

# Assessment of the directional extreme wind speeds of typhoons via the Copula function and Monte Carlo simulation

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**Abstract.** Probabilistic information regarding directional extreme wind speeds is important for the precise estimation of the design wind loads on structures. A joint probability distribution model of directional extreme typhoon wind speeds is established using Monte Carlo simulation and empirical copula function to fully consider the correlations of extreme typhoon wind speeds among the different directions. With this model, a procedure for estimating directional extreme wind speeds for given return periods, which ensures that the overall risk is distributed uniformly by direction, is established. Taking 5 typhoon-prone cities in China as examples, the directional extreme typhoon wind speeds for given return periods estimated by the present method are compared with those estimated by the method proposed by Cook and Miller (1999). Two types of directional factors are obtained based on Cook and Miller (1999) and the UK standard's drafting committee (Standard B, 1997), and the directional risks for the given overall risks are discussed. The influences of the extreme wind speed correlations in the different directions and the simulated typhoon wind speed sample sizes on the estimated extreme wind speeds for a given return period are also discussed.

**Keywords:** directional extreme wind speeds of typhoons; empirical copula function; directional assessment; Monte Carlo simulation

## 1. Introduction

Extreme monsoon wind speeds are generally estimated using observed wind speed records from meteorological stations. Theoretically, if the available observed typhoon wind speed records are adequate, extreme typhoon wind speeds can also be estimated using the same method as that used for monsoon wind speeds. However, typhoons are physically small and strong wind storms with low recurrence rates, and anemometers are vulnerable to damage from typhoons, while typhoon wind speed records are often too short to statistically determine the wind climate of a given area (Harper 1999, Xiao *et al.* 2011a). Therefore, extreme typhoon wind speeds are commonly predicted using numerical modelling (Zhao *et al.* 2013). The numerical models of typhoons in meteorology (Dudhia 1993, Grell *et al.* 1995) considering complex physical processes are based on fluid dynamics and thermodynamics (Ge *et al.* 2003, Xiao *et al.* 2011b), in which the appropriate equations for the conservation of mass, momentum, and energy are strictly solved. The wind field produced with this method is close to reality (Xiao *et al.* 2011b). However, such models are too complicated and computationally

expensive for statistical analysis and prediction of extreme wind speeds in wind engineering. Wind engineering researchers usually prefer to use a series of parameter risk models of typhoons (Chow 1971, Gomes and Vickery 1976, Batts *et al.* 1980, Shapiro 1983, Georgiou 1985, Vickery and Twisdale, 1995a, Meng *et al.* 1995, Thompson and Cardone, 1996, Vickery *et al.* 2000) that are relatively simple, computationally inexpensive and still describe the main features of typhoon wind fields (Ge *et al.* 2003, Xiao *et al.* 2011b).

In wind engineering, Monte Carlo simulation of typhoon wind speeds based on these parametric risk models (referred to hereafter as typhoon simulations) has been accepted as an alternative and feasible approach for typhoon wind speed analysis in the load codes of the United States of America (ASCE7-10 2010) and Australia (SAA 2011). Typhoon simulation is generally conducted as follows (Xiao *et al.* 2011a). (1) Key typhoon parameters are sampled based on their probability distributions, which can be determined by publicly available historical typhoon records. (2) A virtual wind field can be constructed with the sampled key parameters to simulate the wind speeds for a given site. (3) After typhoon wind speed samples for many years are generated by repeatedly simulating the site-specific typhoon wind fields, the extreme typhoon wind speeds for a given return period are estimated using the annual maximum typhoon wind speeds obtained from the generated wind speed samples with a statistical method.

Cook (1983) analysed the directionality of extreme winds for the first time and indicated that the required directional factor should have the standard risk overall, but

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this risk should be distributed uniformly by direction. Cook and Miller (1999) improved this method and showed the geographic variation in the correlation of extreme wind speeds between adjacent sectors. The pioneering work of Cook (1983) and Cook and Miller (1999) resulted in a method (hereafter referred to as the Cook and Miller method) for assessing the difference between the risk in a directional sector and the overall risk in all sectors based on wind speed records from meteorological stations in the UK, while a definite expression of the joint probability distribution of the directional extreme wind speeds has not been obtained. After this pioneering work, researchers (Simiu *et al.* 1985, Kanda and Itoi, 2001, Itoi and Kanda, 2002, Zhang and Chen 2015, Zhang and Chen 2016, Quan *et al.* 2017) analysed the correlations among directional extreme wind speeds using the copula theory based on observed wind speed records from meteorological stations. However, most studies have not considered the influences of typhoons. Although the wind speed data used by Kanda and Itoi (2001) and Itoi and Kanda (2002) comprise mixed wind speeds of typhoons and monsoons, it is difficult to precisely predict the extreme wind speeds for a given return period based on mixed wind speed records because typhoon wind speed records are often too scarce for statistical analysis (Harper 1999, Xiao *et al.* 2011a).

Typhoon simulation can generate enough wind speed samples for statistical analysis. However, the correlations among the extreme typhoon wind speeds in different directions have not been fully considered in previous studies. In the present study, the theory of the joint probability distribution model of directional extreme typhoon wind speeds is established using an empirical copula function to fully consider the correlations of the extreme typhoon wind speeds among the different directions. Directional annual maximum typhoon wind speed samples are obtained with typhoon simulation. Directional extreme typhoon wind speeds with a uniformly distributed risk by direction for a given return period are estimated using the proposed joint probability distribution model and the generated samples. The results are compared with those estimated using the Cook and Miller method. The directional factors, the directional risks, the influences of the correlations of extreme typhoon wind speeds among the different directions and the sample size influence on the estimated results are also discussed.

## 2. Production of the annual maximum typhoon wind speeds for the coastal regions of southeastern China

Since the production of annual maximum wind speeds via typhoon simulation is the first step in establishing the joint probability distribution of directional extreme typhoon wind speeds, the selected typhoon simulation method will be introduced first. The analytical wind field model proposed by Meng *et al.* (1995) has a high computational efficiency and sufficient precision (Ge *et al.* 2003). The model is very suitable for simulating a large number of typhoons. Since a large number of annual maximum wind speed samples are required to establish an empirical copula model, this model is chosen for the present study.

In Meng *et al.* (1995), the typhoon-induced mean wind speed is expressed by addition of the gradient wind  $v^g$  in the free atmosphere and component  $v'$ , which is caused by friction at the ground surface. The cylindrical coordinate system is used, and the gradient wind  $v_g$  is expressed as:

$$v_{\theta g} = \frac{c_\theta - fr}{2} + \sqrt{\left(\frac{c_\theta - fr}{2}\right)^2 + \frac{r}{\rho} \frac{\partial P}{\partial r}} \quad (1)$$

$$v_{rg} = 0 \quad (2)$$

and  $v'$  is expressed as:

$$-\left(2\frac{v_{\theta g}}{r} + f\right)v'_\theta = \frac{\partial}{\partial z}\left(k_m \frac{\partial v'_r}{\partial z}\right) \quad (3)$$

$$\left(\frac{\partial v_{\theta g}}{\partial r} + \frac{v_{\theta g}}{r} + f\right)v'_r = \frac{\partial}{\partial z}\left(k_m \frac{\partial v'_\theta}{\partial z}\right) \quad (4)$$

where  $v_{\theta g}$  and  $v'_\theta$  are the tangential components of  $v^g$  and  $v'$ , respectively;  $v_{rg}$  and  $v'_r$  are the radial components of  $v^g$  and  $v'$ , respectively;  $f$  is the Coriolis parameter;  $r$  is the radial distance from the typhoon centre;  $\rho$  is the density of air;  $k_m$  is the eddy viscosity;  $z$  is the vertical coordinate; and  $P$  is the pressure profile, which is usually computed with the following equation, proposed by Holland (1980):

$$P = P_c + \Delta P \exp\left[-\left(r_m / r\right)^B\right] \quad (5)$$

where  $P_c$  is the central pressure;  $\Delta P$  is the central pressure difference;  $r_m$  is the radius of the maximum wind; and  $B$  is the pressure profile constant. The values of  $B$  fall within the range of 0.5-2.5 recommended by Cardone *et al.* (1994).

The method for computing the pressure profile constant  $B$  proposed in the technical manual of the Hazus typhoon disaster analysis software of the US Federal Emergency Management Agency (FEMA 2003) can be used for the coastal regions of southeastern China if data of the radial profile of the air pressure are unavailable (Zhao *et al.* 2013). In the present study, the following expression for the pressure profile constant  $B$ , proposed by FEMA (2003), is adopted:

$$B = 1.38 + 0.00184\Delta P - 0.00309r_m \quad (6)$$

The probabilistic distributions of the key typhoon parameters for several coastal cities in southeastern China were established by Xiao *et al.* (2011a) as follows:

$$Pr(T=t) = \frac{\lambda^t}{t!} e^{-\lambda} \quad (7)$$

where  $t$  is the number of typhoons in a year and  $\lambda$  is the annual occurrence rate of typhoons.

The approach angle  $\theta$  is sampled from the bimodal distribution:

$$f(\theta; \beta, \mu_1, \sigma_1, \mu_2, \sigma_2) = \frac{\beta}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}\left(\frac{\theta-\mu_1}{\sigma_1}\right)^2} + \frac{1-\beta}{\sqrt{2\pi}\sigma_2} e^{-\frac{1}{2}\left(\frac{\theta-\mu_2}{\sigma_2}\right)^2} \quad \theta \in [-180, 180], \quad (8)$$

where  $\beta$ ,  $\mu_1$ ,  $\sigma_1$ ,  $\mu_2$  and  $\sigma_2$  are fitting parameters.

The central pressure difference  $\Delta P$ , the translation velocity  $V_{Tr}$  and the distance of the closest approach  $D_{min}$  are sampled from their respective empirical distributions. The empirical distribution is directly based on the data of each variable obtained from relevant typhoon records within a 1000 km diameter circular region that centres on the site of interest (Xiao *et al.* 2011a). There is no need to fit the data since the empirical distribution is chosen. All typhoon records are obtained based on the yearbooks of tropical cyclones.

The exponential filling model is specified in Xiao *et al.* (2011a) as

$$\Delta P(t) = \Delta P_0 \exp(-(at + b)), \quad (9)$$

where  $\Delta P_0$  is the central pressure difference at the time of landfall,  $a = a_0 + a_1 \Delta P_0 + \varepsilon_0$ ;  $\varepsilon_0$  is a normally distributed error term with a mean of zero and a standard deviation; and  $a_0$ ,  $a_1$  and  $b$  are fitting parameters.

The radius of the maximum wind  $r_m$  is calculated with the empirical linear model proposed by Xiao *et al.* (2011a)

$$\ln(r_m) = c_0 + c_1 \Delta p + \varepsilon_1, \quad (10)$$

where  $c_0$  and  $c_1$  are fitting parameters and  $\varepsilon_1$  is a zero mean normal variate. More details regarding these key typhoon parameters and their probability models were specified by Xiao *et al.* (2011a).

The research in this paper is based on the 16-wind-sector system commonly used in wind speed and direction records from the Chinese meteorological department. The 16 directional annual maxima of the 10-min average typhoon wind speeds at a height of 10 m above the countryside for  $10^6$  years,  $[\hat{V}]_{10^6 \times 16}$ , are simulated in the present study with the above models. The appropriateness for setting the number of simulated years to  $10^6$  will be discussed in section 4.5. The virtual typhoon tracks are assumed to be straight lines as typhoons evolve (Georgiou *et al.* 1983, Georgiou 1985, Vickery and Twisdale 1995b, Ou *et al.* 2002, Xiao *et al.* 2011a, Li and Hong 2015). The roughness length  $z_0$  of the country is set to 0.05 m, based on the Chinese wind-resistant design specification for highway bridges (Xiang *et al.* 2004).

### 3. Methodology of the proposed approach

#### 3.1 Correlations of the extreme typhoon wind speeds among different directions

A typhoon can cause high wind speeds in several directions simultaneously, so the extreme typhoon wind speeds with long return periods in the different directions are very likely to be correlated with one another. Therefore, it is important to consider the correlations of extreme typhoon wind speeds among the different directions.

Ganssler and Stute (1987), Fermanian *et al.* (2004) and Tsukahara (2005) provided various conditions for goodness-of-fit testing of copula functions, under which the empirical copula model is a consistent estimator of the true underlying copula (Genest *et al.* 2009). Based on the

definition of the consistent estimator by Devore (2015), it is known that as the sample size increases, it is less likely that there is a deviation between the empirical copula and the true underlying copula. Monte Carlo simulation can provide sufficient samples to establish an accurate empirical copula model, so the joint probability distribution model of directional extreme typhoon wind speeds can be established based on a large number of simulated annual maximum typhoon wind speeds via the empirical copula function with the Sklar theorem (Sklar 1959). The production of annual maximum typhoon wind speed samples in the present study was introduced in section 2, and the establishment of the joint probability distribution model of directional extreme typhoon wind speeds is introduced as follows.

Let  $[\hat{V}]_{M \times N} = [\hat{V}_1, \hat{V}_2, \dots, \hat{V}_n, \dots, \hat{V}_N]$  be the annual maximum typhoon wind speed samples for each wind sector, where  $\hat{V}_n$  are the samples in the  $n^{\text{th}}$  wind sector;  $M$  is the number of simulated years and  $N$  is the number of wind sectors, which are  $10^6$  and 16, respectively, in the present study. The extremal event of the annual maximum wind speed among the  $N$  sectors is given by (Salvadori *et al.* 2007):

$$E^\vee = \{\hat{V}_1 > \hat{v}_1\} \vee \{\hat{V}_2 > \hat{v}_2\} \vee \dots \vee \{\hat{V}_n > \hat{v}_n\} \vee \dots \vee \{\hat{V}_N > \hat{v}_N\}, \quad (11)$$

where  $\hat{V}_n$  denotes the annual maximum wind speeds in the  $n^{\text{th}}$  sector and  $\hat{v}_n$  denotes directional extreme wind speed variables. Then, the probability of this extremal event is

$$p^\vee = Pr(E^\vee) = Pr(\{\hat{V}_1 > \hat{v}_1\} \vee \{\hat{V}_2 > \hat{v}_2\} \vee \dots \vee \{\hat{V}_n > \hat{v}_n\} \vee \dots \vee \{\hat{V}_N > \hat{v}_N\}). \quad (12)$$

The return period  $R$  is

$$R = 1/p^\vee = 1/Pr(\{\hat{V}_1 > \hat{v}_1\} \vee \{\hat{V}_2 > \hat{v}_2\} \vee \dots \vee \{\hat{V}_n > \hat{v}_n\} \vee \dots \vee \{\hat{V}_N > \hat{v}_N\}). \quad (13)$$

The relationship between the return period  $R$  and the joint probability distribution function  $F(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N)$  is

$$\begin{aligned} F(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N) &= 1 - 1/R \\ &= 1 - Pr(\{\hat{V}_1 > \hat{v}_1\} \vee \{\hat{V}_2 > \hat{v}_2\} \vee \dots \vee \{\hat{V}_n > \hat{v}_n\} \vee \dots \vee \{\hat{V}_N > \hat{v}_N\}) \\ &= Pr(\{\hat{V}_1 \leq \hat{v}_1\} \wedge \{\hat{V}_2 \leq \hat{v}_2\} \wedge \dots \wedge \{\hat{V}_n \leq \hat{v}_n\} \wedge \dots \wedge \{\hat{V}_N \leq \hat{v}_N\}). \end{aligned} \quad (14)$$

Let  $F(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N)$  be the joint probability distribution function with the directional extreme wind speed variables,  $(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N)$ , whose corresponding one-dimensional margins are  $F_1(\hat{v}_1), \dots, F_N(\hat{v}_N)$ . According to the Sklar theorem (Sklar 1959), when  $F_1(\hat{v}_1), \dots, F_N(\hat{v}_N)$  are all continuous, there exists a unique copula  $C(p_1, p_2, \dots, p_N)$  such that for all variables  $(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N)$ :

$$F(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N) = C(F_1(\hat{v}_1), F_2(\hat{v}_2), \dots, F_n(\hat{v}_n), \dots, F_N(\hat{v}_N)). \quad (15)$$

Based on the Sklar theorem, modelling the joint distributions of the directional extreme typhoon wind speeds using the empirical copula can be divided into two aspects:

- 1) The one-dimensional marginal distribution functions

$F_1(\hat{v}_1), F_2(\hat{v}_2), \dots, F_n(\hat{v}_n), \dots, F_N(\hat{v}_N)$  in every sector are estimated by their one-dimensional empirical distributions,  $F_{0,1}(\hat{v}_1), F_{0,2}(\hat{v}_2), \dots, F_{0,n}(\hat{v}_n), \dots, F_{0,N}(\hat{v}_N)$ , which are established based on the simulated annual maximum typhoon wind speeds introduced in section 2. The expression for the empirical estimate  $F_{0,n}$  in the  $n^{\text{th}}$  sector is (Coles *et al.* 2001)

$$F_{0,n}(\hat{v}_n) = \frac{i_n}{M+1} \quad \text{for } \hat{v}_n^{(i_n)} \leq \hat{v}_n < \hat{v}_n^{(i_n+1)}, \quad (16)$$

where  $\hat{v}_n$  denotes the independent variable of the one-dimensional marginal empirical distribution function in the  $n^{\text{th}}$  sector and  $\hat{v}_n^{(i_n)}$  is the  $i_n^{\text{th}}$  annual maximum typhoon wind speed of the samples in the  $n^{\text{th}}$  wind sector,  $\hat{v}_n^{\tilde{}}$ , in ascending order ( $\hat{v}_n^{(1)} \leq \hat{v}_n^{(2)} \leq \dots \leq \hat{v}_n^{(M)}$ ).

2) To consider the correlation of extreme typhoon wind speeds among the different sectors, the empirical copula  $C_0(p_1, p_2, \dots, p_N)$  is established with an empirical estimate of the one-dimensional marginal non-exceedance probability of the annual maximum typhoon wind speeds in each sector using Eq. (16). This function is expressed as (Deheuvels 1979)

$$C_0(p_1, p_2, \dots, p_n, \dots, p_N) = \frac{1}{M} \sum_{m=1}^M I(p_{m1} \leq p_1, p_{m2} \leq p_2, \dots, p_{mn} \leq p_n, \dots, p_{mN} \leq p_N), \quad (17)$$

where  $p_n \in [0,1]$  denotes the independent variable of the empirical copula and  $p_{mn}$  represents the empirical estimate of the one-dimensional marginal non-exceedance probability of the  $m^{\text{th}}$  element of the annual maximum typhoon wind speed samples of the  $n^{\text{th}}$  wind sector. The variable is calculated by Eq. (16) as  $p_{mn} = F_{0,n}(\hat{v}_{mn})$ .  $I(\cdot)$  is the indicator function, i.e., if  $p_{mn} \leq p_n$  is always satisfied with  $n \in [1, N]$ , then

$$I(p_{m1} \leq p_1, p_{m2} \leq p_2, \dots, p_{mn} \leq p_n, \dots, p_{mN} \leq p_N) = 1;$$

Otherwise,

$$I(p_{m1} \leq p_1, p_{m2} \leq p_2, \dots, p_{mn} \leq p_n, \dots, p_{mN} \leq p_N) = 0.$$

When the true underlying copula  $C(\cdot)$  in Eq. (15) is estimated by the empirical copula  $C_0(\cdot)$  in Eq. (17), the joint probability distribution function  $F(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N)$  of the directional extreme wind speed variables,  $(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N)$  can be estimated with Eqs. (15)-(17):

$$F(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N) \approx C_0(F_{0,1}(\hat{v}_1), F_{0,2}(\hat{v}_2), \dots, F_{0,N}(\hat{v}_N)) \\ = \frac{1}{M} \sum_{m=1}^M I(p_{m1} \leq F_{0,1}(\hat{v}_1), \dots, p_{mN} \leq F_{0,N}(\hat{v}_N)) \quad (18)$$

Upon combining Eqs. (14) and (18), the relationship between  $(\hat{v}_1, \hat{v}_2, \dots, \hat{v}_n, \dots, \hat{v}_N)$  and the return period  $R$  is:

$$1 - \frac{1}{R} \approx \frac{1}{M} \sum_{m=1}^M I(p_{m1} \leq F_{0,1}(\hat{v}_1), p_{m2} \leq F_{0,2}(\hat{v}_2), \dots, p_{mn} \leq F_{0,n}(\hat{v}_n), \dots, p_{mN} \leq F_{0,N}(\hat{v}_N)). \quad (19)$$

### 3.2 Estimation of the directional extreme typhoon wind speeds with a uniformly distributed overall risk

To be able to conduct a comparison with the Cook and Miller method in section 4 and to avoid the infinite number of solutions of Eq. (19), when the directional extreme typhoon wind speeds, which ensure the  $1/R$  overall annual risk, are estimated for a given  $R$ -year return period, the overall risk is distributed uniformly by the direction (Cook 1983) as with the Cook and Miller method, i.e., it is assumed that the non-exceedance probability  $F_{0,n}(\hat{v}_n)$  of each sector is the same and equal to  $p_{dir,R,all}$ . The risk of each sector with the standard  $1/R$  overall risk is uniformly equal to  $1-p_{dir,R,all}$ . Then, Eq. (19) becomes:

$$1 - 1/R \approx \frac{1}{M} \sum_{m=1}^M I(p_{m1} \leq p_{dir,R,all}, \dots, p_{mN} \leq p_{dir,R,all}) \quad (20)$$

When the return period  $R$  is given, there is only a single unknown variable  $p_{dir,R,all}$  that can be estimated with Eq. (20) via numerical means. The parameter  $p_{dir,R,all}$  is commonly quite different from  $1-1/R$ . Setting the non-exceedance probability in each sector as  $1-1/R$  has a greater unknown overall risk of exceedance (Cook, 1983). The equal risk of each sector is usually not equal to  $1/(R \cdot N)$  because the extremal events of annual maximum wind speeds in the different sectors are commonly not mutually exclusive events, i.e., Eq. (12) is commonly not equal to

$$Pr(\hat{V}_1 > \hat{v}_1) + Pr(\hat{V}_2 > \hat{v}_2) + \dots + Pr(\hat{V}_n > \hat{v}_n) + \dots + Pr(\hat{V}_N > \hat{v}_N).$$

After the non-exceedance probability of the extreme typhoon wind speed in each sector,  $p_{dir,R,all}$ , is obtained, the directional extreme typhoon wind speed  $\hat{v}_{n,R,all}$ , which ensures that the  $1/R$  overall annual risk is distributed uniformly by direction, can be calculated with:

$$p_{dir,R,all} = F_{0,n}(\hat{v}_{n,R,all}) \quad \text{for } n=1, 2, \dots, N, \quad (21)$$

where  $F_{0,n}(\cdot)$  is the one-dimensional marginal empirical probability distribution in the  $n^{\text{th}}$  sector, i.e., Eq. (16).

## 4. Results and discussion

With the annual maxima of the simulated typhoon wind speeds, the non-directional extreme typhoon wind speeds, the directional extreme typhoon wind speeds, the directional factors, the influences of the correlations of extreme typhoon wind speeds among the different directions and the sample size influence are calculated and discussed here.

### 4.1 The non-directional extreme typhoon wind speed

The non-directional extreme typhoon wind speed, irrespective of the direction, is obtained from the one-dimensional empirical distribution, which is expressed as (Coles *et al.* 2001)

$$F_0(\hat{v}) = \frac{i}{M+1} \quad \text{for } \hat{v}_{all}^{(i)} \leq \hat{v} < \hat{v}_{all}^{(i+1)}, \quad (22)$$



Fig. 1 Locations of the five typhoon-prone cities in the southeastern China coastal region and the typical tropical cyclone track directions (the clockwise direction from the north is taken as positive) of the cities

Table 1 Comparison of the non-directional extreme typhoon wind speeds (m/s),  $\hat{v}_{all,R,all}$ , reported by different studies for the 50- and 100-year return periods

Cities	Return period	Chinese code (Xiang <i>et al.</i> 2004)*	Ou <i>et al.</i> (2002)* $\hat{v}_{all,R,all}$ is modeled using		Li and Hong(2015)* $\Delta P$ is modeled using		The present study
			Weibull	Gumbel	lognormal	Weibull	
shanghai	50-year	31.3	28.77	30.8	29	28	30.44
	100-year	33.8	33.41	35.2	32	31	33.28
Hongkong	50-year	38.4	30.28	36.15	38	36	36.61
	100-year	39.5	34.04	40.78	42	38	40.52
Guangzhou	50-year	28.6	30.29	--	30	28	26.98
	100-year	31.3	34.46	--	33	30	29.29
Shenzhen	50-year	35	--	--	37	34	33.58
	100-year	38.4	--	--	41	36	36.7
Ningbo	50-year	28.6	29.24	--	31	30	30.5
	100-year	31.3	33.76	--	35	33	33.18

\*Chinese code (Xiang *et al.* 2004): the reported  $\hat{v}_{all,R,all}$  values are obtained from the Gumbel distribution directly fitted by the observed extreme wind speed data.

\*Ou *et al.* (2002): two values are provided for each city. The values are obtained from Weibull and Gumbel distributions fitted by the annual maximum wind speeds sampled by typhoon simulation.

\*Li and Hong (2015): two values are provided for each city based on lognormal and Weibull distributions fitted by  $\Delta P$  samples. The data for fitting key typhoon parameters are obtained from relevant typhoon records within a 500 km diameter radius. The diameter of this circular region is different from the 1000km diameter proposed in Xiao *et al.* (2011a) and adopted in this paper. The distributions of key typhoon parameters of these different papers are different due to the different sampling ranges

where  $\hat{v}$  denotes the independent variable of the one-dimensional empirical distribution function and  $\hat{v}_{all}^{(i)}$  is the  $i^{th}$  element of the annual maximum values in ascending order of the simulated typhoon wind speeds,  $\vec{\hat{v}}_{all}$  ( $\hat{v}_{all}^{(1)} \leq \hat{v}_{all}^{(2)} \leq \dots \leq \hat{v}_{all}^{(m)} \dots \leq \hat{v}_{all}^{(M)}$ ).  $\vec{\hat{v}}_{all}$  contains the maxima of the annual maximum typhoon wind speed samples in all sectors for each year, i.e.,  $\vec{\hat{v}}_{all}$  consists of the maxima of each row of  $[\hat{V}]_{M \times N}$ .

The non-directional extreme typhoon wind speed for a given  $R$ -year return period,  $\hat{v}_{all,R,all}$ , is obtained from:

$$1 - 1/R = F_0(\hat{v}_{all,R,all}) \quad (23)$$

Taking five typhoon-prone cities in the southeastern China coastal region as examples,  $\hat{v}_{all,R,all}$  for the 50- and 100-year return periods reported by the Chinese code (Xiang *et al.* 2004), Ou *et al.* (2002), Li and Hong (2015) and the present study are compared in Table 1. All of these values are calculated for the countryside with  $z_0=0.05$ . Fig. 1 shows the locations of the five cities and the typical tropical cyclone track directions, which are represented by the expectation of the approach angle  $\theta$ .

As shown in Table 1, the estimated values of the non-directional extreme typhoon wind speeds in the present

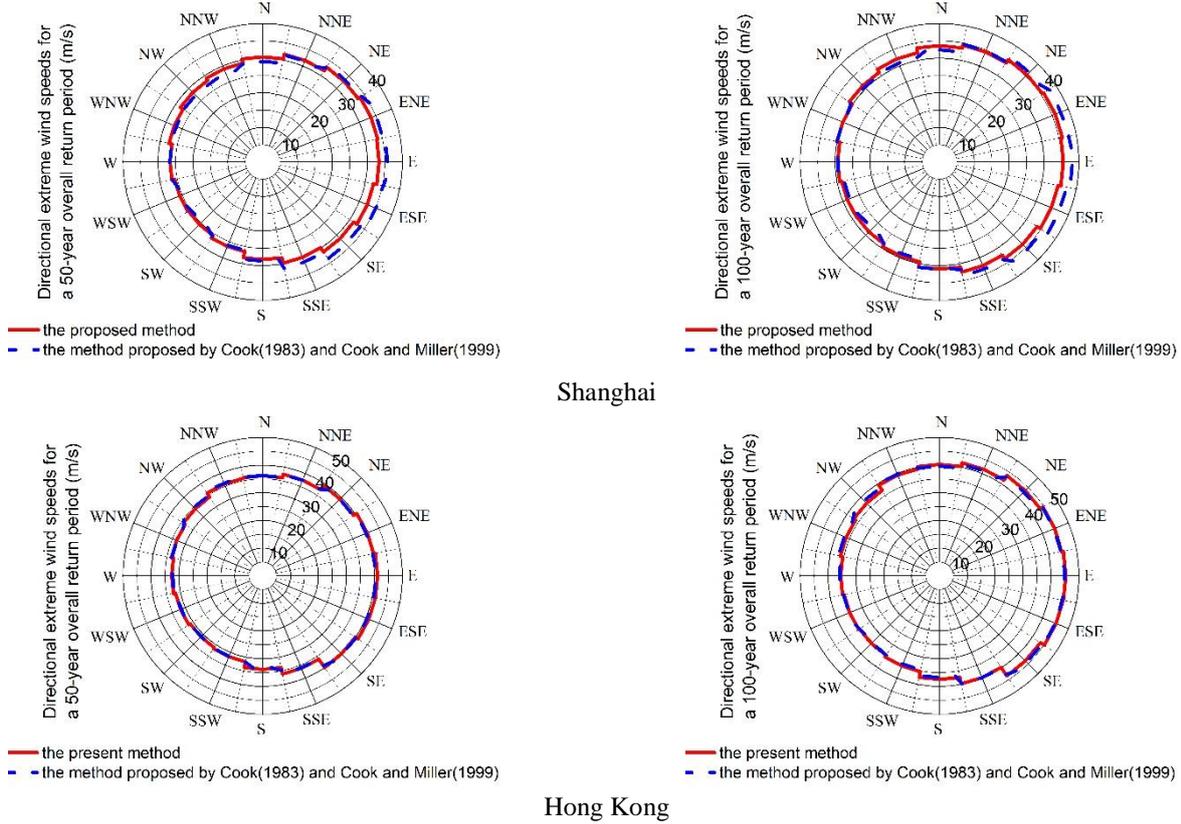


Fig. 2 The directional extreme typhoon wind speeds that ensure that the overall risk is distributed uniformly by direction, estimated by both the present method and the Cook and Miller method

study are within the results of previous studies, except for the results of the 50- and 100-year return periods for Guangzhou and the 50-year return period for Shenzhen. In these exceptional cases, the present study results are close to the results of Li and Hong (2015) with a Weibull distribution for  $\Delta P$ . The differences between the results are approximately 1 m/s.

#### 4.2 The directional extreme typhoon wind speed

In the Cook and Miller method for estimating  $\hat{v}_{n,R,all}$ , which ensures that the  $1/R$  overall annual risk is distributed uniformly by direction, scaling parameter  $k_{\hat{v},n}$  is defined as:

$$k_{\hat{v},n} = \hat{v}_{n,R,all} / \hat{v}_{n,R,n}, \quad (24)$$

where  $\hat{v}_{n,R,n}$  is the extreme wind speed with  $1/R$  annual risk in the  $n^{th}$  sector, irrespective of the other directions.

The Cook and Miller method is also used for the typhoon climate here, while the one-dimensional empirical distribution is utilized instead of the Fisher-Tippett Type I distribution for estimating directional extreme typhoon wind speeds to avoid any potential error due to fitting of the selected distribution model. The annual maximum typhoon wind speeds are simulated with the following process. (1)  $c_n = \hat{v}_{n,R,n} / \hat{v}_{all,R,all}$  for each sector is calculated. (2) The original simulated annual maximum typhoon wind speed

samples are transformed into new samples,  $\vec{\hat{v}}_{n,new} = \vec{\hat{v}}_n / c_n$ .

(3) The  $R$ -year return period value  $\hat{v}_{all,new,R,all}$ , is calculated irrespective of the direction, with the transformed samples  $\vec{\hat{v}}_{n,new}$  in all directions. (4) The scaling parameter  $k_{\hat{v},n} = \hat{v}_{all,new,R,all} / \hat{v}_{all,R,all}$  is estimated. (5) The directional extreme typhoon wind speeds, which ensure that the  $1/R$  overall annual risk is distributed uniformly by direction, are estimated as  $\hat{v}_{n,R,all-Cook} = k_{\hat{v},n} \hat{v}_{n,R,n}$ .

In the present method, the directional extreme typhoon wind speeds are estimated using Eqs. (20) and (21) with the simulated annual maximum typhoon wind speeds. The results estimated by the present method for the five cities are compared with those estimated with the Cook and Miller method, as shown in Table 2. For Hongkong, Guangzhou and Shenzhen, the results estimated by the two methods generally match well, whereas the differences are significantly larger for Shanghai and Ningbo. The results for Shanghai and Hongkong are shown in Fig. 2 as examples.

To determine what causes the differences,  $k_{\hat{v},n}$  ( $n=1, 2, \dots, N$ ) values for the 50- and 100-year return periods are directly obtained by Eq. (24) after  $\hat{v}_{n,R,all}$  and  $\hat{v}_{n,R,n}$  are estimated by the present method. The calculated  $k_{\hat{v},n}$  and mean values and the variation coefficient values are shown in Table 3 for the five typhoon-prone cities.

Then the  $k_{\hat{v},n}$  values estimated by the Cook and Muller

Table 2 The directional extreme typhoon wind speeds (m/s) that ensure that the overall risk is distributed uniformly by direction, estimated by both the present method and the Cook and Miller method

Cities	Method	Direction															
		N	N N E	N E	E N E	E	E S E	S E	S S E	S	S S W	S W	W S W	W	W N W	N W	N N W
50-year return period																	
Shang-hai	Present	30.3	31.7	32.9	33.6	33.3	32.4	31.3	29.8	28.2	26.4	25.6	25.7	26.5	27.4	28.3	29.0
	Cook and Miller	29.0	31.5	33.5	35.1	35.7	35.1	33.6	31.3	28.6	26.1	24.8	25.4	26.6	26.9	27.2	27.6
Hong Kong	Present	36.1	37.4	39.2	40.5	41.1	41.1	39.9	36.2	33.8	31.7	31.2	31.8	32.6	33.3	34.2	35.9
	Cook and Miller	36.2	37.3	39.0	40.2	40.6	40.6	40.1	36.1	33.4	31.5	31.2	31.7	32.4	33.2	34.9	36.3
Guang-zhou	Present	25.4	26.7	27.9	28.9	29.4	29.2	28.6	27.6	26.5	25.6	24.7	24.3	24.4	24.5	24.7	24.9
	Cook and Miller	25.5	26.9	28.6	29.7	30.0	29.7	28.8	27.4	25.9	24.5	23.7	23.8	24.1	24.5	24.8	25.1
Shen-zhen	Present	32.2	34.4	36.3	37.1	36.9	36.3	35.4	34.0	31.9	29.8	28.9	29.2	29.7	30.3	30.8	31.0
	Cook and Miller	33.3	34.8	36.5	37.3	37.1	36.4	34.9	33.4	31.1	29.2	28.7	28.9	29.3	29.9	30.7	31.6
Ning-bo	Present	30.2	31.5	32.7	33.7	34.0	33.7	32.8	31.3	29.4	27.6	26.4	26.2	26.5	26.9	27.3	28.4
	Cook and Miller	30.2	32.8	34.7	35.4	35.6	35.2	34.0	32.1	29.8	27.4	25.6	25.0	25.9	26.1	26.6	27.9
100-year return period																	
Shang-hai	Present	33.5	34.4	35.5	35.9	35.5	34.7	33.6	32.4	30.9	29.4	28.6	28.6	29.2	30.1	31.1	32.2
	Cook and Miller	32.3	34.6	36.5	37.7	38.2	37.2	35.7	33.6	31.0	28.7	27.4	28.0	29.4	30.0	30.5	30.9
Hong Kong	Present	40.2	41.5	43.4	44.6	45.2	45.1	43.5	39.7	37.4	35.3	34.5	34.9	35.5	36.2	37.0	39.3
	Cook and Miller	39.6	41.0	42.9	44.4	45.0	45.0	44.2	39.9	36.8	34.6	34.3	35.1	35.9	36.9	38.2	39.8
Guang-zhou	Present	27.6	29.0	30.2	31.1	31.5	31.4	30.9	30.0	29.1	28.2	27.5	26.9	26.7	26.6	26.6	26.8
	Cook and Miller	27.6	29.0	30.6	31.8	32.3	32.1	31.2	29.9	28.4	27.2	26.2	26.0	26.3	26.6	27.0	27.2
Shen-zhen	Present	34.7	37.0	38.8	39.6	39.6	39.2	38.5	37.2	35.3	33.2	32.0	31.9	32.4	32.9	33.4	33.6
	Cook and Miller	35.6	37.7	39.8	40.7	40.5	39.5	38.1	36.5	34.1	31.8	31.1	31.6	32.1	32.7	33.5	34.0
Ning-bo	Present	32.6	33.8	35.1	36.0	36.4	36.1	35.2	33.8	32.0	30.3	29.1	28.9	29.1	29.4	29.9	31.0
	Cook and Miller	33.1	35.3	36.6	37.5	37.9	37.5	36.4	34.6	32.2	29.9	28.2	27.6	28.2	28.7	29.4	30.7

method, which are the ratios of the non-directional extreme values of the transformed simulated typhoon wind speeds,  $\hat{v}_{all\_new,R\_all}$ , to the non-directional extreme values of the original simulated typhoon wind speeds,  $\hat{v}_{all,R\_all}$ , are also shown in Table 3. The underlying assumption is that the  $k_{\hat{v},n}$  values of each sector are the same because  $\hat{v}_{all\_new,R\_all}/\hat{v}_{all,R\_all}$  does not vary with the direction. The directional variations in  $k_{\hat{v},n}$  ( $n=1, 2, \dots, N$ ) in Shanghai and Ningbo are significantly greater than those in Hong Kong, Guangzhou and Shenzhen, as shown in Table 3. The larger the directional variations in  $k_{\hat{v},n}$  ( $n=1, 2, \dots, N$ ) are, the larger the error caused by the underlying assumption. Therefore, the differences in the results obtained by the two methods in Shanghai and Ningbo are significantly greater than those in Hongkong, Guangzhou and Shenzhen.

For each case, the mean values of  $k_{\hat{v},n}$  ( $n=1, 2, \dots, N$ ) in the present method match well with the  $k_{\hat{v},n}$  values obtained as  $\hat{v}_{all\_new,R\_all}/\hat{v}_{all,R\_all}$  in the Cook and Miller method. If  $k_{\hat{v},n}$  ( $n=1, 2, \dots, N$ ) does not vary significantly

with the direction, then the Cook and Miller method is sufficiently concise and precise.

#### 4.3 The directional factors and the directional risks

The directional factor in the  $n^{th}$  sector is defined by (Cook and Miller 1999)

$$\gamma_{n,R\_all} = \hat{v}_{n,R\_all} / \hat{v}_{all,R\_all} \quad (25)$$

The results of  $\gamma_{n,R\_all}$  for the 50- and 100-year return periods for the five typhoon-prone cities are shown in Table 4 for each sector.

The directional factors  $\gamma_{n,R\_all}$  calculated by the proposed method are larger than 1 in some directions of the stronger winds, as shown in Table 4, which are similar to those calculated by Cook (1983) and Cook and Miller (1999) for the UK. The directional risks can be adopted to explain this phenomenon.

Table 3 The scaling parameters  $k_{\hat{v},n}$  ( $n=1,2,\dots,N$ ) for typhoon climates obtained by Eq. (24) directly based on the present method and those obtained as  $\hat{v}_{all\_new,R\_all}/\hat{v}_{all,R\_all}$  based on the Cook and Miller method

Cities	Shanghai		Hongkong		Guangzhou		Shenzhen		Ningbo		
Return period	50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year	
Direction	N	1.4018	1.3240	1.2979	1.2746	1.2338	1.2172	1.2449	1.2033	1.2338	1.3118
	NNE	1.3490	1.2667	1.3023	1.2726	1.2317	1.2182	1.2700	1.2133	1.1966	1.2616
	NE	1.3140	1.2397	1.3078	1.2711	1.2113	1.1986	1.2771	1.2064	1.1982	1.2379
	ENE	1.2813	1.2153	1.3100	1.2625	1.2065	1.1923	1.2789	1.2010	1.1991	1.2502
	E	1.2494	1.1870	1.3157	1.2625	1.2130	1.1891	1.2808	1.2070	1.2008	1.2524
	ESE	1.2375	1.1910	1.3165	1.2601	1.2192	1.1942	1.2797	1.2268	1.2048	1.2595
	SE	1.2491	1.2031	1.2927	1.2351	1.2311	1.2048	1.3053	1.2479	1.2094	1.2658
	SSE	1.2762	1.2336	1.3053	1.2518	1.2468	1.2250	1.3094	1.2567	1.2224	1.2797
	S	1.3195	1.2727	1.3146	1.2769	1.2720	1.2470	1.3200	1.2791	1.2413	1.2957
	SSW	1.3602	1.3094	1.3074	1.2803	1.2944	1.2661	1.3141	1.2928	1.2661	1.3261
	SW	1.3850	1.3304	1.2989	1.2643	1.2951	1.2790	1.2963	1.2698	1.2900	1.3579
	WSW	1.3587	1.3069	1.3039	1.2480	1.2663	1.2592	1.2969	1.2496	1.3078	1.3742
	W	1.3360	1.2698	1.3104	1.2384	1.2537	1.2394	1.3024	1.2444	1.2913	1.3428
	WNW	1.3650	1.2776	1.3073	1.2334	1.2433	1.2179	1.3026	1.2427	1.2808	1.3542
	NW	1.3947	1.3049	1.2745	1.2160	1.2325	1.1992	1.2910	1.2321	1.2702	1.3467
	NNW	1.4125	1.3292	1.2839	1.2403	1.2285	1.2003	1.2611	1.2177	1.2615	1.3354
Mean	1.3306	1.2663	1.3031	1.2555	1.2424	1.2217	1.2894	1.2369	1.2421	1.3033	
Variation coefficient	0.04	0.04	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.03	
$\frac{\hat{v}_{all\_new,R\_all}}{\hat{v}_{all,R\_all}}$	1.3407	1.2763	1.3007	1.2551	1.2405	1.2178	1.2858	1.2352	1.2495	1.3132	

Table 4 The directional factors  $\gamma_{n,R\_all}$  ( $dir=1, 2, \dots, 16$ ) for the five typhoon-prone cities for the 50- and 100-year return periods

Cities	Shanghai		Hongkong		Guangzhou		Shenzhen		Ningbo		
return period	50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year	
Direction	N	1.00	1.01	0.99	0.99	0.94	0.94	0.96	0.95	0.99	0.98
	NNE	1.04	1.03	1.02	1.03	0.99	0.99	1.02	1.01	1.03	1.02
	NE	1.08	1.07	1.07	1.07	1.04	1.03	1.08	1.06	1.07	1.06
	ENE	1.10	1.08	1.11	1.10	1.07	1.06	1.10	1.08	1.10	1.09
	E	1.09	1.07	1.12	1.12	1.09	1.08	1.10	1.08	1.11	1.10
	ESE	1.06	1.04	1.12	1.11	1.08	1.07	1.08	1.07	1.11	1.09
	SE	1.03	1.01	1.09	1.07	1.06	1.05	1.05	1.05	1.08	1.06
	SSE	0.98	0.97	0.99	0.98	1.02	1.03	1.01	1.01	1.03	1.02
	S	0.93	0.93	0.92	0.92	0.98	0.99	0.95	0.96	0.96	0.96
	SSW	0.87	0.88	0.86	0.87	0.95	0.96	0.89	0.91	0.91	0.91
	SW	0.84	0.86	0.85	0.85	0.92	0.94	0.86	0.87	0.87	0.88
	WSW	0.85	0.86	0.87	0.86	0.90	0.92	0.87	0.87	0.86	0.87
	W	0.87	0.88	0.89	0.88	0.90	0.91	0.88	0.88	0.87	0.88
	WNW	0.90	0.90	0.91	0.89	0.91	0.91	0.90	0.90	0.88	0.89
	NW	0.93	0.94	0.93	0.91	0.91	0.91	0.92	0.91	0.89	0.90
	NNW	0.95	0.97	0.98	0.97	0.92	0.92	0.92	0.91	0.93	0.94

Table 5 The directional risks of exceeding  $\hat{v}_{n,R\_all}$  and  $\hat{v}_{all,R\_all}$  in each sector for the 50- and 100-year return periods for the five typhoon-prone cities

Cities		Shanghai		Hongkong		Guangzhou		Shenzhen		Ningbo	
return period		50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year
The directional risk of exceeding $\hat{v}_{all,R\_all}$	N	0.0037	0.0019	0.0038	0.0018	0.0024	0.0011	0.0026	0.0009	0.0034	0.0014
	NNE	0.0051	0.0025	0.0049	0.0023	0.0037	0.0018	0.0049	0.0020	0.0053	0.0022
	NE	0.0068	0.0035	0.0066	0.0033	0.0055	0.0026	0.0076	0.0036	0.0072	0.0032
	ENE	0.0083	0.0041	0.0082	0.0042	0.0076	0.0035	0.0089	0.0045	0.0089	0.0044
	E	0.0087	0.0039	0.0089	0.0046	0.0087	0.0041	0.0086	0.0043	0.0096	0.0048
	ESE	0.0070	0.0029	0.0089	0.0046	0.0083	0.0040	0.0073	0.0036	0.0089	0.0044
	SE	0.0050	0.0020	0.0079	0.0037	0.0066	0.0032	0.0059	0.0029	0.0072	0.0034
	SSE	0.0032	0.0014	0.0039	0.0016	0.0048	0.0024	0.0044	0.0020	0.0048	0.0021
	S	0.0020	0.0009	0.0023	0.0009	0.0035	0.0018	0.0028	0.0012	0.0028	0.0011
	SSW	0.0013	0.0006	0.0014	0.0005	0.0027	0.0014	0.0016	0.0007	0.0016	0.0006
	SW	0.0010	0.0004	0.0011	0.0003	0.0022	0.0012	0.0012	0.0005	0.0011	0.0004
	WSW	0.0011	0.0004	0.0011	0.0003	0.0019	0.0011	0.0011	0.0004	0.0010	0.0003
	W	0.0013	0.0005	0.0013	0.0003	0.0018	0.0009	0.0012	0.0003	0.0010	0.0004
	WNW	0.0016	0.0007	0.0017	0.0004	0.0017	0.0008	0.0014	0.0004	0.0012	0.0004
	NW	0.0022	0.0009	0.0021	0.0006	0.0016	0.0007	0.0017	0.0005	0.0014	0.0006
NNW	0.0028	0.0013	0.0036	0.0014	0.0018	0.0008	0.0018	0.0006	0.0020	0.0009	
The directional risk of exceeding $\hat{v}_{n,R\_all}$		0.0038	0.0018	0.0042	0.0019	0.0040	0.0019	0.0040	0.0018	0.0038	0.0017

Table 6 The scaled directional factors for the five typhoon-prone cities for the 50- and 100-year return periods

Cities		Shanghai		Hongkong		Guangzhou		Shenzhen		Ningbo	
return period		50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year	50-year	100-year
Direction	N	0.91	0.94	0.88	0.88	0.86	0.87	0.87	0.88	0.89	0.89
	NNE	0.95	0.95	0.91	0.92	0.91	0.92	0.93	0.94	0.93	0.93
	NE	0.98	0.99	0.96	0.96	0.95	0.95	0.98	0.98	0.96	0.96
	ENE	1.00	1.00	0.99	0.98	0.98	0.98	1.00	1.00	0.99	0.99
	E	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	ESE	0.96	0.96	1.00	0.99	0.99	0.99	0.98	0.99	1.00	0.99
	SE	0.94	0.94	0.97	0.96	0.97	0.97	0.95	0.97	0.97	0.96
	SSE	0.89	0.90	0.88	0.88	0.94	0.95	0.92	0.94	0.93	0.93
	S	0.85	0.86	0.82	0.82	0.90	0.92	0.86	0.89	0.86	0.87
	SSW	0.79	0.81	0.77	0.78	0.87	0.89	0.81	0.84	0.82	0.83
	SW	0.76	0.80	0.76	0.76	0.84	0.87	0.78	0.81	0.78	0.80
	WSW	0.77	0.80	0.78	0.77	0.83	0.85	0.79	0.81	0.77	0.79
	W	0.79	0.81	0.79	0.79	0.83	0.84	0.80	0.81	0.78	0.80
	WNW	0.82	0.83	0.81	0.79	0.83	0.84	0.82	0.83	0.79	0.81
	NW	0.85	0.87	0.83	0.81	0.83	0.84	0.84	0.84	0.80	0.82
NNW	0.86	0.90	0.88	0.87	0.84	0.85	0.84	0.84	0.84	0.85	

To distribute the overall risk uniformly by direction, the directional risks of exceeding  $\hat{v}_{n,R\_all}$  in each sector are assumed to be the same, namely, equal to the uniformly distributed overall annual risk  $1-p_{dir,R\_all}$ . The directional

risks of exceeding  $\hat{v}_{all,R\_all}$  in each sector are calculated by  $1-F_{0,n}(\hat{v}_{all,R\_all})$  with Eq. (16). The results of  $1-p_{dir,R\_all}$  and  $1-F_{0,n}(\hat{v}_{all,R\_all})$  are shown in Table 5 for the 50- and 100-year return periods for the five typhoon-prone cities.

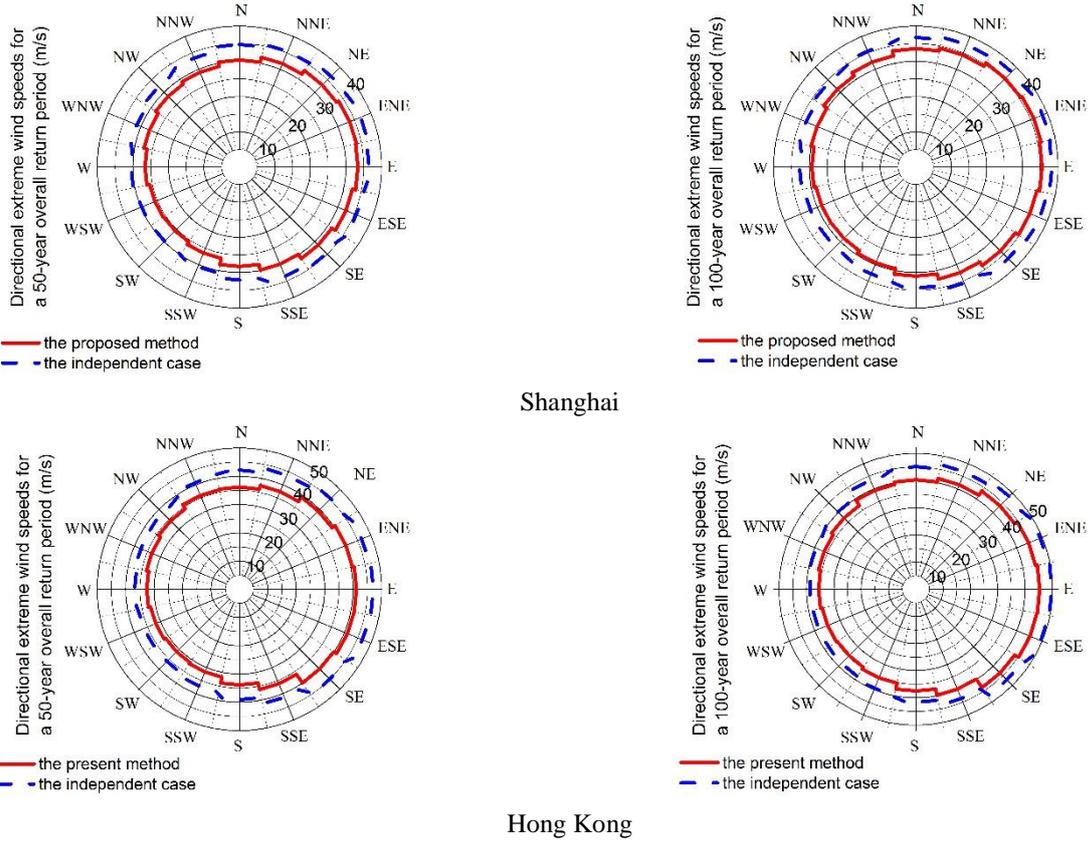


Fig. 3 The estimated directional extreme typhoon wind speeds by the present method and the independent case

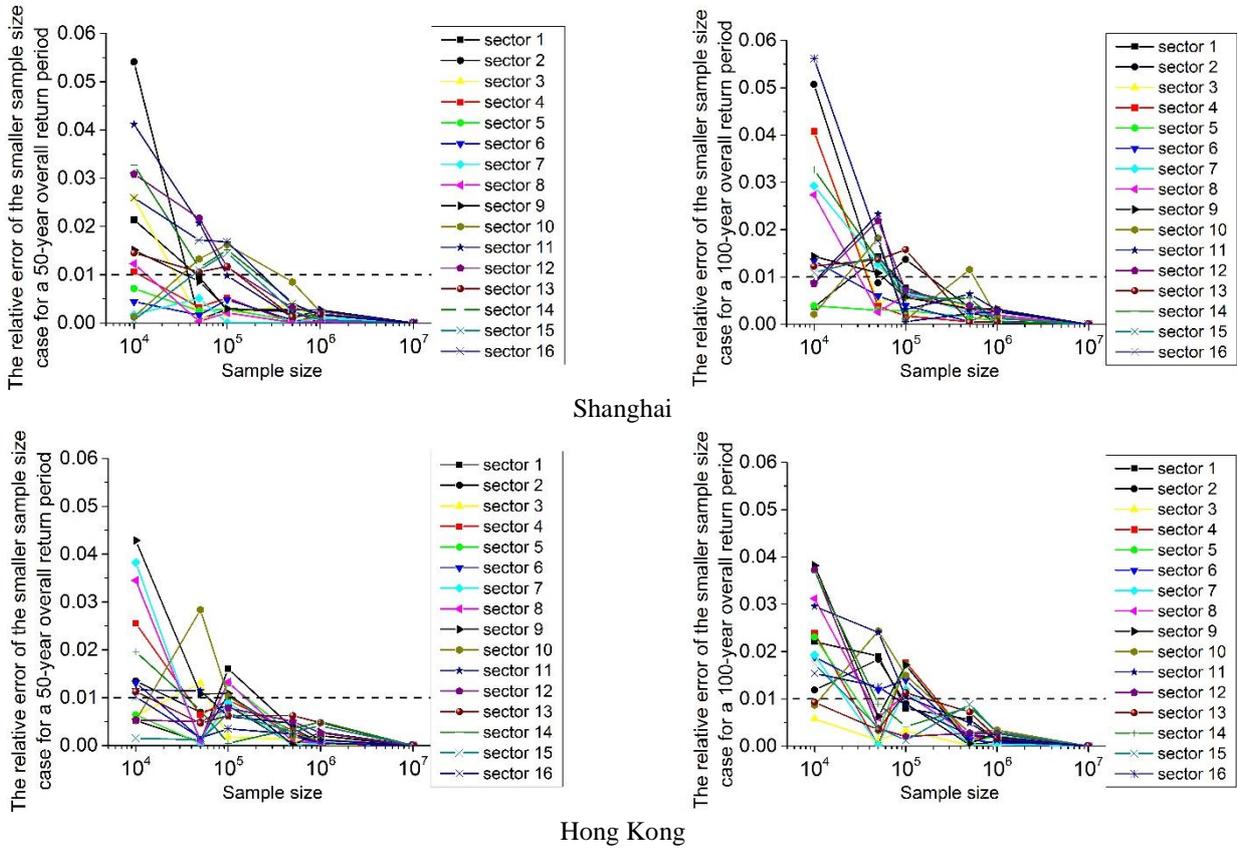


Fig. 4 The relative errors of the results caused by relatively small annual maximum wind speed sample sizes

The sums of the directional risks in all sectors are not equal to the  $1/R$  overall risk in Table 5 because the extremal events of the annual maximum wind speeds in the different sectors are not mutually exclusive events for the five typhoon-prone cities. From Tables 4-5, it can be concluded that the directional risks that exceed  $\hat{v}_{all,R,all}$  are larger than the directional risks that exceed  $\hat{v}_{n,R,all}$  in some directions of the stronger winds. The higher the directional risk in a sector is, the lower the estimated extreme typhoon wind speed in this sector is. Therefore, in some directions of the stronger winds, the estimated non-directional extreme typhoon wind speed  $\hat{v}_{all,R,all}$ , irrespective of the direction, is lower than the estimated directional extreme typhoon wind speed  $\hat{v}_{n,R,all}$  with an equal directional risk. This phenomenon causes the directional factors  $\gamma_{n,R,all}$  to be larger than 1 in these directions.

From the perspective of distributing the overall risk uniformly by direction, the obtained directional factors seem strict. However, the wind-induced response estimated by the sector-by-sector method with the directional factors, which ensure the overall risk is distributed uniformly by direction, may result in predictions exceeding those obtained irrespective of the direction (Holmes and Bekele 2015). Further research is needed to find an appropriate method instead of the sector-by-sector method to connect the wind-induced responses and these directional factors. The drafting committee for the UK standard (Standard, B, 1997) chose a pragmatic view that the use of directional factors should not alter the highest design loads obtained irrespective of the direction, so they scaled the original directional factors to be unity in the direction of the strongest winds (Cook and Miller 1999). The directional factors in Table 4 are scaled and listed in Table 6. The scaled directional factors are not strict in theory. Similar to the scaled directional factors for the UK standard (Standard, B, 1997), Table 5 is an approximate suggestion for the Chinese code (GB50009, 2012).

#### 4.4 Influence of the correlations of extreme typhoon wind speeds among the different directions

If the extreme wind speeds among the different sectors are independent of each other, the relationship between one-dimensional margins and the return period  $R$  is:

$$1 - 1/R = \prod_{n=1}^N F_{0,n}(\hat{v}_n) \quad (26)$$

To distribute the overall risk uniformly by direction (Cook 1983, Cook and Miller 1999), the non-exceedance probability of each sector is assumed to be the same. The extreme typhoon wind speed of the independent case in each sector,  $\hat{v}_{n,R,all,ind}$ , for a given  $R$ -year return period is obtained from:

$$F_{0,n}(\hat{v}_{n,R,all,ind}) = (1 - 1/R)^{1/N} \quad \text{for } n=1, 2, \dots, N, \quad (27)$$

where  $F_{0,n}(\cdot)$  is the one-dimensional marginal empirical probability distribution in the  $n^{\text{th}}$  sector, i.e., Eq. (16).

The directional extreme typhoon wind speeds estimated considering the correlations of extreme wind speeds among

the different sectors by the present method,  $\hat{v}_{n,R,all}$ , are compared with those for the independent case,  $\hat{v}_{n,R,all,ind}$ , for the 50- and 100-year return periods. The results for the two cities are given in Fig. 3 as examples. These  $\hat{v}_{n,R,all}$  and  $\hat{v}_{n,R,all,ind}$  values are notably different. The results are clearly overestimated by the independent case approach, and the overestimations in some sectors are even larger than 15% and 11% for  $\hat{V}_{n,R,all}$  for the 50- and 1-year return periods, respectively. The correlations for extreme typhoon wind speeds among the different directions are not negligible.

#### 4.5 Influence of the sample size

Since the empirical copula is a consistent estimator of the true underlying copula, as the sample size increases, it become increasingly less likely that there is a deviation between the empirical copula and the true underlying copula (Ganssler and Stute 1987; Fermanian *et al.* 2004; Tsukahara 2005; Devore 2015). However, the sample size cannot actually be infinite. To discuss the influence of the sample size on the results and explain the appropriateness for setting the sample size of the annual maximum wind speeds in each sector to  $10^6$  in section 2, several groups of samples are generated using Monte Carlo simulation as described in section 2, and the different groups have different sample sizes. Then, the directional extreme typhoon wind speeds with a uniformly distributed overall risk for a given return period are estimated using the method proposed in section 3 for each sample group. If the estimated results hardly change as the sample size increases beyond a certain value, taking this value as the sample size is considered to be adequate. To investigate the influence of the sample size, the tested sample size is set as  $1 \times 10^4$ ,  $5 \times 10^4$ ,  $1 \times 10^5$ ,  $5 \times 10^5$ ,  $1 \times 10^6$  and  $1 \times 10^7$ . Six groups of samples are generated with six different sample sizes.

For the 50- and 100-year return periods, six groups of the directional extreme wind speeds with a uniformly distributed overall risk are estimated with the six groups of generated samples. The relative errors of the results from a relatively small sample size are then obtained when the results based on the  $1 \times 10^7$  year samples are considered exact values. The relative errors for two cities are given in Fig. 4 as examples, which decrease with the increase in sample size on the whole. For both the 50- and 100-year return periods for each city, the relative errors between the estimated results for a sample size of  $1 \times 10^6$  and those for the large sample size of  $1 \times 10^7$  are less than 1% in every sector, which is negligible in civil engineering. It is completely adequate to set the sample size to  $1 \times 10^6$  for estimating the directional extreme wind speeds for the 50- and 100-year return periods

## 5. Conclusions

Fully considering the correlations of extreme typhoon wind speeds among the different directions with the empirical copula approach and typhoon simulation, the directional extreme typhoon wind speeds that ensure that

the overall risk is distributed uniformly by direction are estimated for the 50- and 100-year return periods for five typhoon-prone cities in the coastal region of southeastern China. The estimated results are compared with those estimated by the Cook and Miller method, and then the directional factors and directional risks are discussed. The influences of the directional correlation and sample size of the simulated annual maximum typhoon wind speed samples on the estimated results are also discussed. Several conclusions are obtained as follows:

- The non-directional extreme typhoon wind speeds, irrespective of the direction, for a given return period estimated by the proposed method are similar to those reported in previous studies.

- The scaling parameters, which are the ratios of the directional extreme wind speeds with a  $1/R$  overall annual risk to those with a  $1/R$  annual risk in each sector, are different for the various directions in some typhoon-prone cities, which make the directional extreme typhoon wind speeds with a uniformly distributed risk by direction calculated by the present method different from those estimated by the Cook and Miller method.

- In some directions of stronger winds, the directional risks of exceeding the non-directional extreme typhoon wind speed are higher than the uniformly distributed overall annual risk for the given return periods. This phenomenon causes the directional factors, which ensure that the overall risk is distributed uniformly by direction, to be larger than 1 in these directions.

- The directional extreme typhoon wind speeds with a uniformly distributed risk by direction for a given return period are notably overestimated when the correlations of extreme typhoon wind speeds among the different directions are ignored.

- The larger the sample size of simulated annual maximum wind speeds in each sector is, the more accurate the results of the estimated directional extreme typhoon wind speeds with a uniformly distributed risk by direction are. It is completely adequate to set the sample size to  $1 \times 10^6$  for estimating the directional extreme wind speeds for the 50- and 100-year return periods.

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