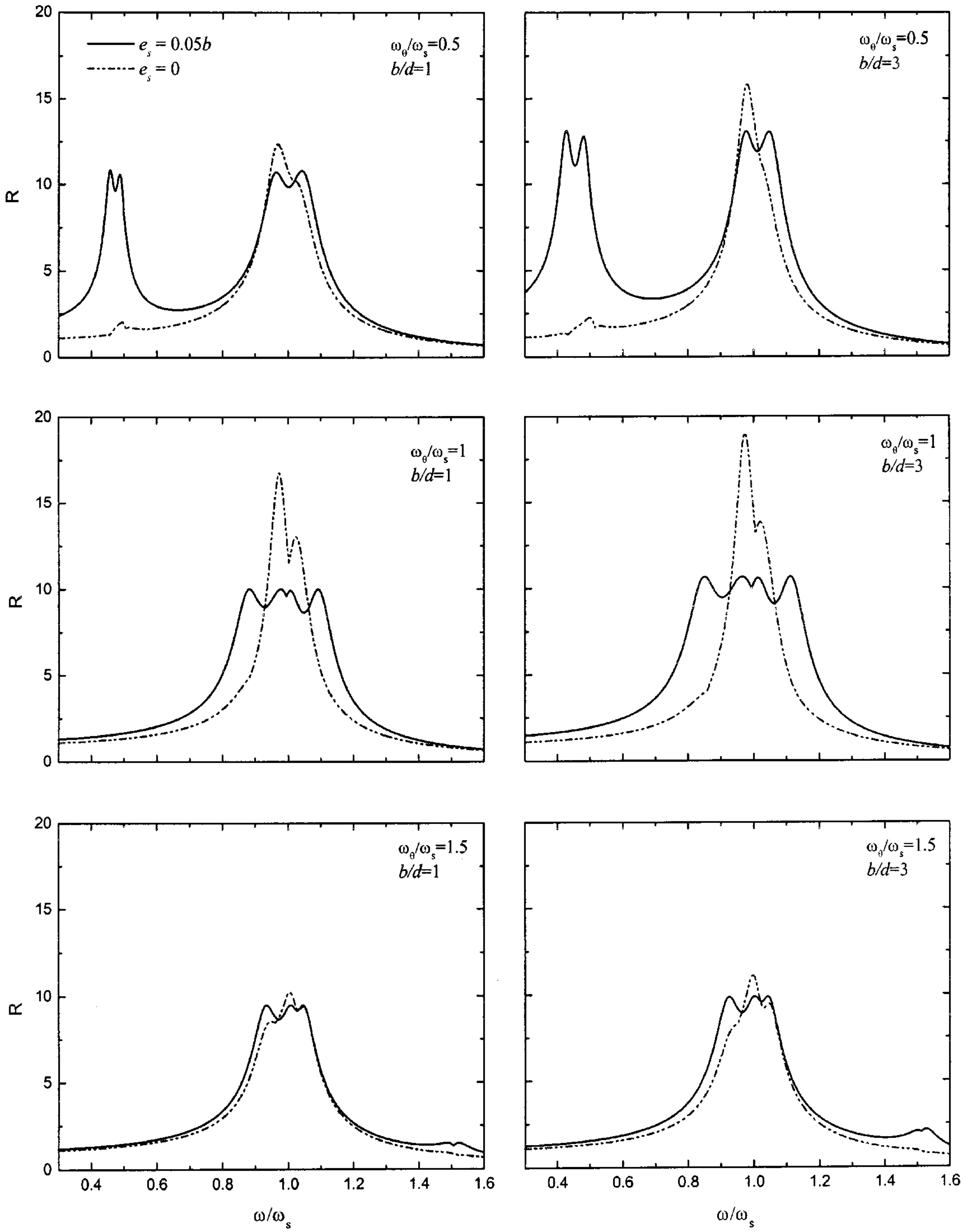

Fig. 5 Effects of eccentricity on the response of torsionally coupled system attached with optimum two TMDs system

3.2. Two identical tuned mass dampers

It is to be noted that the optimum placing of the single TMD for torsionally coupled system is at the corners (expect for $\omega_\theta/\omega_s = 1$ and $b/d = 1$). Such arrangement of TMD system will lead to torsion in the system when there is no eccentricity in the main system. Hence, an alternative approach in the form of divided TMDs (i.e., two TMDs having identical properties placed at two corners) has been proposed for reduction of vibration. The optimum parameters of divided TMDs system are evaluated and are presented in Table 1. It is observed that this system is quite effective for strong torsionally coupled system ($\omega_\theta/\omega_s = 1$). A variation of normalized maximum displacement of main system with optimum single TMD and divided TMD is shown in Fig. 3. The figure indicates that the both TMD systems are almost same effective for torsionally flexible and stiff system. However, the optimally designed divided single TMD is relatively more effective for strong torsionally coupled system.

3.3. Two Independent tuned mass dampers

Since a torsionally coupled system has two natural frequencies, as a result, two TMDs are required to control the response of the system. The optimum parameters of the two TMDs (i.e.,

Table 3 Optimum parameters for two divided TMDs system (ξ_s=0.02 and μ=0.02)

Main System				TMD System						R	
				TMD 1 and 2			TMD 3 and 4				
$\frac{\omega_\theta}{\omega_s}$	b/d	$\frac{\varpi_1}{\omega_s}$	$\frac{\varpi_2}{\omega_s}$	f_1, f_2	μ_1, μ_2	ξ_1, ξ_2	f_3, f_4	μ_3, μ_4	ξ_3, ξ_4	e_s=0.05b	e_s=0
Approach - I											
0.5	1	0.480	1.010	0.4722	0.005	0.1656	0.9869	0.005	0.0646	11.4	9.33
0.5	2	0.468	1.015	0.4478	0.005	0.1356	0.9869	0.005	0.0646	12.5	9.37
0.5	3	0.464	1.017	0.4423	0.005	0.1306	0.9869	0.005	0.0646	12.9	9.39
1.0	1	0.937	1.059	0.9044	0.005	0.1006	0.9869	0.005	0.0646	10.3	8.39
1.0	2	0.919	1.075	0.8757	0.005	0.1063	0.9869	0.005	0.0646	10.3	8.61
1.0	3	0.914	1.079	0.8660	0.005	0.1094	0.9869	0.005	0.0646	10.3	8.67
1.5	1	0.994	1.504	0.9500	0.005	0.0894	0.9869	0.005	0.0646	8.38	7.83
1.5	2	0.991	1.506	0.9396	0.005	0.0925	0.9869	0.005	0.0646	8.77	7.98
1.5	3	0.989	1.507	0.9356	0.005	0.0919	0.9869	0.005	0.0646	8.88	8.04
Approach - II											
0.5	1	0.480	1.010	0.4562	0.0030	0.0650	0.9991	0.0070	0.0750	9.64	9.02
0.5	2	0.468	1.015	0.4493	0.0048	0.0925	0.9991	0.0052	0.0725	11.3	10.14
0.5	3	0.464	1.017	0.4392	0.0054	0.1125	1.0056	0.0046	0.0600	11.7	10.96
1.0	1	0.937	1.059	0.8996	0.0060	0.0775	1.0302	0.0040	0.0800	9.02	8.93
1.0	2	0.919	1.075	0.8866	0.0072	0.0975	1.0389	0.0028	0.0725	9.30	9.78
1.0	3	0.914	1.079	0.8867	0.0080	0.1050	1.0522	0.0020	0.0775	9.43	10.57
1.5	1	0.994	1.504	0.9224	0.0042	0.0520	1.0137	0.0058	0.0625	8.02	7.20
1.5	2	0.991	1.506	0.9222	0.0046	0.0525	1.0134	0.0054	0.0625	8.05	7.34
1.5	3	0.989	1.507	0.9247	0.0050	0.0575	1.0124	0.0050	0.0600	8.18	7.39

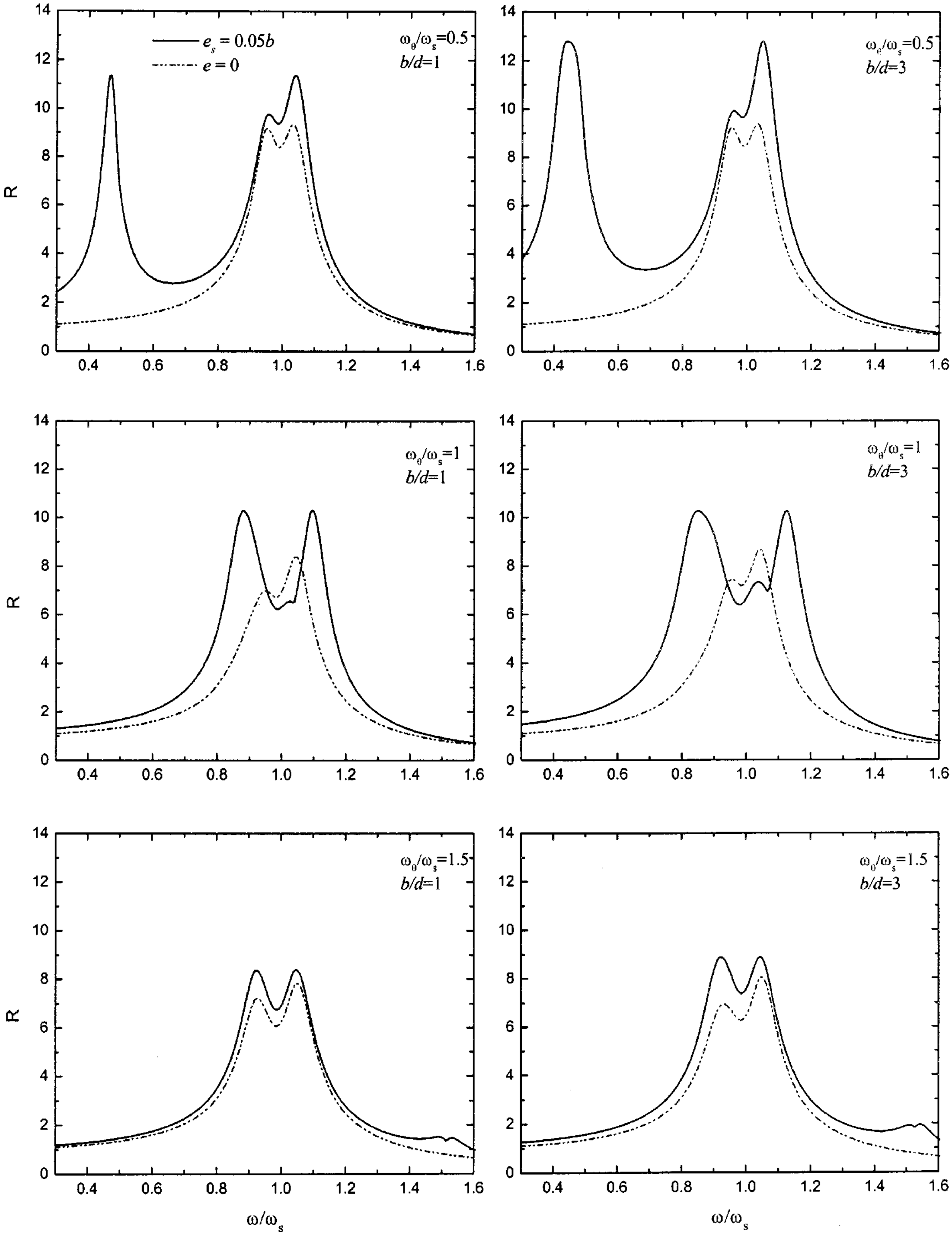

Fig. 6 Effects of eccentricity on the response of torsionally coupled system attached with optimum divided two TMDs system with Approach-I

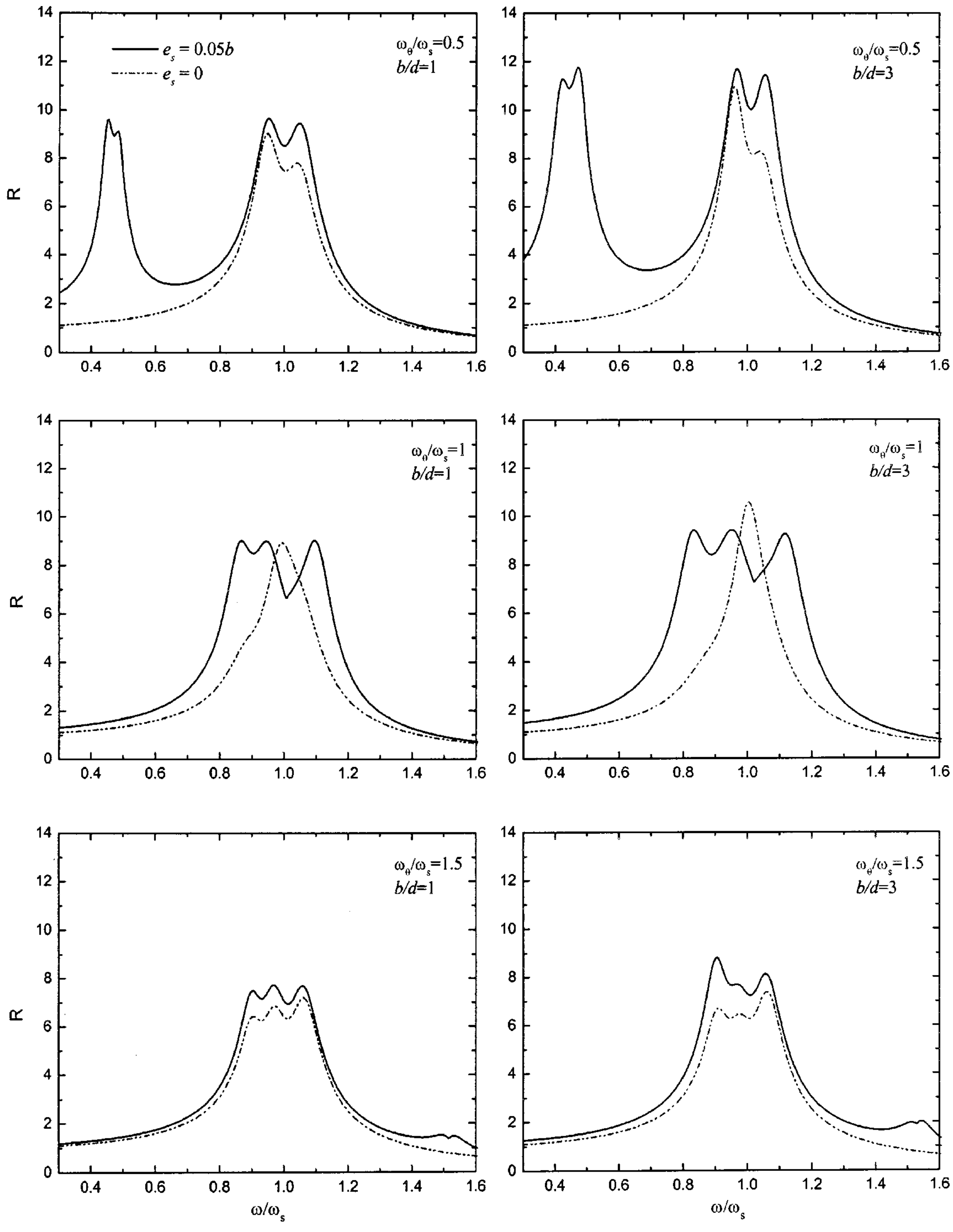

Fig. 7 Effects of eccentricity on the response of torsionally coupled system attached with optimum divided two TMDs system with Approach-II

f_1, μ_1, ξ_1 and y_1; f_2, μ_2, ξ_2 and y_2) for a torsionally coupled system with different system properties are given Table 2. By comparing the Tables 1 and 2, it is found that the optimally designed two TMDs system is more effective approach than the single TMD system for controlling the response of torsionally coupled system. In Fig. 4, the variation of normalized maximum displacement of the main system is plotted against excitation frequencies. It is observed that two TMDs system is much more effective in reducing the response of the structure for both torsionally flexible as well as strong torsionally coupled system. The optimum tuning for two TMDs system is found in the vicinity of the natural frequencies of the main system for torsionally flexible and strong torsionally coupled system. However, for torsionally stiff system the optimum tuning of two TMDs is found in the vicinity of uncoupled lateral frequency of the main system. A comparison

The performance of optimally designed two TMDs system for torsionally coupled and uncoupled system is shown in Fig. 5. The figure indicates that the effectiveness of two TMDs is reduced if there is no eccentricity in the system. These effects are more pronounced for strong torsionally coupled system as compared to torsionally flexible and stiff system. However, using the divided two TMDs system can circumvent this obstacle.

3.4. Four tuned mass dampers

The optimum parameters for two divided TMDs (i.e., total four TMDs in which two each are identical but placed on opposite corners) are presented in Table 3 for Approach-I and II. In Approach-I, all the TMDs have the same mass and two TMDs are tuned to uncoupled lateral frequency of the system (i.e., $f_3=f_4=0.9869$, $\xi_3=\xi_4=0.0646$) and the optimum parameters of the remaining two TMDs are searched for minimum displacement of the system. On the other hand, in Approach-II, the optimum parameters of the both set of TMDs with variable mass are obtained using the numerical searching technique. Variation of normalized maximum displacement of the main system with optimally designed two TMDs is shown in Figs. 6 and 7 for Approach-I and -II, respectively. It is found that for both the approaches the optimally designed two TMDs are found to be same or more effective for torsionally uncoupled system. The Approach-I is found to be quite effective for both torsionally flexible as well as strong torsionally coupled system. However, the approach-II is relatively more effective for torsionally flexible systems.

4. Conclusions

The steady-state response of torsionally coupled system with tuned mass dampers subjected to external wind-induced harmonic excitation is investigated. The performance of optimally designed tuned mass dampers with and without considering the eccentricity in the main system is studied. In addition, the effectiveness of various arrangements of TMDs system for vibration control of torsionally coupled system is investigated. The optimum parameters for different arrangements of TMDs are also evaluated using the numerical searching technique which can be used for the effective design of TMDs for suppressing the coupled lateral-torsional response of a torsionally coupled main system. From the trends of the results present study, following conclusions may be drawn :

1. The optimum single TMD system, designed by neglecting the effect of torsion, is found to be ineffective in reducing the response of the torsionally coupled system.
2. The design of TMD system by ignoring the torsional coupling is justified for torsionally stiff system. Thus, to avoid the effects of torsional coupling on the performance of TMDs the layout of the main system should be such that the torsional frequency is greater than the 1.5 times the lateral frequency.
3. An optimally designed single TMD for torsionally coupled system is found to be less effective in comparison to the corresponding uncoupled system. This effect is found to be more pronounced for torsionally flexible systems.
4. At least two TMDs are required for effective vibration control of torsionally coupled systems.
5. The use of divided TMDs system placed on each corner of the structure is found to be very effective for strong torsionally coupled system.
6. The use of two divided TMDs is found to be most effective for controlling the response of torsionally coupled system. The performance of such arrangement is found to be less sensitive to the change in the eccentricity of the main system.

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