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Stochastic along-wind response of nonlinear structures to quadratic wind pressure

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Abstract. The effects of the nonlinear (quadratic) term in wind pressure have been analyzed in many papers with reference to linear structural models. The present paper addresses the problem of the response of nonlinear structures to stochastic nonlinear wind pressure. Adopting a single-degree-of-freedom structural model with polynomial nonlinearity, the solution is obtained by means of the moment equation approach in the context of Itô's stochastic differential calculus. To do so, wind turbulence is idealized as the output of a linear filter excited by a Gaussian white noise. Response statistical moments are computed for both the equivalent linear system and the actual nonlinear one. In the second case, since the moment equations form an infinite hierarchy, a suitable iterative procedure is used to close it. The numerical analyses regard a Duffing oscillator, and the results compare well with Monte Carlo simulation.

Key words: quadratic wind pressure; nonlinear structures; wind response; Itô's calculus; moment equation approach; iterative closure method.

1. Introduction

In last years great attention has been paid to nonlinear response to wind action as arising from the interaction between wind and structure, but in general a linear elastic structural model has been adopted. Among the studies in which this problem has been considered in a stochastic context by means of analytical non simulative methods, we recall: Saul *et al*. (1976), Soize (1978), Grigoriu (1986), Bartoli and Spinelli (1993), Caddemi and Di Paola (1995), Floris (1995, 1996), Kareem and his collaborators (1994, 1995, 1997), Gusella and Materazzi (1998), Benfratello *et al*. (1996, 1997, 1999). On the contrary, the response of nonlinear structures to random wind pressure has not had equal attention. Nevertheless, many structures exhibit nonlinearities under wind action: Bernoulli-Navier beams with second order effects, stayed structures, large antennas, and so on. Thus, a stochastic study on the response of nonlinear structures to wind pressure has its own importance.

According to the usual model for representing the wind (Davenport 1961, Simiu and Scanlan 1996), wind pressure is proportional to the square of the sum of a mean (static) speed U, and a

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dynamic oscillation around the mean, the turbulence u. The turbulence u is assumed to be a stationary zero mean Gaussian process. Thus, it is fully characterized by its power spectral density (PSD) $S_{uu}(\omega)$ (Dryden 1961, Davenport 1967, Solari 1987, Mann 1998). In this way, wind pressure is no more a Gaussian process. Strictly speaking, even the response of linear structures is no longer Gaussian, and should be characterized by the infinite hierarchy of the statistical moments or cumulants. At least, the first four are required to have an estimate of the non-Gaussianity of the response. The computation of response statistical moments of order larger than two is a heavy task when the response is not Gaussian. For that reason, only few authors carried out the computation of moments of order larger than two (Kareem *et al.* 1994, 1995, 1997, Benfratello *et al.* 1996, 1997, 1999, Floris and his collaborators 1996, 2001, Gusella and Materazzi 1998).

The computational charge is clearly augmented when the structure is nonlinear. The only method of general applicability is Monte Carlo simulation, which is notoriously very onerous from a computational point of view. Another method is the Volterra series approach, which has been applied to wind response of linear structures by Kareem and Zhao (1994), Kareem *et al.* (1995, 1997), and by Benfratello *et al.* (1997). It has some restrictions, and requires the evaluation of lengthy multifold integrals. The extension of this method to nonlinear structures has been presented by Tognarelli *et al.* (1997). Thus, there is the necessity of a method more attractive from a computational point of view.

The moment equation approach of Itô's stochastic differential calculus (Itô 1951a, b, Di Paola 1993), which avoids the evaluation of multifold integrals, proved to be a very useful tool to do that in the case of linear structures (Benfratello *et al.* 1996, 1999, Floris 1996, Floris *et al.* 2001). This approach is extended herein to nonlinear structures. Since the primary excitation of an Itô differential system must be a Gaussian white noise, and the turbulence has a colored PSD, this is approximated as the output of an appropriate linear filter excited by a Gaussian white noise, as has been already done for linear structures.

This study is accomplished for single-degree-of-freedom (SDOF) structures with linear viscous damping and nonlinear restoring force given by a polynomial of the structural displacement. These assumptions are based on the following reasons: (1) in many cases, the dynamical response to wind is dominated by the first mode, and Solari (1983a,b) gave a rationale for reducing a complex structure to a SDOF model; (2) a polynomial restoring force can be seen as a truncated Taylor series expansion of a more general non-linearity (Tognarelli *et al.* 1997). (3) The assumption of linear damping is made for simplicity's sake. In fact, a nonlinear damping mechanism analitically expressed by a polynomial function, such as both Van Der Pol's and aerodynamic damping, can be equally dealt with in the context of the moment equation approach of Itô's calculus.

The equation of motion and the filter equation are transformed into a system of Itô's stochastic differential equations. In the first step, the equation of motion is linearized as regards the restoring force. Then, the actual nonlinear system is analyzed, for which the equations for the response statistical moments constitute an infinite hierarchy (Di Paola 1993). Thus, a suitable closure scheme is adopted, which operates iteratively, and profits from the computations made on the linearized system. The applications regard a Duffing oscillator, and the results of the analytical methods are verified by numerical simulation.

2. Mathematical formulation

2.1. Preliminary concepts

The equation of motion of an SDOF structure with polynomial nonlinearity in the restoring force is written as

$$\ddot{X}(t) + \beta_0 \dot{X}(t) + \sum_1^m {}_i a_i X^i(t) = Q_d(t) \tag{1}$$

where the superimposed dots mean derivative with respect to time, β_0 is the coefficient of viscous damping, a_i are real constants, m is a positive odd integer number, and $Q_d(t)$ is the random dynamical force caused by wind flow.

The summation in the l.h.s. of (1) represents a conservative restoring force deriving from a potential function of the form $U(X) = \Sigma_{i=1}^m a_i X^{i+1}/ i+1$. Physical reasons require that m is odd: in fact, the restoring force may be asymmetric, but globally it must have the same sign as the displacement. In addition, only positive values of the coefficient a_m of the largest power are implicitly considered. This means a hardening behavior: softening behaviors are encountered in practice, but they may pose problems in a stochastic analysis. In fact, were a_m negative, the solution of the Fokker-Planck equation for Q_d given by a white noise would lose its significance. Thus, a different approach should be used such as that by Sobczyk and Trebicki (2000). This subject is beyond the aim of this paper.

If wind is assumed to blow unidirectionally, and perpendicularly to the area A, on which wind velocity is approximately assumed to be constant, and neglecting the structural velocity, $Q_d(t)$ is given by

$$Q_d(t) = \frac{1}{2}\frac{\rho C_D A}{M}[U + u(t)]^2 \tag{2}$$

In Eq. (2) ρ denotes the air density ($\cong 1.225$ kg/m^3), C_D the drag coefficient, M the structural mass, U the mean wind speed on the area A, and $u(t)$ the turbulence, which, as previously stated, is assumed as a zero mean stationary Gaussian process.

The response variables $X(t)$, $\dot{X}(t)$ are not a Gaussian random process for the two fold reason that the l.h.s. of Eq. (1) is nonlinear, and the Gaussian process $u(t)$ is squared in the r.h.s. A statistical characterization through the response moments is pursued herein. Alternative approaches are the stochastic averaging method (Roberts and Spanos 1986), which was used by Lin and Holmes (1978) for wind excitation, and the Volterra series expansion (Tognarelli *et al*. 1997). Both these methods are very onerous from a computational point of view.

To root the problem in the theory of Markov processes (Bharucha-Reid 1960), and apply Itô's stochastic differential calculus (Itô 1951a & b, Di Paola 1993, Lin and Cai 1995), the primary excitation must be a Gaussian white noise. In this paper, the colored process $u(t)$ is approximately represented by the output of a second order linear filter excited by a Gaussian stationary white noise of unit strength $W(t)$, that is

$$u(t) \cong Y(t) \tag{3a}$$

$$\ddot{Y}(t) + 2\zeta_f\ \omega_f \dot{Y}(t) + \omega_f^2 Y(t) = \sqrt{\pi w_0} W(t) \tag{3b}$$

The constant w_0 is related to the intensity of the excitation, while ζ_f and ω_f determine the shape of the PSD (see Chap. 3, sec. 3.1 and the Appendix).

The augmented system of Eqs. (2) and (3b) is rewritten in Itô's form as

$$dz_1 = z_2\,dt \tag{4a}$$

$$dz_2 = -\left(\beta_0 z_2 + \sum_{i=1}^{m} a_i z_1^i\right)dt + (b_0 + b_1 y_1 + b_2 y_1^2)dt \tag{4b}$$

$$dy_1 = y_2\,dt \tag{4c}$$

$$dy_2 = -(2\zeta_f\,\omega_f y_2 + \omega_f^2 y_1)dt + \sqrt{\pi w_0}\,dB \tag{4d}$$

where the state variables are $z_1 = X$, $z_2 = \dot{X}$, $y_1 = Y$, $y_2 = \dot{Y}$; furthermore, $\gamma = 1/2\rho C_D\ A/M$, $b_0 = \gamma U^2$, $b_1 = 2\gamma U$, $b_2 = \gamma$ and dB is the increment of a unit Wiener process, which is related to the white noise by the formal relation $dB(t)/dt = W(t)$.

Expressing the differential of the non-anticipating function $\phi = z_1^p z_2^q$ by means of Itô's differential rule, applying the averaging operator $E[\bullet]$, and dividing by dt, the differential equations ruling the response moments $\mu_{pq} = E[z_1^p z_2^q] = E[X^p \dot{X}^q]$ are obtained as

$$\dot{\mu}_{pq} = p\mu_{p-1,\,q+1} - q\beta_0\mu_{pq} - q\sum_{i=1}^{m}(a_i\mu_{p+i,\,q-1}) + qb_0\mu_{p,\,q-1}$$
$$+ qb_1E[z_1^p z_2^{q-1} y_1] + qb_2E[z_1^p z_2^{q-1} y_1^2] \tag{5}$$

where the dependence on time is omitted for brevity's sake. In the steady state the l.h.s. vanishes, and Eq. (5) becomes an algebraic equation as is done here. In order to write the equations for the moments of a given order r, the exponents p, q assume all the values for which $p + q = r$, with $p, q \geq 0$. By inspection of Eq. (5), other moments are noted beyond those of order r. In particular, the summation in the r.h.s. has moments of order larger than r, say those with $1 < i \leq m$. These moments are called hierarchical. In other words, to compute the moments of order r, moments till the $r + m - 1$ order are required, which constitutes an infinite hierarchy. A suitable closure scheme is required. Viceversa, the cross-moments between z_1 and y_1 are not hierarchical, as will be shown in next section.

The simplest method of closure is obtained by linearizing the l.h.s. of Eq. (1) (Roberts and Spanos 1990). The response of a linear system to a Gaussian process is Gaussian too. In the present case, the excitation [see Eq. (2)] is not Gaussian, and the response of the linearized system is not such as well. For some values of the parameters this method yields acceptable results. Otherwise, a higher order closure is needed.

2.2. Equivalent linear system

Using the method of the equivalent stochastic linearization, Eq. (1) is replaced by

$$\ddot{X}(t) + \beta_0\dot{X}(t) + \omega_e^2 X(t) = Q_d(t) \tag{6}$$

where the linearization parameter ω_e^2 is determined by minimizing the error that is made by using Eq. (6) instead of Eq. (1) in some statistical sense.

Herein, two methods have been used to determine ω_e^2: (a) minimization of the mean square error (Roberts and Spanos 1990); (b) minimization of the mean square difference between the potential function $U(X) = 1/2\,X^2$ of Eq. (6) and that of Eq. (1) (Falsone and Elishakoff 1994). However, preference has been given to (a) since the latter criterion is more onerous computationally. In the case of a nonlinear damping mechanism $g(\dot{X})$ in Eq. (1), another linearization parameter β_e would be present in Eq. (6), and this would be computed simultaneously to ω_e^2 minimizing a total error measure.

The response moments of X are computed resorting to the stochastic differential calculus. Then, Eqs. (4b), (5) are replaced by, respectively

$$dz_2 = -(\beta_0 z_2 + \omega_e^2 z_1)dt + (b_0 + b_1 y_1 + b_2 y_1^2)dt \tag{7}$$

$$\dot{\mu}_{pq} = p\mu_{p-1,q+1} - q\beta_0\mu_{pq} - q\omega_e^2\mu_{p+1,q-1} + qb_0\mu_{p,q-1} + qb_1E[z_1^p z_2^{q-1} y_1] + qb_2E[z_1^p z_2^{q-1} y_1^2] \tag{8}$$

Now, we have a linear system excited by a polynomial form of a filtered Gaussian process. Some authors have shown that the response statistical moments of this type of dynamic systems are computed exactly (Grigoriu and Ariaratnam 1988, Krenk and Gluver 1988, Muscolino 1995, Di Paola 1997). The different proofs of this statement lead to different methods of computation. The methods by Muscolino (1995) and Di Paola (1997) have been adapted to the present case.

By inspection of Eq. (8), it is noted that it contains cross-moments among the variables X_i and Y_i, that is $E[z_1^p z_2^{q-1} y_1]$ and $E[z_1^p z_2^{q-1} y_1^2]$. To exemplify, the equations for the second moments are:

$$\dot{\mu}_{20} = 2\mu_{11} \tag{9a}$$

$$\dot{\mu}_{11} = \mu_{02} - \beta_0\mu_{11} - \omega_e^2\mu_{20} + b_0\mu_{10} + b_1E[z_1 y_1] + b_2E[z_1 y_1^2] \tag{9b}$$

$$\dot{\mu}_{02} = -2\beta_0\mu_{02} - 2\omega_e^2\mu_{11} + 2b_0\mu_{01} + 2b_1E[z_2 y_1] + 2b_2E[z_2 y_1^2] \tag{9c}$$

where the cross-moments are $E[z_i\, y_k](i, k = 1, 2)$, and $E[z_i\, y_1^r\, y_2^s](i = 1, 2;\ r + s = 2)$. By applying Itô's differential rule to the functions $z_i y_k$ and $z_i y_1^r y_2^s$, two sets of equations for these moments are obtained. It can be shown that these sets of equations do not recall other moments apart from the moments of third and fourth order of the vector $\boldsymbol{y} = \{y_1\ y_2\}'$, but these moments are computable separately as the moments of a Gaussian vector. In this way, the computation of the response second order moments requires the solution of three sets of linear equations, and the knowledge of the moments of $\boldsymbol{y}$. This finding is general, and can be demonstrated true for every order of moment (Muscolino 1995).

The availability of a symbolic manipulator (MAPLE V 1991) allows to perform all the operations from the generation of the equations to their solution automatically. It is recalled that in the steady state the l.h.s. of Eqs. (8), (9) vanishes and the problem becomes algebraic.

According to Di Paola (1997), the following coordinate transformation is adopted: $v_1 = X$, $v_2 = Y^2$, $v_3 = 2Y$, $v_4 = \dot{X}$, $v_5 = \dot{Y}$, $v_6 = v_3 \cdot v_5$, $v_7 = \dot{Y}^2$. These positions and Eqs. (3b), (6) yield the differentials of the seven state variables as

$$dv_1 = v_4 dt, \quad dv_2 = v_6 dt, \quad dv_3 = 2v_5 dt \tag{10a-c}$$

$$dv_4 = (-\beta_0 v_4 - \omega_e^2 v_1 + b_0 + 1/2 b_1 v_3 + b_2 v_2)dt \tag{10d}$$

$$dv_5 = (-\beta_f v_5 - 1/2\,\omega_f^2 v_3)dt + \sqrt{\pi w_0}dB \tag{10e}$$

$$dv_6 = (2v_7 - \beta_f v_6 - 2\omega_f^2 v_2)dt + \sqrt{\pi w_0}v_3 dB \tag{10f}$$

$$dv_7 = (-2\beta_f v_7 - \omega_f^2 v_6 + \pi w_0)dt + 2\sqrt{\pi w_0}v_5 dB \tag{10g}$$

Now, the equations for the response moments are obtained by applying Itô's differential rule to the function $\phi = v_1^{p_1}v_2^{p_2}v_3^{p_3}v_4^{p_4}v_5^{p_5}v_6^{p_6}v_7^{p_7}$. Another computer program based on symbolic manipulations (MAPLE V 1991) has been set up for this second method. From a computational point of view, this last method can be programmed in a more straightforward way, but gives raise to a more important computational effort inasmuch as the entire set of the equations for the moments of a given order must be solved in a block. The results of the two methods are coincident, numerical imprecisions apart.

2.3. Actual nonlinear system

Now, let us examine the problem of computing the response moments of the actual nonlinear system. The equations ruling these moments are obtained by specializing Eq. (5). As in the case of the linearized system, the equations for the cross-moments among the system variables z_i and the filter variables y_i must be added. In all sets of equations there are hierarchical terms, say the equations for the r-order moments recall higher order moments. A suitable closure scheme is needed to close the infinite hierarchy.

The most popular closure method is the cumulant neglect closure method (Wu and Lin 1984, Ibrahim *et al*. 1985), by which the higher order moments are expressed in terms of the lower order moments by setting the corresponding cumulants equal to zero. To writers' knowledge this closure scheme has never been applied for closing the moment equations of a nonlinear system excited by a polynomial form of a filtered process, which would result cumbersome. In fact, to get the closure at a given order, cumulants of different order should be set to zero.

Therefore, another closure scheme has been adopted. This is due to Di Paola, Floris, and Sandrelli, and is yet unpublished. The method is based on an iterative procedure that takes advantage of the computations already performed on the linearized system. The procedure is organized through the following steps:

(1) the first order moments (statistical averages) are computed by giving the higher order (hierarchical) moments the values previously obtained on the linearized system. The second order moments are computed by using the first order moments so obtained and the higher order moments of the linearized system. The moments of the orders 2, 3, .. till the closure order n are computed in the same way.

(2) In the second iteration, the first order moments are calculated again, while giving the higher order moments the values obtained in step (1). Then the moments of order 2, 3, ... n are successively computed. It is outlined that the nth order moments of the nonlinear system depend on the lower order moments and on higher order moments that are always those obtained on the linearized system.

(3) Repeat step (2) till the convergence is achieved.

This method requires less computer time than the cumulant neglect closure method. In fact, the latter method introduces nonlinear relationships among the moments, while in the proposed approach all the equations are linear.

3. Applications

3.1. Structural and wind models

In the numerical applications a Duffing oscillator has been analyzed. In this case, it is set $m = 3$ in Eq. (1), which is recast as

$$\ddot{X}(t) + \beta_0\dot{X}(t) + \omega_0^2[X(t) + \varepsilon X^3(t)] = b_0 + b_1 Y(t) + b_2 Y^2(t) \tag{11}$$

The Duffing equation governs the motion of many lumped SDOF structural systems. In particular, the restoring force of cable-stayed structures can be reduced to a cubic polynomial; even accounting for second order effects for a Bernoulli-Navier beam vibrating in its first mode leads to a cubic restoring force. The structural model is in Fig. 1.

In the analyses the nonlinearity parameter ε is kept constant and equal to 0.5, which means a moderately strong nonlinearity. The constant of viscous damping is given as $\beta_0 = 2\zeta_0\omega_0$. The ratio of critical damping ζ_0 assumes the values 0.02 and 0.05, while the nominal pulsation ω_0 varies from π to 4π rad/s, say the nominal period $T_0 = 2\pi/\omega_0$ varies between 0.5 and 2.0 s. The other structural parameters in Eq. (2) (see also Fig. 1) are: $M = 2161$ kg, $A = 20$ m^2, $H = 20$ m, $C_D = 1$, $\rho = 1.25$ kg/m^2.

The mean wind speed obeys a logarithmic profile (Simiu 1973)

$$U(z) = 2.5u_*\ln\frac{z}{z_0} \tag{12}$$

Two cases are considered: in the former the shear velocity u_* is 1.77 m/s, while in the latter is 3.80 m/s; the roughness length z_0 is 0.018 m in both. The mean speed in the centroid of the area A (Fig. 1) is worth 31.04 and 40 m/s in the two cases, respectively.

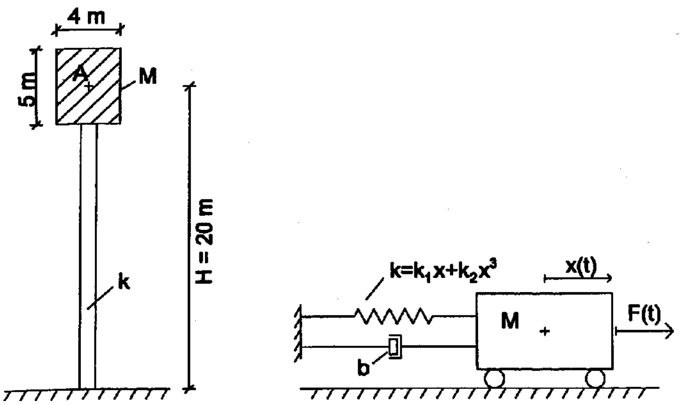

Fig. 1 Point-like structure (left), and its mechanical model (right)

Being the turbulence $u(t)$ a zero mean stationary Gaussian process, it is fully characterized in the frequency domain by its PSD $S_{uu}(\omega)$. From a theoretical point of view, not every spectral function might be assumed as turbulence PSD since there are some requirements that it must obey (see Simiu and Scanlan 1996, Chap. 2). Nevertheless, many usual relationships do not satisfy part of all these requirements: as an example, Davenport's PSD (1967) is zero in the origin, is not monotonically decreasing, and goes to zero as $\omega^{-4/3}$, while the turbulence PSD must be different from zero in $\omega = 0$, monotonically decreasing, and infinitesimal of order 5/3 for large frequencies.

On the other hand, in order to apply the stochastic calculus, and in general the Markov methods of stochastic dynamics, the primary excitation must be a Gaussian white noise. A colored excitation such as $u(t)$ can be obtained by means of a cascade of linear filters, which can be put in the form

$$\dot{\boldsymbol{Y}} = \boldsymbol{AY} + \boldsymbol{DW} \tag{13}$$

where $\boldsymbol{Y}$ is the vector of filter variables, $\boldsymbol{W}$ is a vector of unit strength white noise processes, which can reduce to a scalar, while $\boldsymbol{A}$ and $\boldsymbol{D}$ are deterministic matrices of constants that are to be determined in such a way to fit the theoretical or experimental colored PSD in the best way.

The simplest solution is the scalar counterpart of (13), that is

$$\dot{Y}(t) + aY(t) = dW(t) \tag{14}$$

which is known as Langevin's equation. The PSD of $Y(t)$ is

$$S_{YY}(\omega) = \frac{c}{a^2 + \omega^2} \quad (c = ad / \pi) \tag{15}$$

which in turbulence theory has been proposed by Dryden (1961). Eqs. (14) and (15) have been discarded since there are two parameters only for fitting a non rational curve, even if Floris *et al.* (2001) have shown that they yield quite acceptable results for linear structures. The other equations proposed in literature (Solari 1987, Simiu and Scanlan 1996, Mann 1998) are not rational functions, and require a second order filter at least to get a good or acceptable fitting, that is the vector $\boldsymbol{Y}$ in Eq. (13) must have dimensions (2,1). Davenport's PSD admits a very good rational approximation (Benfratello *et al.* 1996, 1999), but it has not been chosen because of the above mentioned theoretical drawbacks.

In this study the proposal by Kaimal *et al.* (1972) is adopted, which is given as

$$S_{uu}(\omega) = \frac{200 u_*^2 z}{2\pi U(z)\left[1 + \dfrac{25 z\omega}{\pi U(z)}\right]^{5/3}} \qquad (\omega \geq 0) \tag{16}$$

Eq. (16) yields a turbulence variance σ_u^2 which agrees with the relation $6u_*^2$. In applying the numerical simulation, wind histories are generated by using Eq. (16).

According to Eqs. (3), the rational approximation of Eq. (16) and the variance are, respectively

$$S_{YY}(\omega) = \frac{w_0}{(\omega_f^2 - \omega^2)^2 + 4\zeta_f^2\omega_f^2\omega^2} \tag{17}$$

$$\sigma_Y^2 = \frac{\pi w_0}{4\zeta_f\,\omega_f^3} \tag{18}$$

First, the filter parameters must hold the target variance $\sigma_u^2 \cong 6u_*^2$. Unfortunately, this is the only condition that one can impose to find the fitting PSD. To solve the problem, a nonlinear least square fitting has been applied, based on Levenberg-Marquardt algorithm (Marquardt 1963). In general and particularly in this case, the least square method has several admissible solutions: in this study, it has been renounced to find a general approximation of Eq. (16) determining some rational curves expressed by Eq. (17) that were reasonably close to Eq. (16) in an interval containing the structural frequency. More information is in the Appendix.

3.2. Results

The results for the first four response moments have been obtained by means of the following methods: (1) Monte Carlo simulation for the sake of comparison; (2) stochastic calculus of the equivalent linear system [Eqs. (6,8)]; (3) stochastic calculus for the actual nonlinear system (Eq. 11) by applying the proposed closure method with the two approaches that cannot be distinguished; (4) as moments of an approximate response PDF of X given as

$$\tilde{p}(x,\dot{x}) = C\exp\left[-\frac{2\beta_0}{\pi w_d}\left(-\mu x + \omega_0^2\left(\frac{x^2}{2} + \varepsilon\frac{x^4}{4}\right) + \frac{\dot{x}^2}{2}\right)\right] \tag{19}$$

where C is a normalization constant.

This PDF satisfies the Fokker-Planck-Kolmogorov equation associated with Eq. (11) having the approximate excitation $\tilde{Q}_d(t) = \mu + \sqrt{\pi w_d}\,W(t)$, where $W(t)$ is a unit strength Gaussian white noise, $\mu = \gamma(U^2 + \sigma_u^2)$, and $w_d = S_{uu}(\omega_e)$, that is the actual excitation is replaced by an equivalent white noise.

The values of the first four response moments obtained by means of the different approaches are listed in Tables 1, 2 for $U = 31.04$ m/s, $\zeta_0 = 0.02$, and for $U = 40$ m/s, $\zeta_0 = 0.05$, respectively. The moments of second and higher order are plotted against the nominal period of vibration $T_0 = 2\pi/\omega_0$ in Figs. 2-4 for the lowest U, and in Figs. 5-7 for the highest U. In the tables there are also the moments of the corresponding linear system, say with $\varepsilon = 0$ in Eq. (11). These moments are computed by using the actual turbulence PSD of Eq. (16): the first and second moments are exact, while the third and fourth ones are obtained by simulation to avoid lengthy multifold integrations. The moments of the corresponding linear system are always higher than those of the nonlinear one, and particularly for T_0 = 2.0 s. Therefore, the nonlinear behavior of a dynamical system must be properly considered.

A general examination of the results reveals that for T_0 = 0.5 s the structure is stiff with respect to wind excitation that has very little power in this range of frequency (see Fig. 9). Thus, the response is quasi-static, and the results of the various approaches are close together, being the effect of the nonlinearity of limited importance. As T_0 increases, and for the highest value of U, the results of the different methods scatter, and will be examined in detail. The simulation results are considered as exact, and are assumed as a basis of comparison: the percent error in the tables is computed as $100[(m_{theor}/m_{simul})-1]$, being m a generic moment.

As regards the case $U = 31.04$ m/s, $\zeta_0 = 0.02$, the approximate PDF (Eq. 19) is not completely adequate, but in some instances the errors are acceptable. The equivalent linearization and the higher

Table 1 Response moments for U = 31.04 m/s, ζ_0 = 0.02

Mom.	Simulation	SEL	Er.	HOC	Er.	PDF	Er.	LS
				T_0 = 0.50 s				
$m[1]$	0.358533E-1	0.359399E-1	0.2	0.359470E-1	0.3	0.367060E-1	2.4	0.35981E-1
$m[2]$	0.153056E-2	0.148050E-2	-3.3	0.148049E-2	-3.3	0.151222E-2	-1.2	0.15426E-2
$m[3]$	0.734065E-4	0.675694E-4	-8.0	0.675325E-4	-8.0	0.676084E-4	-7.9	0.74174E-4
$m[4]$	0.386204E-5	0.335809E-5	-13.1	0.335447E-5	-13.1	0.322925E-5	-16.4	0.39253E-5
				T_0 = 1.0 s				
$m[1]$	0.141207	0.140900	-0.2	0.141605	0.3	0.143372	1.5	0.143923
		0.141813*	0.4	0.141605*	0.3			
$m[2]$	0.248575E-1	0.241376E-1	-2.9	0.241918E-1	-2.7	0.244200E-1	-1.8	0.25989E-1
		0.244511E-1*	-1.6	0.241965E-1*	-2.7			
$m[3]$	0.494924E-2	0.466359E-2	-5.8	0.463534E-2	-6.3	0.460344E-2	-7.0	0.53715E-2
		0.475469E-2*	-3.9	0.463616E-2*	-6.3			
$m[4]$	0.108382E-2	0.992696E-3	-8.4	0.976498E-3	-9.9	0.940477E-3	-13.2	0.12208E-2
		0.101863E-2*	-6.0	0.978767E-3*	-9.7			
				T_0 = 1.50 s				
$m[1]$	0.298956	0.288129	-3.6	0.300518	0.5	0.294398	-1.5	0.323826
$m[2]$	0.113511	0.106614	-6.1	0.111883	-1.4	0.107336	-5.4	0.136738
$m[3]$	0.484286E-1	0.448157E-1	-7.5	0.464664E-1	-4.1	0.434620E-1	-10.3	0.663023E-1
$m[4]$	0.226076E-1	0.209357E-1	-7.4	0.211635E-1	-6.4	0.191710E-1	-15.2	0.353995E-1
				T_0 = 2.0 s				
$m[1]$	0.475767	0.471526	-0.9	0.493931	3.8	0.439875	-7.5	0.575690
$m[2]$	0.289940	0.269236	-7.1	0.279660	-3.6	0.248482	-14.3	0.445756
$m[3]$	0.196594	0.173238	-11.9	0.174652	-11.2	0.155059	-21.1	0.396466
$m[4]$	0.145125	0.122726	-15.4	0.152718	5.2	0.105743	-27.1	0.389948

KEYS : $m[i] = E[X^i]$ (i = 1, ..4); SEL = stochastic equivalent linearization; HOC = higher order closure; PDF = approximate PDF of Eq. (19); LS = linear system (same T_0 and $\varepsilon = 0$); Er. = percent error with respect to simulation. Values in $(meter)^i$. * Falsone-Elishakoff's linearization procedure.

order closure are substantially close together for the two lowest nominal periods, while for the other two the higher order closure matches the simulation better. Anyhow, the errors are quite acceptable. It is worth noting that Falsone-Elishakoff's linearization procedure estimates the moments slightly better, but, when used as a basis for the higher order closure, the moments of this are not improved or worsen .

When the average wind speed is increased to 40 m/s, even the turbulence is increased according to the relation $6u_*^2$. In this way, system nonlinearity is more excited. The approximate PDF is generally quite in error. In this case, the equivalent linearization shows a good performance for all the statistical averages, yielding quite acceptable errors for the other moments of the first two periods, and more important errors for the two larger periods. The errors of the equivalent linearization are on the unsafe side in some cases, while in others are overestimations. Even Falsone and Elishakoff's linearization procedure has been applied to T_0 = 1.0 and 2.0 s. For T_0 = 1.0 s, the moment estimates are better, but, if used as a basis for the higher order closure, this does not become better. For T_0 = 2.0 s, it improves the mean square value and the third moment considerably at the expense of a worsening in the estimates of the average and the fourth moment, being the latter greatly

Table 2 Response moments for U = 40.0 m/s, ζ_0 = 0.05

Mom.	Simulation	SEL	Er.	HOC	Er.	PDF	Er.	LS
				T_0 = 0.50 s				
$m[1]$	0.612208E-1	0.615073E-1	0.5	0.615837E-1	0.6	0.627357E-1	2.5	0.61798E-1
$m[2]$	0.500585E-2	0.474120-E-2	-5.3	0.474188E-2	-5.3	0.454882E-2	-9.1	0.50959E-2
$m[3]$	0.484345E-3	0.429638E-3	-11.3	0.428116E-3	-11.6	0.362046E-3	-25.3	0.49444E-3
$m[4]$	0.534634E-4	0.443628E-4	-17.0	0.441432E-4	-17.4	0.310412E-4	-41.9	0.55157E-4
				T_0 = 1.0 s				
$m[1]$	0.232414	0.228129	-1.8	0.235223	1.2	0.230563	-0.8	0.247191
		0.231915*	-0.2	0.235198*	1.2			
$m[2]$	0.739292E-1	0.678985E-1	-8.2	0.703989E-1	-4.8	0.656201E-1	-11.2	0.85882E-1
		0.698942E-1*	-5.5	0.707238E-1*	-4.3			
$m[3]$	0.272435E-1	0.239422E-1	-12.1	0.239355E-1	-12.1	0.207817E-1	-23.7	0.35145E-1
		0.249257E-1*	-8.5	0.236350E-1*	-13.3			
$m[4]$	0.113018E-1	0.966845E-2	-14.5	0.939342E-2	-16.9	0.717660E-2	-36.5	0.16563E-1
		0.101704E-1*	-10.0	0.920607E-2*	-18.5			
				T_0 = 1.50 s				
$m[1]$	0.456072	0.455963	-0.02	0.483006	5.9	0.412227	-9.6	0.556180
$m[2]$	0.281003	0.250264	-10.9	0.286460	1.9	0.220149	-21.7	0.451424
$m[3]$	0.195018	0.158735	-18.6	0.194911	-0.1	0.130037	-33.3	0.427590
$m[4]$	0.149796	0.113516	-24.2	0.155059	3.5	0.0841323	-43.8	0.468460
				T_0 = 2.0 s				
$m[1]$	0.678216	0.667447	-1.6	0.679483	0.2	0.544487	-19.7	0.988765
		0.706763*	4.2	0.693878*	2.3			
$m[2]$	0.614914	0.535106	-13.0	0.664775	8.1	0.318648	-48.2	1.469127
		0.600004*	-2.4	0.705542*	14.7			
$m[3]$	0.616024	0.495154	-19.6	0.618868	0.5	0.322786	-47.6	2.534465
		0.587911*	-4.6	0.559083*	-9.2			
$m[4]$	0.375928	0.516171	37.3	0.331109	-11.9	0.286323	-23.8	5.041769
		0.648968*	72.6	0.325256*	-13.5			

KEYS: as in Table 1. *Falsone-Elishakoff's linearization procedure

overestimated. On the average, the higher order closure proposed in this paper matches the simulation results quite acceptably, even if the errors are over a 10% for some moments.

The principal statistical functions of the response, such as joint and marginal PDF, mean upcrossing rate functions, and so on, can be constructed starting from the response moments. Herein, two PDF $p_X(x)$ of the displacement X are shown in Fig. 8 to demonstrate the marked non-Gaussianity of the response. They are obtained by inversion of the characteristic function (Roberts and Spanos 1990, Chap. 3):

$$\Psi(\omega) = \exp\left[\sum_{j=1}^{+\infty} (i^j / j!)\, \kappa_{Xj} \omega^j\right] \qquad (i = \sqrt{-1}\,), \tag{20}$$

where κ_{Xj} is the cumulant of order j of X (it is recalled that the cumulants are directly expressed in terms of moments). The series expansion in previous expression is truncated at the fourth or sixth cumulant.

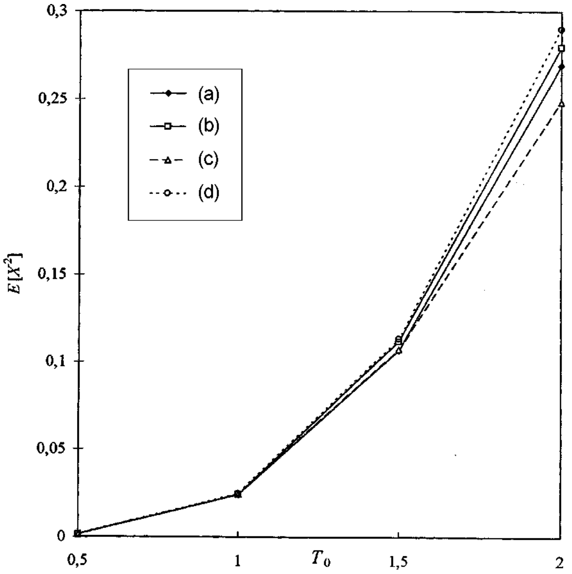

Fig. 2 Plot of response mean square value $E[X^2]$ against $T_0 = 2\pi / \omega_0$ for $U = 31.04$ m/s, $\zeta_0 = 0.02$: (a) stochastic equivalent linearization; (b) higher order closure; (c) approximate PDF [Eq. (19)]; (d) simulation

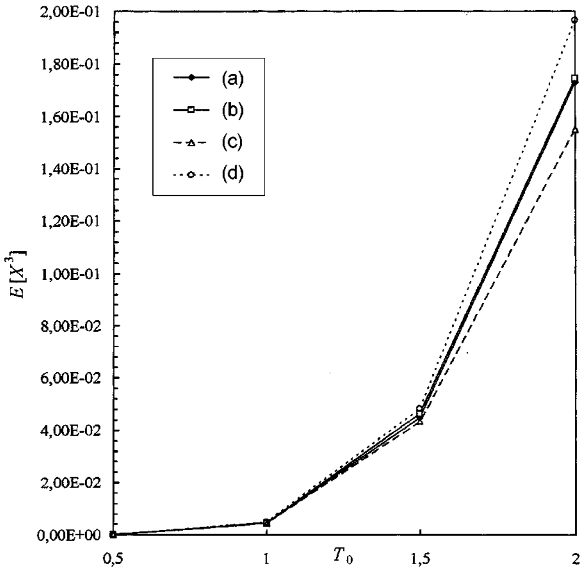

Fig. 3 Plot of response third moment $E[X^3]$ against $T_0 = 2\pi / \omega_0$ for $U = 31.04$ m/s, $\zeta_0 = 0.02$: keys as in Fig. 2

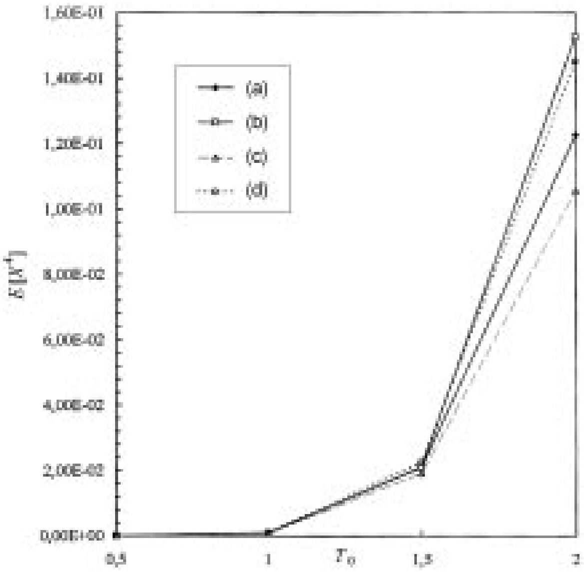

Fig. 4 Plot of response fourth moment $E[X^4]$ against $T_0 = 2\pi / \omega_0$ for $U = 31.04$ m/s, $\zeta_0 = 0.06$: keys as in Fig. 2

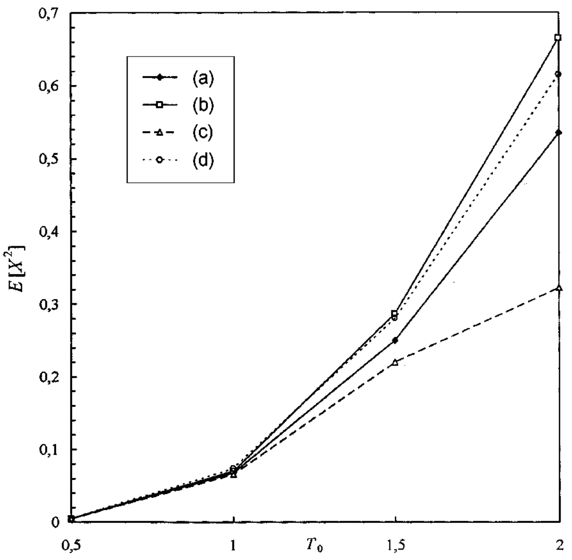

Fig. 5 Plot of response mean square value $E[X^2]$ against $T_0 = 2\pi / \omega_0$ for $U = 40.0$ m/s, $\zeta_0 = 0.05$: (a) stochastic equivalent linearization; (b) higher order closure; (c) approximate PDF [Eq. (19)]; (d) simulation

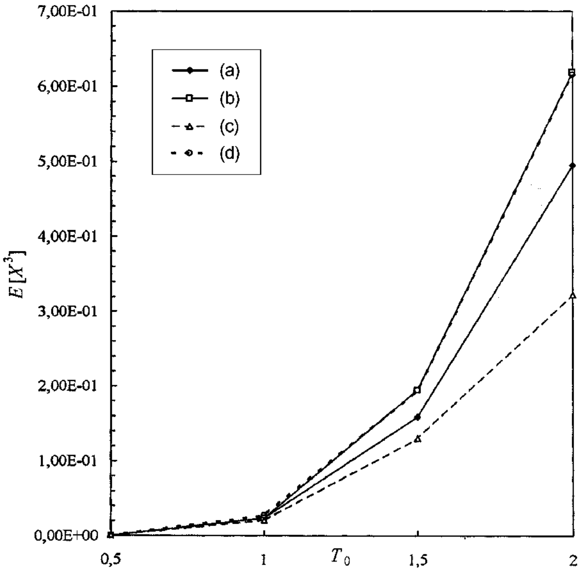

Fig. 6 Plot of response third moment $E[X^3]$ against $T_0 = 2\pi / \omega_0$ for $U = 40.0$ m/s, $\zeta_0 = 0.05$: keys as in Fig. 5

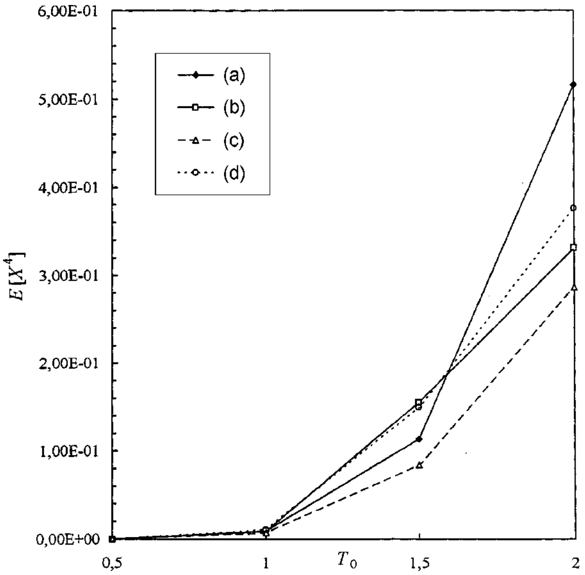

Fig. 7 Plot of response fourth moment $E[X^4]$ against $T_0 = 2\pi / \omega_0$ for $U = 40.0$ m/s, $\zeta_0 = 0.05$: keys as in Fig. 5

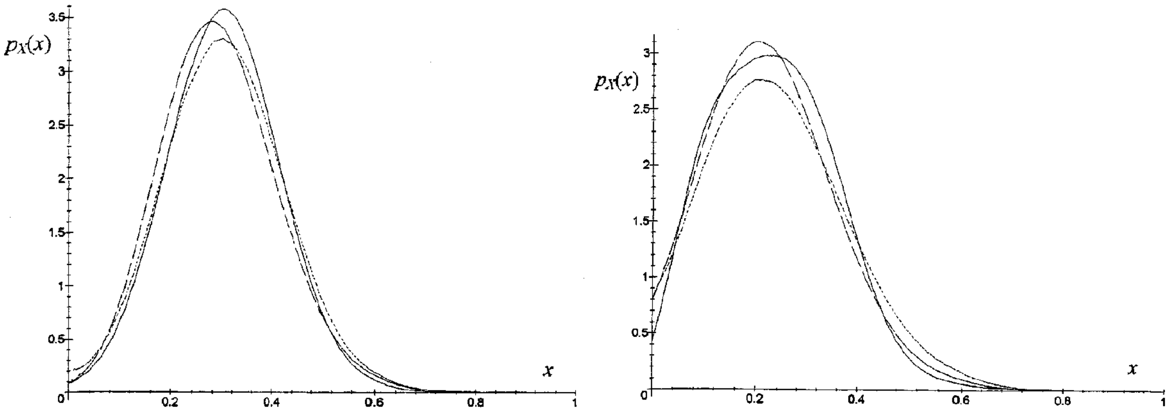

Fig. 8 Plot of the response PDF $p_X(x)$ for $T_0 = 1.5$ s, $\zeta_0 = 0.02$, $U = 31.04$ m/s (top); and for $T_0 = 1.0$ s, $\zeta_0 = 0.05$, $U = 40.0$ m/s (bottom). — stochastic equivalent linearization; higher order closure; little dots simulation

The agreement of the PDF of both the linearization and the higher order closure with the simulation is acceptable. The important feature of these plots is the marked non-Gaussianity of the response, which is testified by the asymmetric aspect with a long tail towards the large values. This is important in reliability estimates.

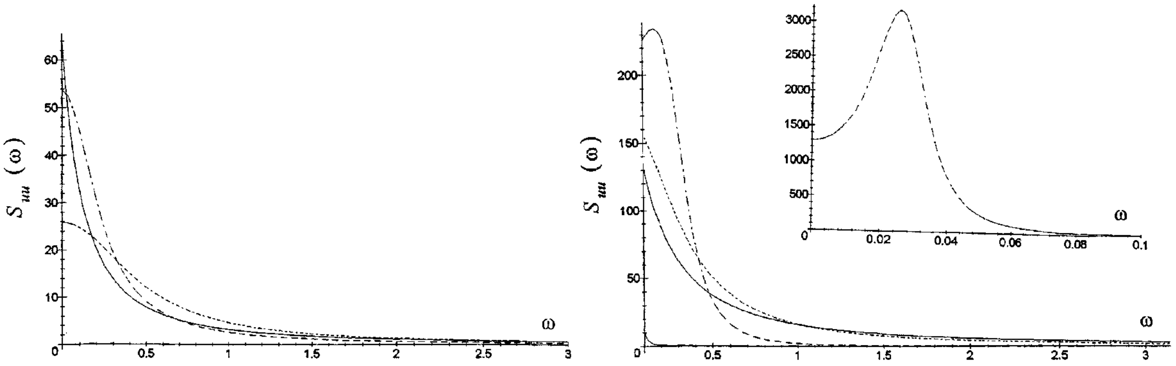

Fig. 9 Comparison among Kaimal's turbulence PSD (continuous line) and approximating PSD. Top: U = 31.04 m/s, dotted line = filter LUNO-96, dashed-dotted line = filter LUDI-96. Bottom: U = 40.0 m/s, dotted line = filter QU1-97, dashed-dotted line = filter QU2-97, in the detail filter QU1-98

4. Conclusions

The problem of the response of nonlinear systems to quadratic random wind pressure is addressed herein. Attention is focused on single-degree-of-freedom oscillators with linear damping and nonlinear restoring force with polynomial form so that the problem is doubly nonlinear, which causes the response to be markedly non Gaussian.

The statistical characterization of a nonlinear dynamic system excited by a non-Gaussian agency is obtainable at the expense of a notable computational effort. In fact the only methods that presently are applicable are the Monte Carlo simulation and the Volterra series expansion. The simulation requires the generation of several thousands of motion histories, in each of which the equations governing the problem are numerically evaluated. The Volterra series approach operates under some restrictions: the damping must be linear, and the restoring force a polynomial. In practice, this cannot have a degre larger than three in order to truncate the series at the second term. With two terms in the series, a moment of n-th order is obtained by evaluating an integral with n dimensions.

The principal motivation of the present paper is presenting a method of analysis more appealing from a computational point of view. Analogously to the determination of the non-Gaussian response of a linear system subjected to wind pressure, the problem is framed in the field of Markov methods of stochastic dynamics. These are applicable only if the excitation is a Gaussian white noise, which is not the case of wind turbulence. The simplest approximation replaces the turbulence with a white noise. In this way, the Fokker-Planck equation in the JPDF of the problem variables has an analytical solution, but the computed response moments are not quite in accord with those obtained by simulation for all vibration periods and mean wind speeds.

Thus, resort is made to the moment equation approach of Itô's stochastic differential calculus. This requires that the turbulence PSD is idealized as the output of one or more linear filters excited by a Gaussian white noise. Including the filter variables in the analysis, Itô's differential rule allows writing the differential equations ruling the response moments. In the steady state, these equations become algebraic.

As a first approximation, the system is linearized retaining the nonlinear excitation with the square of the turbulence. In this way, we have a linear system excited by a quadratic polynomial of a filtered Gaussian process. The response moments are computed exactly, and the non Gaussian features are preserved. This approach yields estimates of the response moments that are acceptably close to those obtained by simulation in most cases, but in few others they are affected by more significant errors.

Thus, the nonlinearities of both the oscillator and the excitation are hold, and a higher order closure is considered. In this way, the moment equations form an infinite hierarchy. In order to close this, an iterative procedure is proposed, which takes advantage of the moment estimates obtained for the linearized system.

The numerical applications, which regard a hardening Duffing oscillator, prove the validity of the proposed approach notwithstanding the rational representation of the turbulence PSD has not been optimized. The results compare well with those of the numerical simulation.

Finally, it is recalled that the present approach can be easily adapted to other problems in which the dynamic system has polynomial nonlinearities in both damping and restoring force, while the excitation is a polynomial of a filtered Gaussian process. Nonlinear systems analyzed by perturbation or pseudo-force method can be reduced to this case, while wave forces can be expressed by means of a polynomial.

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Appendix : Approximation of wind turbulence spectrum with a rational function

In order to approximate Eqs. (16) by means of Eq. (17), the filter parameters w_0, ζ_f and ω_f must be properly selected. Lacking a theoretical basis to do that, one must resort to a least square fit. The problem can be stated as

$$\sum_{1}^{N}{}_{i}\,[S_{uu}^{(T)}(\omega_i) - S_{uu}^{(A)}(\omega_i)\,]^2 = \text{minimum} \tag{A.1}$$

where $S_{uu}^{(T)}(\omega_i)$ is the target PSD, that is Eq. (16), while $S_{uu}^{(A)}(\omega_i)$ is the approximating one, and N denotes the number of points that are chosen for the fit. The minimization of (A.1) is constrained by the condition

$$\int_0^{+\infty} S_{uu}^{(A)}(\omega)d\omega \;=\; \int_0^{+\infty} \frac{w_0}{(\omega_f^2-\omega^2)^2 + 4\zeta_f^2\omega_f^2\omega^2}d\omega \;=\; 6u_*^2 \tag{A.2}$$

Levenberg-Marquardt's algorithm (Marquardt 1963) is used in this study.

The parameters of the filters that have been used in the analyses are listed in Table 3. The comparisons among Kaimal's and approximating PSD are shown in Fig. 9 for both wind speeds. As regards $U = 31.04$ m / s, the agreement is good except for $\omega < 1$ rad / s. For $U = 40.0$ m / s, the filters QU1-97 and QU2-97 show the

Table 3 Parameters of the fitting PSD

Spectrum	U (m/s)	w_0 ($m^2 \cdot s^{-5}$)	ω_f (rad / s)	ζ_f
LUNO-96	31.04	0.204688E+06	9.4355	10.181
LUDI-96	31.04	0.209036E+05	4.4428	9.9596
QU1-97	40.0	0.1982174E+08	18.4150	28.6264
QU2-97	40.0	1.93	0.307936	0.596094
QU1-98	40.0	0.935956E-03	0.292115E-01	0.338634

same trend as for $U = 31.04$, while the filter QU1-98 appears to have a notable mismatch. However, this disappears for $\omega > \pi$. In fact, it has been used only for $T_0 = 2.0$ s ($\omega_0 = \pi$ rad / s), giving the best results. It is emphasized that the fitting has been searched in the vicinity of nominal structural frequency ω_0 only. This is why more fitting PSD have been used.

From a theoretical point of view, the turbulence could be approximated by means of a cascade of linear filters, that is with $m > 2$ in Eq. (13). This way has been tested with $m = 4$, that is two second order filters, but the results have been disappointing. This is probably due to the fact that the approximating PSD is infinitesimal of order 2 m as $\omega \to +\infty$, while Eq. (16) tends to zero as $\omega^{-5/3}$. Presently, the method of the analytical continuation as proposed by Roy and Spanos (1993) is under study.

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