Wind and Structures, Vol. 5, No. 1 (2002) 15-24
DOI: http://dx.doi.org/10.12989/was.2002.5.1.015

A numerical study of the turbulent fluctuating flow around a square cylinder for different inlet shear

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Abstract. This paper reports the numerical calculations of uniform turbulent shear flow around a square cylinder. The predictions are obtained by solving the two-dimensional unsteady Navier-Stokes equations in a finite volume technique. The turbulent fluctuations are simulated by the standard k-ε model and one of its variant which takes care of the realizability constraint in order to suppress the excessive generation of turbulence in a stagnation condition. It has been found that the Strouhal number and the mean drag coefficient are almost unaffected by the shear parameter but the mean lift coefficient is increased. The present predictions are compared with available experimental data.

Key words: vortex shedding; shear flow; k-ε variant; square cylinder.

1. Introduction

Flow past a bluff body constitutes a classical problem where flow separates over the surface of the body evolving periodic shear layer. This periodic shear layer generates vortices causing a fluctuating surface pressure and vortex shedding and these issues are of great practical importance in the field of structural design (Saha *et al*. 1999, Murakami and Mochida 1995); flame stabilisation in combustors (Bailly, Champion and Garreton 1995) etc. A number of experimental and numerical studies are found in the literature which deal with the cases of uniform (plug) flow over a circular and rectangular/square cylinder (Rodi 1997). However, very little information is available for shear flow around a square cylinder. Kiya *et al.* (1980) and Kwon *et al*. (1992) presented experimental results for uniform shear flow around a circular cylinder at moderate Reynolds number ($Re = U_0H / \nu = 35$–1600) when the shear parameter was varied from 0 to 0.25. The shear parameter, S is defined as $S = \lambda H / U_0$ where $\lambda = dU / dy$ is the velocity gradient, H is the cylinder height, U_0 is the mean streamwise velocity at the level of cylinder centreline (see Fig. 1) and ν is the kinematic viscosity of the fluid. These studies demonstrated that the vortex shedding frequency, f is increased and the drag coefficient is slightly decreased as S is increased. The subsequent numerical study by Hwang and Sue (1997) showed that this kind of flow is dependent on the Reynolds number and presented

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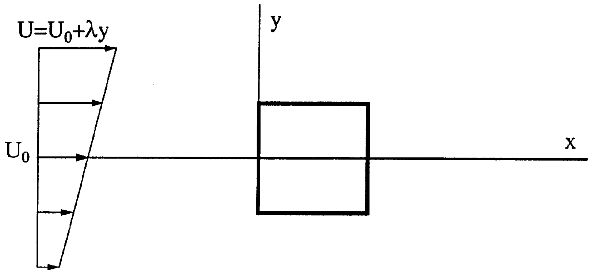

Fig. 1 Definition and sketch of uniform shear flow approaching a square cylinder

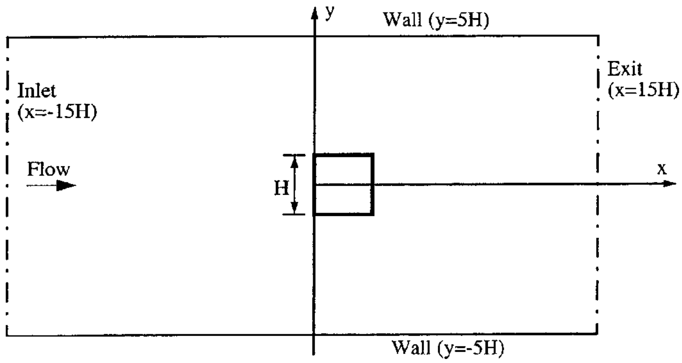

Fig. 2 Computational domain and coordinates

results of up to $Re = 1500$. Ayukawa *et al.* (1985) considered a rectangular cylinder in a uniform shear flow at a high $Re = 20{,}000$ at four different shear parameters of up to $S = 0.064$. The results demonstrate that unlike flows in moderate *Re*, the Strouhal number, St $(= fH / U_0)$ and drag coefficient are almost unaffected by shear parameter (in fact the results of Hwang and Sue (1997) show an exponential decay of the rate of increase of drag coefficient with *Re*), but the lift coefficient is found to increase. Later Ayukawa *et al.* (1993) presented a numerical study using the discrete vortex method for a square cylinder at $Re = 4000$ for a wide range of shear parameters ($0<S<0.15$). This study revealed that the vortex shedding frequency, and drag coefficient are unaffected by the shear parameter, whereas the lift coefficient is changed in a rather peculiar manner. The lift coefficient became maximum at $S = 0.05$ and decreased to some negative values at higher values of *S*. But this study did not provide detail explanation for this behaviour.

The present study considers the experimental test case of Ayukawa *et al.* (1985) to study the flow phenomena at different shear parameters of up to $S = 0.15$. The experiment was conducted in a wind tunnel with a blockage ratio of 10% and the aspect ratio of the cylinder length/height was 10. The inlet boundary was 17.5 H upstream of the cylinder where a shear generator was placed. The turbulence intensity after the shear generator was $T_u = 2.5\%$. Only the time mean integral parameters are reported in the paper of Ayukawa *et al.* (1985) for four different shear parameters of up to $S = 0.064$. The equivalent numerical set up (considered in the present work) of the above experiment is shown schematically in Figs. 1 and 2 and explained in more detail in section 3.

2. Computational details

2.1. Governing equations

The equations solved are the two-dimensional, ensemble-averaged continuity and momentum equations for constant property flow written in Cartesian tensor notation, as :

$$\frac{\partial U_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial U_i}{\partial t} + \frac{\partial}{\partial x_j} U_i U_j = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\upsilon \frac{\partial U_i}{\partial x_j} - \langle u_i u_j \rangle\right] \tag{2}$$

where the correlations $-\langle u_i u_j \rangle$ between the fluctuating velocities represent the turbulent Reynolds stresses and need to be modelled to close the equations. The two-equation k-ε model is still the most popular in industrially relevant applications and is used in the present study. In the k-ε model the unknown Reynolds stresses are obtained from the conventional Boussinesq linear stress-strain relationship viz. :

$$-\langle u_i u_j \rangle = \nu_t \left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right] - \frac{2}{3} k \delta_{ij} \tag{3}$$

The eddy viscosity, ν_t is related to the kinetic energy, k and its dissipation rate, ε as :

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \tag{4}$$

The modelled transport equations for k and ε are :

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j} U_j k = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G - \varepsilon \tag{5}$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial x_j} U_j \varepsilon = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + \frac{C_{\varepsilon 1} G - C_{\varepsilon 2}\varepsilon}{\tau} \tag{6}$$

where,

$$G = -\langle u_i u_j \rangle \left(\frac{\partial U_i}{\partial x_j}\right) \tag{7}$$

is the production of turbulence kinetic energy and $\tau = k/\varepsilon$ is the turbulent time scale. The empirical coefficients appearing in the above equations are assigned their standard high Reynolds number values, viz., $C_\mu = 0.09$; $C_{\varepsilon 1} = 1.44$; $C_{\varepsilon 2} = 1.9$; $\sigma_k = 1.0$; $\sigma_\varepsilon = 1.3$.

One of the inherent shortcomings of the standard k-ε model is the generation of very high turbulence intensity in a stagnating condition which originates from the fact that generation term featuring in the k and ε equation is calculated via the mean velocity gradients only. A modification which addresses this stagnation flow anomaly is due to Durbin (1996) who suggested to impose the 'realizability' constraint $2k \geq \langle u_i u_i \rangle \geq 0$ in the calculation of G via a bound on the time scale, τ.

Durbin proposed the following bound on the time scale :

$$\tau = \min\left[\frac{k}{\varepsilon}, \frac{2\alpha}{3C_\mu\sqrt{2|S|^2}}\right] \tag{8}$$

where, $|S|^2 = S_{ij}S_{ji}$ and S_{ij} is the rate of strain tensor given by :

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{9}$$

The other coefficient, α is a model parameter which should have a value $\alpha \leq 1.0$ when the eddy viscosity is defined as $\nu_t = C_\mu k\tau$ (Hereafter this variant will be referred to as the D model). This model has been successfully applied to the prediction of steady jet impingement on a plane surface as shown by Behnia *et al*. (1996) and unsteady flow past a square cylinder in a uniform flow field by Islam (1997). Both of these studies found that $\alpha = 0.5$ is an optimum value. In the current calculation, this value of $\alpha = 0.5$ has been used for all the calculations.

2.2. The flow solver

A computer code (Little and Manners 1993), developed for 2D non-orthogonal body fitted co-ordinate system employing co-located cells and primitive solution variables, is used in the present study. A finite volume method was used to discretise the differential equations. Second order central differencing was used for all terms except the convection terms which used the central/upwind Hybrid scheme. The discretised equations were solved using the SIMPLE pressure correction method. For time discretisation, a fully implicit first order Euler scheme was used. This provides high stability but requires small time-steps to obtain accurate solutions. The resulting difference equations were solved iteratively by a tri-diagonal-matrix algorithm.

3. Computational domain and boundary conditions

Calculations were conducted for a square cylinder immersed in a uniform shear flow (Fig. 1) at Re = 20,000 while the shear parameter was varied up to S = 0.15. The shear parameter was changed by changing the velocity gradient λ at the inlet. The inlet boundary of the flow domain was set at 15 H upstream of the cylinder (see Fig. 2) where the profile of the inlet streamwise velocities, U was specified for a particular shear parameter. A uniform turbulent kinetic energy, k_0 was prescribed at the inlet which was equal to $10^{-4}T_u^2U_0^2$. Here T_u is the measured percentage of free stream turbulence intensity (2.5%) after the shear generator. The ε value was set at the inlet from the equilibrium condition for homogeneous shear flow ($\varepsilon_0 = k_0\lambda / 4.82$) as mentioned by Speziale (1991).

The top and bottom boundaries of the computational domain were placed at a distance of 5 H from the cylinder centre and were treated as no-slip boundaries. The exit boundary was placed 15 H downstream of the cylinder where a zero gradient condition was imposed for all the variables. The cylinder surfaces were treated by standard wall functions. A total of 107×88 grid nodes (in the axial and transverse directions) were used, with the grid lines concentrated near the four cylinder walls. Distance of the first point from the wall was 0.0125 H. A non-dimensional time step, $\Delta\tau(= U_0\Delta t / H)$ of 0.005 was used, requiring about 1500 time steps for a complete vortex shedding cycle. These grid arrangements and time-step were selected on the basis of an elaborate grid density and time-step testing of the uniform flow over a square cylinder as documented in Islam (1997). It may be worthwhile to note that no attempt was made in using low-*Re* models due to the fact that this particular flow is likely to be little dominated by wall boundary layers (Hasan and McGuirk, 2001) and that these models are still unpopular among the industrial users.

4. Results and discussion

The time histories of the lift coefficient C_L at different shear parameters predicted by the k-ε model and the Durbin model (or D model) are presented in Figs. 3 and 4 respectively. The lift coefficients are normalised by the dynamic pressure $0.5\rho U_0^2 H$. The Strouhal number (shown next to each C_L curve) is evaluated by analysing the time histories of the lift coefficient using an FFT package. It can be seen that steady state solutions have been reached for both the models and the lift coefficient shows a sinusoidal variation. Although the Strouhal numbers show very little sensitivity to the shear parameter, the amplitudes of the lift coefficient are found to vary

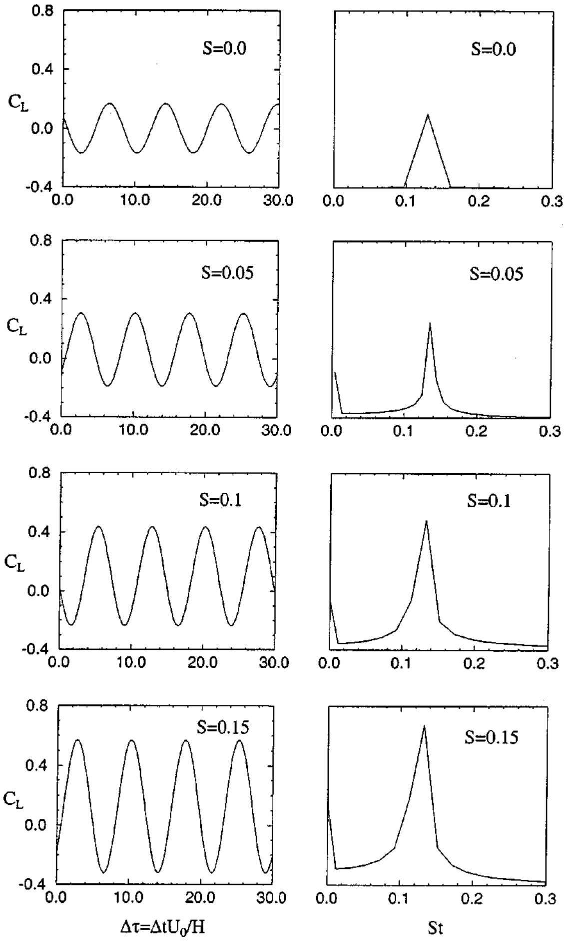

Fig. 3 Time histories of the lift coefficient at different shear parameter k-ε model predicton

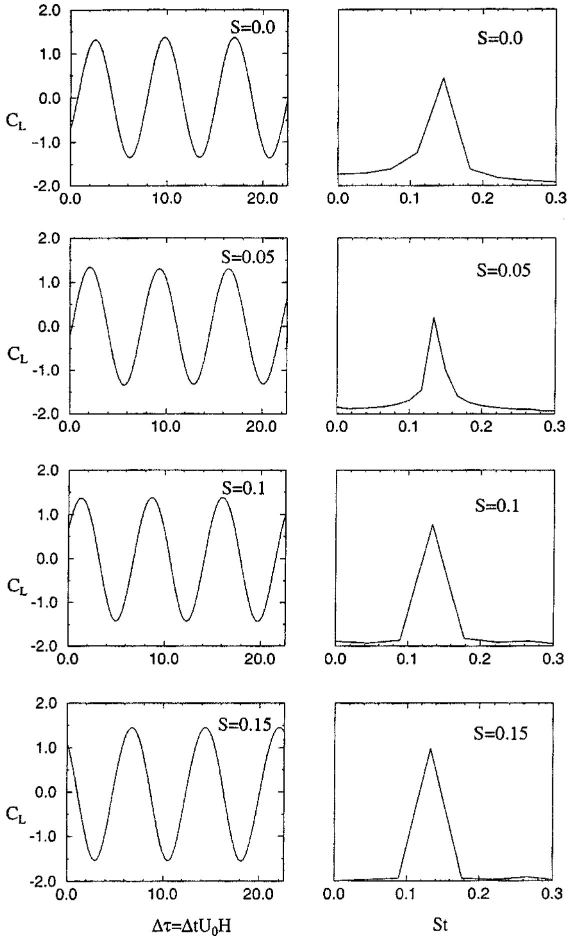

Fig. 4 Time histories of the lift coefficient at different shear parameter Durbin's model predicton

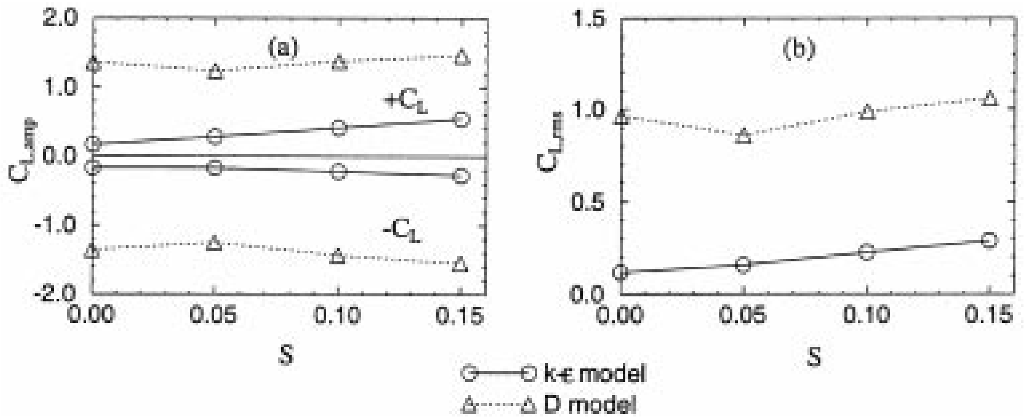

Fig. 5 Effect of shear on (a) the amplitudes and (b) the rms value of lift coefficient

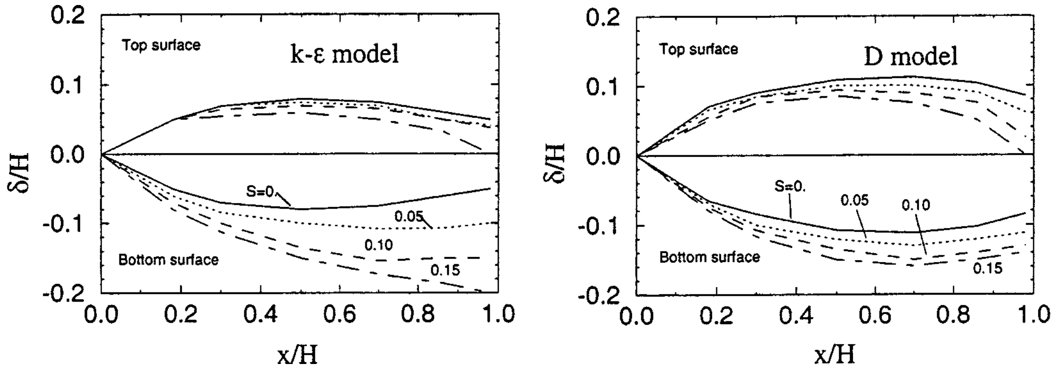

Fig. 6 Shear layer thickness on the top and bottom surfaces of cylinder at different shear parameter

significantly due to model variation. Figs. 5a-b show the variations of amplitudes and rms values of the lift coefficient for different shear parameters. For both the models, the rms values and amplitudes are found to increase with shear parameter. A closer look further reveals that the D model predicts almost symmetric amplitudes and thus results in almost zero lift coefficient. On the other hand, for the k-ε model the positive amplitude of C_L increases more than that of the negative amplitude and thus results in an overall increase in the mean lift coefficient with shear as will be shown in Fig. 7 during data comparison.

Fig. 6 shows the variation of the thickness of the shear layer on the top and bottom surfaces of the cylinder for different shear parameters. This thickness was calculated as the normal distance from the cylinder surface where the U velocity is zero. The top surface shear layer becomes thinner with shear and at $S = 0.15$ it reattaches on the surface near the downstream corner. The bottom surface shear layer shows opposite trend and becomes thicker with shear. The shear layer thickness predicted by the D model is bigger than the k-ε model for $S = 0$. The high viscosity produced by the k-ε model makes the shear layer thinner than the D model. It may be interesting to note that the rate of variation of the shear layer thickness is comparatively more pronounced for the k-ε model than the D model. Since the magnitude of the turbulent viscosity for the k-ε model is 2-3 times higher

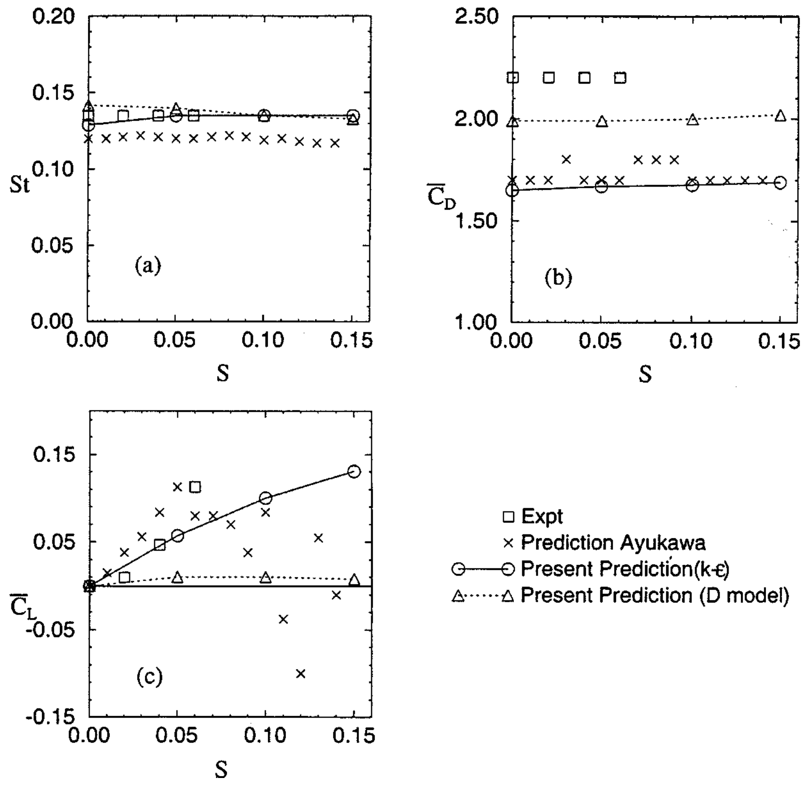

Fig. 7 Effect of shear on (a) Strouhal number (b) Drag coefficient and (c) Lift coefficient

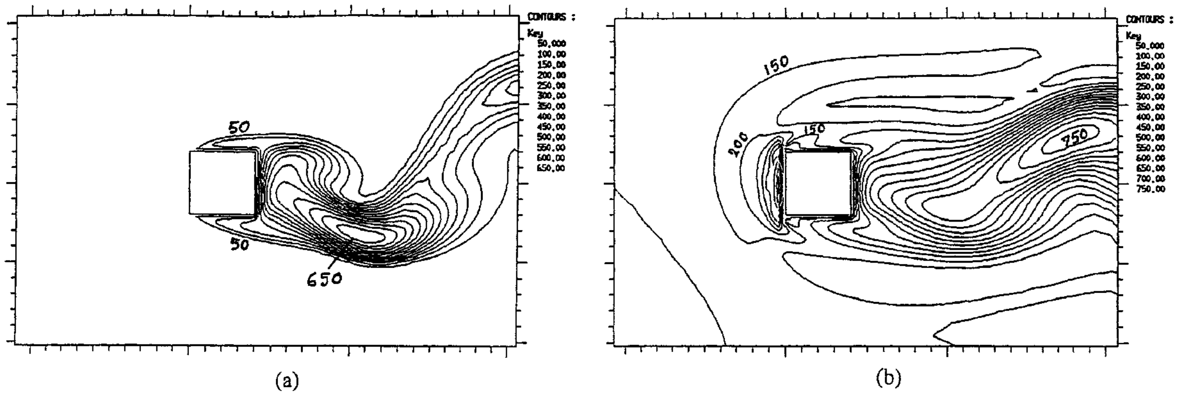

Fig. 8 Contours of normalised viscosity (ν_t/ν) at zero phase angle for $S = 0.05$. Predictions by k-ε model (a) $Tu = 0.1\%$ (b) $Tu = 2.5\%$

than that of the *D* model (see Figs. 8b and 9), the relative change of the magnitude is also higher; resulting in the comparatively larger (sparse) variation in the mixing characteristics around the cylinder for this case.

Figs. 7a-c compare the predicted results of Strouhal number and mean drag and lift coefficients

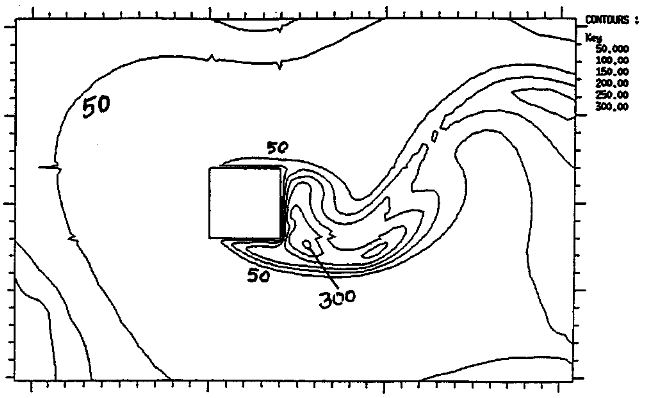

Fig. 9 Contours of normalised viscosity (ν_t / ν) at zero degree phase angle for $S = 0.05$. Predictions by *D* model ($Tu = 2.5\%$)

for various shear parameters with the measured data of Ayukawa *et al*. (1985). Similar to experimental observations, the predicted Strouhal number and the mean drag coefficient show little sensitivity to the shear parameter. Although both of the standard k-ε model and the D-model are found to under predict the mean drag coefficient, the latter shows significantly better agreement with experiment.

As indicated before, the mean lift coefficient predicted by the k-ε model increases with shear (Fig. 7c). On the other hand, the D model results in almost zero lift and the reason for this deficiency will be explained later. The present prediction by the k-ε model shows a gradual increase of mean lift coefficient with shear rate. In contrast, the prediction of Ayukawa *et al.* (1993) shows that it increases up to $S = 0.05$ and then it oscillates (even showing some negative values). To explain this Ayukawa *et al*. (1993) mentioned that it was impossible to get enough data to average due to the limitation of the computer capacity.

In order to investigate the effect of inlet turbulence intensity, calculations were done with different turbulence intensities from 0.1% to 3% for $S = 0.05$ with the standard k-ε model. The calculated integral parameters for these turbulence intensities are given in Table 1. A marked influence is seen in the calculated integral parameters at low turbulence intensity. At the lowest turbulence intensity, the Strouhal number and the mean drag coefficient are increased by 5% and 9% respectively from the values at $T_u = 2.5\%$ whereas the mean lift coefficient is reduced to 0.019 from 0.057, a three-

Table 1 Effect of inlet turbulence intensity on integral parameters for $S = 0.05$; k-ε model predictions

T_u (%)	St	$\overline{C}_D$	$\overline{C}_L$	$C_{L,amp}$
0.1	0.138	1.81	0.019	+0.751 -0.720
0.5	0.137	1.78	0.029	+0.635 -0.582
2.0	0.133	1.67	0.056	+0.304 -0.189
2.5	0.132	1.66	0.057	+0.283 -0.167
3.0	0.130	1.66	0.057	+0.205 -0.097

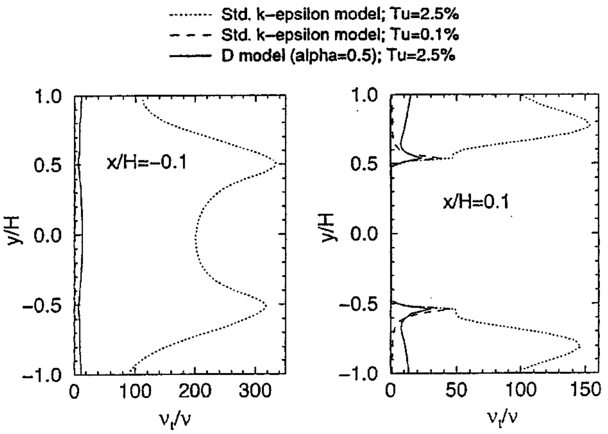

Fig. 10 Time mean viscosity profiles at two axial positions for $S = 0.05$

fold decrease. This marked reduction of the mean lift coefficient is probably due to the generation of low turbulence intensity around the cylinder which is shown in Fig. 8 at phase angle 0^0 (i.e., it refers to the time when the amplitude of lift coefficient becomes positive maximum). From this observation, it can be inferred that a positive mean lift coefficient may be experienced by a square cylinder in a shear flow if there exists sufficient viscosity around the cylinder.

The viscosity field (at phase angle 0^0) predicted by the *D* model for $S = 0.05$ is shown in Fig. 9. The peak viscosity is about 300 and it occurs near the downstream region of the cylinder. The viscosity around the top and bottom surfaces is also very low. Fig. 10 shows the viscosity profiles at two axial positions just-before and just-after the leading surface of the cylinder predicted by the *D* model with $T_u = 2.5\%$ and the k-ε model with $T_u = 0.1\%$ and 2.5%. The very low levels of viscosity predicted by the *D* model may be the reason for very low mean lift coefficient in the shear flow as it is indicated by the k-ε model prediction with $T_u = 0.1\%$. It is possible that the time scale restriction imposed by this variant of the k-ε model has reduced the turbulence to a ‘too low’ level. However, the actual dynamics involved in the process i.e., how the lift coefficient (via the pressure force) is correlated to the turbulence field is a matter for further detailed study and is beyond the scope of the current exercise.

5. Conclusions

Shear flow around a square cylinder is predicted by the standard k-ε model and a variant of this attributable to Durbin (1996). Both the models predict that the Strouhal number and mean drag coefficient remain unaffected by the shear and is consistent with the measured data. The mean lift coefficient increases with shear, which is predicted by the standard k-ε model but not by the *D* model. The ‘excessive’ suppression of k values upstream of the cylinder by the *D* model may be the reason for this failure. The thickness of the shear layer at the top and bottom surfaces of the cylinder are affected by the shear rate. The top surface shear layer becomes thinner and the bottom surface shear layer becomes thicker with shear. At $S = 0.15$ the shear layer reattaches on the downstream edge of the top surface of the cylinder.

Acknowledgements

The authors would like to thank Prof. J. J. McGuirk for his helpful discussion during the course of the work. The fellowship granted by the Association of Commonwealth Universities to the first author (AKMSI) is gratefully acknowledged.

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