Wind and Structures, *Vol. 32, No. 1 (2021) 11-17*
DOI: https://doi.org/10.12989/was.2021.32.1.11

Application of differential transformation method for free vibration analysis of wind turbine

Kanat Burak Bozdoğan*[1] and Farshid Khosravi Maleki[2]

[1]*Çanakkale Onsekiz Mart University, Engineering Faculty, Civil Engineering Department, Çanakkale, Turkey*
[2]*Bartin University, Faculty of Engineering, Architecture and Design, Mechanical Engineering Department, Bartin, Turkey*

(*Received March 3, 2020, Revised May 9, 2020, Accepted December 22, 2020*)

Abstract. In recent years, there has been a tendency towards renewable energy sources considering the damages caused by non-renewable energy resources to nature and humans. One of the renewable energy sources is wind and energy is obtained with the help of wind turbines. To determine the behavior of wind turbines under earthquake loads, dynamic characteristics are required. In this study, the differential transformation method is proposed to determine the free vibration analysis of wind turbines with a variable cross-section. The wind turbine is modeled as an equivalent variable continuous flexural beam and blade weight is considered as a point mass at the top of the structures. The differential equation representing the free vibration of the wind turbine is transformed into an algebraic equation with the help of differential transformation method and the angular frequencies and the mode shapes of the wind turbine are obtained by the help of the differential transformation method. In the study, a sample taken from the literature was solved with the presented method and the suitability of the method was investigated. The same wind turbine example also modeled by finite element modelling software, ABAQUS. Results of the finite element model and differential transformation method are compared with each other and the results are in good agreement.

Keywords: differential transformation method; wind turbine; finite element modelling; free vibration; flexural beam

1. Introduction

In recent years, the use of wind turbines has become widespread in both offshore and onshore structures and has a significant place among renewable energies and extensive studies have been done in recent decades (Anup *et al.* 2020, Liu *et al.* 2020, Malz *et al.* 2020, Schaffarczyk 2020, Yusof and Mohamed 2020). Wind turbines not only must have high resistance to static loading but also dynamic loading. The behavior of wind turbines under earthquake loads must be accurately determined for the wind turbines to continue their functions immediately after the earthquakes. It is important to determine dynamic parameters that determine the behavior of the wind turbine under earthquake loads and tornadoes (AbuGazia *et al.* 2020, Wang and Ishihara 2020). There are many studies in the literature on the dynamic behavior of wind turbines, which some of them are summarized below.

Makarios and Baniotopoulos 2014, 2015 were studied the role of a fully and partially fixed foundation on modal analysis of a prototype wind energy tower by applying the continuous model method.

Adhikari, Bhattacharya 2012 were proposed a simplified approach for the free vibration analysis of wind turbines on a flexible foundation. They derived a closed-form expression of the characteristic equation governing all the natural frequencies of the system based on Euler-Bernoulli beam-column with elastic end supports. Manenti and Petrini (2010) introduced a finite element model for coupled wind-waves analysis and the results of the dynamic behavior of a monopole-type support structure for offshore wind turbines were given. Wang *et al.* (2017) considered the nacelle as a mass with 2 degrees of freedom the tower as a beam with elastic end support. They used the Euler–Lagrangian approach and also applied interaction effects between the nacelle and the tower and the effects between the tower and the foundation during model formulation.

The transfer matrix method (TMM) is another analytical method to analyses the buckling and vibration of turbine wind towers (Dong *et al.* 2012, Rezaei *et al.* 2012, Meng *et al.* 2013, Feyzollahzadeh and Mahmoodi 2016, Rui *et al.* 2018). (Feyzollahzadeh and Mahmoodi, 2016) presented TMM for vibrations analysis of wind turbine towers. In another research, Feyzollahzadeh *et al.* (2016) used TMM to determine the wind load response of wind turbines. They modeled wind turbine foundation using CS, DS and AF models and axial force are modeled as a variable force along with the tower. They compared results with FM results in the literature.

On the other hand, the differential transformation method (DTM) is an alternative method for solving differential equations. DTM was first introduced by Zhou (1986). A number of studies on the method have been presented in the literature (Abualnaja 2020, Adeleye *et al.* 2020, Bekiryazici *et al.* 2020, Gireesha and Sowmya 2020, Hamada 2020, Sobamowo *et al.* 2020). Dado (2005) investigated prismatic as well as non-prismatic beams for large deformations under several loading conditions. It was based on representation of rotation angle by a polynomial of

*Corresponding author, Professor
E-mail: kbbozdogan@comu.edu.tr

http://www.techno-press.com/journals/was&subpage=7 ISSN: 1226-6116 (Print), 1598-6225 (Online)

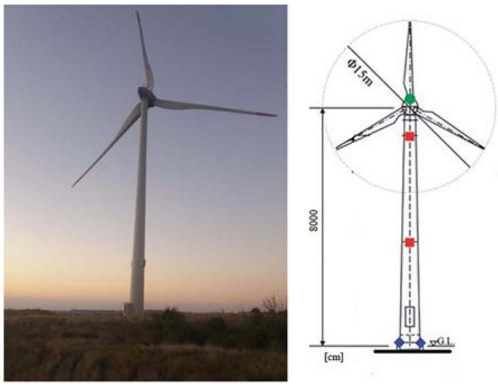

Fig. 1 Overview and schematic of the investigated wind turbine in analysis

the position along the deformed axis of beam. Mutyalarao *et al.* (2010) considered a uniform cantilever beam for large deformation, subjected to a concentrated load at tip and having an inclination normal to the deflected beam axis. Mawphlang *et al.* (2020) investigated the buckling behavior of nonuniform carbon nanotubes using the differential transformation method. The accuracy of the results revealed that differential transformation method is useful and convenient for studying the buckling behavior of nonuniform carbon nanotubes for various boundary conditions compared to other analytical methods.

In this study, a formulation for the free vibration analysis of wind turbines is proposed by the differential transformation method. The following assumptions have been made in the derivation of the wind turbine formulation and its modelling as a variable cross-section bending beam:

- The shear deformations are neglected.
- The material is assumed to be linear elastic.
- Geometric nonlinear effects are neglected.

At the end of the study an example which taken from the literature was solved with the presented method and the results are compared by the literature and finite element modelling software, ABAQUS. Results of the finite element model and DTM are compared with each other and the results are in good agreement.

2. Physical and mathematical modelling

Multi-story buildings and tower-type buildings can be modeled as an equivalent beam under certain assumptions. This type of modelling can be used safely in the preliminary stage. At the same time, with this type of modelling, the structural behavior that is overlooked due to the high number of elements in the finite element method, can be easily understood with a few parameters. Wind turbine type structures can be represented as an equivalent cantilever beam under earthquake loads. Such structures can be modeled as an equivalent cantilever Timoshenko beam, including rotary inertia and shear displacements effects. However, such structures can be modeled as an equivalent Euler-Bernoulli beam, as the rotary inertia effects and shear displacements effects are negligible (Jahanghiry *et al.* 2016).

Fig. 1 shows a typical wind turbine built in Çanakkale Geyikli (Turkey).

In this study, the wind turbine can be modeled as a continuous flexural beam with a variable cross-section. The wing weight is applied to the top of the tower as a point mass in the model.

In this system, the lateral and the torsional vibration are uncoupled because the center of mass coincides with the shear center.

a) Lateral Vibration

The differential equation of the variable cross-section flexural beam shown in Fig. 1 in the case of lateral free vibration is written as follows (Makarios and Baniotopoulos 2014, Makarios *et al.* 2016).

$$\frac{d^2}{dz^2}[EI(z)\frac{d^2y}{dz^2}]-\rho A(z)\omega^2 y=0 \tag{1}$$

where E is the modulus of elasticity, $y(z)$ is the lateral displacement function. Moment of inertia, $I(z)$, and distributed mass, $\rho A(z)$, variations along the height of the structure are considered parabolic as given below.

$$I(z)=a-bz+cz^2 \tag{2}$$

$$\rho A(z)=s-nz+lz^2 \tag{3}$$

The moment inertia function at the base of the wind turbine is written as follows.

$$I(0)=a \tag{4}$$

Similarly, the value of the mass distribution function on the wind turbine base is found as follows.

$$\rho A(0)=s \tag{5}$$

If the Eqs. (2) and (3) are placed in (1), the differential Eq. (6) is obtained.

$$\frac{d^2}{dz^2}[E(a-bz+cz^2)\frac{d^2y}{dz^2}]-[s-nz+lz^2]\omega^2 y=0 \tag{6}$$

After taking the necessary derivatives in the differential Eq. (6) and making the arrangement, the Eq. (7) is obtained.

$$\begin{aligned}&(a-bz+cz^2)\frac{d^4y}{dz^4}+(-2b+4cz)\frac{d^3y}{dz^3}+\\&2c\frac{d^2y}{dz^2}-(s-nz+lz^2)\frac{\omega^2}{E}y=0\end{aligned} \tag{7}$$

The boundary conditions of Eq. (7) are the displacement and slope at the base and the bending moment and shear force at the top of the wind turbine tower are zero which mathematically are shown below.

$$y(0)=0 \tag{8}$$

$$\left.\frac{dy}{dz}\right|_{z=0}=0 \tag{9}$$

$$[E(a-bz+cz^2)\frac{d^2y}{dz^2}]\bigg|_{z=H}=0 \quad (10)$$

$$[E(a-bz+cz^2)\frac{d^3y}{dz^3}]+[E(2cz-b)\frac{d^2y}{dz^2}]\bigg|_{z=H}+ M\omega^2y(H)=0 \quad (11)$$

The last term in Eq. (11) indicates the inertia force of the mass of the wings at the peak. The following transformation can be applied for the vertical axis set to make the equations dimensionless.

$$\varepsilon=\frac{z}{H} \quad (12)$$

The differential Eq. (7) was converted to the Eq. (13) using the transformation (12) for ease of calculations.

$$(a-bH\varepsilon+cH^2\varepsilon^2)\frac{d^4y}{d\varepsilon^4}+H(-2b+4cH\varepsilon)\frac{d^3y}{d\varepsilon^3}+ 2cH^2\frac{d^2y}{d\varepsilon^2}-(s-nH\varepsilon+lH^2\varepsilon^2)\frac{H^4\omega^2}{E}y=0 \quad (13)$$

The Eq. (14) is used to shorten the differential Eq. (13). So, Eq. (15) can be delivered.

$$\Gamma=\frac{H^4\omega^2}{E} \quad (14)$$

$$(a-bH\varepsilon+cH^2\varepsilon^2)\frac{d^4y}{d\varepsilon^4}+H(-2b+4cH\varepsilon)\frac{d^3y}{d\varepsilon^3}+ 2cH^2\frac{d^2y}{d\varepsilon^2}-(s-nH\varepsilon+lH^2\varepsilon^2)\Gamma y=0 \quad (15)$$

If the transformation (12) is applied at the (8), (9), (10), (11) and boundary conditions, i.e., Eqs. (16)-(19), we have

$$y(0)=0 \quad (16)$$

$$\frac{1}{H}\frac{dy}{d\varepsilon}_{\varepsilon=0}=0 \quad (17)$$

$$\frac{1}{H^2}[E(a-bH\varepsilon+cH^2\varepsilon^2)\frac{d^2y}{d\varepsilon^2}]\bigg|_{\varepsilon=1}=0 \quad (18)$$

$$[(a-bH\varepsilon+cH^2\varepsilon^2)\frac{d^3y}{d\varepsilon^3}]\bigg|_{\varepsilon=1}+ H[(2cH\varepsilon-b)\frac{d^2y}{d\varepsilon^2}]\bigg|_{\varepsilon=1}+\frac{M\Gamma y(1)}{H}=0 \quad (19)$$

b) Torsional vibration

Free vibration in the state of torsion in the turbine can be written by the following differential equation

$$\frac{d}{dz}[GJ(z)\frac{d\theta}{dz}]+I_0\rho A(z)\omega^2\theta=0 \quad (20)$$

where θ is the torsional rotation function, $GJ(z)$ is the torsion rigidity and I_0 is the mass moment of inertia and calculated as follows:

$$I_0=\frac{J(z)}{A(z)} \quad (21)$$

Using Eqs. (20) and (21) we obtained:

$$\frac{d}{dz}[GJ(z)\frac{d\theta}{dz}]+\rho J(z)\omega^2\theta=0 \quad (22)$$

Eq. (25) is obtained by accepting changes for J (z) and ρJ (z) shown in Eqs. (23) and (24).

$$J(z)=d-ez+fz^2 \quad (23)$$

$$\rho J(z)=t-qz+vz^2 \quad (24)$$

$$\frac{d}{dz}[G(d-ez+fz^2)\frac{d\theta}{dz}]+(t-qz+vz^2)\omega^2\theta=0 \quad (25)$$

If necessary arrangements are made in Eq. (25), Eq. (26) is derived.

$$[(d-ez+fz^2)]\frac{d^2\theta}{dz^2}+[(2fz-e)]\frac{d\theta}{dz}+ (t-qz+vz^2)\frac{\omega^2}{G}\theta=0 \quad (26)$$

The boundary conditions required for the solution of the second-order differential Eq. (26) are given below.

$$\theta(0)=0 \quad (27)$$

$$G(d-ez+fz^2)\frac{d\theta}{dz}_{z=H}-M\frac{J(H)}{A(H)}\omega^2\theta(H)=0 \quad (28)$$

Eq. (29) is obtained after applying transformation given by Eq. (12) in differential Eq. (28).

$$[d-e\varepsilon H+f\varepsilon^2H^2]\frac{d^2\theta}{d\varepsilon^2}+(2f\varepsilon H^2-eH)\frac{d\theta}{d\varepsilon}+ (t-q\varepsilon H+v\varepsilon^2H^2)\frac{\omega^2H^2}{G}\theta=0 \quad (29)$$

After the abbreviations are made, Eq. (31) is obtained.

$$\frac{H^2\omega^2}{G}=\beta \quad (30)$$

$$[d-e\varepsilon H+f\varepsilon^2H^2]\frac{d^2\theta}{d\varepsilon^2}+(2f\varepsilon H^2-eH)\frac{d\theta}{d\varepsilon}+ (t-q\varepsilon H+v\varepsilon^2H^2)\beta\theta=0 \quad (31)$$

Similarly, if the transformation Eq. (12) is applied to (27) and (28) boundary condition Eqs. (32) and (33), we have

$$\theta(0)=0 \tag{32}$$

$$(d-e\varepsilon H+f\varepsilon^2H^2)\frac{d\theta}{d\varepsilon}_{\varepsilon=1}-M\frac{J(H)}{A(H)}\beta\theta(H)=0 \tag{33}$$

3. Solution with differential transform method

Many numerical methods have been developed for the solution of partial and ordinary differential equations in engineering and mathematics (Ghaderi *et al.* 2015, Maleki, Mohammadi 2017, Atangana and Araz 2020, Bermúdez *et al.* 2020, Fryklund *et al.* 2020). One of these methods is the differential transform method, which can be described as a semi-analytical method. Differential transform method is a practical and easy to apply method. For this reason, in this study, the differential transform method was used for the dynamic analysis of wind turbines.

In the differential transformation method, the function whose solution is sought is expressed as a power series and the coefficients of the power series are calculated recursively with an applied transformation.

According to the differential transformation method a function y is written as (Rajasekaran 2009, Bervillier 2012);

$$y(\varepsilon)=\sum\nolimits_{k=0}^{\infty}\varepsilon^kY(\varepsilon) \tag{34}$$

where the Y function is defined as follows:

$$Y[k]=\frac{1}{k!}\left[\frac{d^kY(\varepsilon)}{d\varepsilon^k}\right]_{\varepsilon=0c=0} \tag{35}$$

Derivative expressions can be written as follows according to the differential transformation method

$$DT(\varepsilon^\alpha y^\beta)=\left[\prod_{i=1}^{\beta}(k-\alpha+i)\right]Y[k-\alpha+\beta] \tag{36}$$

$$y^\beta=\frac{d^\beta y}{d\varepsilon^\beta} \tag{37}$$

If the differential transformation method is applied to the differential Eq. (15), the equation given below is obtained.

$$Y[k+4]=\frac{R_1*Y[k+3]-R_2*Y[k+2]}{a*(k+1)*(k+2)*(k+3)*(k+4)}+\frac{R_3*Y[k]-R_4*Y[k-1]+R_5*Y[k-2]}{a*(k+1)*(k+2)*(k+3)*(k+4)} \tag{38}$$

R_1, R_2, R_3, R_4, and R_5 are defined as below

$$R_1=b*H*k*(k+1)*(k+2)*(k+3)+2*b*H*(k+3)*(k+2)*(k+1) \tag{39}$$

$$R_2=c*H^2*(k-1)*k*(k+1)*(k+2)+4cH^2*(k+2)*(k+1)*k+(k+2)*(k+1)2cH^2 \tag{40}$$

$$R_3=s*\Gamma \tag{41}$$

$$R_4=n*H*\Gamma \tag{42}$$

$$R_5=l*H^2*\Gamma \tag{43}$$

If DTM is applied to the boundary conditions (16) and (17), Eqs. (44) and (45) are obtained.

$$Y[0]=0 \tag{44}$$

$$Y[1]=0 \tag{45}$$

If DTM is applied to the boundary conditions (18) and (19), Eqs. (46) and (47) are obtained.

$$\left(a-bH+cH^2\right)\sum_{k=0}^{\infty}(k+2)*(k+1)*Y[k+2]=0 \tag{46}$$

$$\left(a-bH+cH^2\right)*\sum_{k=0}^{\infty}(k+3)*(k+2)*(k+1)*Y[k+3]+H[(2cH-b)*\sum_{k=0}^{\infty}(k+2)*(k+1)*Y[k+2]+\sum\nolimits_{k=0}^{\infty}\frac{M\Gamma Y[k]}{H}=0 \tag{47}$$

If Eq. (38) is applied recursively all unknowns written in $Y[2]$ and $Y[3]$. If the coefficients $Y[k]$ in the equations (46) and (47) are written in terms of $Y[2]$ and $Y[3]$, the matrix Eq. (48) is obtained.

$$\begin{bmatrix}A(1,1) & A(1,2)\\ A(2,1) & A(2,2)\end{bmatrix}\begin{Bmatrix}Y[2]\\ Y[3]\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \tag{48}$$

The $\boldsymbol{\omega}$ values providing the matrix Eq. (48) give the lateral vibration frequencies of the wind turbine. After the natural frequency is found, the mode shape is found using the Eq. (38) and Eq. (34).

In the torsional state, similarly, if DTM is applied in Eq. (31),

$$\Phi[k+2]=\frac{Z_1\Phi[k+1]-Z_2\Phi[k]+Z_3\Phi[k-1]-Z_4\Phi[k-2]}{d*(k+1)*(k+2)} \tag{49}$$

Z_1, Z_2, Z_3, and Z_4 are defined as follows, respectively.

$$Z_1=k(k+1)eH+eH(k+1) \tag{50}$$

$$Z_2=fH^2k(k-1)+2fH^2k+\beta t \tag{51}$$

$$Z_3=\beta qH \tag{52}$$

$$Z_4=\beta vH^2 \tag{53}$$

Eq. (54) is derived by applying DTM to the boundary condition (32),

(54)

If differential transformation is applied in the boundary condition (33), equation (55) is obtained.

$$(d - eH + fH^2)\sum_{k=0}^{\infty}(k+1)*\Phi[k+1] - M\frac{J(H)}{A(H)}\beta\sum_{k=0}^{\infty}\Phi[k] = 0 \quad (55)$$

Φ [1] is selected as unknown and all other unknowns are written in Φ [1] using Eq. (49).

If DTM is applied to the boundary condition (55), the following transcendent equation is obtained.

$$f(\omega)\Phi[1] = 0 \quad (56)$$

4. Wind turbine analysis in case of partial fixed foundation

In the case of a partially rigid foundation, a rotation spring is added under the wind turbine to represent the foundation rotation. In the case of a partially fixed foundation, only the boundary conditions will change in the analysis.

In this case, the boundary condition (9) will change as follows.

$$k_r \left.\frac{dy}{dz}\right|_{z=0} - EI(0)\frac{d^2y}{dz^2} = k_r \left.\frac{dy}{dz}\right|_{z=0} - Ea\frac{d^2y}{dz^2} = 0 \quad (57)$$

This equation can be written in dimensionless form as below.

$$k_r H \left.\frac{dy}{d\varepsilon}\right|_{\varepsilon=0} - EI(0)\frac{d^2y}{d\varepsilon^2} = k_r H \left.\frac{dy}{d\varepsilon}\right|_{\varepsilon=0} - a\frac{d^2y}{d\varepsilon^2} = 0 \quad (58)$$

If differential transformation is applied to equation (58), equation (59) is obtained.

$$k_r HY[1] - 2Ea.Y[2] = 0 \quad (59)$$

Eq. (59) can be written as follows.

$$Y[2] = \frac{Y[1]}{2\Delta} \quad (60)$$

The delta in Eq. (60) is defined below

$$\Delta = \frac{Ea}{k_r} \quad (61)$$

5. Finite element modelling

The finite element method is one of the numerical methods used to solve complex engineering problems. In recent years, due to significant developments in the field of finite element simulation software, this field has been able to easily simulate laboratory work in specialized software environments and even study the behaviour of structures Under different payloads without the need for laboratory models. ABAQUS software is one of the most powerful engineering analysis software in the field of finite element analysis.

A finite element vibration analysis of wind turbine tower

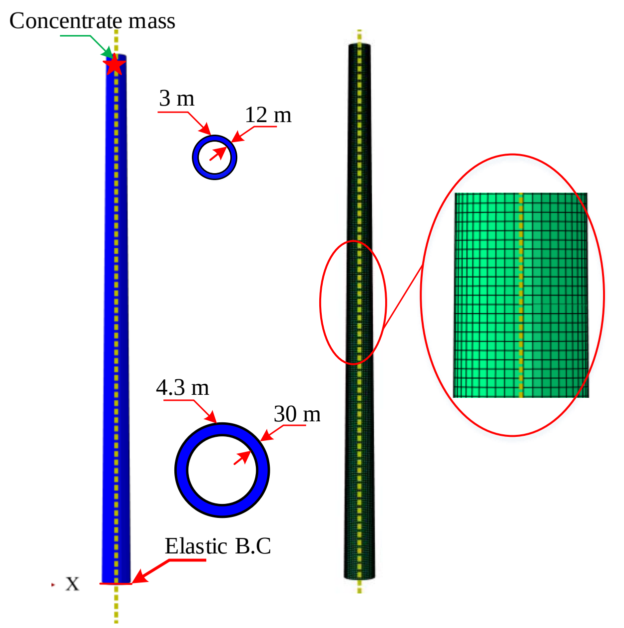

Fig. 2 geometrical properties and ABAQUS model of the wind turbine tower

Table 1 Periods of the tower with different modes

Mode	DTM	SAP2000 (Makarios *et al*. 2016)	Error (%)	FEM
1	2.69 s	2.59 s	3.86	2.59 s
2	0.35 s	0.33 s	6.01	0.38 s
3	0.13 s	0.13 s	0.0	0.14 s
4	0.12 s	0.12 s	0.0	0.12 s

is carried out using ABAQUS software. The tower is modelled by shell elements and S4 quadrangular element was employed in the analyses. The mesh of the shell models was refined until further refinement did not lead to significant variations in the results. Tower height is 80 m and the diameters of the tower are 4.3 m and 3 m and shell thickness is 30 mm and 12 mm at the bottom and top, respectively. The element size is considered as 0.5 m. Modulus of elasticity and density for steel is 210 GPa and 7800 kg/m^3. The self-weight of the tower is 1480 kN. The bottom of the tower is considered clamped as a boundary condition. The self-weight of the blade is applied 108.76 tonnes and 241.78 tonnes in x and y axes as a point mass at the top of the tower, respectively. Fig. 2 shows that the geometrical properties and ABAQUS model of the wind turbine tower.

6. Results and discussions

To investigate the suitability of the method presented in this section, the wind turbine example taken from the literature is solved with the presented method and compared with the literature. Besides, the same example was modelled with the ABAQUS program and the results were compared.

Mw wind turbine sample height is 80m and the total height of the wind turbine with rotor and blades is 125 m.

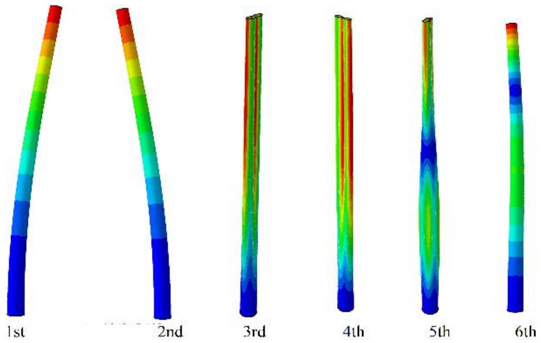

Fig. 3 Model of the prototype wind energy tower and the six first mode shapes

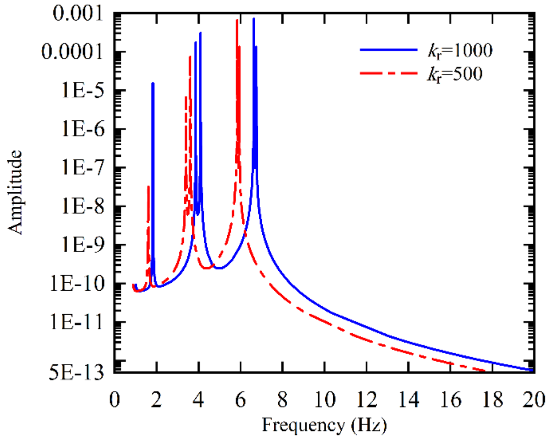

Fig. 4 Effect of foundation stiffness on the values of the natural frequencies of the under study tower

The diameter of the turbine is 4.3 m at the bottom and decreases linearly to 3 m at the top of the structures. The thickness of the turbine decreases linearly along with the construction height from 30 mm to 12 mm. The steel grade is S355 and the modulus of elasticity is E = 210 GPa. The blade weight is 1067 kN and is defined as 0.50 m below the peak of the turbine. The lateral and torsional periods calculated by DTM and finite element modelling are given in Table 1. The first, second and fourth modes are lateral vibration mode and the third mode is torsional vibration mode.

It is seen from the table that the period values found by the method presented in this study are sufficiently converged with the results given in the literature and the results obtained from the ABAQUS program. The range of error is below 5% in four modes.

7. Conclusions

In this study, DTM is applied for free vibration analysis of variable section wind turbines. With the presented method, lateral and torsion frequency values can be determined quickly and practically. At the end of the study, it was observed that DTM method yielded sufficiently close results to the finite element solution. The presented method can be used easily in the preliminary stage. Also, the proposed method was found to be very useful in determining the vibration behavior of the wind turbine with few parameters.

Mode shapes can also be found with the presented method. After the mode shapes are found, dynamic parameters such as modal participation factor effective mass ratio can also be found. The forces and displacements in the wind turbine can be calculated with the help of these dynamic characteristics. The results show that the torsional mode shapes has a significant effect on the mechanical behavior of this turbine and therefore in the design and construction of these turbines, the torsional strength of the structure should also be considered.

The presented approach can also be developed according to the Timoshenko beam model. Also in the presented model, the effect of the foundation on the dynamic behavior of the wind turbine can be taken into account. For this, the rotational and translational effect of the foundation can be taken into account in boundary conditions by using spring stiffness.

References

Abualnaja, K.M. (2020), “Numerical treatment of a physical problem in fluid film flow using the differential transformation method”, *Int. J. Modern Phys. C*, **31**(05), 87-96. https://doi.org/10.1142/S0129183120500679.

AbuGazia, M., El Damatty, A.A., Dai, K., Lu, W. and Ibrahim, A. (2020), “Numerical model for analysis of wind turbines under tornadoes”, *Eng., Struct.*, **223**, 54-67. https://doi.org/10.1016/j.engstruct.2020.111157.

Adeleye, D.O.A., Yusuf, M. and Balogun, O. (2020), “Dynamic analysis of viscoelastic circular diaphragm of a MEMS capacitive pressure sensor using modified differential transformation method”, *Karbala Int. J. Modern Sci.*, **6**(3), 65-78. https://doi.org/10.33640/2405-609X.1706.

Adhikari, S. and Bhattacharya, S. (2012), “Dynamic analysis of wind turbine towers on flexible foundations”, *Shock Vib.*, **19**(1), 37-56. https://doi.org/10.3233/SAV-2012-0615.

Anup, K., Whale, J., Evans S.P. and Clausen, P.D. (2020), “An investigation of the impact of wind speed and turbulence on small wind turbine operation and fatigue loads”, *Renew. Energy*, **146**, 87-98. https://doi.org/10.1016/j.renene.2019.06.124.

Atangana, A. and Araz, S.İ. (2020), “New numerical method for ordinary differential equations: Newton polynomial”, *J. Comput. Appl. Mathem.*, **372**, 112-122. https://doi.org/10.1016/j.cam.2019.112622.

Bekiryazici, Z., Merdan, M. and Kesemen, K. (2020), “Modification of the random differential transformation method and its applications to compartmental models”, *Communications Statistics-Theory Methods*, 1-22. https://doi.org/10.1080/03610926.2020.1713372

Bermúdez, A., Gómez, D., Piñeiro, M. and Salgado, P. (2020), “A novel numerical method for accelerating the computation of the steady-state in induction machines”, *Comput. Mathem. Appl.*, **79**(2), 274-292. https://doi.org/10.1016/j.camwa.2019.06.032.

Bervillier, C. (2012), “Status of the differential transformation method”, *Appl. Mathem. Comput.*, **218**(20), 10158-10170. https://doi.org/10.1016/j.amc.2012.03.094.

Dado, M. and Al-Sadder, S. (2005), “A new technique for large deflection analysis of non-prismatic cantilever beams”, *Mech. Res. Commun.*, **32**(6), 692-703. https://doi.org/10.1016/j.mechrescom.2005.01.004.

Dong, L., Sun L.X. and Qin, M. (2012), “Applying transfer matrix method in mode analysis of wind turbine", *Advan. Mater. Res.*, **512**, 686-689.

Feyzollahzadeh, M. and Mahmoodi, M. (2016), "Dynamic analysis of offshore wind turbine towers with fixed monopile platform using the transfer matrix method", *J. Solid Mech.*, **8**(1), 130-151.

Feyzollahzadeh, M., Mahmoodi, M. Yadavar-Nikravesh, S.M. and Jamali, J. (2016), "Wind load response of offshore wind turbine towers with fixed monopile platform", *J. Wind Eng. Ind. Aerod.*, **158**, 122-138. https://doi.org/10.1016/j.jweia.2016.09.007.

Fryklund, F., Kropinski, M.C.A. and Tornberg, A.K. (2020), "An integral equation-based numerical method for the forced heat equation on complex domains", *Advan. Comput. Mathem.*, **46**(5), 1-36. https://doi.org/10.1007/s10444-020-09804-z.

Ghaderi, M., Ghaffarzadeh, H. and Maleki, V.A. (2015), "Investigation of vibration and stability of cracked columns under axial load", *Earthq. Struct.*, **9**(6), 1181-1192. http://dx.doi.org/10.12989/eas.2015.9.6.1181.

Gireesha, B. and Sowmya, G. (2020), "Heat transfer analysis of an inclined porous fin using differential transform method", *Int. J. Ambient Energy*, **89**, 1-7. https://doi.org/10.1080/01430750.2020.1818619.

Hamada, Y.M. (2020), "Solution of a new model of fractional telegraph point reactor kinetics using differential transformation method", *Appl. Mathem. Modelling*, **78**, 297-321. https://doi.org/10.1016/j.apm.2019.10.001.

Jahanghiry, R., Yahyazadeh, R., Sharafkhani, N. and Maleki, V.A. (2016), "Stability analysis of FGM microgripper subjected to nonlinear electrostatic and temperature variation loadings", *Sci. Eng. Compos. Mater.*, **23**(2), 199-207. https://doi.org/10.1515/secm-2014-0079.

Liu, H., Li, Y., Duan, Z. and Chen, C. (2020), "A review on multi-objective optimization framework in wind energy forecasting techniques and applications", *Energy Convers. Management*, **224**, 11-24. https://doi.org/10.1016/j.enconman.2020.113324.

Makarios, T.K. and Baniotopoulos, C.C. (2014), "Wind energy structures: modal analysis by the continuous model approach", *J. Vib. Control*, **20**(3), 395-405. https://doi.org/10.1177/1077546312463761.

Makarios, T.K. and Baniotopoulos, C.C. (2015), "Modal analysis of wind turbine tower via its continuous model with partially fixed foundation", *Int. J. Innov. Res. Advan. Eng.*, **2**(1), 14-25.

Makarios, T.K., Efthymiou, E. and Baniotopoulos, C.C. (2016), "On the torsional-translational response of wind turbine structures", *Arab. J. Sci. Eng.*, **41**(4), 1241-1249. https://doi.org/10.1007/s13369-015-1911-7.

Maleki, V.A. and Mohammadi, N. (2017), "Buckling analysis of cracked functionally graded material column with piezoelectric patches", *Smart Mater. Struct.*, **26**(3), 035-041. https://doi.org/10.1088/1361-665X/aa5324.

Malz, E.C., Hedenus, F., Göransson, L., Verendel, V. and Gros, S. (2020), "Drag-mode airborne wind energy vs. wind turbines: An analysis of power production, variability and geography", *Energy*, **193**, 116-125. https://doi.org/10.1016/j.energy.2019.116765

Manenti, S. and Petrini, F. (2010), "Dynamic analysis of an offshore wind turbine: wind-waves nonlinear interaction", *Earth Space 2010: Eng Sci., Construct. Operat. Challenging Environm.*, 2014-2026.

Mawphlang, B.R.K.L.L., Ghimire, M.P., Rai, D.P. and Patra, P. K. (2020), "Buckling behavior of nonuniform carbon nanotubes using nonlocal elasticity theory and the differential transformation method", *Int. Nano Lett.*, **43**, 56-79. https://doi.org/10.1007/s40089-020-00319-5.

Meng, C., Su, M. and Wang, S. (2013), "An investigation on dynamic characteristics of a gas turbine rotor using an improved transfer matrix method", *J. Eng. G0as Turbines Power*, **135**(12), 78-90. https://doi.org/10.1115/1.4025234.

Mutyalarao, M., Bharathi, D. and Rao, B.N. (2010), "Large deflections of a cantilever beam under an inclined end load," *Appl. Mathem. Comput.*, **217**(7), 3607-3613. https://doi.org/10.1016/j.amc.2010.09.021.

Rajasekaran, S. (2009), *Structural Dynamics of Earthquake Engneering Theory and Applcation Using Mathematica and Matlab'India*, Woodhead Publshing

Rezaei, E., Tabesh, A. and Ebrahimi, M. (2012), "Dynamic model and control of DFIG wind energy systems based on power transfer matrix", *IEEE Transactions Power Delivery*, **27**(3), 1485-1493. https://doi.org/10.1109/TPWRD.2012.2195685.

Rui, X., Wang, G. and Zhang, J. (2018), *Transfer Matrix Method for Multibody Systems: Theory and Applications*, John Wiley & Sons.

Schaffarczyk, A. (2020), *Types of wind turbines. Introduction to Wind Turbine Aerodynamics*, Springer: 7-25.

Sobamowo, M.G., Yinusa, A.A., Adeleye, O.A., Alozie, S.I., Salawu, S.A. and Salami, M.O. (2020), "On the efficiency of differential transformation method to the solutions of large amplitude nonlinear oscillation systems", *World Scientific News*, **139**(1), 1-60.

Wang, L. and Ishihara, T. (2020), "A study of the effects of foundation uplift on the seismic loading of wind turbine tower and shallow foundation using a new dynamic Winkler model", *Eng. Struct.*, **219**, 110-125. https://doi.org/10.1016/j.engstruct.2020.110745.

Wang, S., Huang, Y., Li, L., Liu, C. and Zhang, D. (2017), "Dynamic analysis of wind turbines including nacelle–tower–foundation interaction for condition of incomplete structural parameters", *Advan. Mech. Eng.*, **9**(3), 43-57. https://doi.org/10.1177/1687814017692940.

Yusof, A. and Mohamed, M. (2020), *Vertical Axis Wind Turbines: An Overview*, InECCE2019, Springer, 821-835.

Zhou, J. (1986), *Differential transformation and its applications for electrical circuits*, Huazhong University Press, Wuhan, China.

CC