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Determination of the Strouhal number based on the aerodynamic behavior of rectangular cylinders

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Abstract. The Strouhal number is an important nondimensional number which is explanatory of aerodynamic instability phenomena. It takes on the different characteristic constant value depending upon the cross-sectional shape of the body being enveloped by the flow. A number of investigations into this subject, especially on the drag test, surface pressure test and hot-wire test, have been carried out under the fixed state of the body in the past. However, almost no investigations concerning the determination of the *St* on wind-induced vibration of the body have been reported in the past even though the aerodynamic behavior of the body is very important because the construction of wind-sensitive structures is recently on the sharp increase. Based on a series of wind tunnel tests, this paper addresses a new method to determine the Strouhal number of rectangular cylinder in the uniform flow. The central idea of the proposed method is that the Strouhal number can be obtained directly by the aerodynamic behaviors of the body through wind-induced vibration test. The validity of proposed method is evaluated by comparing with the results obtained by previous studies in three B/Ds at attack angle 0° and a square cylinder with various attack angles. The values and trends of the proposed Strouhal numbers are in good agreements with values of previous studies. And also, the Strouhal numbers of B/D = 1.5 and 2.0 with various attack angles are obtained by the proposed method and verified by other method. This proposed method is as good as any other previous methods to obtain the Strouhal number.

Key words: determination of the Strouhal number.

1. Introduction

Detailed information regarding to the flow around a rectangular cylinder in a uniform flow is of special interest for the basic understanding of aerodynamics of the body, and is of great importance in the study of aerodynamic instability phenomena such as the galloping, flutter and vortex induced vibration. These phenomena may not only pose significant risk in the structural failure, but also cause discomfort to the occupants and generate serious serviceability problems. To prevent and avoid these phenomena, the physical mechanism of flow around structures should be investigated.

Generally, a fixed bluff body under wind sheds alternating vortices. Thus, the flow separates from

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the body at upwind corners and advances by cyclically alternating vortices that are formed by turns at the top and bottom edges and finally swept downstream (Karman vortex). The regularity of such wake effects was first reported by Strouhal (1878) who pointed out that the vortex-shedding phenomenon is descriable in the term of a nondimensional number which is known as the Strouhal number (St). The number takes a unique characteristic constant value depending upon the cross-sectional shape of the body being enveloped by the flow. Because of this characteristics, the Strouhal number has been recognized as one of the important index to understand the physical mechanism of aerodynamic instability phenomena. Therefore, a great deal of researchers efforts have been devoted to the determination of the Strouhal number. Vickery (1966) suggested to determine the St in a smooth and a turbulent flow through measurements of fluctuating lifts and drags on a long square cylinder under the state of various attack angles. Bearman (1967) also reported the St of a square cylinder in a smooth flow, which was a little different from Vickery's. Nakaguchi (1968) investigated the correlation between the St and various B/D (dimensional ratio) of rectangular cylinder through aerodynamic drag tests at attack angle 0^o, and discovered an interesting fact that there exists the discontinuity of the St at the particular value of B/D = 2.8. Otsuki (1974) also investigated this correlation through the aerodynamic force measurement test, but no attention was given to the effect of attack angles in his study. Lee (1975) examined the effect of turbulence on the St for a square cylinder with various attack angles through surface pressure tests and found that the St is shown to increase to a maximum at the angle at which the mean drag is a minimum, i.e., $\alpha = 15^o$. Okajima (1982) conducted hot-wire test for a series of the experiments on the St of various rectangular cylinders under the state of various Reynolds numbers at attack angle 0^o and found that the region of Reynolds number where the discontinuity occurs in the Strouhal number curve is strongly dependent upon the dimensional ratio of the cylinder.

To determine the Strouhal number of the body, three variables must be given; they are, the vortex shedding frequency (f_s), wind velocity (V), and across-wind dimension (D) of the body. Because the across-wind dimension of the body is already given and the wind velocity can be determined by the wind tunnel operation, the vortex shedding frequency which is the frequency of cyclically alternating vortices becomes the only unknown variable. Therefore, most previous studies to determine the Strouhal number were focused on vortex shedding frequency.

In the majority of previous studies, the determination of the Strouhal number was based on the three basic test results. The first test result is the signal data of aerodynamic drag (C_D) and lift coefficient (C_L) obtained by the aerodynamic force measurement test. The second one is the signal data of pressure coefficient obtained by the surface pressure test and finally, the signal data of fluctuating flow velocity obtained by the hot-wire test. In the method using these three tests, the vortex shedding frequency (f_s) or the Strouhal number under the fixed state of the body is determined by the power spectral density (PSD) based on the signal data.

In the present study, a new method to determine the Strouhal number of rectangular cylinders in a uniform flow based on the aerodynamic behaviors of the body during wind-induced vibration is proposed. Through the wind-induced vibration test, the signal data of the behavior of the body can be obtained and the vortex shedding frequency can be obtained by PSD of this signal data. The validity and effectiveness of the proposed method are confirmed by comparison with those of previous studies

2. Proposed method to determine the strouhal number

2.1. The Strouhal number

Strouhal (1878) suggested that the regularity of vortex shedding phenomenon is descriable in the term of a nondimensional number ;

$$St = \frac{f_s D}{V} \tag{1}$$

where, St : the Strouhal number
f_s : vortex-shedding frequency
D : across-wind dimension of the body
V : mean velocity of the uniform flow

This equation indicates that the Strouhal number (*St*) depends upon the body geometry and the Reynolds number. As the separation point is changed by the Reynolds number (*Re*), the vortex-shedding frequency and the *St* are influenced by *Re*. It has been known that the *St* of circular cylinder has a constant value (about 0.20) because circular section is not affected by attack angles. In the case of a sharp-edged body like a rectangular cylinder, where the separation points are almost fixed at the leading edges, the aerodynamic characteristics are said to be relatively insensitive to Reynolds number. In recent studies, it is reported that the phenomenon of transition from uniform to turbulent flow, which occurs in the flow near the cylinder at certain values of the Reynolds number, should be taken into consideration (Okajima 1982). Fig. 1 shows the relation between the *St* and various B/Ds of rectangular cylinders obtained by aerodynamic drag tests (Nakaguchi 1968). It can be noticed that there is a certain B/D value of rectangular cylinders where the *St* changes abruptly.

It is known that the vortex-shedding frequency is related to the width of the wake (Roshiko 1954). When the separated flows detach themselves suddenly from the surface, which results in the

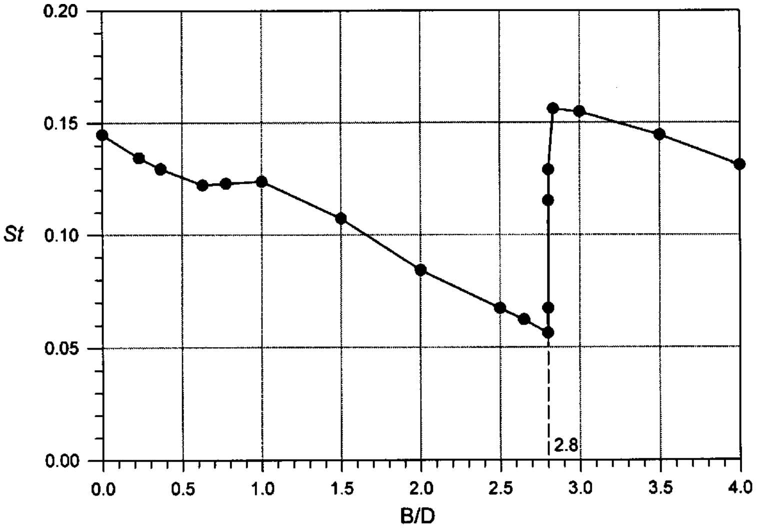

Fig. 1 Relation between the *St* and various B/Ds of rectangular cylinders (Nakaguchi 1968)

widening of the wake, and the vortex spacing will be large longitudinal length if a constant ratio of vortex spacing to wake width is presumed. This would lead to a decrease in the vortex-shedding frequency, and thus to a decrease in the Strouhal number.

2.2. Procedure of the proposed method to determine the Strouhal number

The central idea of the proposed method is that the Strouhal number can also be obtained directly by the aerodynamic behaviors of the body through wind-induced vibration test. In the proposed method, the Lock-in theory in which the vortex-shedding frequency (f_s) approximately coincide with the natural frequency (f_n) of structure ($f_s/f_n \approx 1.0$) in the occurrence range of vortex induced vibration, and the spectral frequency analysis based on the signal data of aerodynamic behaviors of the body are adopted as the two key vehicles to find out the vortex shedding frequency (f_s). The validity of the *St* obtained by the proposed method is clearly shown in the test results hereafter.

Thus, to calculate the Strouhal number, the vortex shedding frequency (f_s) needs be determined first. The following are general procedures to find out the vortex shedding frequency or the Strouhal number by the proposed method ;

1. Perform the wind-induced vibration test to obtain aerodynamic displacements of the body. Based on a series of wind tunnel tests, the vibrating behaviors of the test models were classified into the galloping and the vortex-induced vibration (Choi and Kwon 2000).
2. Execute the spectral frequency analysis for the signal data (displacement history) of aerodynamic behaviors using Fast Fourier Transform (FFT). Power spectral density (PSD) is obtained from the signal data within the occurrence range of wind-induced vibration (galloping or vortex-induced vibration) indicated in Fig. 5. Based on the Lock-in theory, f_n is obtained first by PSD and used for f_s in the case of vortex-induced vibration. If there is no evident VIV in the galloping region ($\alpha = 0\sim10^o$), the vortex-shedding frequency is determined by the assumption that the VIV and

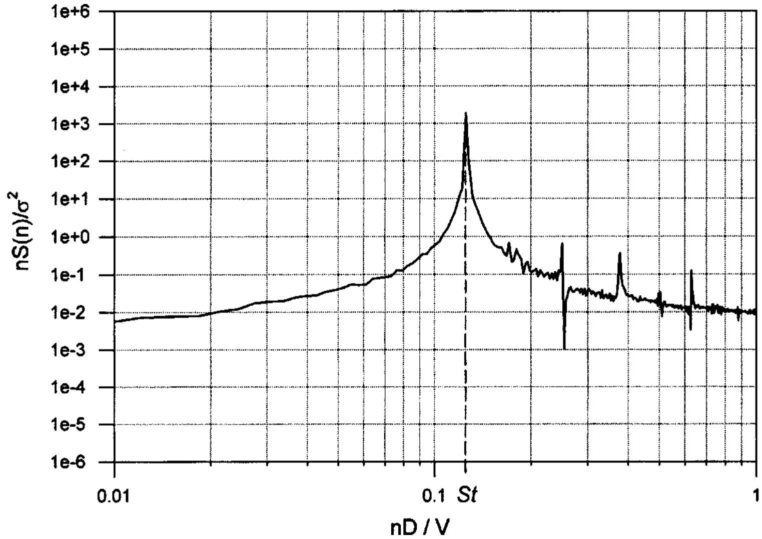

Fig. 2 An example of the nondimensional power spectrum (B/D = 1.0, α = 30^o)

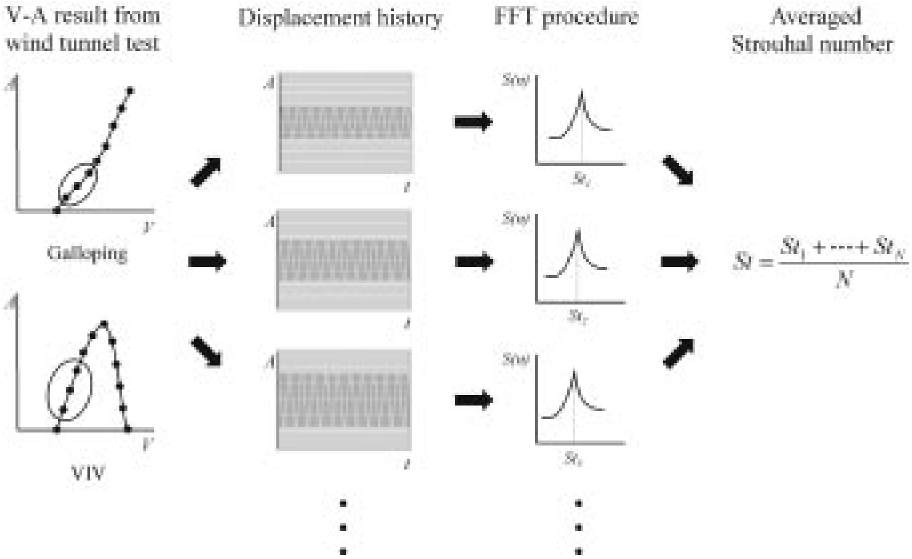

Fig. 3 Procedure of the proposed method to determine the Strouhal number

the galloping will be mixed in a low damping system (Fig. 5).

3. Calculate the Strouhal number by Eq. (1). The *St* can be determined directly from the nondimensional power spectrum using Eq. (1). Fig. 2 shows an example of such a spectrum. A number (N) of the Strouhal numbers can be identified according to N different wind velocities (*V*). Then, the mean value of these the Strouhal numbers is taken as the proposed Strouhal number (Fig. 3).

3. Experiments

3.1. Experimental set-up

The experiments were carried out with the Eiffel type wind tunnel at Korea Advanced Institute of Science and Technology (KAIST) whose test section is 1 m$\times$1 m and the length of test zone is 4 m. The maximum wind velocity is about 17.0m/s and the turbulence intensity is 0.15% (uniform flow state).

To measure the aerodynamic response (displacement) caused by wind-induced vibration, the sectional model tests were carried out in a uniform flow. The test model was set up to have two response modes, i.e., the acrosswind mode and the torsional mode. The movement of the test model in the flow direction is restrained (Fig. 4). The fluctuating displacements were measured on the model through the position sensor, low pass filter (cut off frequency : 20 Hz), A/D converter and computer. The sampling rate was 100 Hz (Δt = 0.01sec.) with sampling time of T = 10.24 sec.(1024 points).

3.2. Test models

The B/Ds of test models are 1.0, 1.5 and 2.0. The aerodynamic responses of B/D = 1.0 were measured under various attack angles (α), i.e., from 0^o to 45^o increased by 5 degrees a step, and in the various wind velocities increased by 0.2 m/s a step. The Reynolds number of this experiment is

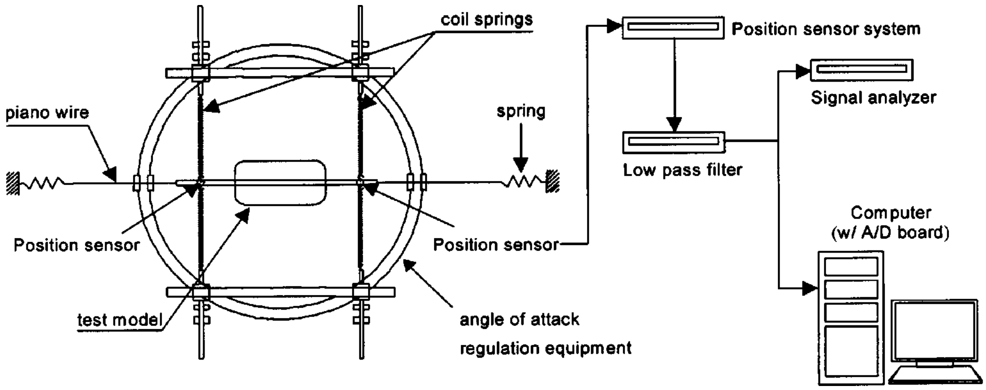

Fig. 4 Experimental setup

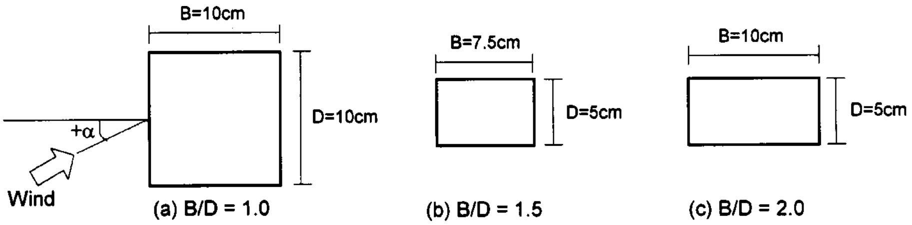

Fig. 5 Crosssectional shapes of test models

Table 1 Wind tunnel test conditions

B/D	Mass	δ_b	δ_t	f_b(Hz)	f_t(Hz)
1.0	2.08	0.0140	0.0148	4.532	9.245
1.5	1.76	0.0150	0.0163	4.532	9.485
2.0	1.96	0.0170	0.0180	4.313	9.750

$Re = V \cdot D/\nu = 6.6 \times 10^2 \sim 4.0 \times 10^4$. The positive direction of attack angle is defined as the upwind side of the model lifts (Fig. 5). Test conditions are given in Table 1.

4. Test results

4.1. The Strouhal number of B/D=1.0, 1.5 and 2.0 with attack angle 0^o

The proposed *St* values of three rectangular cylinders (B/D=1.0, 1.5 and 2.0) with a fixed attack angle 0^o are compared with the results of previous studies (Nakaguchi 1968, Otsuki 1974 and Okajima 1982) to verify the validity of the proposed method. From the wind tunnel test results, it is shown that the common dominant aerodynamic phenomenon of these test models at the attack angle 0^o is galloping (Fig. 6). In Fig. 7, the *St* estimated by current method are compared with those obtained by previous studies. The proposed *St* is only slightly different from Nakaguchi's results at each B/D, and thus these proposed *St* values can be considered in good agreements with the values in the references.

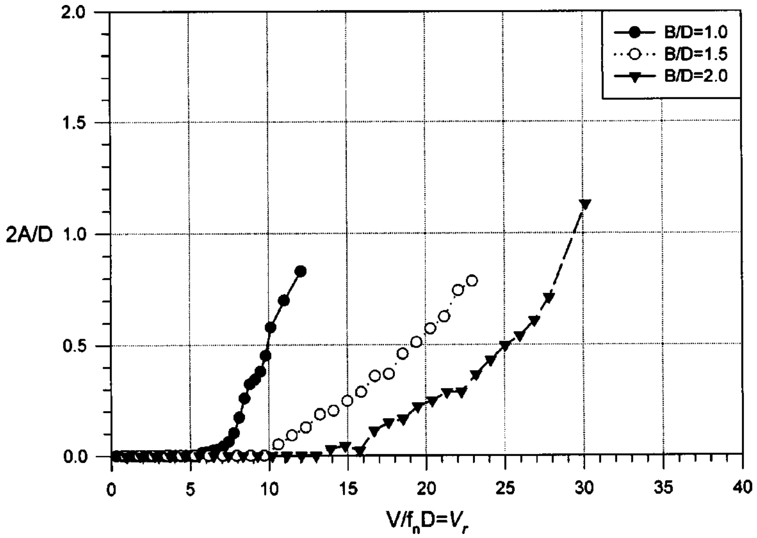

Fig. 6 Aerodynamic behaviors of three rectangular cylinders ($\alpha = 0^o$)

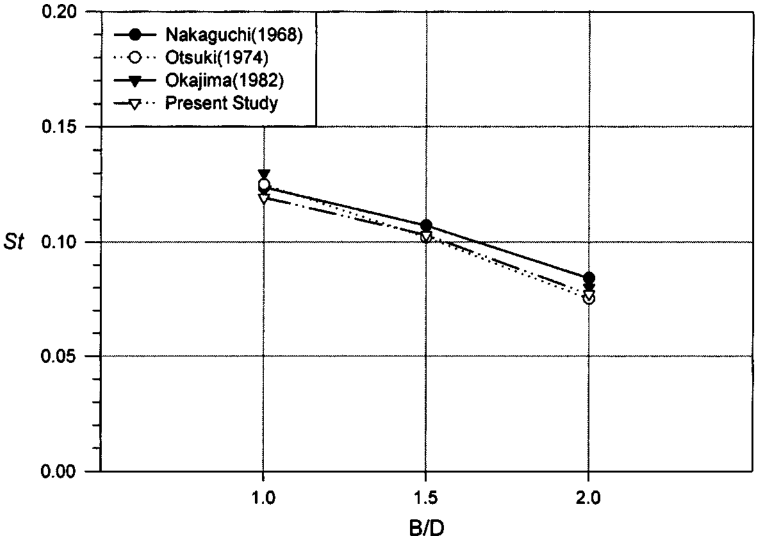

Fig. 7 Comparison with the proposed *St* and other studies

4.2. The Strouhal number of B/D=1.0 with various attack angles

It is shown from the wind-induced vibration test that two important aerodynamic phenomena of a square cylinder (B/D=1.0) are the galloping and the vortex-induced vibration (Fig. 8). When a is larger than 15^o, no galloping phenomena occurred but the occurrence of vortex-induced vibration is observed at each attack angles. This attack angle $\alpha = 15^o$ has been known as the critical occurrence point of the galloping phenomenon.

Fig. 9 shows the *St* values of a square cylinder with various attack angles. Test results from

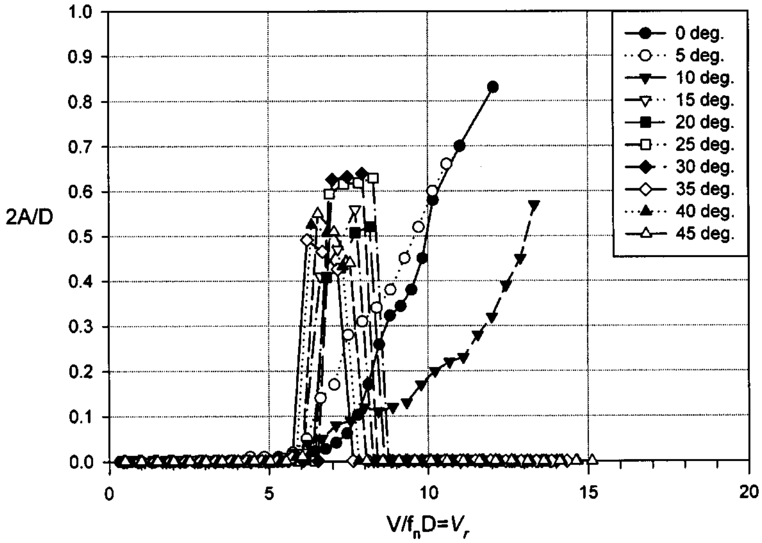

Fig. 8 Aerodynamic behaviors of a square cylinder with various attack angles (B/D=1.0)

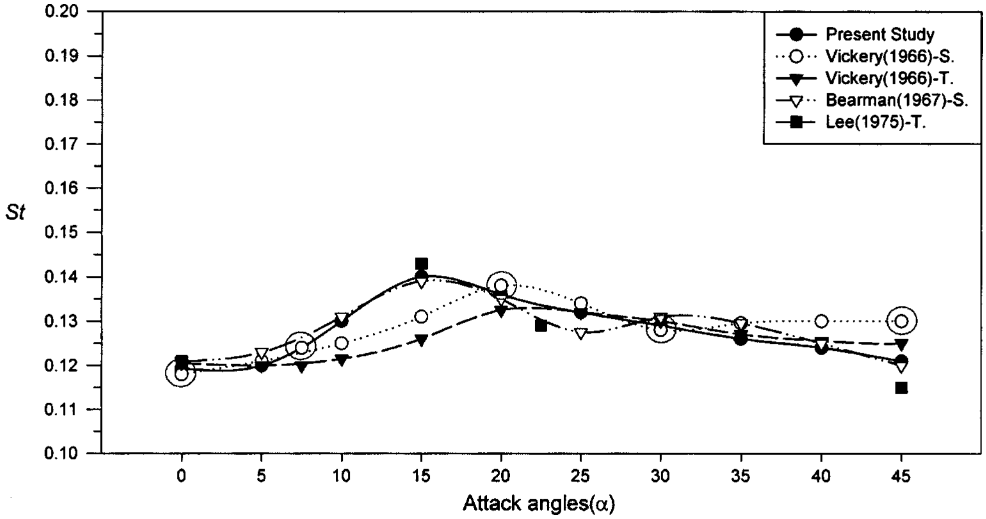

Fig. 9 The Strouhal numbers of a square cylinder (S. : Smooth flow, T. : Turbulent flow)

previous researchers are somewhat scattered at each attack angle. The proposed *St* are plotted in the Fig. 9 to compare with other results, and found to be in the closest agreement with Bearman's results (1967) except the cases of $\alpha = 25^{\circ}$ and 35°. It is also observed that the trend of the proposed *St* is similar to that of Lee's results (1975), i.e., the *St* has the maximum value at $\alpha = 15^{\circ}$ and decreases as α changes upto 45°. However, it should be noted that while the Lee's results were obtained in a turbulent flow, tests in this study were carried out in a uniform flow. Because Vickery's results (1966) were measured at smaller number of attack angles (0°, 7.5°, 20°, 30°, 45° ; indicated by large circles in Fig. 9) and the *St* at other attack angles were obtained by curve-fitting, Vickery's results do not have the important value at $\alpha = 15^{\circ}$. Thus, the proposed *St* values in this study show good agreements with values of other studies for a square cylinder at all attack angles tested.

4.3. The Strouhal number of B/D=1.5 and 2.0 with various attack angles

Fig. 10 and 11 show respectively the aerodynamic behaviors of B/D = 1.5 and 2.0 with various attack angles obtained by wind-induced vibration tests. Based on the aerodynamic behaviors of these test models, the Strouhal numbers of these models with various attack angles are also obtained by the proposed method (Fig. 12). It is observed that the trend of the proposed *St* is similar to that of B/D=1.0, i.e., the *St* has the maximum value at $\alpha = 15^{\circ}$ and decreases as a changes upto 45°. It is

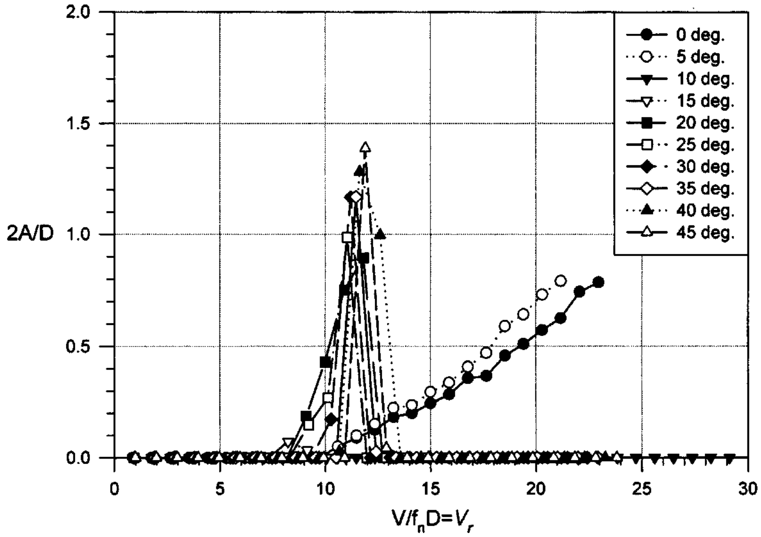

Fig. 10 Aerodynamic behaviors of B/D=1.5 with various attack angles

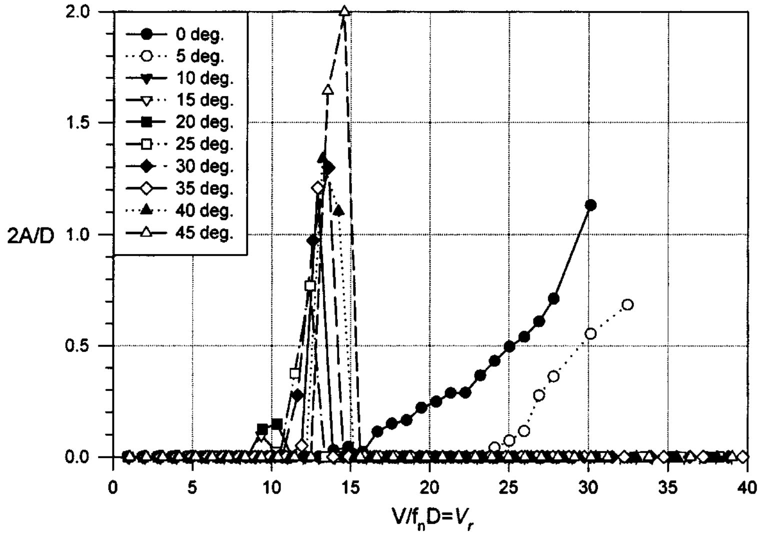

Fig. 11 Aerodynamic behaviors of B/D=2.0 with various attack angles

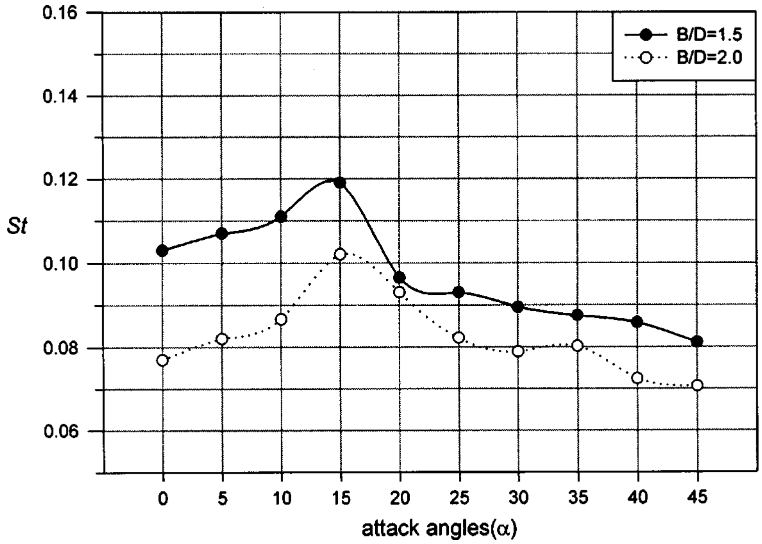

Fig. 12 The Strouhal numbers of B/D=1.5 and 2.0 with various attack angles

shown that these Strouhal numbers obtained by the proposed method are reasonable even though there are no other studies in these cases to compare with present study.

4.4. The Strouhal number of B/D=1.0, 1.5 and 2.0 during vortex-induced vibration

For a quick verification of the validity of the proposed method, an approximate method based on the Shiraishi's early research (1981) is introduced to determine the approximated *St*. Shiraishi suggested that the reduced velocity at the peak amplitude of vortex-induced vibration almost coincides with

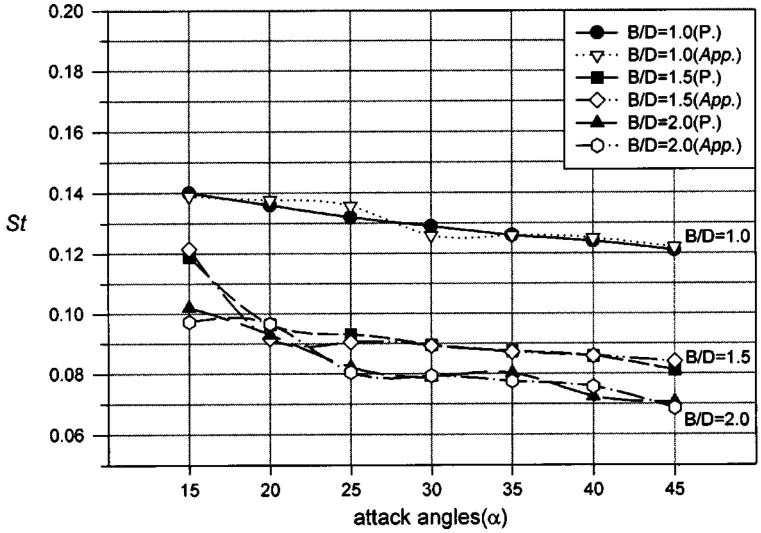

Fig. 13 Comparison with the proposed *St* and the approximated *St*
(P. : the proposed method, *App.* : the approximated method)

the reciprocal of the Strouhal number of the Karman type VIV. As f_s is closely approximated by f_n in the case of Karman type VIV(Lock-in), a reciprocal of V_r can also closely represent the *St* ($St \approx 1/V_r$) (Eq. (2)). Therefore, the approximated *St* within VIV regions can be determined by the reciprocal of V_r at peak amplitude of VIV picked up from the results of wind-induced vibration tests (Fig. 8, 10 and 11). Fig. 13 shows the *St* of the test models (B/D=1.0, 1.5 and 2.0) obtained by two different methods, i.e., cases for the proposed method and the approximated method ($1/V_r$) within a range of attack angles $\alpha = 15^o \sim 45^o$ (or VIV regions). As the *St* values obtained by the proposed method are found to be similar to those obtained by the Shiraishi's approximate method, it can be concluded that the proposed method to evaluate the *St* values is as good as any other methods previously suggested.

$$V_r = \frac{V}{f_n D} \approx \frac{1}{St} \qquad \text{(when Karman type VIV occurs)} \tag{2}$$

(P. : the proposed method, *App.* : the approximated method)

5. Conclusions

A new approach is suggested to determine the Strouhal number of rectangular cylinders (B/D=1.0, 1.5 and 2.0) in a uniform flow based on the aerodynamic behaviors of the body. This proposed method has two important merits. First, the *St* can be obtained in the state of vibration of the body even though a concept of the *St* is derived from the fixed state of the body. From aerodynamic behavior of the body obtained by wind-induced vibration test, the *St* can be determined directly by the proposed method. Second, once the aerodynamic behavior of the body is obtained by wind-induced vibration test, no other tests are necessary to find out the *St*. These merits may indicate that this proposed method is effective to determine the *St* when the wind-induced vibration test of the body is carried out.

The validity of the proposed method is evaluated by comparing with the results obtained by previous studies with three different B/Ds at attack angle 0^o. In comparison with the *St* values of a square cylinder with various attack angles, the values and trends of the proposed *St* are in good agreements with values of previous studies. And also, the proposed *St* values of three test models within the range of $\alpha = 15 \sim 45^o$ are compared favorably with the approximated *St* obtained by experimental value of V_r. This proposed method based on the aerodynamic behavior of the body through wind-induced vibration test is simple yet produces the accurate and stable the Strouhal numbers at all test cases and therefore, it is concluded that the new method is as good as any previously suggested methods to obtain the Strouhal number.

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